

Package ‘lmom’

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Title L-moments

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Description Functions related to L-moments: computation of L-moments of distributions and data samples; parameter estimation; L-moment ratio diagram; plot vs. quantiles of an extreme-value distribution.

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lmom-package	<i>The lmom package</i>
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Description

R functions for use with the method of L -moments

Details

L -moments are measures of the location, scale, and shape of probability distributions or data samples. They are based on linear combinations of order statistics. Hosking (1990) and Hosking and Wallis (1997, chap. 2) give expositions of the theory of L -moments and L -moment ratios. Hosking and Wallis (1997, Appendix) give, for many distributions in common use, expressions for the L -moments of the distributions and algorithms for estimating the parameters of the distributions by equating sample and population L -moments (the “method of L -moments”). This package contains R functions that should facilitate the use of L -moment-based methods.

For each of 13 probability distributions, the package contains functions to evaluate the cumulative distribution function and quantile function of the distribution, to calculate the L -moments given the parameters and to calculate the parameters given the low-order L -moments. These functions are as follows.

`cdf` . . . computes the cumulative distribution function of the distribution.

`qua` . . . computes the quantile function (inverse cumulative distribution function) of the distribution.

`lmsr` . . . calculates the L -moment ratios of the distribution given its parameters.

`pel` . . . calculates the parameters of the distribution given its L -moments. When the L -moments are the sample L -moments of a set of data, the resulting parameters are of course the “method of L -moments” estimates of the parameters.

Here . . . is a three-letter code used to identify the distribution, as given in the table below. For example the cumulative distribution function of the gamma distribution is `cdfgam`.

<code>exp</code>	exponential
<code>gam</code>	gamma
<code>gev</code>	generalized extreme-value
<code>glo</code>	generalized logistic
<code>gpa</code>	generalized Pareto
<code>gno</code>	generalized normal
<code>gum</code>	Gumbel (extreme-value type I)

kap	kappa
ln3	lognormal
nor	normal
pe3	Pearson type III
wak	Wakeby
wei	Weibull

The following functions are also contained in the package.

`samlmu` computes the sample L -moments of a data vector.

`lmrp` and `lmrq` compute the L -moments of a probability distribution specified by its cumulative distribution function (for function `lmrp`) or its quantile function (for function `lmrq`). The computation uses numerical integration applied to a general expression for the L -moments of a distribution. Functions `lmrp` and `lmrq` can be used for any univariate distribution. They are slower and usually less accurate than the computations carried out for specific distributions by the `lmr . . .` functions.

`pelp` and `pelq` compute the parameters of a probability distribution as a function of the L -moments. The computation uses function `lmrp` or `lmrq` to compute L -moments and numerical optimization to find parameter values for which the sample and population L -moments are equal. Functions `pelp` and `pelq` can be used for any univariate distribution. They are slower and usually less accurate than the computations carried out for specific distributions by the `pel . . .` functions.

`lmsd` draws an L -moment ratio diagram.

`lmsdpoints` and `lmsdlines` add points, or connected line segments, respectively, to an L -moment ratio diagram.

`evplot` draws an “extreme-value plot”, i.e. a quantile-quantile plot in which the horizontal axis is the quantile of an extreme-value type I (Gumbel) distribution.

`evpoints`, `evdistp`, and `evdistq` add, respectively, a set of points, a cumulative distribution function, and a quantile function to an extreme-value plot.

Parameters of cumulative distribution functions and quantile functions

The functions `cdf . . .` (cumulative distribution functions) and `qua . . .` (quantile functions) expect the distribution parameters to be specified as a single vector. This differs from the standard R convention, in which each parameter is a separate argument. There are two reasons for this. First, the single-vector parameterization is consistent with the Fortran routines on which these R functions are based. Second, the single-vector parametrization is often easier to use. For example, consider computing the 80th and 90th percentiles of a normal distribution fitted to a set of L -moments stored in a vector `lmom`. In the single-vector parametrization, this is achieved by

```
quanor( c(.8,.9), pelnor(lmom) )
```

The separate-arguments parametrization would need a more complex expression, such as

```
do.call( qnorm, c( list(.8,.9), pelnor(lmom) ) )
```

In functions (`lmrp`, `lmrq`, `pelp`, `pelq`, `evplot`, `evdistp`, `evdistq`) that take a cumulative distribution function or a quantile function as an argument, the cumulative distribution function or quantile function can use either form of parametrization.

Relation to the LMOMENTS Fortran package

Functions `cdf...`, `qua...`, `lmr...`, `pe1...`, and `samlmu` are analogous to Fortran routines from the LMOMENTS package, version 3.04, available from StatLib at <http://lib.stat.cmu.edu/general/lmoments>. Functions `cdfwak` and `samlmu`, and all the `lmr...` and `pe1...` functions, internally call Fortran code that is derived from the LMOMENTS package.

Author(s)

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References

Hosking, J. R. M. (1990). *L*-moments: analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society, Series B*, **52**, 105-124.

Hosking, J. R. M., and Wallis, J. R. (1997). *Regional frequency analysis: an approach based on L-moments*. Cambridge University Press.

cdfexp	<i>Exponential distribution</i>
--------	---------------------------------

Description

Distribution function and quantile function of the exponential distribution.

Usage

```
cdfexp(x, para = c(0, 1))
quaexp(f, para = c(0, 1))
```

Arguments

<code>x</code>	Vector of quantiles.
<code>f</code>	Vector of probabilities.
<code>para</code>	Numeric vector containing the parameters of the distribution, in the order ξ, α (location, scale).

Details

The exponential distribution with parameters ξ (lower bound) and α (mean) has distribution function

$$F(x) = 1 - \exp\{-(x - \xi)/\alpha\}$$

for $x \geq 0$, and quantile function

$$x(F) = \xi - \alpha \log(1 - F).$$

Value

`cdfexp` gives the distribution function; `quaexp` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R functions `pexp` and `qexp`.

See Also

`pexp` for the standard R version of the exponential distribution. `cdfgam` for the gamma distribution, `cdfgpa` for the generalized Pareto distribution, `cdfkap` for the kappa distribution, `cdfpe3` for the Pearson type III distribution, and `cdfwak` for the Wakeby distribution, all of which generalize the exponential distribution.

Examples

```
# Random sample from the exponential distribution
# with lower bound 0 and mean 3.
quaexp(runif(100), c(0,3))
```

`cdfgam`

Gamma distribution

Description

Distribution function and quantile function of the gamma distribution.

Usage

```
cdfgam(x, para = c(1, 1))
quagam(f, para = c(1, 1))
```

Arguments

<code>x</code>	Vector of quantiles.
<code>f</code>	Vector of probabilities.
<code>para</code>	Numeric vector containing the parameters of the distribution, in the order α, β (shape, scale).

Details

The gamma distribution with shape parameter α and scale parameter β has probability density function

$$f(x) = \frac{x^{\alpha-1} \exp(-x/\beta)}{\beta^\alpha \Gamma(\alpha)}$$

for $x \geq 0$, where $\Gamma(\cdot)$ is the gamma function.

Value

cdfgam gives the distribution function; quagam gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R functions pgamma and qgamma.

See Also

[gamma](#) for the gamma function.

[pgamma](#) for the standard R version of the gamma distribution.

[cdfpe3](#) for the Pearson type III distribution, which generalizes the gamma distribution.

Examples

```
# Random sample from the gamma distribution
# with shape parameter 4 and mean 1.
quagam(runif(100), c(4,1/4))
```

cdfgev

Generalized extreme-value distribution

Description

Distribution function and quantile function of the generalized extreme-value distribution.

Usage

```
cdfgev(x, para = c(0, 1, 0))
quagev(f, para = c(0, 1, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ξ, α, k (location, scale, shape).

Details

The generalized extreme-value distribution with location parameter ξ , scale parameter α and shape parameter k has distribution function

$$F(x) = \exp\{-\exp(-y)\}$$

where

$$y = -k^{-1} \log\{1 - k(x - \xi)/\alpha\},$$

with x bounded by $\xi + \alpha/k$ from below if $k < 0$ and from above if $k > 0$, and quantile function

$$x(F) = \xi + \frac{\alpha}{k} \{1 - (-\log F)^k\}.$$

Extreme-value distribution types I, II and III (Gumbel, Frechet, Weibull) correspond to shape parameter values $k = 0$, $k < 0$ and $k > 0$ respectively.

Value

`cdfgev` gives the distribution function; `quagev` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

See Also

[cdfgum](#) for the Gumbel (extreme-value type I) distribution.

[cdfkap](#) for the kappa distribution, which generalizes the generalized extreme-value distribution.

[cdfwei](#) for the Weibull distribution,

Examples

```
# Random sample from the generalized extreme-value distribution
# with parameters xi=0, alpha=1, k=-0.5.
quagev(runif(100), c(0,1,-0.5))
```

`cdfglo`

Generalized logistic distribution

Description

Distribution function and quantile function of the generalized logistic distribution.

Usage

```
cdfglo(x, para = c(0, 1, 0))
quaglo(f, para = c(0, 1, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ξ, α, k (location, scale, shape).

Details

The generalized logistic distribution with location parameter ξ , scale parameter α and shape parameter k has distribution function

$$F(x) = 1/\{1 + \exp(-y)\}$$

where

$$y = -k^{-1} \log\{1 - k(x - \xi)/\alpha\},$$

with x bounded by $\xi + \alpha/k$ from below if $k < 0$ and from above if $k > 0$, and quantile function

$$x(F) = \xi + \frac{\alpha}{k} \left\{ 1 - \left(\frac{1-F}{F} \right)^k \right\}.$$

The logistic distribution is the special case $k = 0$.

Value

`cdfglo` gives the distribution function; `quaglo` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

See Also

[cdfkap](#) for the kappa distribution, which generalizes the generalized logistic distribution.

Examples

```
# Random sample from the generalized logistic distribution
# with parameters xi=0, alpha=1, k=-0.5.
quaglo(runif(100), c(0,1,-0.5))
```

cdfgno *Generalized normal distribution*

Description

Distribution function and quantile function of the generalized normal distribution.

Usage

```
cdfgno(x, para = c(0, 1, 0))
quagno(f, para = c(0, 1, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ξ, α, k (location, scale, shape).

Details

The generalized normal distribution with location parameter ξ , scale parameter α and shape parameter k has distribution function

$$F(x) = \Phi(y)$$

where

$$y = -k^{-1} \log\{1 - k(x - \xi)/\alpha\}$$

and $\Phi(y)$ is the distribution function of the standard normal distribution, with x bounded by $\xi + \alpha/k$ from below if $k < 0$ and from above if $k > 0$.

The generalized normal distribution contains as special cases the usual three-parameter lognormal distribution, corresponding to $k < 0$, with a finite lower bound and positive skewness; the normal distribution, corresponding to $k = 0$; and the reverse lognormal distribution, corresponding to $k > 0$, with a finite upper bound and negative skewness. The two-parameter lognormal distribution, with a lower bound of zero and positive skewness, is obtained when $k < 0$ and $\xi + \alpha/k = 0$.

Value

cdfgno gives the distribution function; quagno gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions pnorm, qnorm, etc.

See Also

[cdfln3](#) for the **lmom** package's version of the three-parameter lognormal distribution.

[cdfnor](#) for the **lmom** package's version of the normal distribution.

[pnorm](#) for the standard R version of the normal distribution.

[plnorm](#) for the standard R version of the two-parameter lognormal distribution.

Examples

```
# Random sample from the generalized normal distribution
# with parameters xi=0, alpha=1, k=-0.5.
quagno(runif(100), c(0,1,-0.5))

# The generalized normal distribution with parameters xi=1, alpha=1, k=-1,
# is the standard lognormal distribution. An illustration:
fval<-seq(0.1,0.9,by=0.1)
cbind(fval, lognormal=qlnorm(fval), g.normal=quagno(fval, c(1,1,-1)))
```

cdfgpa

Generalized Pareto distribution

Description

Distribution function and quantile function of the generalized Pareto distribution.

Usage

```
cdfgpa(x, para = c(0, 1, 0))
quagpa(f, para = c(0, 1, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ξ, α, k (location, scale, shape).

Details

The generalized Pareto distribution with location parameter ξ , scale parameter α and shape parameter k has distribution function

$$F(x) = 1 - \exp(-y)$$

where

$$y = -k^{-1} \log\{1 - k(x - \xi)/\alpha\},$$

with x bounded by $\xi + \alpha/k$ from below if $k < 0$ and from above if $k > 0$, and quantile function

$$x(F) = \xi + \frac{\alpha}{k} \{1 - (1 - F)^k\}.$$

The exponential distribution is the special case $k = 0$. The uniform distribution is the special case $k = 1$.

Value

`cdfgpa` gives the distribution function; `quagpa` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

See Also

[cdfexp](#) for the exponential distribution.

[cdfkap](#) for the kappa distribution and [cdfwak](#) for the Wakeby distribution, which generalize the generalized Pareto distribution.

Examples

```
# Random sample from the generalized Pareto distribution
# with parameters xi=0, alpha=1, k=-0.5.
quagpa(runif(100), c(0,1,-0.5))
```

`cdfgum`

Gumbel (extreme-value type I) distribution

Description

Distribution function and quantile function of the Gumbel distribution.

Usage

```
cdfgum(x, para = c(0, 1))
quagum(f, para = c(0, 1))
```

Arguments

<code>x</code>	Vector of quantiles.
<code>f</code>	Vector of probabilities.
<code>para</code>	Numeric vector containing the parameters of the distribution, in the order ξ, α (location, scale).

Details

The Gumbel distribution with location parameter ξ and scale parameter α has distribution function

$$F(x) = \exp[-\exp\{-(x - \xi)/\alpha\}]$$

and quantile function

$$x(F) = \xi - \alpha \log(-\log F).$$

Value

`cdfgum` gives the distribution function; `quagum` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

See Also

[cdfgev](#) for the generalized extreme-value distribution, which generalizes the Gumbel distribution.

Examples

```
# Random sample from the Gumbel distribution with parameters xi=0, alpha=3.
quagum(runif(100), c(0,3))
```

cdfkap

Kappa distribution

Description

Distribution function and quantile function of the kappa distribution.

Usage

```
cdfkap(x, para = c(0, 1, 0, 0))
quakap(f, para = c(0, 1, 0, 0))
```

Arguments

<code>x</code>	Vector of quantiles.
<code>f</code>	Vector of probabilities.
<code>para</code>	Numeric vector containing the parameters of the distribution, in the order ξ, α, k, h (location, scale, shape, shape).

Details

The kappa distribution with location parameter ξ , scale parameter α and shape parameters k and h has quantile function

$$x(F) = \xi + \frac{\alpha}{k} \left\{ 1 - \left(\frac{1 - F^h}{h} \right)^k \right\}.$$

Its special cases include the generalized logistic ($h = -1$), generalized extreme-value ($h = 0$), generalized Pareto ($h = 1$), logistic ($k = 0, h = -1$), Gumbel ($k = 0, h = 0$), exponential ($k = 0, h = 1$), and uniform ($k = 1, h = 1$) distributions.

Value

`cdfkap` gives the distribution function; `quakap` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

References

Hosking, J. R. M. (1994). The four-parameter kappa distribution. *IBM Journal of Research and Development*, **38**, 251-258.

Hosking, J. R. M., and Wallis, J. R. (1997). *Regional frequency analysis: an approach based on L-moments*, Cambridge University Press, Appendix A.10.

See Also

[cdfglo](#) for the generalized logistic distribution, [cdfgev](#) for the generalized extreme-value distribution, [cdfgpa](#) for the generalized Pareto distribution, [cdfgum](#) for the Gumbel distribution, [cdfexp](#) for the exponential distribution.

Examples

```
# Random sample from the kappa distribution
# with parameters xi=0, alpha=1, k=-0.5, h=0.25.
quakap(runif(100), c(0,1,-0.5,0.25))
```

`cdfln3`

Three-parameter lognormal distribution

Description

Distribution function and quantile function of the three-parameter lognormal distribution.

Usage

```
cdfln3(x, para = c(0, 0, 1))
qualn3(f, para = c(0, 0, 1))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ζ, μ, σ (lower bound, mean on log scale, standard deviation on log scale).

Details

The three-parameter lognormal distribution with lower bound ζ , mean on log scale μ , and standard deviation on log scale σ has distribution function

$$F(x) = \Phi(y),$$

$x > 0$, where

$$y = \{\log(x - \zeta) - \mu\} / \sigma$$

and $\Phi(y)$ is the distribution function of the standard normal distribution.

Value

`cdfln3` gives the distribution function; `qualn3` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

See Also

[cdfgno](#) for the generalized normal distribution, a more general form of the three-parameter lognormal distribution.

[qlnorm](#) for the standard R version of the two-parameter lognormal distribution.

Examples

```
# Random sample from three-parameter lognormal distribution
# with parameters zeta=0, mu=1, sigma=0.5.
qualn3(runif(100), c(0,1,0.5))

## Functions for the three-parameter lognormal distribution can
## also be used with the two-parameter lognormal distribution
# Generate a random sample from a standard lognormal distribution
xx <- qualn3(runif(50), c(0,0,1))
# Fit 2-parameter LN distribution
pelln3(samlmu(xx), bound=0)
```

```
# Fit 2-parameter LN distribution "in log space", i.e fit
# normal distribution to log-transformed data
pelnor(samlmu(log(xx)))
```

cdfnor	<i>Normal distribution</i>
--------	----------------------------

Description

Distribution function and quantile function of the normal distribution.

Usage

```
cdfnor(x, para = c(0, 1))
quanor(f, para = c(0, 1))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order μ, σ (location, scale).

Details

The normal distribution with location parameter μ and scale parameter σ has probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\{-(x - \mu)^2/(2\sigma^2)\}.$$

Value

cdfnor gives the distribution function; quanor gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions pnorm and qnorm.

See Also

[pnorm](#) for the standard R version of the normal distribution.
[cdfgno](#) for the generalized normal distribution, which generalizes the normal distribution.

Examples

```
# Random sample from the normal distribution
# with mean 0 and standard deviation 3.
quanor(runif(100), c(0,3))
```

cdfpe3 *Pearson type III distribution*

Description

Distribution function and quantile function of the Pearson type III distribution

Usage

```
cdfpe3(x, para = c(0, 1, 0))
quape3(f, para = c(0, 1, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order μ, σ, γ (location, scale, shape).

Details

The Pearson type III distribution contains as special cases the usual three-parameter gamma distribution (a shifted version of the gamma distribution) with a finite lower bound and positive skewness; the normal distribution, and the reverse three-parameter gamma distribution, with a finite upper bound and negative skewness. The distribution's parameters are the first three (ordinary) moment ratios: μ (the mean, a location parameter), σ (the standard deviation, a scale parameter) and γ (the skewness, a shape parameter).

If $\gamma \neq 0$, let $\alpha = 4/\gamma^2$, $\beta = \frac{1}{2}\sigma|\gamma|$, $\xi = \mu - 2\sigma/\gamma$. The probability density function is

$$f(x) = \frac{|x - \xi|^{\alpha-1} \exp(-|x - \xi|/\beta)}{\beta^\alpha \Gamma(\alpha)}$$

with x bounded by ξ from below if $\gamma > 0$ and from above if $\gamma < 0$. If $\gamma = 0$, the distribution is a normal distribution with mean μ and standard deviation σ .

The Pearson type III distribution is usually regarded as consisting of just the case $\gamma > 0$ given above, and is usually parametrized by α, β and ξ . Our parametrization extends the distribution to include the usual Pearson type III distributions, with positive skewness and lower bound ξ , reverse Pearson type III distributions, with negative skewness and upper bound ξ , and the Normal distribution, which is included as a special case of the distribution rather than as the unattainable limit $\alpha \rightarrow \infty$. This enables the Pearson type III distribution to be used when the skewness of the observed data may be negative. The parameters μ, σ and γ are the conventional moments of the distribution.

The gamma distribution is obtained when $\gamma > 0$ and $\mu = 2\sigma/\gamma$. The normal distribution is the special case $\gamma = 0$. The exponential distribution is the special case $\gamma = 2$.

Value

cdfpe3 gives the distribution function; quape3 gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

References

Hosking, J. R. M. and Wallis, J. R. (1997). *Regional frequency analysis: an approach based on L-moments*, Cambridge University Press, Appendix A.10.

See Also

[cdfgam](#) for the gamma distribution.

[cdfnor](#) for the normal distribution.

Examples

```
# Random sample from the Pearson type III distribution
# with parameters mu=1, alpha=2, gamma=3.
quape3(runif(100), c(1,2,3))

# The Pearson type III distribution with parameters
# mu=12, sigma=6, gamma=1, is the gamma distribution
# with parameters alpha=4, beta=3. An illustration:
fval<-seq(0.1,0.9,by=0.1)
cbind(fval, qgamma(fval, shape=4, scale=3), quape3(fval, c(12,6,1)))
```

cdfwak

Wakeby distribution

Description

Distribution function and quantile function of the Wakeby distribution.

Usage

```
cdfwak(x, para = c(0, 1, 0, 0, 0))
quawak(f, para = c(0, 1, 0, 0, 0))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order $\xi, \alpha, \beta, \gamma, \delta$.

Details

The Wakeby distribution with parameters ξ , α , β , γ and δ has quantile function

$$x(F) = \xi + \frac{\alpha}{\beta} \{1 - (1 - F)^\beta\} - \frac{\gamma}{\delta} \{1 - (1 - F)^{-\delta}\}.$$

The parameters are restricted as in Hosking and Wallis (1997, Appendix A.11):

- either $\beta + \delta > 0$ or $\beta = \gamma = \delta = 0$;
- if $\alpha = 0$ then $\beta = 0$;
- if $\gamma = 0$ then $\delta = 0$;
- $\gamma \geq 0$;
- $\alpha + \gamma \geq 0$.

The distribution has a lower bound at ξ and, if $\delta < 0$, an upper bound at $\xi + \alpha/\beta - \gamma/\delta$.

The generalized Pareto distribution is the special case $\alpha = 0$ or $\gamma = 0$. The exponential distribution is the special case $\beta = \gamma = \delta = 0$. The uniform distribution is the special case $\beta = 1$, $\gamma = \delta = 0$.

Value

`cdfwak` gives the distribution function; `quawak` gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions `pnorm`, `qnorm`, etc.

References

Hosking, J. R. M. and Wallis, J. R. (1997). *Regional frequency analysis: an approach based on L-moments*, Cambridge University Press, Appendix A.11.

See Also

[cdfgpa](#) for the generalized Pareto distribution.

[cdfexp](#) for the exponential distribution.

Examples

```
# Random sample from the Wakeby distribution
# with parameters xi=0, alpha=30, beta=20, gamma=1, delta=0.3.
quawak(runif(100), c(0,30,20,1,0.3))
```

cdfwei	<i>Weibull distribution</i>
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Description

Distribution function and quantile function of the Weibull distribution.

Usage

```
cdfwei(x, para = c(0, 1, 1))  
quawei(f, para = c(0, 1, 1))
```

Arguments

x	Vector of quantiles.
f	Vector of probabilities.
para	Numeric vector containing the parameters of the distribution, in the order ζ, β, δ (location, scale, shape).

Details

The Weibull distribution with location parameter ζ , scale parameter β and shape parameter δ has distribution function

$$F(x) = 1 - \exp[-\{(x - \zeta)/\beta\}^\delta]$$

for $x > \zeta$.

Value

cdfwei gives the distribution function; quawei gives the quantile function.

Note

The functions expect the distribution parameters in a vector, rather than as separate arguments as in the standard R distribution functions pnorm, qnorm, etc.

See Also

[cdfgev](#) for the generalized extreme-value distribution, of which the Weibull (reflected through the origin) is a special case.

Examples

```

# Random sample from a 2-parameter Weibull distribution
# with scale parameter 2 and shape parameter 1.5.
quawei(runif(100), c(0,2,1.5))

# Illustrate the relation between Weibull and GEV distributions.
# weifit() fits a Weibull distribution to data and returns
#   quantiles of the fitted distribution
# gevfit() fits a Weibull distribution as a "reverse GEV",
#   i.e. fits a GEV distribution to the negated data,
#   then computes negated quantiles
weifit <- function(qval, x) quawei(qval, pelwei(samlmu(x)))
gevfit <- function(qval, x) -quagev(1-qval, pelgev(samlmu(-x)))
# Compare on Ozone data
data(airquality)
weifit(c(0.2,0.5,0.8), airquality$Ozone)
gevfit(c(0.2,0.5,0.8), airquality$Ozone)

```

evplot

Extreme-value plot

Description

evplot draws an “extreme-value plot”, i.e. a quantile-quantile plot in which the horizontal axis is the quantile of an extreme-value type I (Gumbel) distribution.

evdistp adds the cumulative distribution function of a distribution to an extreme-value plot.

evdistq adds the quantile function of a distribution to an extreme-value plot.

evpoints adds a set of data points to an extreme-value plot.

Usage

```
evplot(y, ...)
```

```
## Default S3 method:
```

```
evplot(y, qfunc, para, npoints = 101, plim, xlim = c(-2, 5),
       ylim, type,
       xlab = expression("Reduced variate, " * -log(-log(italic(F)))),
       ylab = "Quantile", rp.axis = TRUE, ...)
```

```
evdistp(pfunc, para, npoints = 101, ...)
```

```
evdistq(qfunc, para, npoints = 101, ...)
```

```
evpoints(y, ...)
```

Arguments

<code>y</code>	Numeric vector. The data values in the vector are plotted on the extreme-value plot.
<code>qfunc</code>	A quantile function. The function is drawn as a curve on the extreme-value plot.
<code>pfunc</code>	A cumulative distribution function. The function is drawn as a curve on the extreme-value plot.
<code>para</code>	Distribution parameters for the quantile function <code>qfunc</code> or cumulative distribution function <code>pfunc</code> . If <code>pfunc</code> or <code>qfunc</code> is the standard R form of quantile function, <code>para</code> should be a list. If <code>pfunc</code> or <code>qfunc</code> is the <code>qua...</code> form of quantile function used throughout the lmom package, <code>para</code> should be a numeric vector. In <code>evplot</code> , <code>para</code> is not used if <code>qfunc</code> is omitted.
<code>npoints</code>	Number of points to use in drawing the quantile function. The points are equally spaced along the x axis. Not used if <code>qfunc</code> is omitted.
<code>plim</code>	X axis limits, specified as probabilities.
<code>xlim</code>	X axis limits, specified as values of the Gumbel reduced variate $-\log(-\log F)$, where F is the nonexceedance probability. Not used if <code>plim</code> is specified.
<code>ylim</code>	Y axis limits.
<code>type</code>	Plot type. Determines how the data values in the <code>y</code> parameter are plotted. Interpreted in the same way as the <code>type</code> plotting parameter, i.e. "p" for points, "b" for points connected by lines, etc.
<code>xlab</code>	X axis label.
<code>ylab</code>	Y axis label.
<code>rp.axis</code>	Logical. Whether to draw the "Return period" axis, a secondary horizontal axis.
<code>...</code>	Additional parameters are passed to the plotting routine.

Arguments of cumulative distribution functions and quantile functions

`pfunc` and `qfunc` can be either the standard R form of cumulative distribution function or quantile function (i.e. for a distribution with r parameters, the first argument is the variate x or the probability p and the next r arguments are the parameters of the distribution) or the `cdf...` or `qua...` forms used throughout the **lmom** package (i.e. the first argument is the variate x or probability p and the second argument is a vector containing the parameter values).

Note

Data points are plotted at the Gringorten plotting position, i.e. the i th smallest of n data points is plotted at the horizontal position corresponding to nonexceedance probability $(i-0.44)/(n+0.12)$.

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

Examples

```
# Extreme-value plot of Ozone from the airquality data
data(airquality)
evplot(airquality$Ozone)

# Fit a GEV distribution and add it to the plot
evdistq(quagev, pelgev(samlmu(airquality$Ozone)))

# Not too good -- try a kappa distribution instead
evdistq(quakap, pelkap(samlmu(airquality$Ozone)), col="red")
```

lmr-functions

L-moments of specific probability distributions

Description

Computes the *L*-moments of a probability distribution given its parameters. The following distributions are recognized:

lmrexp	exponential
lmgam	gamma
lmrgev	generalized extreme-value
lmglo	generalized logistic
lmgpa	generalized Pareto
lmgno	generalized normal
lmgum	Gumbel (extreme-value type I)
lmrkap	kappa
lmrln3	three-parameter lognormal
lmrnor	normal
lmrpe3	Pearson type III
lmrwak	Wakeby
lmrwei	Weibull

Usage

```
lmrexp(para = c(0, 1), nmom = 2)
lmgam(para = c(1, 1), nmom = 2)
lmrgev(para = c(0, 1, 0), nmom = 3)
lmglo(para = c(0, 1, 0), nmom = 3)
lmgno(para = c(0, 1, 0), nmom = 3)
lmgpa(para = c(0, 1, 0), nmom = 3)
lmgum(para = c(0, 1), nmom = 2)
lmrkap(para = c(0, 1, 0, 0), nmom = 4)
lmrln3(para = c(0, 0, 1), nmom = 3)
lmrnor(para = c(0, 1), nmom = 2)
lmrpe3(para = c(0, 1, 0), nmom = 3)
```

```
lmrwak(para = c(0, 1, 0, 0, 0), nmom = 5)
lmrwei(para = c(0, 1, 1), nmom = 3)
```

Arguments

`para` Numeric vector containing the parameters of the distribution.
`nmom` The number of L -moments to be calculated.

Details

Numerical methods and accuracy are as described in Hosking (1996, pp. 8–9).

Value

Numeric vector containing the L -moments.

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

References

Hosking, J. R. M. (1996). Fortran routines for use with the method of L -moments, Version 3. Research Report RC20525, IBM Research Division, Yorktown Heights, N.Y.

See Also

[lmrp](#) to compute L -moments of a general distribution specified by its cumulative distribution function or quantile function.

[sam1mu](#) to compute L -moments of a data sample.

[pe1exp](#), etc., to compute the parameters of a distribution given its L -moments.

For individual distributions, see their cumulative distribution functions:

cdfexp	exponential
cdfgam	gamma
cdfgev	generalized extreme-value
cdfglo	generalized logistic
cdfgpa	generalized Pareto
cdfgno	generalized normal
cdfgum	Gumbel (extreme-value type I)
cdfkap	kappa
cdf1n3	three-parameter lognormal
cdfnor	normal
cdfpe3	Pearson type III
cdfwak	Wakeby
cdfwei	Weibull

Examples

```
# Compare sample L-moments of Ozone from the airquality data
# with the L-moments of a GEV distribution fitted to the data
data(airquality)
smom <- samlmu(airquality$Ozone, nmom=6)
gevpar <- pelgev(smom)
pmom <- lmrgev(gevpar, nmom=6)
print(smom)
print(pmom)
```

 lmrd

L-moment ratio diagram

Description

Draws an *L*-moment ratio diagram.

Usage

```
lmrd(x, y, distributions = "GLO GEV GPA GNO PE3", twopar,
     xlim, ylim, pch=3, cex, col, lty, lwd=1,
     legend.lmrd = TRUE, xlegend, ylegend,
     xlab = expression(italic(L) * "-skewness"),
     ylab = expression(italic(L) * "-kurtosis"), ...)
```

Arguments

- | | |
|---------------|--|
| x | Numeric vector of <i>L</i> -skewness values.
Alternatively, if argument y is omitted, x can be an object that contains both <i>L</i> -skewness and <i>L</i> -kurtosis values. It can be a vector with elements named "t_3" and "t_4" (or "tau_3" and "tau_4"), a matrix or data frame with columns named "t_3" and "t_4" (or "tau_3" and "tau_4"), or an object of class "regdata" (as defined in package lmomRFA). |
| y | Numeric vector of <i>L</i> -kurtosis values. |
| distributions | Indicates the three-parameter distributions whose <i>L</i> -skewness– <i>L</i> -kurtosis relations are to be plotted as lines on the diagram. The following distribution identifiers are recognized, in upper or lower case: |
| GLO | generalized logistic |
| GEV | generalized extreme-value |
| GPA | generalized Pareto |
| GNO | generalized normal |
| PE3 | Pearson type III |
| WAK.LB | lower bound of <i>L</i> -kurtosis for given <i>L</i> -skewness, for the Wakeby distribution. |
| ALL.LB | lower bound of <i>L</i> -kurtosis for given <i>L</i> -skewness, for all distributions. |

The argument should be either a character vector each of whose elements is one of the above abbreviations or a character string containing one or more of the abbreviations separated by blanks. The specified L -skewness– L -kurtosis curves will be plotted.

If no three-parameter distributions are to be plotted, specify distributions to be FALSE or the empty string, "".

twopar

Two-parameter distributions whose (L -skewness, L -kurtosis) values are to be plotted as points on the diagram. The following distribution identifiers are recognized, in upper or lower case:

E or EXP	exponential
G or GUM	Gumbel
L or LOG	logistic
N or NOR	normal
U or UNI	uniform

The argument should be either a character vector each of whose elements is one of the above abbreviations or a character string containing one or more of the abbreviations separated by blanks. L -skewness– L -kurtosis points for the specified distributions will be plotted and given one-character labels.

The default is to plot the two-parameter distributions that are special cases of the three-parameter distributions specified in the distributions parameter. Thus for example if distributions="GPA PE3", the default for twopar is "EXP NOR UNI": NOR is a special case of PE3, UNI of GPA, EXP of both GPA and PE3.

If no two-parameter distributions are to be plotted, specify twopar to be FALSE or the empty string, "".

xlim

x axis limits. Default: c(0, 0.6), expanded if necessary to cover the range of the data.

ylim

y axis limits. Default: c(0, 0.4), expanded if necessary to cover the range of the data.

pch

Plotting character to be used for the plotted (L -skewness, L -kurtosis) points.

cex

Symbol size for plotted points, like graphics parameter cex.

col

Vector specifying the colors. If it is of length 1 and x is present, it will be used for the plotted points. Otherwise it will be used for the lines on the plot. For the default colors for the lines, see the description of argument lty below.

lty

Vector specifying the line types to be used for the lines on the plot.

By default, colors and line types are matched to the distributions given in the distributions parameter, as follows:

GLO	blue, solid line
GEV	green, solid line
GPA	red, solid line
GNO	black, solid line
PE3	cyan, solid line
WAK.LB	red, dashed line
ALL.LB	black, dashed line

	The green and cyan colors are less bright than the standard "green" and "cyan"; they are defined to be "#00C000" and "#00E0E0", respectively.
lwd	Vector specifying the line widths to be used for the lines on the plot.
legend.lmr	Controls whether a legend, identifying the L -skewness– L -kurtosis relations of the three-parameter distributions, is plotted. Either logical, indicating whether a legend is to be drawn, or a list specifying arguments to the legend function. Default arguments include <code>bty="n"</code> , which must be overridden if a legend box is to be drawn; other arguments set by default are <code>x</code> , <code>y</code> , <code>legend</code> , <code>col</code> , <code>lty</code> , and <code>lwd</code> . Not used if <code>distributions=FALSE</code> .
xlegend	<code>x</code> coordinate of the upper left corner of the legend. Default: the minimum <code>x</code> value. Not used if <code>distributions=FALSE</code> or <code>legend.lmr=FALSE</code> .
ylegend	<code>y</code> coordinate of the upper left corner of the legend. Default: the maximum <code>y</code> value. Not used if <code>distributions=FALSE</code> or <code>legend.lmr=FALSE</code> .
xlab	X axis label.
ylab	Y axis label.
...	Additional arguments are passed to the <code>matplot</code> function that draws the axis box and the lines for three-parameter distributions.

Details

`lmr` calls a sequence of graphics functions: `matplot` for the axis box and the curves for three-parameter distributions; `points` for the points for two-parameter distributions and `text` for their labels; `legend` for the legend; and `points` for the (x, y) data points.

Note that the only graphics parameters passed to `points` are `col` (if of length 1), `cex`, and `pch`. If more complex features are required, such as different colors for different points, follow `lmr` by an additional call to `points`, e.g. follow `lmr(t3, t4)` by `points(t3, t4, col=c("red", "green"))`.

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

Examples

```
data(airquality)
lmr(samlmu(airquality$Ozone))
# Tweaking a few graphics parameters makes the graph look better
# (in the author's opinion)
lmr(samlmu(airquality$Ozone), xaxs="i", yaxs="i", las=1)

# An example that illustrates the sampling variability of L-moments
#
# Generate 50 random samples of size 30 from the Gumbel distribution
# - stored in the rows of matrix mm
mm <- matrix(quagum(runif(1500)), nrow=50)
#
# Compute the first four sample L-moments of each sample
```

```
# - stored in the rows of matrix aa
aa <- apply(mm, 1, samlmu)
#
# Plot the L-skewness and L-kurtosis values on an L-moment ratio
# diagram that also shows (only) the population L-moment ratios
# of the Gumbel distribution
lmerd(t(aa), dist="", twopar="G", col="red")
```

lmerdpoints

Add points or lines to an L-moment ratio diagram

Description

`lmerdpoints` adds points, and `lmerdlines` adds connected line segments, to an *L*-moment ratio diagram.

Usage

```
lmerdpoints(x, y=NULL, type="p", ...)
lmerdlines(x, y=NULL, type="l", ...)
```

Arguments

<code>x</code>	Numeric vector of <i>L</i> -skewness values.
<code>y</code>	Numeric vector of <i>L</i> -kurtosis values. May be omitted: see “Details” below.
<code>type</code>	Character indicating the type of plotting. Can be any valid value for the <code>type</code> argument of <code>plot.default</code> .
<code>...</code>	Further arguments (graphics parameters), passed to <code>points</code> or <code>lines</code> .

Details

The functions `lmerdpoints` and `lmerdlines` are equivalent to `points` and `lines` respectively, except that if argument `y` is omitted, `x` is assumed to be an object that contains both *L*-skewness and *L*-kurtosis values. As in `lmerd`, it can be a vector with elements named “`t_3`” and “`t_4`” (or “`tau_3`” and “`tau_4`”), a matrix or data frame with columns named “`t_3`” and “`t_4`” (or “`tau_3`” and “`tau_4`”), or an object of class “`regdata`” (as defined in package **lmomRFA**).

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

See Also

[lmerd](#), [points](#), [lines](#).

Examples

```

# Plot L-moment ratio diagram of Wind from the airquality data set
data(airquality)
lmsd(samlmu(airquality$Wind), xlim=c(-0.2, 0.2))
# Sample L-moments of each month's data
( lmom.monthly <- with(airquality,
  t(sapply(5:9, function(mo) samlmu(Wind[Month==mo])))) )
# Add the monthly values to the plot
lmsdpoints(lmom.monthly, pch=19, col="blue")

# Draw an L-moment ratio diagram and add a line for the
# Weibull distribution
lmsd()
weimom <- sapply( seq(0, 0.9, by=0.01),
  function(tau3) lmsdwei(pelwei(c(0,1,tau3)), nmom=4) )
lmsdlines(t(weimom), col='darkgreen', lwd=2)

```

lmp

L-moments of a general probability distribution

Description

Computes the *L*-moments of a probability distribution given its cumulative distribution function (for function `lmp`) or quantile function (for function `lmq`).

Usage

```
lmp(pfunc, ..., bounds = c(-Inf, Inf), symm = FALSE, order = 1:4,
  acc = 1e-6, subdiv = 100, verbose = FALSE)
```

```
lmq(qfunc, ..., symm = FALSE, order = 1:4, acc = 1e-6,
  subdiv = 100, verbose = FALSE)
```

Arguments

<code>pfunc</code>	Cumulative distribution function.
<code>qfunc</code>	Quantile function.
<code>...</code>	Arguments to <code>pfunc</code> or <code>qfunc</code> .
<code>bounds</code>	Either a vector of length 2, containing the lower and upper bounds of the distribution, or a function that calculates these bounds given the distribution parameters as inputs.
<code>symm</code>	For <code>lmq</code> , a logical value indicating whether the distribution is symmetric about its median.

For `lmp`, either the logical value `FALSE` to indicate that the distribution is not symmetric, or a numeric value to indicate that the distribution is symmetric and that the specified value is the center of symmetry.

If the distribution is symmetric, odd-order L -moment ratios are exactly zero and the symmetry is used to slightly speed up the computation of even-order L -moments.

<code>order</code>	Orders of the L -moments and L -moment ratios to be computed.
<code>acc</code>	Requested accuracy. The function will try to achieve this level of accuracy, as relative error for L -moments and absolute error for L -moment ratios.
<code>subdiv</code>	Maximum number of subintervals used in numerical integration.
<code>verbose</code>	Logical. If <code>FALSE</code> , only the values of the L -moments and L -moment ratios are returned. If <code>TRUE</code> , more details of the numerical integration are returned: see “Value” section below.

Details

L -moments are computed by numerical integration, using the R function `integrate`. `lmp` uses the expression $\lambda_r = \int_0^1 Q(u)P_{r-1}^*(u)du$ where $Q(u)$ is the quantile function and $P_{r-1}^*(u)$ is a shifted Legendre polynomial of order r (Hosking, 1990, eq. (2.2)). `lmp` uses the expression $\lambda_r = \int Z_r(F(x))dx$ where $F(x)$ is the cumulative distribution function and Z_r is a polynomial of degree r defined in Hosking (2007, eq. (3.5)).

Value

If `verbose=FALSE`, a numeric vector containing the L -moments (of orders 1 and 2) and L -moment ratios (of orders 3 and higher).

If `verbose=TRUE`, a data frame with columns as follows:

<code>value</code>	L -moments (for orders 1 and 2) and L -moment ratios (for orders 3 and higher).
<code>abs.error</code>	Estimate of the absolute error in the computed value.
<code>message</code>	“OK” or a character string giving the error message resulting from the numerical integration.

Arguments of cumulative distribution functions and quantile functions

`pfunc` and `qfunc` can be either the standard R form of cumulative distribution function or quantile function (i.e. for a distribution with r parameters, the first argument is the variate x or the probability p and the next r arguments are the parameters of the distribution) or the `cdf...` or `qua...` forms used throughout the **lmom** package (i.e. the first argument is the variate x or probability p and the second argument is a vector containing the parameter values). Even for the R form, however, starting values for the parameters are supplied as a vector `start`.

If `bounds` is a function, its arguments must match the distribution parameter arguments of `pfunc`: either a single vector, or a separate argument for each parameter.

Warning

Arguments `bounds`, `symm`, `order`, `acc`, `subdiv`, and `verbose` cannot be abbreviated and must be specified by their full names (if abbreviated, the names would be matched to the arguments of `pfunc` or `qfunc`).

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

References

Hosking, J. R. M. (1990). *L*-moments: analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society, Series B*, **52**, 105-124.

Hosking, J. R. M. (2007). Distributions with maximum entropy subject to constraints on their *L*-moments or expected order statistics. *Journal of Statistical Planning and Inference*, **137**, 2870-2891.

See Also

[lmrexp](#) to compute *L*-moments of specific distributions.

[samlm](#) to compute *L*-moments of a data sample.

[pel](#) and [pelexp](#), to compute the parameters of a distribution given its *L*-moments.

Examples

```
## Generalized extreme-value (GEV) distribution
## - three ways to get its L-moments
lmp(cdfgev, c(2,3,-0.2))
lmp(qagev, c(2,3,-0.2))
lmpgev(c(2,3,-0.2), nmom=4)

## GEV bounds specified as a vector
##
lmp(cdfgev, c(2,3,-0.2), bounds=c(-13,Inf))
##
## GEV bounds specified as a function -- single vector of parameters
##
gevbounds <- function(para) {
  k <- para[3]
  b <- para[1]+para[2]/k
  c(ifelse(k<0, b,- Inf), ifelse(k>0, b, Inf))
}
lmp(cdfgev, c(2,3,-0.2), bounds=gevbounds)
##
## GEV bounds specified as a function -- separate parameters
##
pgev <- function(x, xi, alpha ,k)
  pmin(1, pmax(0, exp(-((1-k*(x-xi)/alpha)^(1/k))))))
pgevbounds <- function(xi,alpha,k) {
  b <- xi+alpha/k
```

```

    c(ifelse(k<0, b, -Inf), ifelse(k>0, b, Inf))
  }
  lmrp(pgev, xi=2, alpha=3, k=-0.2, bounds=pgevbounds)

## Normal distribution
  lmrp(pnorm)
  lmrp(pnorm, symm=0)
  lmrp(pnorm, mean=2, sd=3, symm=2)
  # For comparison, the exact values
  lmrnor(c(2,3), nmom=4)

# Many L-moment ratios of the exponential distribution
# This may warn that "the integral is probably divergent"
  lmrq(qexp, order=3:20)

# ... nonetheless the computed values seem accurate:
# compare with the exact values, tau_r = 2/(r*(r-1)):
  cbind(exact=2/(3:20)/(2:19), lmrq(qexp, order=3:20, verbose=TRUE))

# Of course, sometimes the integral really is divergent
## Not run:
  lmrq(function(p) (1-p)^(-1.5))

## End(Not run)

# And sometimes the integral is divergent but that's not what
# the warning says (at least on the author's system)
  lmrq(qcauchy)

# Works for discrete distributions too, but often requires
# a larger-than-default value of subdiv
  lmrp(pbinom, size=20, prob=0.8, bounds=c(0,20), subdiv=1000)

```

pel-functions *Parameter estimation for specific distributions by the method of L-moments*

Description

Computes the parameters of a probability distribution as a function of the L -moments. The following distributions are recognized:

pelexp	exponential
pelgam	gamma
pelgev	generalized extreme-value
pelglo	generalized logistic
pelgpa	generalized Pareto
pelgno	generalized normal
pelgum	Gumbel (extreme-value type I)

pelkap	kappa
pelln3	three-parameter lognormal
pelnor	normal
pelpe3	Pearson type III
pelwak	Wakeby
pelwei	Weibull

Usage

```

pelexp(lmom)
pelgam(lmom)
pelgev(lmom)
pelglo(lmom)
pelgno(lmom)
pelgpa(lmom, bound = NULL)
pelgum(lmom)
pelkap(lmom)
pelln3(lmom, bound = NULL)
pelnor(lmom)
pelpe3(lmom)
pelwak(lmom, bound = NULL, verbose = FALSE)
pelwei(lmom, bound = NULL)

```

Arguments

lmom	Numeric vector containing the L -moments of the distribution or of a data sample.
bound	Lower bound of the distribution. If NULL (the default), the lower bound will be estimated along with the other parameters.
verbose	Logical: whether to print a message when not all parameters of the distribution can be computed.

Details

Numerical methods and accuracy are as described in Hosking (1996, pp. 8–9).

Value

A numeric vector containing the parameters of the distribution.

Author(s)

J. R. M. Hosking <hosking@us.ibm.com>

References

Hosking, J. R. M. (1996). Fortran routines for use with the method of L -moments, Version 3. Research Report RC20525, IBM Research Division, Yorktown Heights, N.Y.

See Also

[peIp](#) for parameter estimation of a general distribution specified by its cumulative distribution function or quantile function.

[lmrexp](#), etc., to compute the L -moments of a distribution given its parameters.

For individual distributions, see their cumulative distribution functions:

cdfexp	exponential
cdfgam	gamma
cdfgev	generalized extreme-value
cdfglo	generalized logistic
cdfgpa	generalized Pareto
cdfgno	generalized normal
cdfgum	Gumbel (extreme-value type I)
cdfkap	kappa
cdfln3	three-parameter lognormal
cdfnor	normal
cdfpe3	Pearson type III
cdfwak	Wakeby
cdfwei	Weibull

Examples

```
# Sample L-moments of Ozone from the airquality data
data(airquality)
lmom <- samlmu(airquality$Ozone)

# Fit a GEV distribution
peIgev(lmom)
```

peIp	<i>Parameter estimation for a general distribution by the method of L-moments</i>
------	---

Description

Computes the parameters of a probability distribution as a function of the L -moments.

Usage

```
peIp(lmom, pfunc, start, bounds = c(-Inf, Inf),
     type = c("n", "s", "ls", "lss"), method = "nlm", acc = 1e-5, ...)
```

```
peIq(lmom, qfunc, start, type = c("n", "s", "ls", "lss"),
     method = "nlm", acc = 1e-5, ...)
```

Arguments

lmom	Numeric vector containing the L -moments of the distribution or of a data sample.
pfunc	Cumulative distribution function of the distribution.
qfunc	Quantile function of the distribution.
start	Vector of starting values for the parameters.
bounds	Either a vector of length 2, containing the lower and upper bounds of the distribution, or a function that calculates these bounds given the distribution parameters as inputs.
type	Type of distribution, i.e. how it is parametrized. Must be one of the following: "1s" The distribution has a location parameter and a scale parameter. "1ss" The distribution has a location parameter and a scale parameter, and is symmetric about its median. "s" The distribution has a scale parameter but not a location parameter. "n" The distribution has neither a location parameter nor a scale parameter. For more details, see the "Distribution type" section below.
method	Method used to estimate the shape parameters (i.e. all parameters other than the location and scale parameters, if any). Valid values are "nlm" (the default), "uniroot" (which is valid only if the distribution has at most one shape parameter), and any of the values of the method argument of function <code>optim</code> . See the "Details" section below.
acc	Requested accuracy for the estimated parameters. This will be absolute accuracy for shape parameters, relative accuracy for a scale parameter, and absolute accuracy of the location parameter divided by the scale parameter for a location parameter.
...	Further arguments will be passed to the optimization function (<code>nlm</code> , <code>uniroot</code> , or <code>optim</code>).

Details

For shape parameters, numerical optimization is used to find parameter values for which the population L -moments or L -moment ratios are equal to the values supplied in `lmom`. Numerical optimization uses R functions `nlm` (if `method="nlm"`), `uniroot` (if `method="uniroot"`), or `optim` with the specified method (for the other values of `method`). Function `uniroot` uses one-dimensional root-finding, while functions `nlm` and `optim` try to minimize a criterion function that is the sum of squared differences between the population L -moments or L -moment ratios and the values supplied in `lmom`. Location and scale parameters are then estimated noniteratively. In all cases, the calculation of population L -moments and L -moment ratios is performed by function `lmp` or `lmq` (when using `pe1p` or `pe1q` respectively).

This approach is very crude. Nonetheless, it is often effective in practice. As in all numerical optimizations, success may depend on the way that the distribution is parametrized and on the particular choice of starting values for the parameters.

Value

A list with components:

para	Numeric vector containing the estimated parameters of the distribution.
code	An integer indicating the result of the numerical optimization used to estimate the shape parameters. It is 0 if there are no shape parameters. In general, values 1 and 2 indicate successful convergence of the iterative procedure, a value of 3 indicates that the iteration may not have converged, and values of 4 or more indicate that the iteration did not converge. Specifically, code is: For method "nlm", the code component of the return value from nlm. For method "uniroot", 1 if the estimated precision of the shape parameter is less than or equal to acc, and 4 otherwise. For the other methods, the convergence component of the return value from optim.

Further details of arguments

The length of `lmom` should be (at least) the highest order of L -moment used in the estimation procedure. For a distribution with r parameters this is $2r - 2$ if `type="lss"` and r otherwise.

`pfunc` and `qfunc` can be either the standard R form of cumulative distribution function or quantile function (i.e. for a distribution with r parameters, the first argument is the variate x or the probability p and the next r arguments are the parameters of the distribution) or the `cdf...` or `qua...` forms used throughout the **lmom** package (i.e. the first argument is the variate x or probability p and the second argument is a vector containing the parameter values). Even for the R form, however, starting values for the parameters are supplied as a vector `start`.

If `bounds` is a function, its arguments must match the distribution parameter arguments of `pfunc`: either a single vector, or a separate argument for each parameter.

It is assumed that location and scale parameters come first in the set of parameters of the distribution. Specifically: if `type="ls"` or `type="lss"`, it is assumed that the first parameter is the location parameter and that the second parameter is the scale parameter; if `type="s"` it is assumed that the first parameter is the scale parameter.

It is important that the length of `start` be equal to the number of parameters of the distribution. Starting values for location and scale parameters should be included in `start`, even though they are not used. If `start` has the wrong length, it is possible that meaningless results will be returned without any warning being issued.

Distribution type

The `type` argument affects estimation as follows. We assume that location and scale parameters are ξ and α respectively, and that the shape parameters (if there are any) are collectively designated by θ .

If `type="ls"`, then the L -moment ratios τ_3, τ_4, \dots depend only on the shape parameters. If there are any shape parameters, they are estimated by equating the sample L -moment ratios of orders 3, 4, etc., to the population L -moment ratios and solving the resulting equations for the shape parameters (using as many equations as there are shape parameters). The L -moment λ_2 is a multiple of α , the multiplier being a function only of θ . α is estimated by dividing the second sample L -moment by

the multiplier function evaluated at the estimated value of θ . The L -moment λ_1 is ξ plus a function of α and θ . ξ is estimated by subtracting from the first sample L -moment the function evaluated at the estimated values of α and θ .

If type="lss", then the L -moment ratios of odd order, τ_3, τ_5, \dots , are zero and the L -moment ratios of even order, τ_4, τ_6, \dots , depend only on the shape parameters. If there are any shape parameters, they are estimated by equating the sample L -moment ratios of orders 4, 6, etc., to the population L -moment ratios and solving the resulting equations for the shape parameters (using as many equations as there are shape parameters). Parameters α and ξ are estimated as in case when type="ls".

If type="s", then the L -moments divided by λ_1 , i.e. $\lambda_2/\lambda_1, \lambda_3/\lambda_1, \dots$, depend only on the shape parameters. If there are any shape parameters, they are estimated by equating the sample L -moments (divided by the first sample L -moment) of orders 2, 3, etc., to the corresponding population L -moments (divided by the first population L -moment) and solving the resulting equations (as many equations as there are shape parameters). The L -moment λ_1 is a multiple of α , the multiplier being a function only of θ . α is estimated by dividing the first sample L -moment by the multiplier function evaluated at the estimated value of θ .

If type="n", then all parameters are shape parameters. They are estimated by equating the sample L -moments of orders 1, 2, etc., to the population L -moments and solving the resulting equations for the parameters (using as many equations as there are parameters).

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See Also

[pelexp](#) for parameter estimation of specific distributions.

Examples

```
## Gamma distribution -- rewritten so that its first parameter
## is a scale parameter
my.pgamma <- function(x, scale, shape) pgamma(x, shape=shape, scale=scale)
pelp(c(5,2), my.pgamma, start=c(1,1), bounds=c(0,Inf), type="s")
# We can also do the estimation suppressing our knowledge
# that one parameter is a shape parameter.
pelp(c(5,2), my.pgamma, start=c(1,1), bounds=c(0,Inf), type="n")
rm(my.pgamma)

## Kappa distribution -- has location, scale and 2 shape parameters
# Estimate via pelq
pel.out <- pelq(c(10,5,0.3,0.15), quakap, start=c(0,1,0,0), type="ls")
pel.out
# Check that L-moments of estimated distribution agree with the
# L-moments input to pelq()
lmrkap(pel.out$para)
# Compare with the distribution-specific routine pelkap
pelkap(c(10,5,0.3,0.15))
rm(pel.out)

# Similar results -- what's the advantage of the specific routine?
```

```

system.time(pelq(c(10,5,0.3,0.15), quakap, start=c(0,1,0,0), type="ls"))
system.time(pelkap(c(10,5,0.3,0.15)))

# Caution -- pelq() will not check that estimates are reasonable
lmom <- c(10,5,0.2,0.25)
pel.out <- pelq(lmom, quakap, start=c(0,1,0,0), type="ls")
pel.out
lmrkap(pel.out$para) # should be close to lmom, but tau_3 and tau_4 are not
# What happened? pelkap will tell us
try(pelkap(lmom))
rm(lmom, pel.out)

## Inverse Gaussian -- don't have explicit estimators for this
## distribution, but can use numerical methods
#
# CDF of inverse gaussian distribution
pig <- function(x, mu, lambda) {
  temp <- suppressWarnings(sqrt(lambda/x))
  xx <- pnorm(temp*(x/mu-1))+exp(2*lambda/mu+pnorm(temp*(x/mu+1),
    lower.tail=FALSE, log.p=TRUE))
  out <- ifelse(x<=0, 0, xx)
  out
}
# Fit to ozone data
data(airquality)
(lmom<-samlmu(airquality$Ozone))
pel.out <- pelp(lmom[1:2], pig, start=c(10,10), bounds=c(0,Inf))
pel.out
# First four L-moments of fitted distribution,
# for comparison with sample L-moments
lmp(pig, pel.out$para[1], pel.out$para[2], bounds=c(0,Inf))
rm(pel.out)

## A Student t distribution with location and scale parameters
#
qstu <- function(p, xi, alpha, df) xi + alpha * qt(p, df)
# Estimate parameters. Distribution is symmetric: use type="lss"
pelq(c(3,5,0,0.2345), qstu, start=c(0,1,10), type="lss")
# Doesn't converge (at least on the author's system) --
# try a different parametrization
qstu2 <- function(p, xi, alpha, shape) xi + alpha * qt(p, 1/shape)
# Now it converges
pelq(c(3,5,0,0.2345), qstu2, start=c(0,1,0.1), type="lss")
# Or try a different optimization method
pelq(c(3,5,0,0.2345), qstu, start=c(0,1,10), type="lss",
  method="uniroot", lower=2, upper=100)
rm(qstu,qstu2)

```

Description

Computes the “unbiased” sample L -moments and L -moment ratios of a data vector.

Usage

```
sam1mu(x, nmom = 4, sort.data = TRUE)
sam1mu.s(x, nmom = 4, sort.data = TRUE)
```

Arguments

x	A numeric vector.
nmom	Number of L -moments to be found.
sort.data	Whether the x vector needs to be sorted.

Details

sam1mu and sam1mu.s are functionally identical. sam1mu calls a Fortran routine internally, and is usually faster. sam1mu.s is written entirely in the S language; it is provided so that users can conveniently see how the calculations are done.

Value

A numeric vector containing the L -moments and L -moment ratios, in the order ℓ_1, ℓ_2, t_3, t_4 , etc.

Note

Sample L -moments are defined in Hosking (1990). Calculations use the algorithm given in Hosking (1996, p.14).

Author(s)

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References

Hosking, J. R. M. (1990). L -moments: analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society, Series B*, **52**, 105-124.

Hosking, J. R. M. (1996). Fortran routines for use with the method of L -moments, Version 3. Research Report RC20525, IBM Research Division, Yorktown Heights, N.Y.

Examples

```
data(airquality)
sam1mu(airquality$Ozone,6)
```

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