Package ‘bssn’

March 17, 2016

Type Package

Title Birnbaum-Saunders Model Based on Skew-Normal Distribution

Version 0.7

Date 2016-03-16

Author Rocio Paola Maehara <rmaeharaa@gmail.com> and Luis Benites Sanchez <lbenitesanchez@gmail.com>

Maintainer Rocio Paola Maehara <rmaeharaa@gmail.com>

Imports sn

Description It provides the density, distribution function, quantile function, random number generator, reliability function, failure rate, likelihood function,
moments and EM algorithm for Maximum Likelihood estimators, also empirical quantile and generated envelope for a given sample, all this for the three parameter
Birnbaum-Saunders model based on Skew-Normal Distribution.
Additionally, it provides the random number generator for the mixture of Birnbaum-Saunders model based on Skew-Normal distribution.

License GPL (>= 2)

Repository CRAN

NeedsCompilation no

Date/Publication 2016-03-17 23:19:57

R topics documented:

`bssn-package` *Birnbaum-Saunders model based on Skew-Normal distribution*

Description

It provides the density, distribution function, quantile function, random number generator, reliability function, failure rate, likelihood function, moments and EM algorithm for Maximum Likelihood estimators, also empirical quantile and generated envelope for a given sample, all this for the three parameter Birnbaum-Saunders model based on Skew-Normal Distribution. Additionally, it provides the random number generator for the mixture of Birbaum-Saunders model based on Skew-Normal distribution.

Details

Package:	bssn
Type:	Package
Version:	0.6
Date:	2015-09-07
License:	GPL (>=2)

Author(s)

Rocio Maehara <rmaeharaa@gmail.com> and Luis Benites <lbenitesanchez@gmail.com>

References

Vilca, Filidor; Santana, L. R.; Leiva, Victor; Balakrishnan, N. (2011). Estimation of extreme percentiles in Birnbaum Saunders distributions. Computational Statistics & Data Analysis (Print), 55, 1665-1678.

Santana, Lucia; Vilca, Filidor; Leiva, Victor (2011). Influence analysis in skew-Birnbaum Saunders regression models and applications. Journal of Applied Statistics, 38, 1633-1649.

See Also

bssn, EMbssn, momentsbssn, ozone, reliabilitybssn

Examples

```
#See examples for the bssnEM function linked above.
```

bssn *Birnbaum-Saunders model based on Skew-Normal distribution*

Description

It provides the density, distribution function, quantile function, random number generator, likelihood function, moments and EM algorithm for Maximum Likelihood estimators for a given sample, all this for the three parameter Birnbaum-Saunders model based on Skew-Normal Distribution. Also, we have the random number generator for the mixture of Birbaum-Saunders model based on Skew-Normal distribution. Finally, the function mmmeth() is used to find the initial values for the parameters alpha and beta using modified-moment method.

Usage

```
dbssn(ti, alpha=0.5, beta=1, lambda=1.5)
pbssn(q,  alpha=0.5, beta=1, lambda=1.5)
qbssn(p,  alpha=0.5, beta=1, lambda=1.5)
rbssn(n,  alpha=0.5, beta=1, lambda=1.5)
rmixbssn(n,alpha,beta,lambda,pii)
mmmeth(ti)
```

Arguments

ti	vector of observations.
q	vector of quantiles.
p	vector of probabilities.
n	number of observations.
alpha	shape parameter.
beta	scale parameter.
lambda	skewness parameter.
pii	Are weights adding to 1. Each one of them (alpha, beta and lambda) must be a vector of length g if you want to generate a random numbers from a mixture distribution BSSN.

Details

If `alpha`, `sigma` or `lambda` are not specified they assume the default values of 0.5, 1 and 1.5, respectively, belonging to the Birnbaum-Saunders model based on Skew-Normal distribution denoted by $BSSN(0.5, 1, 1.5)$.

As discussed in Filidor et. al (2011) we say that a random variable T is distributed as an BSSN with shape parameter $\alpha > 0$, scale parameter $\beta > 0$ and skewness parameter λ in R, if its probability density function (pdf) is given by

$$f(t) = 2\phi(a(t;\alpha,\beta))\Phi(\lambda a(t;\alpha,\beta))A(t;\alpha,\beta), t > 0$$

where $\phi(.)$ and $\Phi(.)$ are the standard normal density and cumulative distribution function respectively. Also $a(t;\alpha,\beta) = (1/\alpha)(\sqrt{t/\beta} - \sqrt{\beta/t})$ and $A(t;\alpha,\beta) = t^{-3/2}(t+\beta)/(2\alpha\beta^{1/2})$

Value

dbssn gives the density, pbssn gives the distribution function, qbssn gives the quantile function, rbssn generates a random sample and rmixbssn genrates a mixture random sample.

The length of the result is determined by n for rbssn, and is the maximum of the lengths of the numerical arguments for the other functions dbssn, pbssn and qbssn.

Author(s)

Rocio Maehara <rmaeharaa@gmail.com> and Luis Benites <lbenitesanchez@gmail.com>

References

Vilca, Filidor; Santana, L. R.; Leiva, Victor; Balakrishnan, N. (2011). Estimation of extreme percentiles in Birnbaum Saunders distributions. Computational Statistics & Data Analysis (Print), 55, 1665-1678.

Santana, Lucia; Vilca, Filidor; Leiva, Victor (2011). Influence analysis in skew-Birnbaum Saunders regression models and applications. Journal of Applied Statistics, 38, 1633-1649.

See Also

EMbssn, momentsbssn, ozone, reliabilitybssn

Examples

```
## Not run:
## Let's plot an Birnbaum-Saunders model based on Skew-Normal distribution!

## Density
 sseq <- seq(0,3,0.01)
 dens <- dbssn(sseq,alpha=0.2,beta=1,lambda=1.5)
 plot(sseq, dens,type="l", lwd=2,col="red", xlab="x", ylab="f(x)", main="BSSN Density function")

# Differing densities on a graph
# positive values of lambda
 y   <- seq(0,3,0.01)
 f1  <- dbssn(y,0.2,1,1)
 f2  <- dbssn(y,0.2,1,2)
 f3  <- dbssn(y,0.2,1,3)
 f4  <- dbssn(y,0.2,1,4)
 den <- cbind(f1,f2,f3,f4)

 matplot(y,den,type="l", col=c("deepskyblue4", "firebrick1", "darkmagenta", "aquamarine4"), ylab
 ="Density function",xlab="y",lwd=2,sub="(a)")

 legend(1.5,2.8,c("BSSN(0.2,1,1)", "BSSN(0.2,1,2)", "BSSN(0.2,1,3)","BSSN(0.2,1,4)"),
 col = c("deepskyblue4", "firebrick1", "darkmagenta", "aquamarine4"), lty=1:4,lwd=2,
 seg.len=2,cex=0.8,box.lty=0,bg=NULL)


#negative values of lambda
```

```
y   <- seq(0,3,0.01)
f1  <- dbssn(y,0.2,1,-1)
f2  <- dbssn(y,0.2,1,-2)
f3  <- dbssn(y,0.2,1,-3)
f4  <- dbssn(y,0.2,1,-4)
den <- cbind(f1,f2,f3,f4)

matplot(y,den,type="l", col=c("deepskyblue4", "firebrick1", "darkmagenta", "aquamarine4"),
ylab ="Density function",xlab="y",lwd=2,sub="(a)")
legend(1.5,2.8,c("BSSN(0.2,1,-1)", "BSSN(0.2,1,-2)","BSSN(0.2,1,-3)", "BSSN(0.2,1,-4)"),
col=c("deepskyblue4","firebrick1", "darkmagenta","aquamarine4"),lty=1:4,lwd=2,seg.len=2,
cex=1,box.lty=0,bg=NULL)


## Distribution Function
sseq <- seq(0.1,6,0.05)
df   <- pbssn(q=sseq,alpha=0.75,beta=1,lambda=3)
plot(sseq, df, type = "l", lwd=2, col="blue", xlab="x", ylab="F(x)",
main = "BSSN Distribution  function")
abline(h=1,lty=2)


#Inverse Distribution Function
prob <- seq(0,1,length.out = 1000)
idf  <- qbssn(p=prob,alpha=0.75,beta=1,lambda=3)
plot(prob, idf, type="l", lwd=2, col="gray30", xlab="x", ylab =
expression(F^{-1}~(x)), mgp=c(2.3,1,.8))
title(main="BSSN Inverse Distribution function")
abline(v=c(0,1),lty=2)


#Random Sample Histogram
sample <- rbssn(n=10000,alpha=0.75,beta=1,lambda=3)
hist(sample,breaks = 70,freq = FALSE,main="")
title(main="Histogram and True density")
sseq   <- seq(0,8,0.01)
dens   <- dbssn(sseq,alpha=0.75,beta=1,lambda=3)
lines(sseq,dens,col="red",lwd=2)


##Random Sample Histogram for Mixture of BSSN
alpha=c(0.55,0.25);beta=c(1,1.5);lambda=c(3,2);pii=c(0.3,0.7)
sample <- rmixbssn(n=1000,alpha,beta,lambda,pii)
hist(sample$y,breaks = 70,freq = FALSE,main="")
title(main="Histogram and True density")
temp   <- seq(min(sample$y), max(sample$y), length.out=1000)
lines(temp, (pii[1]*dbssn(temp, alpha[1], beta[1],lambda[1]))+(pii[2]*dbssn(temp, alpha[2]
, beta[2],lambda[2])), col="red", lty=3, lwd=3) # the theoretical density
lines(temp, pii[1]*dbssn(temp, alpha[1], beta[1],lambda[1]), col="blue", lty=2, lwd=3)
# the first component
lines(temp, pii[2]*dbssn(temp, alpha[2], beta[2],lambda[2]), col="green", lty=2, lwd=3)
# the second component
```

```
## End(Not run)
```

`EMbssn` *EM Algorithm Birnbaum-Saunders model based on Skew-Normal distribution*

Description

Performs the EM algorithm for Birnbaum-Saunders model based on Skew-Normal distribution.

Usage

```
EMbssn(ti,alpha,beta,delta,loglik=F,accuracy=1e-8,
show.envelope="FALSE",iter.max=500)
```

Arguments

`ti`	the response vector of dimension n where n is the total of observations.
`alpha,beta,delta`	an numeric of initial estimates.
`loglik`	showvalue of the log-likelihood (V) or not (F).
`accuracy`	the convergence maximum error.
`show.envelope`	TRUE or FALSE. Indicates if envelope graph should be built for the fitted model. Default is FALSE.
`iter.max`	The maximum number of iterations of the EM algorithm

Value

The function returns a list with 11 elements detailed as

`iter`	number of iterations.
`alpha`	estimated shape parameter.
`beta`	estimated scale parameter.
`lambda`	estimate skewness parameter.
`SE`	Standard Error estimates.
`table`	Table containing the inference for the estimated parameters.
`loglik`	Log-likelihood value.
`AIC`	Akaike information criterion.
`BIC`	Bayesian information criterion.
`HQC`	Hannan-Quinn information criterion.
`time`	processing time.

Author(s)

Rocio Maehara <rmaeharaa@gmail.com> and Luis Benites <lbenitesanchez@gmail.com>

References

Vilca, Filidor; Santana, L. R.; Leiva, Victor; Balakrishnan, N. (2011). Estimation of extreme percentiles in Birnbaum Saunders distributions. Computational Statistics & Data Analysis (Print), 55, 1665-1678.

Santana, Lucia; Vilca, Filidor; Leiva, Victor (2011). Influence analysis in skew-Birnbaum Saunders regression models and applications. Journal of Applied Statistics, 38, 1633-1649.

See Also

bssn, EMbssn, momentsbssn, ozone, reliabilitybssn

Examples

```
## Not run:
#Using the ozone data

data(ozone)
attach(ozone)

#################################
#The model
 ti        <- dailyozonelevel

#Initial values for the parameters
 initial   <- mmmeth(ti)
 alpha0    <- initial$alpha0ini
 beta0     <- initial$beta0init
 lambda0   <- 0
 delta0    <- lambda0/sqrt(1+lambda0^2)

#Estimated parameters of the model (by default)
 est_param <- EMbssn(ti,alpha0,beta0,delta0,loglik=T,
 accuracy = 1e-8,show.envelope = "TRUE", iter.max=500)

#ML estimates
 alpha     <- est_param$res$alpha
 beta      <- est_param$res$beta
 lambda    <- est_param$res$lambda


#########################################
#A simple output example

---------------------------------------------------------
Birnbaum-Saunders model based on Skew-Normal distribution
---------------------------------------------------------
```

```
Observations = 116
-----------
Estimates
-----------

       Estimate Std. Error z value Pr(>|z|)
alpha   1.26014    0.23673 5.32311  0.00000
beta   14.65730    4.01984 3.64624  0.00027
lambda  1.06277    0.54305 1.95706  0.05034
------------------------
Model selection criteria
------------------------

        Loglik   AIC   BIC   HQC
Value -542.768 4.705 4.741 4.719
-------
Details
-------

Iterations = 415
Processing time = 0.4283214 secs
Convergence = TRUE

## End(Not run)
```

---

momentsbssn *Moments for the Birnbaum-Saunders model based on Skew-Normal distribution*

Description

Mean, variance, skewness and kurtosis for the Birnbaum-Saunders model based on Skew-Normal distribution defined in Filidor et. al (2011).

Usage

```
meanbssn(alpha=0.5,beta=1,lambda=1.5)
varbssn(alpha=0.5,beta=1,lambda=1.5)
skewbssn(alpha=0.5,beta=1,lambda=1.5)
kurtbssn(alpha=0.5,beta=1,lambda=1.5)
```

Arguments

alpha	shape parameter α.
beta	scale parameter β.
lambda	skewness parameter λ.

Value

`meanbssn` gives the mean, `varbssn` gives the variance, `skewbssn` gives the skewness, `kurtbssn` gives the kurtosis.

Author(s)

Rocio Maehara <rmaeharaa@gmail.com> and Luis Benites <lbenitesanchez@gmail.com>

References

Vilca, Filidor; Santana, L. R.; Leiva, Victor; Balakrishnan, N. (2011). Estimation of extreme percentiles in Birnbaum Saunders distributions. Computational Statistics & Data Analysis (Print), 55, 1665-1678.

Santana, Lucia; Vilca, Filidor; Leiva, Victor (2011). Influence analysis in skew-Birnbaum Saunders regression models and applications. Journal of Applied Statistics, 38, 1633-1649.

See Also

`bssn`, `EMbssn`, `momentsbssn`, `ozone`, `reliabilitybssn`

Examples

```
## Let's compute some moments for a Birnbaum-Saunders model based on Skew normal Distribution.
# The well known mean, variance, skewness and kurtosis
meanbssn(alpha=0.5,beta=1,lambda=1.5)
varbssn(alpha=0.5,beta=1,lambda=1.5)
skewbssn(alpha=0.5,beta=1,lambda=1.5)
kurtbssn(alpha=0.5,beta=1,lambda=1.5)
```

---

ozone — *Daily ozone level measurements*

Description

These data correspond to daily ozone level measurements (in $ppb = ppmx1000$) in New York in May-September, 1973, from the New York State Department of Conservation.

Usage

```
data(ozone)
```

Format

ozone is a data frame with 116 cases (rows).

Details

For a complete description of various distributions applied to data concentration of air pollutants see Gokhale and Khare (2004).

Source

Leiva, V., Barros, M., Paula, G. e Sanhueza, A. (2007). Generalized BirnbaumSaunders distribution applied to air pollutant concentration. Environmetrics, 19, 235-249.

Nadarajah, S. (2007). A truncated inverted beta distribution with application to air pollution data. Stoch. Environ. Res. Risk. Assess., 22, 285-289.

Gokhale, S. e Khare, M. (2004) A review of deterministic, stochastic and hybrid vehicular exhaust emission models International. J. Transp. Manag., 2, 59-74.

`reliabilitybssn` *Reliability Function for the Birnbaum-Saunders model based on Skew-Normal distribution*

Description

Two useful descriptors in reliability analysis are the reliability function (rf), and the failure rate (fr) function or hazard function. For a non-negative random variable t with pdf $f(t)$ (and cdf $F(t)$), its distribution can be characterized equally in terms of the rf, or of the fr, which are respectively defined by $R(t) = 1 - F(t)$, and $h(t) = f(t)/R(t)$, for $t > 0$,and $0 < R(t) < 1$.

Usage

```
Rebssn(ti,alpha=0.5,beta=1,lambda=1.5)
Fbssn(ti,alpha=0.5,beta=1,lambda=1.5)
```

Arguments

`ti`	dataset.
`alpha`	shape parameter α.
`beta`	scale parameter β.
`lambda`	skewness parameter λ.

Value

`Rbssn` gives the reliability function, `Fbssn` gives the failure rate or hazard function.

Author(s)

Rocio Maehara <rmaeharaa@gmail.com> and Luis Benites <lbenitesanchez@gmail.com>

References

Leiva, V., Vilca-Labra, F. E., Balakrishnan, N. e Sanhueza, A. (2008). A skewed sinh-normal distribution and its properties and application to air pollution. Comm. Stat. Theoret. Methods. Submetido.

Guiraud, P., Leiva, V., Fierro, R. (2009). A non-central version of the Birnbaum-Saunders distribution for reliability analysis. IEEE Transactions on Reliability 58, 152-160.

See Also

bssn, EMbssn, momentsbssn, ozone, Rebssn

Examples

```
## Let's compute some realiability functions for a Birnbaum-Saunders model based on
## Skew normal Distribution for different values of the shape parameter.

ti  <- seq(0,2,0.01)
f1  <- Rebssn(ti,0.75,1,1)
f2  <- Rebssn(ti,1,1,1)
f3  <- Rebssn(ti,1.5,1,1)
f4  <- Rebssn(ti,2,1,1)
den <- cbind(f1,f2,f3,f4)

matplot(ti,den,type="l", col=c("deepskyblue4","firebrick1","darkmagenta","aquamarine4"),
ylab="S(t)", xlab="t",lwd=2)
legend(1.5,1,c(expression(alpha==0.75), expression(alpha==1), expression(alpha==1.5),
expression(alpha==2)),col= c("deepskyblue4","firebrick1","darkmagenta","aquamarine4"),
lty=1:4,lwd=2,seg.len=2,cex=0.9,box.lty=0,bg=NULL)


## Let's compute some hazard functions for a Birnbaum Saunders model based on
## Skew normal Distribution for different values of the skewness parameter.

ti  <- seq(0,2,0.01)
f1  <- Fbssn(ti,0.5,1,-1)
f2  <- Fbssn(ti,0.5,1,-2)
f3  <- Fbssn(ti,0.5,1,-3)
f4  <- Fbssn(ti,0.5,1,-4)
den <- cbind(f1,f2,f3,f4)
matplot(ti,den,type = "l", col = c("deepskyblue4","firebrick1", "darkmagenta", "aquamarine4"),
ylab = "h(t)" , xlab="t",lwd=2)
legend(0.1,23, c(expression(lambda==-1), expression(lambda==-2), expression(lambda == -3),
expression(lambda == -4)), col = c("deepskyblue4", "firebrick1", "darkmagenta","aquamarine4"),
lty=1:4,lwd=2,seg.len=2,cex=0.9,box.lty=1,bg=NULL)
```

Index