

DirichletReg: Dirichlet Regression for Compositional Data in R

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Abstract

Full R Code for

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Keywords: Dirichlet regression, Dirichlet distribution, multivariate generalized linear model, rates, proportions, rates, compositional data, simplex, R.

4. Application examples

4.1. The Arctic lake (common parametrization)

```
> library("DirichletReg")
> head(ArcticLake)
```

```
   sand  silt  clay depth
1 0.775 0.195 0.030 10.4
2 0.719 0.249 0.032 11.7
3 0.507 0.361 0.132 12.8
4 0.522 0.409 0.066 13.0
5 0.700 0.265 0.035 15.7
6 0.665 0.322 0.013 16.3
```

```
> AL <- DR_data(ArcticLake[, 1:3])
```

```
> AL[1:6, ]
```

```
   sand      silt      clay
1 0.7750000 0.1950000 0.0300000
2 0.7190000 0.2490000 0.0320000
3 0.5070000 0.3610000 0.1320000
4 0.5235707 0.4102307 0.0661986
5 0.7000000 0.2650000 0.0350000
6 0.6650000 0.3220000 0.0130000
```

Code for Fig. ?? (left):

```
> plot(AL, cex = 0.5, a2d = list(colored = FALSE, c.grid = FALSE))
```

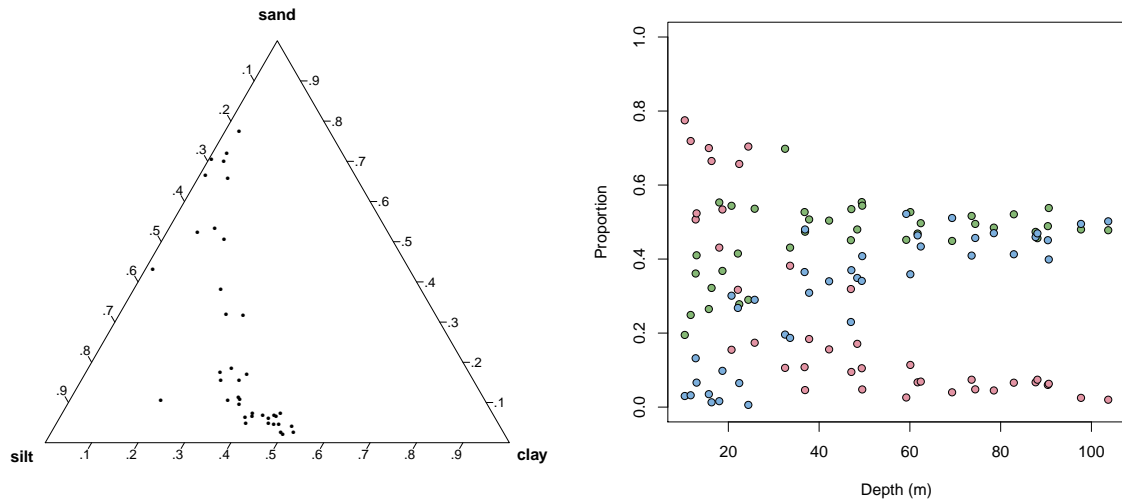


Figure 1: Arctic lake: Ternary plot and depth vs. composition.

Code for Fig. ?? (right):

```
> plot(rep(ArcticLake$depth, 3), as.numeric(AL), pch = 21, bg = rep(c("#E495A5",
+ "#86B875", "#7DB0DD"), each = 39), xlab = "Depth (m)", ylab = "Proportion",
+ ylim = 0:1)
```

```
> lake1 <- DirichReg(AL ~ depth, ArcticLake)
> lake1
```

Call:

```
DirichReg(formula = AL ~ depth, data = ArcticLake)
using the common parametrization
```

Log-likelihood: 101.4 on 6 df (100 BFGS + 1 NR Iterations)

```
-----
Coefficients for variable no. 1: sand
(Intercept)      depth
      0.11662      0.02335
-----
```

```
-----
Coefficients for variable no. 2: silt
(Intercept)      depth
     -0.31060      0.05557
-----
```

```
-----
Coefficients for variable no. 3: clay
(Intercept)      depth
     -1.1520      0.0643
-----
```

```
> coef(lake1)
```

```
$sand
(Intercept)      depth
 0.11662480  0.02335114
```

```
$silt
```

```
(Intercept)      depth
-0.31059591  0.05556745
```

```
$clay
(Intercept)      depth
-1.15195642  0.06430175
```

```
> lake2 <- update(lake1, . ~ . + I(depth^2) | . + I(depth^2) | . + I(depth^2))
> anova(lake1, lake2)
```

Analysis of Deviance Table

```
Model 1: DirichReg(formula = AL ~ depth, data = ArcticLake)
Model 2: DirichReg(formula = AL ~ depth + I(depth^2) | depth + I(depth^2) | depth + I(depth^2),
  data = ArcticLake)
```

	Deviance	N. par	Difference	df	Pr(>Chi)
Model 1	-202.74	6			
Model 2	-217.99	9	15.254	3	0.001612 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> summary(lake2)
```

Call:

```
DirichReg(formula = AL ~ depth + I(depth^2) | depth + I(depth^2) | depth + I(depth^2), data =
ArcticLake)
```

Standardized Residuals:

	Min	1Q	Median	3Q	Max
sand	-1.7647	-0.7080	-0.1786	0.9598	3.0460
silt	-1.1379	-0.5330	-0.1546	0.2788	1.5604
clay	-1.7661	-0.6583	-0.0454	0.6584	2.0152

Beta-Coefficients for variable no. 1: sand

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.4361967	0.8026814	1.789	0.0736 .
depth	-0.0072383	0.0329433	-0.220	0.8261
I(depth^2)	0.0001324	0.0002761	0.480	0.6315

Beta-Coefficients for variable no. 2: silt

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-0.0259705	0.7598827	-0.034	0.9727
depth	0.0717450	0.0343089	2.091	0.0365 *
I(depth^2)	-0.0002679	0.0003088	-0.867	0.3857

Beta-Coefficients for variable no. 3: clay

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-1.7931487	0.7362293	-2.436	0.01487 *
depth	0.1107906	0.0357705	3.097	0.00195 **
I(depth^2)	-0.0004872	0.0003308	-1.473	0.14079

Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-likelihood: 109 on 9 df (162 BFGS + 2 NR Iterations)

AIC: -200, BIC: -185

Number of Observations: 39

Link: Log

Parametrization: common

Code for Fig. ??:

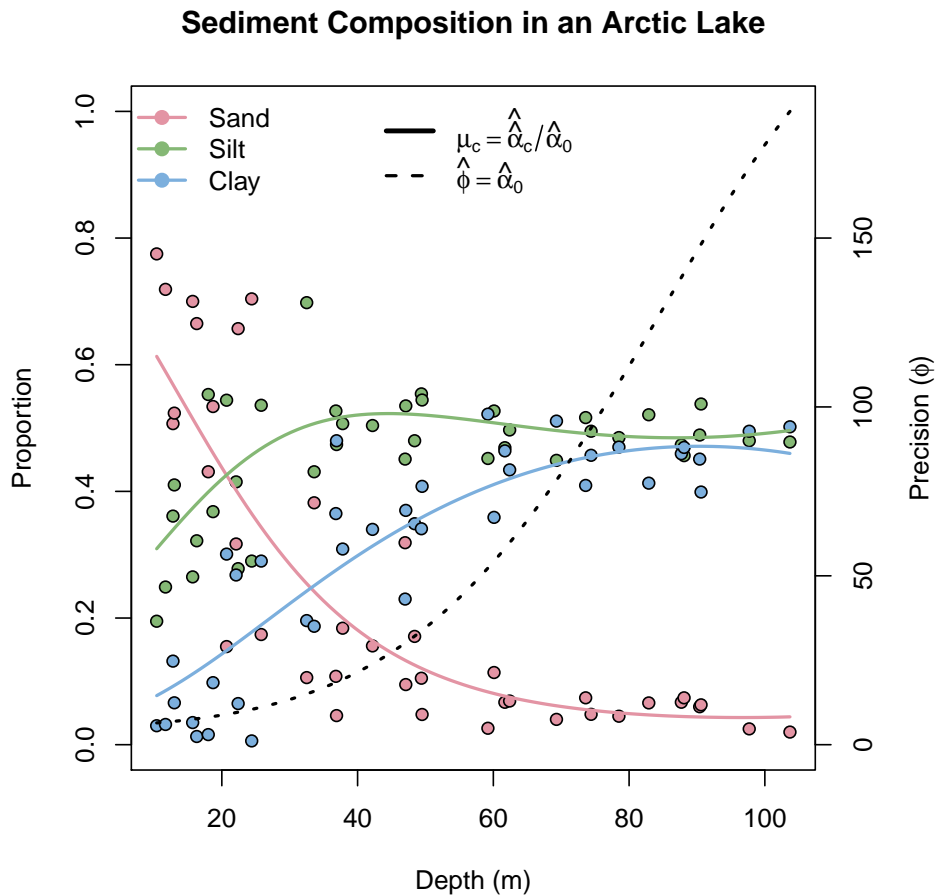


Figure 2: Arctic lake: Fitted values of the quadratic model.

```

> par(mar = c(4, 4, 4, 4) + 0.1)
> plot(rep(ArcticLake$depth, 3), as.numeric(AL), pch = 21, bg = rep(c("#E495A5",
+ "#86B875", "#7DB0DD"), each = 39), xlab = "Depth (m)", ylab = "Proportion",
+ ylim = 0:1, main = "Sediment Composition in an Arctic Lake")
> Xnew <- data.frame(depth = seq(min(ArcticLake$depth), max(ArcticLake$depth),
+ length.out = 100))
> for (i in 1:3) lines(cbind(Xnew, predict(lake2, Xnew)[, i]), col = c("#E495A5",
+ "#86B875", "#7DB0DD")[i], lwd = 2)
> legend("topleft", legend = c("Sand", "Silt", "Clay"), lwd = 2, col = c("#E495A5",
+ "#86B875", "#7DB0DD"), pt.bg = c("#E495A5", "#86B875", "#7DB0DD"), pch = 21,
+ bty = "n")
> par(new = TRUE)
> plot(cbind(Xnew, predict(lake2, Xnew, F, F, T)), lty = "24", type = "l", ylim = c(0,
+ max(predict(lake2, Xnew, F, F, T))), axes = F, ann = F, lwd = 2)
> axis(4)
> mtext(expression(paste("Precision (", phi, ")"), sep = ""), 4, line = 3)
> legend("top", legend = c(expression(hat(mu)[c] == hat(alpha)[c]/hat(alpha)[0])),
+ expression(hat(phi) == hat(alpha)[0])), lty = c(1, 2), lwd = c(3, 2), bty = "n")

> AL <- ArcticLake
> AL$AL <- DR_data(ArcticLake[, 1:3])

```

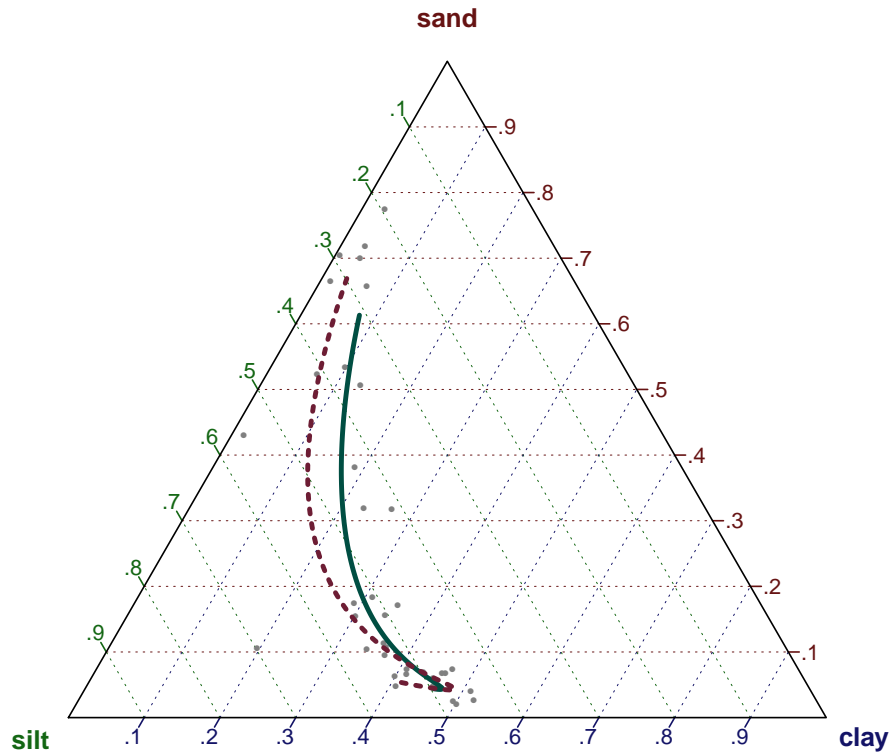


Figure 3: Arctic lake: OLS (dashed) vs. Dirichlet regression (solid) predictions.

```
> dd <- range(ArcticLake$depth)
> X <- data.frame(depth = seq(dd[1], dd[2], length.out = 200))
> pp <- predict(DirichReg(AL ~ depth + I(depth^2), AL), X)
```

Code for Fig. ??:

```
> plot(AL$AL, cex = 0.1, reset_par = FALSE)
> points(toSimplex(AL$AL), pch = 16, cex = 0.5, col = gray(0.5))
> lines(toSimplex(pp), lwd = 3, col = c("#6E1D34", "#004E42")[2])
> Dols <- log(cbind(ArcticLake[, 2]/ArcticLake[, 1], ArcticLake[, 3]/ArcticLake[,
+ 1]))
> ols <- lm(Dols ~ depth + I(depth^2), ArcticLake)
> p2 <- predict(ols, X)
> p2m <- exp(cbind(0, p2[, 1], p2[, 2]))/rowSums(exp(cbind(0, p2[, 1], p2[, 2])))
> lines(toSimplex(p2m), lwd = 3, col = c("#6E1D34", "#004E42")[1], lty = "21")
```

4.2. Blood samples (alternative parametrization)

```
> Bld <- BloodSamples
> Bld$Smp <- DR_data(Bld[, 1:4])
```

```
> blood1 <- DirichReg(Smp ~ Disease | 1, Bld, model = "alternative", base = 3)
> blood2 <- DirichReg(Smp ~ Disease | Disease, Bld, model = "alternative", base = 3)
> anova(blood1, blood2)
```

Analysis of Deviance Table

```
Model 1: DirichReg(formula = Smp ~ Disease | 1, data = Bld, model = "alternative", base = 3)
Model 2: DirichReg(formula = Smp ~ Disease | Disease, data = Bld, model = "alternative", base = 3)
```

	Deviance	N. par	Difference	df	Pr(>Chi)
Model 1	-303.86	7			
Model 2	-304.61	8	0.7587	1	0.3837

```
> summary(blood1)
```

Call:

```
DirichReg(formula = Smp ~ Disease | 1, data = Bld, model = "alternative", base = 3)
```

Standardized Residuals:

	Min	1Q	Median	3Q	Max
Albumin	-2.1310	-0.9307	-0.1234	0.8149	2.8429
Pre.Albumin	-1.0687	-0.4054	-0.0789	0.1947	1.5691
Globulin.A	-2.0503	-1.0392	0.1938	0.7927	2.2393
Globulin.B	-1.8176	-0.5347	0.1488	0.5115	1.3284

MEAN MODELS:

```
-----
Coefficients for variable no. 1: Albumin
      Estimate Std. Error z value Pr(>|z|)
(Intercept)  1.11639    0.09935  11.237 <2e-16 ***
DiseaseB     -0.07002    0.13604  -0.515  0.607
-----
```

```
-----
Coefficients for variable no. 2: Pre.Albumin
      Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.5490    0.1082   5.076 3.86e-07 ***
DiseaseB     -0.1276    0.1493  -0.855  0.393
-----
```

```
-----
Coefficients for variable no. 3: Globulin.A
- variable omitted (reference category) -
-----
```

```
-----
Coefficients for variable no. 4: Globulin.B
      Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.4863    0.1094   4.445 8.8e-06 ***
DiseaseB     0.1819    0.1472   1.236  0.216
-----
```

PRECISION MODEL:

```
-----
      Estimate Std. Error z value Pr(>|z|)
(Intercept)  4.2227    0.1475  28.64 <2e-16 ***
-----
```

```
Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Log-likelihood: 151.9 on 7 df (44 BFGS + 1 NR Iterations)
```

```
AIC: -289.9, BIC: -280
```

```
Number of Observations: 30
```

```
Links: Logit (Means) and Log (Precision)
```

```
Parametrization: alternative
```

Code for Fig. ??:

```
> par(mfrow = c(1, 4), mar = c(4, 4, 4, 2) + 0.25)
> for (i in 1:4) {
+   boxplot(Bld$Smp[, i] ~ Bld$Disease, ylim = range(Bld$Smp[, 1:4]), main = paste(names(Bld)[i]),
```

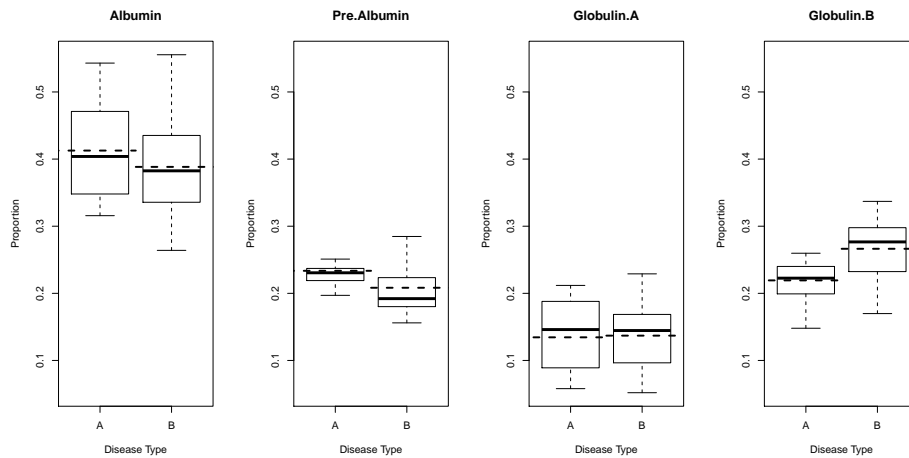


Figure 4: Blood samples: Box plots and fitted values (dashed lines indicate the fitted values for each group).

```
+       xlab = "Disease Type", ylab = "Proportion")
+       segments(c(-5, 1.5), unique(fitted(blood2)[, i]), c(1.5, 5), unique(fitted(blood2)[,
+       i]), lwd = 2, lty = 2)
+ }
```

```
> alpha <- predict(blood2, data.frame(Disease = factor(c("A", "B"))), F, T, F)
> L <- sapply(1:2, function(i) ddirichlet(DR_data(Bld[31:36, 1:4]), unlist(alpha[i,
+   ])))
> LP <- L/rowSums(L)
> dimnames(LP) <- list(paste("C", 1:6), c("A", "B"))
> print(data.frame(round(LP * 100, 1), pred. = as.factor(ifelse(LP[, 1] > LP[,
+   2], "==" A", "==" B))), print.gap = 2)
```

	A	B	pred.
C 1	59.4	40.6	==> A
C 2	43.2	56.8	==> B
C 3	38.4	61.6	==> B
C 4	43.8	56.2	==> B
C 5	36.6	63.4	==> B
C 6	70.2	29.8	==> A

Code for Fig. ??:

```
> B2 <- DR_data(BloodSamples[, c(1, 2, 4)])
> plot(B2, cex = 0.001, reset_par = FALSE)
> div.col <- colorRampPalette(c("#023FA5", "#c0c0c0", "#8E063B"))(100)
> temp <- (alpha/rowSums(alpha))[, c(1, 2, 4)]
> points(toSimplex(temp/rowSums(temp)), pch = 22, bg = div.col[c(1, 100)], cex = 2,
+   lwd = 0.25)
> temp <- B2[1:30, ]
> points(toSimplex(temp/rowSums(temp)), pch = 21, bg = (div.col[c(1, 100)])[BloodSamples$Disease[1:30]],
+   cex = 0.5, lwd = 0.25)
> temp <- B2[31:36, ]
> points(toSimplex(temp/rowSums(temp)), pch = 21, bg = div.col[round(100 * LP[,
+   2], 0)], cex = 1, lwd = 0.5)
> legend("topright", bty = "n", legend = c("Disease A", "Disease B", NA, "Expected Values"),
+   pch = c(21, 21, NA, 22), pt.bg = c(div.col[c(1, 100)], NA, "white"))
```

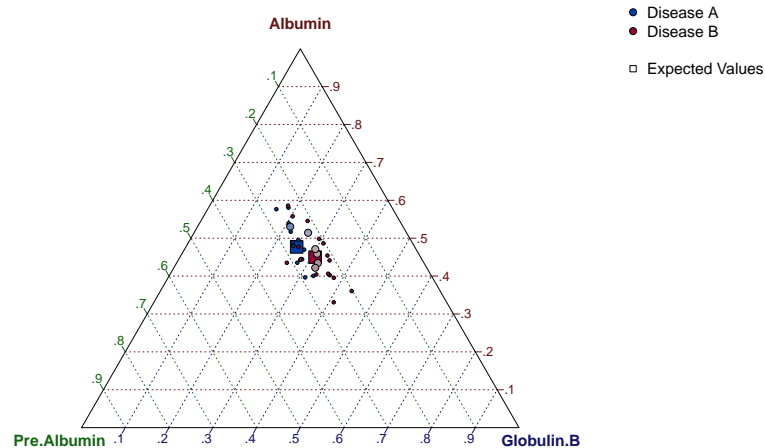


Figure 5: Blood samples: Observed values and predictions

4.3. Reading skills data (alternative parametrization)

```
> RS <- ReadingSkills
> RS$acc <- DR_data(RS$accuracy)
> RS$dyslexia <- C(RS$dyslexia, treatment)
> rs1 <- DirichReg(acc ~ dyslexia * iq | dyslexia * iq, RS, model = "alternative")
> rs2 <- DirichReg(acc ~ dyslexia * iq | dyslexia + iq, RS, model = "alternative")
> anova(rs1, rs2)
```

Analysis of Deviance Table

```
Model 1: DirichReg(formula = acc ~ dyslexia * iq | dyslexia * iq, data = RS, model = "alternative")
Model 2: DirichReg(formula = acc ~ dyslexia * iq | dyslexia + iq, data = RS, model = "alternative")
```

	Deviance	N. par	Difference	df	Pr(>Chi)
Model 1	-133.47	8			
Model 2	-131.80	7	1.6645	1	0.197

Code for Fig. ??:

```
> g.ind <- as.numeric(RS$dyslexia)
> g1 <- g.ind == 1
> g2 <- g.ind != 1
> par(mar = c(4, 4, 4, 4) + 0.25)
> plot(accuracy ~ iq, RS, pch = 21, bg = c("#E495A5", "#39BEB1")[3 - g.ind], cex = 1.5,
+      main = "Dyslexic (Red) vs. Control (Green) Group", xlab = "IQ Score", ylab = "Reading Accuracy",
+      xlim = range(ReadingSkills$iq))
> x1 <- seq(min(RS$iq[g1]), max(RS$iq[g1]), length.out = 200)
> x2 <- seq(min(RS$iq[g2]), max(RS$iq[g2]), length.out = 200)
> n <- length(x1)
> X <- data.frame(dyslexia = factor(rep(0:1, each = n), levels = 0:1, labels = c("no",
+      "yes")), iq = c(x1, x2))
> pv <- predict(rs2, X, TRUE, TRUE, TRUE)
> lines(x1, pv$mu[1:n, 2], col = c("#E495A5", "#39BEB1")[2], lwd = 3)
> lines(x2, pv$mu[(n + 1):(2 * n), 2], col = c("#E495A5", "#39BEB1")[1], lwd = 3)
> a <- RS$accuracy
> logRa_a <- log(a/(1 - a))
> rlr <- lm(logRa_a ~ dyslexia * iq, RS)
```

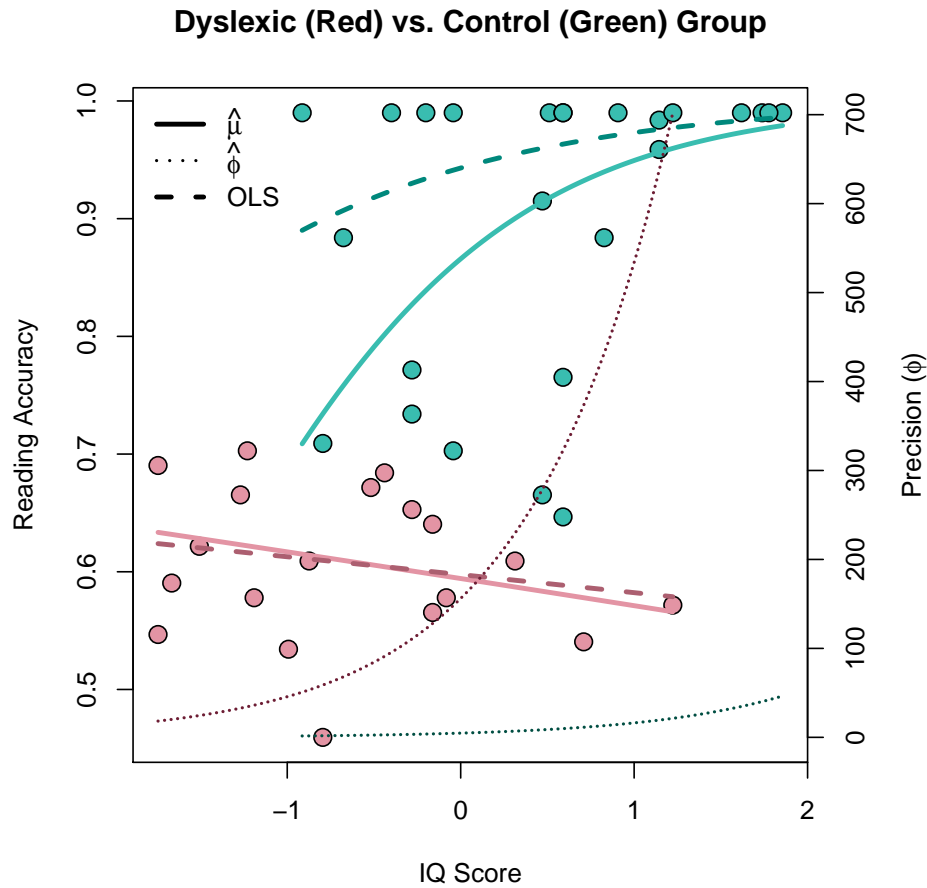



Figure 6: Reading skills: Predicted values of Dirichlet regression and OLS regression.

```

> ols <- 1/(1 + exp(-predict(rlr, X)))
> lines(x1, ols[1:n], col = c("#AD6071", "#00897D")[2], lwd = 3, lty = 2)
> lines(x2, ols[(n + 1):(2 * n)], col = c("#AD6071", "#00897D")[1], lwd = 3, lty = 2)
> par(new = TRUE)
> plot(x1, pv$phi[1:n], col = c("#6E1D34", "#004E42")[2], lty = "11", type = "l",
+      ylim = c(0, max(pv$phi)), axes = F, ann = F, lwd = 2, xlim = range(RS$iq))
> lines(x2, pv$phi[(n + 1):(2 * n)], col = c("#6E1D34", "#004E42")[1], lty = "11",
+      type = "l", lwd = 2)
> axis(4)
> mtext(expression(paste("Precision (", phi, ")"), sep = "")), 4, line = 3)
> legend("topleft", legend = c(expression(hat(mu)), expression(hat(phi)), "OLS"),
+      lty = c(1, 3, 2), lwd = c(3, 2, 3), bty = "n")

> a <- RS$accuracy
> logRa_a <- log(a/(1 - a))
> rlr <- lm(logRa_a ~ dyslexia * iq, RS)
> summary(rlr)

```

Call:

```
lm(formula = logRa_a ~ dyslexia * iq, data = RS)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2.66405	-0.37966	0.03687	0.40887	2.50345

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.8067	0.2822	9.944	2.27e-12 ***
dyslexiayes	-2.4113	0.4517	-5.338	4.01e-06 ***
iq	0.7823	0.2992	2.615	0.0125 *
dyslexiayes:iq	-0.8457	0.4510	-1.875	0.0681 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.2 on 40 degrees of freedom
 Multiple R-squared: 0.6151, Adjusted R-squared: 0.5862
 F-statistic: 21.31 on 3 and 40 DF, p-value: 2.083e-08

> *summary(rs2)*

Call:

DirichReg(formula = acc ~ dyslexia * iq | dyslexia + iq, data = RS, model = "alternative")

Standardized Residuals:

	Min	1Q	Median	3Q	Max
1 - accuracy	-1.5661	-0.8204	-0.5112	0.5211	3.4334
accuracy	-3.4334	-0.5211	0.5112	0.8204	1.5661

MEAN MODELS:

 Coefficients for variable no. 1: 1 - accuracy
 - variable omitted (reference category) -

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.8649	0.2991	6.235	4.52e-10 ***
dyslexiayes	-1.4833	0.3029	-4.897	9.74e-07 ***
iq	1.0676	0.3359	3.178	0.001482 **
dyslexiayes:iq	-1.1625	0.3452	-3.368	0.000757 ***

PRECISION MODEL:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	1.5579	0.3336	4.670	3.01e-06 ***
dyslexiayes	3.4931	0.5880	5.941	2.83e-09 ***
iq	1.2291	0.4596	2.674	0.00749 **

 Significance codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log-likelihood: 65.9 on 7 df (56 BFGS + 2 NR Iterations)
 AIC: -117.8, BIC: -105.3
 Number of Observations: 44
 Links: Logit (Means) and Log (Precision)
 Parametrization: alternative

> *confint(rs2)*

95% Confidence Intervals (original form)

- Beta-Parameters:
 Variable: 1 - accuracy
 variable omitted

```
Variable: accuracy
      2.5% Est. 97.5%
(Intercept) 1.279 1.86 2.451
dyslexiayes -2.077 -1.48 -0.890
iq          0.409 1.07 1.726
dyslexiayes:iq -1.839 -1.16 -0.486
```

```
- Gamma-Parameters
      2.5% Est. 97.5%
(Intercept) 0.904 1.56 2.21
dyslexiayes 2.341 3.49 4.65
iq          0.328 1.23 2.13
```

```
> confint(rs2, exp = TRUE)
```

```
95% Confidence Intervals (exponentiated)
```

```
- Beta-Parameters:
Variable: 1 - accuracy
variable omitted
```

```
Variable: accuracy
      2.5% exp(Est.) 97.5%
(Intercept) 3.592 6.455 11.601
dyslexiayes 0.125 0.227 0.411
iq          1.506 2.908 5.618
dyslexiayes:iq 0.159 0.313 0.615
```

```
- Gamma-Parameters
      2.5% exp(Est.) 97.5%
(Intercept) 2.47 4.75 9.13
dyslexiayes 10.39 32.89 104.12
iq          1.39 3.42 8.41
```

Code for Fig. ??:

```
> gcol <- c("#E495A5", "#39BEB1")[3 - as.numeric(RS$dyslexia)]
> tmt <- c(-3, 3)
> par(mfrow = c(3, 2), cex = 0.8)
> qqnorm(residuals(rlr, "pearson"), ylim = tmt, xlim = tmt, pch = 21, bg = gcol,
+       main = "Normal Q-Q-Plot: OLS Residuals", cex = 0.75, lwd = 0.5)
> abline(0, 1, lwd = 2)
> qqline(residuals(rlr, "pearson"), lty = 2)
> qqnorm(residuals(rs2, "standardized")[, 2], ylim = tmt, xlim = tmt, pch = 21,
+       bg = gcol, main = "Normal Q-Q-Plot: DirichReg Residuals", cex = 0.75, lwd = 0.5)
> abline(0, 1, lwd = 2)
> qqline(residuals(rs2, "standardized")[, 2], lty = 2)
> plot(ReadingSkills$iq, residuals(rlr, "pearson"), pch = 21, bg = gcol, ylim = c(-3,
+ 3), main = "OLS Residuals", xlab = "IQ", ylab = "Pearson Residuals", cex = 0.75,
+  lwd = 0.5)
> abline(h = 0, lty = 2)
> plot(ReadingSkills$iq, residuals(rs2, "standardized")[, 2], pch = 21, bg = gcol,
+  ylim = c(-3, 3), main = "DirichReg Residuals", xlab = "IQ", ylab = "Standardized Residuals",
+  cex = 0.75, lwd = 0.5)
> abline(h = 0, lty = 2)
> plot(fitted(rlr), residuals(rlr, "pearson"), pch = 21, bg = gcol, ylim = c(-3,
+ 3), main = "OLS Residuals", xlab = "Fitted", ylab = "Pearson Residuals",
+  cex = 0.75, lwd = 0.5)
> abline(h = 0, lty = 2)
> plot(fitted(rs2)[, 2], residuals(rs2, "standardized")[, 2], pch = 21, bg = gcol,
+  ylim = c(-3, 3), main = "DirichReg Residuals", xlab = "Fitted", ylab = "Standardized Residuals",
```

```
+      cex = 0.75, lwd = 0.5)  
> abline(h = 0, lty = 2)
```

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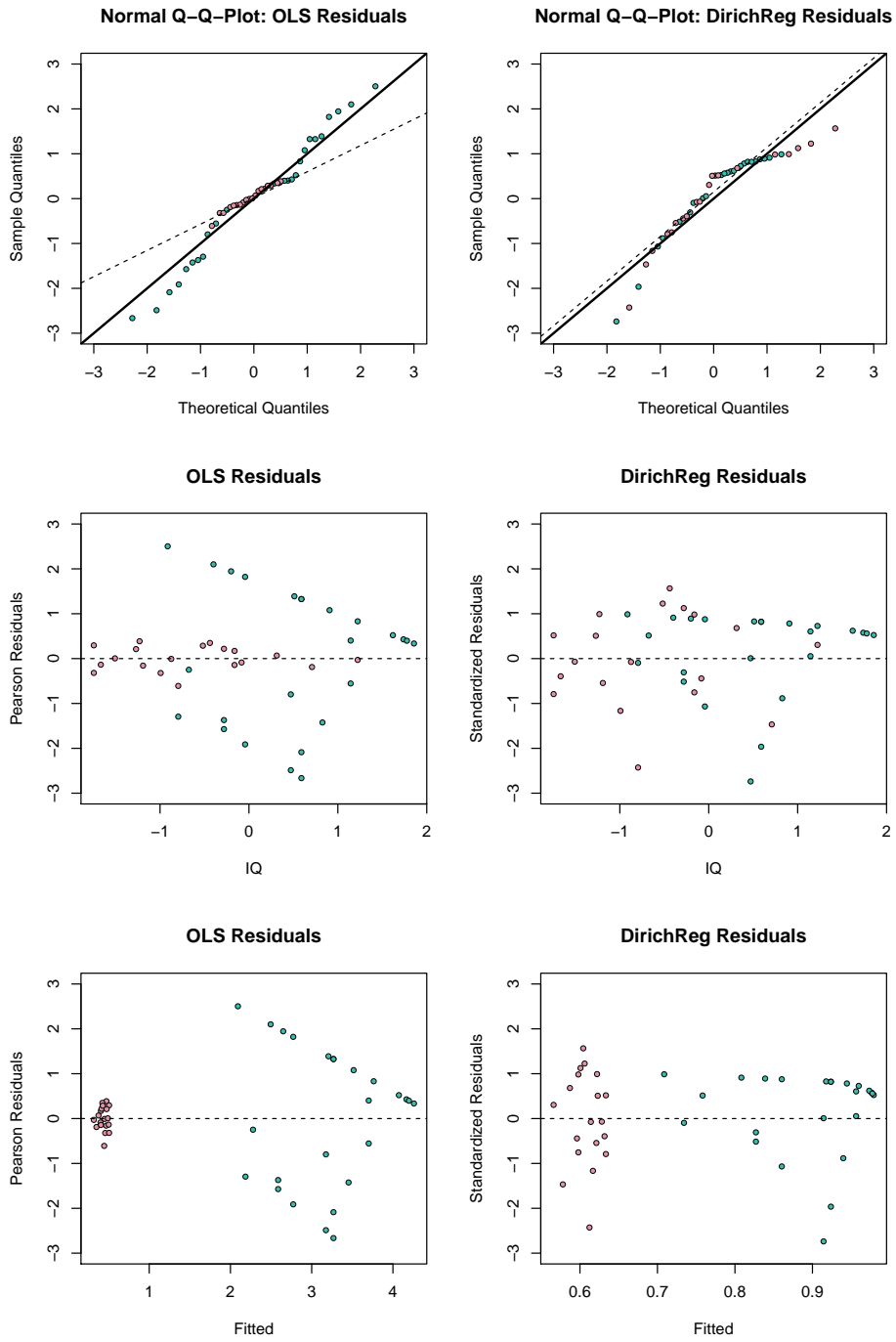


Figure 7: Reading skills: residual plots of OLS and Dirichlet regression models.