Package ‘powerMediation’

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Title Power/Sample size calculation for mediation analysis

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Depends R (>= 2.9.0), stats

Description The package contains functions for calculating power,sample size, and minimal detectable mediation effect for testing mediation effect in linear, logistic, poisson, or cox regression. The package also contains functions for calculating power, sample size, and minimal detectable slope for testing the slope in a simple linear regression (only one predictor).

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R topics documented:

`minEffect.SLR` *Minimum detectable slope*

Description

Calculate minimal detectable slope given sample size and power for simple linear regression.

Usage

```
minEffect.SLR(n, power, sigma.x, sigma.y, alpha = 0.05, verbose = TRUE)
```

Arguments

`n`	sample size.
`power`	power for testing if $\lambda = 0$ for the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`sigma.x`	standard deviation of the predictor.
`sigma.y`	standard deviation of the outcome.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing minimum absolute detectable effect; `FALSE` means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Value

`lambda.a`	minimum absolute detectable effect.
`res.uniroot`	results of optimization to find the optimal minimum absolute detectable effect.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Dupont, W.D. and Plummer, W.D.. Power and Sample Size Calculations for Studies Involving Linear Regression. *Controlled Clinical Trials*. 1998;19:589-601.

See Also

power.SLR, power.SLR.rho, ss.SLR, ss.SLR.rho.

Examples

```
minEffect.SLR(n=100, power=0.8, sigma.x=0.2, sigma.y=0.5,
  alpha = 0.05, verbose = TRUE)
```

---

minEffect.VSMc *Minimum detectable slope for mediator in linear regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate minimal detectable slope for mediator given sample size and power in simple linear regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
minEffect.VSMc(n, power, sigma.m, sigma.e, corr.xm, alpha = 0.05, verbose = TRUE)
```

Arguments

n	sample size.
power	power for testing $b_2 = 0$ for the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
sigma.m	standard deviation of the mediator.
sigma.e	standard deviation of the random error term in the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
corr.xm	correlation between the predictor x and the mediator m.
alpha	type I error rate.
verbose	logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation `corr.xm` between the primary predictor and mediator is non-zero.

The full model is

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

The reduced model is

$$y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`b2`	minimum absolute detectable effect.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`powerMediation.VSMc`, `ssMediation.VSMc`

Examples

```
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# minimum effect is =0.1
minEffect.VSMc(n = 863, power = 0.8, sigma.m = 1,
  sigma.e = 1, corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

`minEffect.VSMc.cox` *Minimum detectable slope for mediator in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate minimal detectable slope for mediator given sample size and power in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
minEffect.VSMc.cox(n, power, sigma.m, psi, corr.xm, alpha = 0.05, verbose = TRUE)
```

Arguments

`n`	sample size.
`power`	power for testing $b_2 = 0$ for the cox regression $\log(\lambda) = \log(\lambda_0) + b1x_i + b2m_i$, where λ is the hazard function and λ_0 is the baseline hazard function.
`sigma.m`	standard deviation of the mediator.
`psi`	the probability that an observation is uncensored, so that the number of event $d = n * psi$, where n is the sample size.
`corr.xm`	correlation between the predictor x and the mediator m.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing minimum absolute detectable effect; `FALSE` means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the cox regressions:

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation `corr.xm` between the primary predictor and mediator is non-zero.

The full model is

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

b2	minimum absolute detectable effect.
res.uniroot	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

powerMediation.VSMc.cox, ssMediation.VSMc.cox

Examples

```
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# minimum effect is = log(1.5) = 0.4054651

minEffect.VSMc.cox(n = 1399, power = 0.7999916,
  sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
  alpha = 0.05, verbose = TRUE)
```

---

minEffect.VSMc.logistic   *Minimum detectable slope for mediator in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate minimal detectable slope for mediator given sample size and power in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
minEffect.VSMc.logistic(n, power, sigma.m, p, corr.xm, alpha = 0.05, verbose = TRUE)
```

Arguments

`n`	sample size.
`power`	power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1-p_i)) = b0 + b1x_i + b2m_i$.
`sigma.m`	standard deviation of the mediator.
`p`	the marginal prevalence of the outcome.
`corr.xm`	correlation between the predictor x and the mediator m.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing minimum absolute detectable effect; `FALSE` means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation `corr.xm` between the primary predictor and mediator is non-zero.

The full model is

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`b2`	minimum absolute detectable effect.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

powerMediation.VSMc.logistic, ssMediation.VSMc.logistic

Examples

```
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# minimum effect is log(1.5)= 0.4054651

minEffect.VSMc.logistic(n = 255, power = 0.8, sigma.m = 1,
  p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```

---

minEffect.VSMc.poisson

Minimum detectable slope for mediator in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method

---

Description

Calculate minimal detectable slope for mediator given sample size and power in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
minEffect.VSMc.poisson(n, power, sigma.m, EY, corr.xm, alpha = 0.05, verbose = TRUE)
```

Arguments

n	sample size.
power	power for testing $b_2 = 0$ for the poisson regression $\log(E(Y_i)) = b0 + b1x_i + b2m_i$.
sigma.m	standard deviation of the mediator.
EY	the marginal mean of the outcome
corr.xm	correlation between the predictor x and the mediator m.
alpha	type I error rate.
verbose	logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation corr.xm between the primary predictor and mediator is non-zero.

The full model is

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`b2`	minimum absolute detectable effect.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`powerMediation.VSMc.poisson`, `ssMediation.VSMc.poisson`

Examples

```
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# minimum effect is = log(1.35) = 0.3001046
minEffect.VSMc.poisson(n = 1239, power = 0.7998578,
  sigma.m = sqrt(0.25 * (1 - 0.25)),
  EY = 0.5, corr.xm = 0.5,
  alpha = 0.05, verbose = TRUE)
```

power.SLR *Power for testing slope for simple linear regression*

Description

Calculate power for testing slope for simple linear regression.

Usage

```
power.SLR(n, lambda.a, sigma.x, sigma.y, alpha = 0.05, verbose = TRUE)
```

Arguments

n	sample size.
lambda.a	regression coefficient in the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
sigma.x	standard deviation of the predictor.
sigma.y	standard deviation of the outcome.
alpha	type I error rate.
verbose	logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Value

power	power for testing if $b_2 = 0$.
delta	$\lambda\sigma_x\sqrt{n}/\sqrt{\sigma_y^2 - (\lambda\sigma_x)^2}$.
s	$\sqrt{\sigma_y^2 - (\lambda\sigma_x)^2}$.
t.cr	$\Phi^{-1}(1-\alpha/2)$, where Φ is the cumulative distribution function of the standard normal distribution.
rho	correlation between the predictor x and outcome $y = \lambda\sigma_x/\sigma_y$.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Dupont, W.D. and Plummer, W.D.. Power and Sample Size Calculations for Studies Involving Linear Regression. *Controlled Clinical Trials*. 1998;19:589-601.

See Also

`minEffect.SLR`, `power.SLR.rho`, `ss.SLR.rho`, `ss.SLR`.

Examples

```
power.SLR(n=100, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5,
  alpha = 0.05, verbose = TRUE)
```

`power.SLR.rho` *Power for testing slope for simple linear regression*

Description

Calculate power for testing slope for simple linear regression.

Usage

```
power.SLR.rho(n, rho2, alpha = 0.05, verbose = TRUE)
```

Arguments

`n`	sample size.
`rho2`	square of the correlation between the outcome and the predictor.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing power; `FALSE` means not printing power.

Details

The power is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Value

`power`	power for testing if $b_2 = 0$.
`delta`	$\sqrt{n}/\sqrt{1/\rho^2 - 1}$.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Dupont, W.D. and Plummer, W.D.. Power and Sample Size Calculations for Studies Involving Linear Regression. *Controlled Clinical Trials*. 1998;19:589-601.

See Also

`minEffect.SLR`, `power.SLR`, `ss.SLR.rho`, `ss.SLR`.

Examples

```
power.SLR.rho(n=100, rho2=0.6, alpha = 0.05, verbose = TRUE)
```

---

powerMediation.Sobel *Power for testing mediation effect (Sobel's test)*

Description

Calculate power for testing mediation effect based on Sobel's test.

Usage

```
powerMediation.Sobel(n, theta.1a, lambda.a, sigma.x, sigma.m,
  rho2.mx, sigma.epsilon, alpha = 0.05, verbose = TRUE)
```

Arguments

`n`	sample size.
`theta.1a`	regression coefficient for the predictor in the linear regression linking the predictor x to the mediator m ($m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$).
`lambda.a`	regression coefficient for the mediator in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
`sigma.x`	standard deviation of the predictor.
`sigma.m`	standard deviation of the mediator.
`rho2.mx`	square of the correlation between the predictor and the mediator.
`sigma.epsilon`	standard deviation of the random error term in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
`alpha`	type I error.
`verbose`	logical. `TRUE` means printing power; `FALSE` means not printing power.

Details

The power is for testing the null hypothesis $\theta_1\lambda = 0$ versus the alternative hypothesis $\theta_{1a}\lambda_a \neq 0$ for the linear regressions:

$$m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$$

$$y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

Test statistic is based on Sobel's (1982) test:

$$Z = \frac{\hat{\theta}_1\hat{\lambda}}{\hat{\sigma}_{\theta_1\lambda}}$$

where $\hat{\sigma}_{\theta_1\lambda}$ is the estimated standard deviation of the estimate $\hat{\theta}_1\hat{\lambda}$ using multivariate delta method:

$$\sigma_{\theta_1\lambda} = \sqrt{\theta_1^2\sigma_\lambda^2 + \lambda^2\sigma_{\theta_1}^2}$$

and $\sigma_{\theta_1}^2 = \sigma_e^2/(n\sigma_x^2)$ is the variance of the estimate $\hat{\theta}_1$, and $\sigma_\lambda^2 = \sigma_\epsilon^2/(n\sigma_m^2(1-\rho_{mx}^2))$ is the variance of the estimate $\hat{\lambda}$, $sigma_m^2$ is the variance of the mediator m_i.

From the linear regression $m_i = \theta_0 + \theta_1 x_i + e_i$, we have the relationship $\sigma_e^2 = \sigma_m^2(1-\rho_{mx}^2)$. Hence, we can simply the variance $\sigma_{\theta_1,\lambda}$ to

$$\sigma_{\theta_1\lambda} = \sqrt{\theta_1^2\frac{\sigma_\epsilon^2}{n\sigma_m^2(1-\rho_{mx}^2)} + \lambda^2\frac{\sigma_m^2(1-\rho_{mx}^2)}{n\sigma_x^2}}$$

Value

`power`	power of the test for the parameter $\theta_1\lambda$
`delta`	$\theta_1\lambda/(sd(\hat{\theta}_1)sd(\hat{\lambda}))$

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Sobel, M. E. Asymptotic confidence intervals for indirect effects in structural equation models. *Sociological Methodology*. 1982;13:290-312.

See Also

`ssMediation.Sobel`, `testMediation.Sobel`

Examples

```
powerMediation.Sobel(n=248, theta.1a=0.1701, lambda.a=0.1998,
  sigma.x=0.57, sigma.m=0.61, rho2.mx=0.3, sigma.epsilon=0.2,
  alpha = 0.05, verbose = TRUE)
```

`powerMediation.VSMc` *Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
powerMediation.VSMc(n, b2, sigma.m, sigma.e, corr.xm, alpha = 0.05,
  verbose = TRUE)
```

Arguments

`n`	sample size.
`b2`	regression coefficient for the mediator m in the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`sigma.m`	standard deviation of the mediator.
`sigma.e`	standard deviation of the random error term in the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`corr.xm`	correlation between the predictor x and the mediator m.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing power; `FALSE` means not printing power.

Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

The reduced model is

$$y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

power
: power for testing if $b_2 = 0$.

delta
: $b_2\sigma_m\sqrt{1-\rho_{xm}^2}/\sigma_e$, where σ_m is the standard deviation of the mediator m, ρ_{xm} is the correlation between the predictor x and the mediator m, and σ_e is the standard deviation of the random error term in the linear regression.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

minEffect.VSMc, ssMediation.VSMc

Examples

```
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# power=0.8
powerMediation.VSMc(n = 863, b2 = 0.1, sigma.m = 1, sigma.e = 1,
  corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

---

powerMediation.VSMc.cox

Power for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method

---

Description

Calculate Power for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
powerMediation.VSMc.cox(n, b2, sigma.m, psi, corr.xm, alpha = 0.05,
  verbose = TRUE)
```

Arguments

n	sample size.
b2	regression coefficient for the mediator m in the cox regression $\log(\lambda) = \log(\lambda_0) + b1x_i + b2m_i$, where λ is the hazard function and λ_0 is the baseline hazard function.
sigma.m	standard deviation of the mediator.
psi	the probability that an observation is uncensored, so that the number of event $d = n * psi$, where n is the sample size.
corr.xm	correlation between the predictor x and the mediator m.
alpha	type I error rate.
verbose	logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the cox regressions:

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i$$

where λ is the hazard function and λ_0 is the baseline hazard function.

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(\lambda) = \log(\lambda_0) + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

power	power for testing if $b_2 = 0$.
delta	$b_2\sigma_m\sqrt{(1-\rho_{xm}^2)psi}$

, where σ_m is the standard deviation of the mediator m, ρ_{xm} is the correlation between the predictor x and the mediator m, and psi is the probability that an observation is uncensored, so that the number of event $d = n * psi$, where n is the sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

minEffect.VSMc.cox, ssMediation.VSMc.cox

Examples

```
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# power = 0.7999916
powerMediation.VSMc.cox(n = 1399, b2 = log(1.5),
  sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
  alpha = 0.05, verbose = TRUE)
```

---

powerMediation.VSMc.logistic

Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method

---

Description

Calculate Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
powerMediation.VSMc.logistic(n, b2, sigma.m, p, corr.xm, alpha = 0.05,
  verbose = TRUE)
```

Arguments

n	sample size.
b2	regression coefficient for the mediator m in the logistic regression $\log(p_i/(1-p_i)) = b0 + b1x_i + b2m_i$.
sigma.m	standard deviation of the mediator.
p	the marginal prevalence of the outcome.
corr.xm	correlation between the predictor x and the mediator m.
alpha	type I error rate.
verbose	logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1-p_i)) = b0 + b1x_i + b2m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(p_i/(1-p_i)) = b_0 + b_1x_i + b_2m_i$$

The reduced model is

$$\log(p_i/(1-p_i)) = b_0 + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`power`	power for testing if $b_2 = 0$.
`delta`	$b_2\sigma_m\sqrt{(1-\rho_{xm}^2)p(1-p)}$

, where σ_m is the standard deviation of the mediator m, ρ_{xm} is the correlation between the predictor x and the mediator m, and p is the marginal prevalence of the outcome.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`minEffect.VSMc.logistic`, `ssMediation.VSMc.logistic`

Examples

```
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# power = 0.8005793
powerMediation.VSMc.logistic(n = 255, b2 = log(1.5), sigma.m = 1,
  p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```

`powerMediation.VSMc.poisson`

Power for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method

---

Description

Calculate Power for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
powerMediation.VSMc.poisson(n, b2, sigma.m, EY, corr.xm, alpha = 0.05,
  verbose = TRUE)
```

Arguments

`n`	sample size.
`b2`	regression coefficient for the mediator m in the poisson regression $\log(E(Y_i)) = b0 + b1x_i + b2m_i$.
`sigma.m`	standard deviation of the mediator.
`EY`	the marginal mean of the outcome.
`corr.xm`	correlation between the predictor x and the mediator m.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing power; `FALSE` means not printing power.

Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b0 + b1x_i + b2m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`power` power for testing if $b_2 = 0$.

`delta` $b_2 \sigma_m \sqrt{(1 - \rho_{xm}^2) EY}$

, where σ_m is the standard deviation of the mediator m, ρ_{xm} is the correlation between the predictor x and the mediator m, and EY is the marginal mean of the outcome.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`minEffect.VSMc.poisson`, `ssMediation.VSMc.poisson`

Examples

```
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# power = 0.7998578
powerMediation.VSMc.poisson(n = 1239, b2 = log(1.35),
  sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
  alpha = 0.05, verbose = TRUE)
```

---

ss.SLR *Sample size for testing slope for simple linear regression*

Description

Calculate sample size for testing slope for simple linear regression.

Usage

```
ss.SLR(power, lambda.a, sigma.x, sigma.y, n.lower = 2.01, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)
```

Arguments

`power`	power for testing if $\lambda = 0$ for the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`lambda.a`	regression coefficient in the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`sigma.x`	standard deviation of the predictor.
`sigma.y`	standard deviation of the outcome.
`n.lower`	lower bound for the sample size.
`n.upper`	upper bound for the sample size.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing sample size; `FALSE` means not printing sample size.

Details

The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Value

`n`	sample size.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu `<stwxq@channing.harvard.edu>`

References

Dupont, W.D. and Plummer, W.D.. Power and Sample Size Calculations for Studies Involving Linear Regression. *Controlled Clinical Trials*. 1998;19:589-601.

See Also

`minEffect.SLR`, `power.SLR`, `power.SLR.rho`, `ss.SLR.rho`.

Examples

```
ss.SLR(power=0.8, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5,
  alpha = 0.05, verbose = TRUE)
```

ss.SLR.rho *Sample size for testing slope for simple linear regression based on R2*

Description

Calculate sample size for testing slope for simple linear regression based on R2.

Usage

```
ss.SLR.rho(power, rho2, n.lower = 2.01, n.upper = 1e+30,
    alpha = 0.05, verbose = TRUE)
```

Arguments

power	power.
rho2	square of the correlation between the outcome and the predictor.
n.lower	lower bound of the sample size.
n.upper	upper bound o the sample size.
alpha	type I error rate.
verbose	logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Value

n	sample size.
res.uniroot	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Dupont, W.D. and Plummer, W.D.. Power and Sample Size Calculations for Studies Involving Linear Regression. *Controlled Clinical Trials*. 1998;19:589-601.

See Also

minEffect.SLR, power.SLR, power.SLR.rho, ss.SLR.

Examples

```
ss.SLR.rho(power=0.8, rho2=0.6, alpha = 0.05, verbose = TRUE)
```

---

ssMediation.Sobel *Sample size for testing mediation effectd (Sobel's test)*

Description

Calculate sample size for testing mediation effect based on Sobel's test.

Usage

```
ssMediation.Sobel(power, theta.1a, lambda.a, sigma.x, sigma.m,
  rho2.mx, sigma.epsilon, n.lower = 1, n.upper = 1e+30,
  alpha = 0.05, verbose = TRUE)
```

Arguments

power	power of the test.
theta.1a	regression coefficient for the predictor in the linear regression linking the predictor x to the mediator m ($m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$).
lambda.a	regression coefficient for the mediator in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
sigma.x	standard deviation of the predictor.
sigma.m	standard deviation of the mediator.
rho2.mx	square of the correlation between the predictor and the mediator.
sigma.epsilon	standard deviation of the random error term in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
n.lower	lower bound of the sample size.
n.upper	upper bound of the sample size.
alpha	type I error rate.
verbose	logical. TRUE means printing power; FALSE means not printing power.

Details

The sample size is for testing the null hypothesis $\theta_1\lambda = 0$ versus the alternative hypothesis $\theta_{1a}\lambda_a \neq 0$ for the linear regressions:

$$m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$$
$$y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

Test statistic is based on Sobel's (1982) test:

$$Z = \frac{\hat{\theta}_1\hat{\lambda}}{\hat{\sigma}_{\theta_1\lambda}}$$

where $\hat{\sigma}_{\theta_1\lambda}$ is the estimated standard deviation of the estimate $\hat{\theta}_1\hat{\lambda}$ using multivariate delta method:

$$\sigma_{\theta_1\lambda} = \sqrt{\theta_1^2\sigma_\lambda^2 + \lambda^2\sigma_{\theta_1}^2}$$

and $\sigma_{\theta_1}^2 = \sigma_e^2/(n\sigma_x^2)$ is the variance of the estimate $\hat{\theta}_1$, and $\sigma_\lambda = \sqrt{\sigma_\epsilon^2/(n\sigma_m^2(1-\rho_{mx}^2))}$ is the variance of the estimate $\hat{\lambda}$, $sigma_m^2$ is the variance of the mediator m_i.
From the linear regression $m_i = \theta_0 + \theta_1 x_i + e_i$, we have the relationship $\sigma_e^2 = \sigma_m^2(1-\rho_{mx}^2)$. Hence, we can simply the variance $\sigma_{\theta_1,\lambda}$ to

$$\sigma_{\theta_1\lambda} = \sqrt{\theta_1^2\frac{\sigma_\epsilon^2}{n\sigma_m^2(1-\rho_{mx}^2)} + \lambda^2\frac{\sigma_m^2(1-\rho_{mx}^2)}{n\sigma_x^2}}$$

Value

`n`	sample size.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Sobel, M. E. Asymptotic confidence intervals for indirect effects in structural equation models. *Sociological Methodology*. 1982;13:290-312.

See Also

`powerMediation.Sobel`, `testMediation.Sobel`

Examples

```
ssMediation.Sobel(power=0.8, theta.1a=0.1701, lambda.a=0.1998,
  sigma.x=0.57, sigma.m=0.61, rho2.mx=0.3, sigma.epsilon=0.2,
  alpha = 0.05, verbose = TRUE)
```

`ssMediation.VSMc` *Sample size for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate sample size for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
ssMediation.VSMc(power, b2, sigma.m, sigma.e, corr.xm,
  n.lower=1, n.upper=1e+30, alpha = 0.05, verbose=TRUE)
```

Arguments

`power`	power for testing $b_2 = 0$ for the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`b2`	regression coefficient for the mediator m in the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`sigma.m`	standard deviation of the mediator.
`sigma.e`	standard deviation of the random error term in the linear regression $y_i = b0 + b1x_i + b2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
`corr.xm`	correlation between the predictor x and the mediator m.
`n.lower`	lower bound for the sample size.
`n.upper`	upper bound for the sample size.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing sample size; `FALSE` means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

The reduced model is

$$y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

n	sample size.
res.uniroot	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

minEffect.VSMc, powerMediation.VSMc

Examples

```
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# n=863
ssMediation.VSMc(power = 0.80, b2 = 0.1, sigma.m = 1, sigma.e = 1,
  corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

---

ssMediation.VSMc.cox *Sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
ssMediation.VSMc.cox(power, b2, sigma.m, psi, corr.xm,
  n.lower=1, n.upper=1e+30, alpha = 0.05, verbose=TRUE)
```

Arguments

`power`	power for testing $b_2 = 0$ for the cox regression $\log(\lambda) = \log(\lambda_0) + b1x_i + b2m_i$, where λ is the hazard function and λ_0 is the baseline hazard function.
`b2`	regression coefficient for the mediator m in the cox regression $\log(\lambda) = \log(\lambda_0) + b1x_i + b2m_i$, where λ is the hazard function and λ_0 is the baseline hazard function.
`sigma.m`	standard deviation of the mediator.
`psi`	the probability that an observation is uncensored, so that the number of event $d = n * psi$, where n is the sample size.
`corr.xm`	correlation between the predictor x and the mediator m.
`n.lower`	lower bound for the sample size.
`n.upper`	upper bound for the sample size.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing sample size; `FALSE` means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the cox regressions:

$$\log(\lambda) = \log(\lambda_0) + b1x_i + b2m_i$$

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$$

The reduced model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`n`	sample size.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu `<stwxq@channing.harvard.edu>`

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`minEffect.VSMc.cox`, `powerMediation.VSMc.cox`

Examples

```
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# n = 1399
ssMediation.VSMc.cox(power = 0.7999916, b2 = log(1.5),
  sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
  alpha = 0.05, verbose = TRUE)
```

---

ssMediation.VSMc.logistic

Sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method

---

Description

Calculate sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
ssMediation.VSMc.logistic(power, b2, sigma.m, p, corr.xm,
  n.lower=1, n.upper=1e+30, alpha = 0.05, verbose=TRUE)
```

Arguments

`power`	power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1-p_i)) = b0 + b1x_i + b2m_i$.
`b2`	regression coefficient for the mediator m in the logistic regression $\log(p_i/(1-p_i)) = b0 + b1x_i + b2m_i$.
`sigma.m`	standard deviation of the mediator.
`p`	the marginal prevalence of the outcome.
`corr.xm`	correlation between the predictor x and the mediator m.
`n.lower`	lower bound for the sample size.
`n.upper`	upper bound for the sample size.
`alpha`	type I error rate.
`verbose`	logical. `TRUE` means printing sample size; `FALSE` means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(p_i/(1-p_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

`n`	sample size.
`res.uniroot`	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

`minEffect.VSMc.logistic`, `powerMediation.VSMc.logistic`

Examples

```
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# n=255

ssMediation.VSMc.logistic(power = 0.80, b2 = log(1.5), sigma.m = 1, p = 0.5,
  corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```

ssMediation.VSMc.poisson

Sample size for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method

Description

Calculate sample size for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```
ssMediation.VSMc.poisson(power, b2, sigma.m, EY, corr.xm,
  n.lower=1, n.upper=1e+30, alpha = 0.05, verbose=TRUE)
```

Arguments

power	power for testing $b_2 = 0$ for the poisson regression $\log(E(Y_i)) = b0 + b1x_i + b2m_i$.
b2	regression coefficient for the mediator m in the poisson regression $\log(E(Y_i)) = b0 + b1x_i + b2m_i$.
sigma.m	standard deviation of the mediator.
EY	the marginal mean of the outcome.
corr.xm	correlation between the predictor x and the mediator m.
n.lower	lower bound for the sample size.
n.upper	upper bound for the sample size.
alpha	type I error rate.
verbose	logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

n	sample size.
res.uniroot	results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Vittinghoff, E. and Sen, S. and McCulloch, C.E.. Sample size calculations for evaluating mediation. *Statistics In Medicine*. 2009;28:541-557.

See Also

minEffect.VSMc.poisson, powerMediation.VSMc.poisson

Examples

```
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# n = 1239
ssMediation.VSMc.poisson(power = 0.7998578, b2 = log(1.35),
  sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
  alpha = 0.05, verbose = TRUE)
```

---

testMediation.Sobel *P-value and confidence interval for testing mediation effect (Sobel's test)*

Description

Calculate p-value and confidence interval for testing mediation effect based on Sobel's test.

Usage

```
testMediation.Sobel(theta.1.hat, lambda.hat,
  sigma.theta1, sigma.lambda, alpha=0.05)
```

Arguments

theta.1.hat	estimated regression coefficient for the predictor in the linear regression linking the predictor x to the mediator m ($m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$).
lambda.hat	estimated regression coefficient for the mediator in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
sigma.theta1	standard deviation of $\hat{\theta}_1$ in the linear regression linking the predictor x to the mediator m ($m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$).
sigma.lambda	standard deviation of $\hat{\lambda}$ in the linear regression linking the predictor x and the mediator m to the outcome y ($y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$).
alpha	significance level of a test.

Details

The test is for testing the null hypothesis $\theta_1 \lambda = 0$ versus the alternative hypothesis $\theta_{1a}\lambda_a \neq 0$ for the linear regressions:

$$m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma_e^2)$$

$$y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

Test statistic is based on Sobel's (1982) test:

$$Z = \frac{\hat{\theta}_1 \hat{\lambda}}{\hat{\sigma}_{\theta_1 \lambda}}$$

where $\hat{\sigma}_{\theta_1 \lambda}$ is the estimated standard deviation of the estimate $\hat{\theta}_1 \hat{\lambda}$ using multivariate delta method:

$$\sigma_{\theta_1 \lambda} = \sqrt{\theta_1^2 \sigma_\lambda^2 + \lambda^2 \sigma_{\theta_1}^2}$$

and $\hat{\sigma}_{\theta_1}$ is the estimated standard deviation of the estimate $\hat{\theta}_1$, and $\hat{\sigma}_\lambda$ is the estimated standard deviation of the estimate $\hat{\lambda}$.

Value

pval	p-value for testing the null hypothesis $\theta_1 \lambda = 0$ versus the alternative hypothesis $\theta_{1a}\lambda_a \neq 0$.
CI.low	Lower bound of the $100(1-\alpha)\%$ confidence interval for the parameter $\theta_1 \lambda$.
CI.upp	Upper bound of the $100(1-\alpha)\%$ confidence interval for the parameter $\theta_1 \lambda$.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

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References

Sobel, M. E. Asymptotic confidence intervals for indirect effects in structural equation models. *Sociological Methodology*. 1982;13:290-312.

See Also

powerMediation.Sobel, ssMediation.Sobel

Examples

```
testMediation.Sobel(theta.1.hat=0.1701, lambda.hat=0.1998,
  sigma.theta1=0.01, sigma.lambda=0.02, alpha=0.05)
```

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