Package ‘ARDL’

May 16, 2023

Type Package

Title ARDL, ECM and Bounds-Test for Cointegration

Description Creates complex autoregressive distributed lag (ARDL) models and constructs the underlying unrestricted and restricted error correction model (ECM) automatically, just by providing the order. It also performs the bounds-test for cointegration as described in Pesaran et al. (2001) <doi:10.1002/jae.616> and provides the multipliers and the cointegrating equation. The validity and the accuracy of this package have been verified by successfully replicating the results of Pesaran et al. (2001) in Natsiopoulos and Tzeremes (2022) <doi:10.1002/jae.2919>.

Version 0.2.3

BugReports https://github.com/Natsiopoulos/ARDL/issues

License GPL-3

URL https://github.com/Natsiopoulos/ARDL

Encoding UTF-8

LazyData true

Depends R (>= 3.5.0)

Suggests qpcR, sandwich, testthat (>= 3.0.0)

Imports aod, dplyr, dynlm, gridExtra, ggplot2, lmtest, msm, stringr, zoo

RoxygenNote 7.2.0

Config/testthat/edition 3

NeedsCompilation no

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Repository CRAN

Date/Publication 2023-05-16 12:36:06 UTC
**R topics documented:**

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**Description**

A simple way to construct complex ARDL specifications providing just the model order additional to the model formula. It uses `dynlm` under the hood. `ardl` is a generic function and the default method constructs an 'ardl' model while the other method takes a model of `class 'uecm'` and converts it into an 'ardl'.

**Usage**

```r
ardl(...)  

## S3 method for class 'uecm'
ardl(object, ...)

## Default S3 method:
ardl(formula, data, order, start = NULL, end = NULL, ...)
```

**Arguments**

- `...`: Additional arguments to be passed to the low level regression fitting functions.
- `object`: An object of `class 'uecm'`.
- `formula`: A "formula" describing the linear model. Details for model specification are given under 'Details'.
ardl

data  A time series object (e.g., "ts", "zoo" or "zooreg") or a data frame containing the variables in the model. In the case of a data frame, it is coerced into a ts object with start = 1, end = nrow(data) and frequency = 1. If not found in data, the variables are NOT taken from any environment.

order  A specification of the order of the ARDL model. A numeric vector of the same length as the total number of variables (excluding the fixed ones, see 'Details'). It should only contain positive integers or 0. An integer could be provided if all variables are of the same order.

start  Start of the time period which should be used for fitting the model.

end  End of the time period which should be used for fitting the model.

Details

The formula should contain only variables that exist in the data provided through data plus some additional functions supported by dynlm (i.e., trend()).

You can also specify fixed variables that are not supposed to be lagged (e.g. dummies etc.) simply by placing them after |. For example, y ~ x1 + x2 | z1 + z2 where z1 and z2 are the fixed variables and should not be considered in order. Note that the | notion should not be confused with the same notion in dynlm where it introduces instrumental variables.

Value

ardl returns an object of class c("dynlm", "lm", "ardl"). In addition, attributes 'order', 'data', 'parsed_formula' and 'full_formula' are provided.

Mathematical Formula

The general form of an ARDL($p,q_1,...,q_k$) is:

\[ y_t = c_0 + c_1 t + \sum_{i=1}^{p} b_{y,i} y_{t-i} + \sum_{j=1}^{k} \sum_{l=0}^{q_j} b_{j,l} x_{j,t-l} + \epsilon_t \]

Author(s)

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See Also

uecm, recm

Examples

data(denmark)

## Estimate an ARDL(3,1,3,2) model -------------------------------------

ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
summary(ardl_3132)
## Add dummies or other variables that should stay fixed ---------------

d_74Q1_75Q3 <- ifelse(time(denmark) >= 1974 & time(denmark) <= 1975.5, 1, 0)

# the date can also be setted as below

d_74Q1_75Q3_ <- ifelse(time(denmark) >= "1974 Q1" & time(denmark) <= "1975 Q3", 1, 0)

identical(d_74Q1_75Q3, d_74Q1_75Q3_)

den <- cbind(denmark, d_74Q1_75Q3)

ardl_3132_d <- ardl(LRM ~ LRY + IBO + IDE | d_74Q1_75Q3,
                     data = den, order = c(3,1,3,2))

summary(ardl_3132_d)

compare <- data.frame(AIC = c(AIC(ardl_3132), AIC(ardl_3132_d)),
                       BIC = c(BIC(ardl_3132), BIC(ardl_3132_d)))
rownames(compare) <- c("no dummy", "with dummy")

compare

## Estimate an ARDL(3,1,3,2) model with a linear trend ---------------

ardl_3132_tr <- ardl(LRM ~ LRY + IBO + IDE + trend(LRM),
                      data = denmark, order = c(3,1,3,2))

# Alternative time trend specifications:
# time(LRM) 1974 + (0, 1, ..., 55)/4 time(data)
# trend(LRM) (1, 2, ..., 55)/4 (1:n)/freq
# trend(LRM, scale = FALSE) (1, 2, ..., 55) 1:n

## Subsample ARDL regression (start after 1975 Q4) ---------------------

ardl_3132_sub <- ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                       order = c(3,1,3,2), start = "1975 Q4")

# the date can also be setted as below

ardl_3132_sub2 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                        order = c(3,1,3,2), start = c(1975,4))

identical(ardl_3132_sub, ardl_3132_sub2)

summary(ardl_3132_sub)

## Ease of use ---------------------------------------------------------

# The model specification of the ardl_3132 model can be created as easy as order=c(3,1,3,2)
# or else, it could be done using the dynlm package as:

library(dynlm)
m <- dynlm(LRM ~ L(LRM, 1) + L(LRM, 2) + L(LRM, 3) + LRY + L(LRY, 1) + IBO + L(IBO, 1) + L(IBO, 2) + L(IBO, 3) + IDE + L(IDE, 1) + L(IDE, 2), data = denmark)

identical(m$coefficients, ardl_3132$coefficients)

# The full formula can be extracted from the ARDL model, and this is equal to

ardl_3132$full_formula

m2 <- dynlm(ardl_3132$full_formula, data = ardl_3132$data)

identical(m$coefficients, m2$coefficients)
**auto_ardl**

---

### Automatic ARDL model selection

**Description**

It searches for the best ARDL order specification, according to the selected criterion, taking into account the constraints provided.

**Usage**

```r
auto_ardl(
  formula,
  data,
  max_order,
  fixed_order = -1,
  starting_order = NULL,
  selection = "AIC",
  selection_minmax = c("min", "max"),
  grid = FALSE,
  search_type = c("horizontal", "vertical"),
  start = NULL,
  end = NULL,
  ...
)
```

**Arguments**

- **formula**: A "formula" describing the linear model. Details for model specification are given under 'Details' in the help file of the `ardl` function.
- **data**: A time series object (e.g., "ts", "zoo" or "zooreg") or a data frame containing the variables in the model. In the case of a data frame, it is coerced into a `ts` object with `start = 1`, `end = nrow(data)` and `frequency = 1`. If not found in data, the variables are NOT taken from any environment.
- **max_order**: It sets the maximum order for each variable where the search is taking place. A numeric vector of the same length as the total number of variables (excluding the fixed ones, see 'Details' in the help file of the `ardl` function). It should only contain positive integers. An integer could be provided if the maximum order for all variables is the same.
- **fixed_order**: It allows setting a fixed order for some variables. The algorithm will not search for any other order than this. A numeric vector of the same length as the total number of variables (excluding the fixed ones). It should contain positive integers or 0 to set as a constraint. A -1 should be provided for any variable that should not be constrained. `fixed_order` overrides the corresponding `max_order` and `starting_order`.
starting_order Specifies the order for each variable from which each search will start. It is a numeric vector of the same length as the total number of variables (excluding the fixed ones). It should contain positive integers or 0 or only one integer could be provided if the starting order for all variables is the same. Default is set to NULL. If unspecified (NULL) and grid = FALSE, then all possible ARDL(p) models are calculated (constraints are taken into account), where p is the minimum value in max_order. Note that where starting_order is provided, its first element will be the minimum value of p that the searching algorithm will consider (think of it like a 'minimum p order' restriction) (see 'Searching algorithm' below). If grid = TRUE, only the first argument (p) will have an effect.

selection A character string specifying the selection criterion according to which the candidate models will be ranked. Default is AIC. Any other selection criterion can be used (a user specified or a function from another package) as long as it can be applied as selection(model). The preferred model is the one with the smaller value of the selection criterion. If the selection criterion works the other way around (the bigger the better), selection_minmax = "max" should also be supplied (see 'Examples' below).

selection_minmax A character string that indicates whether the criterion in selection is supposed to be minimized (default) or maximized.

grid If FALSE (default), the stepwise searching regression algorithm will search for the best model by adding and subtracting terms corresponding to different ARDL orders. If TRUE, the whole set of all possible ARDL models (accounting for constraints) will be evaluated. Note that this method can be very time-consuming in case that max_order is big and there are many independent variables that create a very big number of possible combinations.

search_type A character string describing the search type. If "horizontal" (default), the searching algorithm increases or decreases by 1 the order of each variable in each iteration. When the order of the last variable has been accessed, it begins again from the first variable until it converges. If "vertical", the searching algorithm increases or decreases by 1 the order of a variable until it converges. Then it continues the same for the next variable. The two options result to very similar top orders. The default ("horizontal"), sometimes is a little more accurate, but the "vertical" is almost 2 times faster. Not applicable if grid = TRUE.

start Start of the time period which should be used for fitting the model.

end End of the time period which should be used for fitting the model.

... Additional arguments to be passed to the low level regression fitting functions.

Value

auto_ardl returns a list which contains:

best_model An object of class c("dynlm", "lm", "ardl")
best_order A numeric vector with the order of the best model selected
top_orders A data.frame with the orders of the top 20 models
Searching algorithm

The algorithm performs the optimization process starting from multiple starting points concerning the autoregressive order \( p \). The searching algorithm will perform a complete search, each time starting from a different starting order. These orders are presented in the tables below, for \( \text{grid} = \text{FALSE} \) and different values of \( \text{starting\_order} \).

\[
\begin{align*}
\text{starting\_order} & = \text{NULL}: \\
\text{ARDL}(p) & \rightarrow p \quad q_1 \quad q_2 \ldots \quad q_k \\
\text{ARDL}(1) & \rightarrow 1 \quad 1 \quad 1 \ldots \quad 1 \\
\text{ARDL}(2) & \rightarrow 2 \quad 2 \quad 2 \ldots \quad 2 \\
\vdots & \rightarrow : \quad : \quad : \quad : \\
\text{ARDL}(P) & \rightarrow P \quad P \quad P \ldots \quad P
\end{align*}
\]

\[
\begin{align*}
\text{starting\_order} & = c(3, 0, 1, 2): \\
p & q_1 \quad q_2 \quad q_3 \\
3 & 0 \quad 1 \quad 2 \\
4 & 0 \quad 1 \quad 2 \\
\vdots & \vdots \quad \vdots \quad \vdots \\
P & 0 \quad 1 \quad 2
\end{align*}
\]

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See Also

ardl

Examples

data(denmark)

## Find the best ARDL order --------------------------------------------
# Up to 5 for the autoregressive order (p) and 4 for the rest (q1, q2, q3)
# Using the defaults search_type = "horizontal", grid = FALSE and selection = "AIC"
# ("Not run" indications only for testing purposes)
# Not run:
model1 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                    max_order = c(5,4,4,4))
model1$top_orders

## Same, with search_type = "vertical" -------------------------------
model1_h <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                      max_order = c(5,4,4,4), search_type = "vertical")
model1_h$top_orders
## Find the global optimum ARDL order ----------------------------------

# It may take more than 10 seconds
model_grid <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                      max_order = c(5,4,4,4), grid = TRUE)

## Different selection criteria ----------------------------------------

# Using BIC as selection criterion instead of AIC
model1_b <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                      max_order = c(5,4,4,4), selection = "BIC")
model1_b$top_orders

# Using other criteria like adjusted R squared (the bigger the better)
adjr2 <- function(x) { summary(x)$adj.r.squared }
model1_adjr2 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                          max_order = c(5,4,4,4), selection = "adjr2",
                          selection_minmax = "max")
model1_adjr2$top_orders

# Using functions from other packages as selection criteria
if (requireNamespace("qpcR", quietly = TRUE)) {
    library(qpcR)
    model1_aicc <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                          max_order = c(5,4,4,4), selection = "AICc")
    model1_aicc$top_orders
    adjr2 <- function(x){ Rsq.ad(x) }
    model1_adjr2 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                              max_order = c(5,4,4,4), selection = "adjr2",
                              selection_minmax = "max")
    model1_adjr2$top_orders
}

## Different starting order --------------------------------------------

# The searching algorithm will start from the following starting orders:
# p q1 q2 q3
# 1 1 3 2
# 2 1 3 2
# 3 1 3 2
# 4 1 3 2
# 5 1 3 2
model1_so <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark,
                        max_order = c(5,4,4,4), starting_order = c(1,1,3,2))

# Starting from p=3 (don't search for p=1 and p=2)
# Starting orders:
# p q1 q2 q3
# 3 1 3 2
# 4 1 3 2
# 5 1 3 2
model1_so_3 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark, 
                   max_order = c(5,4,4,4), starting_order = c(3,1,3,2))

# If starting_order = NULL, the starting orders for each iteration will be:
# p  q1  q2  q3
# 1   1   1
# 2   2   2
# 3   3   3
# 4   4   4
# 5   5   5

## Add constraints -----------------------------------------------

# Restrict only the order of IBO to be 2
model1_ibo2 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark, 
                           max_order = c(5,4,4,4), fixed_order = c(-1,-1,2,-1))

model1_ibo2$top_orders

# Restrict the order of LRM to be 3 and the order of IBO to be 2
model1_lrm3_ibo2 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark, 
                               max_order = c(5,4,4,4), fixed_order = c(3,-1,2,-1))

model1_lrm3_ibo2$top_orders

## Set the starting date for the regression (data starts at "1974 Q1") -

# Set regression starting date to "1976 Q1"
model1_76q1 <- auto_ardl(LRM ~ LRY + IBO + IDE, data = denmark, 
                         max_order = c(5,4,4,4), start = "1976 Q1")

start(model1_76q1$best_model)

## End(Not run)

---

**bounds_f_test**

*Bounds Wald-test for no cointegration*

**Description**

`bounds_f_test` performs the Wald bounds-test for no cointegration *Pesaran et al. (2001)*. It is a Wald test on the parameters of a UECM (Unrestricted Error Correction Model) expressed either as a Chisq-statistic or as an F-statistic.

**Usage**

```r
bounds_f_test(
  object, 
  case, 
  alpha = NULL, 
  pvalue = TRUE, 
)```

---
\begin{verbatim}
    bounds_f_test

    exact = FALSE,
    R = 40000,
    test = c("F", "Chisq"),
    vcov_matrix = NULL
  

Arguments

object       An object of class 'ardl' or 'uecm'.
case         An integer from 1-5 or a character string specifying whether the 'intercept' and/or the 'trend' have to participate in the short-run or the long-run relationship (cointegrating equation) (see section 'Cases' below).
alpha        A numeric value between 0 and 1 indicating the significance level of the critical value bounds. If NULL (default), no critical value bounds for a specific level of significance are provide, only the p-value. See section 'alpha, bounds and p-value' below for details.
pvalue       A logical indicating whether you want the p-value to be provided. The default is TRUE. See section 'alpha, bounds and p-value' below for details.
exact        A logical indicating whether you want asymptotic (T = 1000) or exact sample size critical value bounds and p-value. The default is FALSE for asymptotic. See section 'alpha, bounds and p-value' below for details.
R             An integer indicating how many iterations will be used if exact = TRUE. Default is 40000.
test         A character vector indicating whether you want the Wald test to be expressed as 'F' or as 'Chisq' statistic. Default is "F".
vcov_matrix  The estimated covariance matrix of the random variable that the test uses to estimate the test statistic. The default is vcov(object) (when vcov_matrix = NULL), but other estimations of the covariance matrix of the regression's estimated coefficients can also be used (e.g., using vcovHC or vcovHAC). Only applicable if the input object is of class "uecm".

Value

A list with class "htest" containing the following components:

method       a character string indicating what type of test was performed.
alternative  a character string describing the alternative hypothesis.
statistic    the value of the test statistic.
null.value   the value of the population parameters k (the number of independent variables) and T (the number of observations) specified by the null hypothesis.
data.name     a character string giving the name(s) of the data.
parameters    numeric vector containing the critical value bounds.
p.value       the p-value of the test.

A numerical vector containing the critical value bounds as presented by Pesaran et al. (2001). See section 'alpha, bounds and p-value' below for details.
\end{verbatim}
data.frame containing the statistic, the critical value bounds, the alpha level of significance and the p-value.

Hypothesis testing

\[ \Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{j=1}^k \pi_j x_{j,t-1} + \sum_{i=1}^{p-1} \psi_{y,i} \Delta y_{t-i} + \sum_{j=1}^k \sum_{l=1}^{q_j-1} \psi_{j,l} \Delta x_{j,t-l} + \sum_{j=1}^k \sum_{l=1}^{q_j-1} \omega_j \Delta x_{j,t-l} + \epsilon_t \]

Cases 1, 3, 5:

\[ H_0 : \pi_y = \pi_1 = \cdots = \pi_k = 0 \]
\[ H_1 : \pi_y \neq \pi_1 \neq \cdots \neq \pi_k \neq 0 \]

Case 2:

\[ H_0 : \pi_y = \pi_1 = \cdots = \pi_k = c_0 = 0 \]
\[ H_1 : \pi_y \neq \pi_1 \neq \cdots \neq \pi_k \neq c_0 \neq 0 \]

Case 4:

\[ H_0 : \pi_y = \pi_1 = \cdots = \pi_k = c_1 = 0 \]
\[ H_1 : \pi_y \neq \pi_1 \neq \cdots \neq \pi_k \neq c_1 \neq 0 \]

**alpha, bounds and p-value**

In this section it is explained how the critical value bounds and p-values are obtained.

- If `exact = FALSE`, then the asymptotic (T = 1000) critical value bounds and p-value are provided.
- Only the asymptotic critical value bounds and p-values, and only for k <= 10 are precalculated, everything else has to be computed.
- Precalculated critical value bounds and p-values were simulated using `set.seed(2020)` and `R = 70000`.
- Precalculated critical value bounds exist only for `alpha` being one of the 0.005, 0.01, 0.025, 0.05, 0.075, 0.1, 0.15 or 0.2, everything else has to be computed.
- If `alpha` is one of the 0.1, 0.05, 0.025 or 0.01 (and `exact = FALSE` and k <= 10), `PSS2001parameters` shows the critical value bounds presented in Pesaran et al. (2001) (less precise).
Cases

According to Pesaran et al. (2001), we distinguish the long-run relationship (cointegrating equation) (and thus the bounds-test and the Restricted ECMs) between 5 different cases. These differ in terms of whether the 'intercept' and/or the 'trend' are restricted to participate in the long-run relationship or they are unrestricted and so they participate in the short-run relationship.

Case 1:  
- No intercept and no trend.
  - case inputs: 1 or "n" where "n" stands for none.

Case 2:  
- Restricted intercept and no trend.
  - case inputs: 2 or "rc" where "rc" stands for restricted constant.

Case 3:  
- Unrestricted intercept and no trend.
  - case inputs: 3 or "uc" where "uc" stands for unrestricted constant.

Case 4:  
- Unrestricted intercept and restricted trend.
  - case inputs: 4 or "ucrt" where "ucrt" stands for unrestricted constant and restricted trend.

Case 5:  
- Unrestricted intercept and unrestricted trend.
  - case inputs: 5 or "ucut" where "ucut" stands for unrestricted constant and unrestricted trend.

Note that you can't restrict (or leave unrestricted) a parameter that doesn't exist in the input model. For example, you can't compute `recm(object,case=3)` if the object is an ARDL (or UECM) model with no intercept. The same way, you can't compute `bounds_f_test(object, case=5)` if the object is an ARDL (or UECM) model with no linear trend.

References


Author(s)

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See Also

`bounds_t_test ardl uecm`

Examples

data(denmark)

## How to use cases under different models (regarding deterministic terms)

## Construct an ARDL(3,1,3,2) model with different deterministic terms -

# Without constant
ardl_3132_n <- ardl(LRM ~ LRY + IBO + IDE -1, data = denmark, order = c(3,1,3,2))

# With constant
bounds_f_test

ardl_3132_c <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))

# With constant and trend
ardl_3132_ct <- ardl(LRM ~ LRY + IBO + IDE + trend(LRM), data = denmark, order = c(3,1,3,2))

## F-bounds test for no level relationship (no cointegration) ------

# For the model without a constant
bounds_f_test(ardl_3132_n, case = 1)
# or
bounds_f_test(ardl_3132_n, case = "n")

# For the model with a constant
# Including the constant term in the long-run relationship (restricted constant)
bounds_f_test(ardl_3132_c, case = 2)
# or
bounds_f_test(ardl_3132_c, case = "rc")

# Including the constant term in the short-run relationship (unrestricted constant)
bounds_f_test(ardl_3132_c, case = "uc")
# or
bounds_f_test(ardl_3132_c, case = 3)

# For the model with constant and trend
# Including the constant term in the short-run and the trend in the long-run relationship
# (unrestricted constant and restricted trend)
bounds_f_test(ardl_3132_ct, case = "ucrt")
# or
bounds_f_test(ardl_3132_ct, case = 4)

# For the model with constant and trend
# Including the constant term and the trend in the short-run relationship
# (unrestricted constant and unrestricted trend)
bounds_f_test(ardl_3132_ct, case = "ucut")
# or
bounds_f_test(ardl_3132_ct, case = 5)

## Note that you can't restrict a deterministic term that doesn't exist

## Asymptotic p-value and critical value bounds (assuming T = 1000) ----

# F-statistic is larger than the I(1) bound (for a=0.05) as expected (p-value < 0.05)
bft <- bounds_f_test(ardl_3132_c, case = 2, alpha = 0.05)
bounds_t_test

**Bounds t-test for no cointegration**

### Description

`bounds_t_test` performs the t-bounds test for no cointegration Pesaran et al. (2001). It is a t-test on the parameters of a UECM (Unrestricted Error Correction Model).

### Usage

```r
bounds_t_test(
  object,
  case,
  alpha = NULL,
  pvalue = TRUE,
  exact = FALSE,
  R = 40000,
  vcov_matrix = NULL
)
```
Arguments

object An object of class `ardl` or `uecm`.
case An integer (1, 3 or 5) or a character string specifying whether the 'intercept' and/or the 'trend' have to participate in the short-run relationship (see section 'Cases' below). Note that the t-bounds test can’t be applied for cases 2 and 4.
alpha A numeric value between 0 and 1 indicating the significance level of the critical value bounds. If NULL (default), no critical value bounds for a specific level of significance are provide, only the p-value. See section 'alpha, bounds and p-value' below for details.
pvalue A logical indicating whether you want the p-value to be provided. The default is TRUE. See section 'alpha, bounds and p-value' below for details.
exact A logical indicating whether you want asymptotic (T = 1000) or exact sample size critical value bounds and p-value. The default is FALSE for asymptotic. See section 'alpha, bounds and p-value' below for details.
R An integer indicating how many iterations will be used if exact = TRUE. Default is 40000.
vcov_matrix The estimated covariance matrix of the random variable that the test uses to estimate the test statistic. The default is vcov(object) (when vcov_matrix = NULL), but other estimations of the covariance matrix of the regression’s estimated coefficients can also be used (e.g., using vcovHC or vcovHAC). Only applicable if the input object is of class "uecm".

Value

A list with class "htest" containing the following components:

method a character string indicating what type of test was performed.
alternative a character string describing the alternative hypothesis.
statistic the value of the test statistic.
null.value the value of the population parameters k (the number of independent variables) and T (the number of observations) specified by the null hypothesis.
data.name a character string giving the name(s) of the data.
parameters numeric vector containing the critical value bounds.
p.value the p-value of the test.
PSS2001parameters numeric vector containing the critical value bounds as presented by Pesaran et al. (2001). See section 'alpha, bounds and p-value' below for details.
tab data.frame containing the statistic, the critical value bounds, the alpha level of significance and the p-value.

Hypothesis testing

\[
\Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{j=1}^{k} \pi_j x_{j,t-1} + \sum_{i=1}^{p-1} \psi_{y,i} \Delta y_{t-i} + \sum_{j=1}^{k} \sum_{i=1}^{q_j-1} \psi_{j,i} \Delta x_{j,t-l} + \sum_{j=1}^{k} \omega_j \Delta x_{j,t} + \epsilon_t
\]
\[ H_0 : \pi_y = 0 \]
\[ H_1 : \pi_y \neq 0 \]

alpha, bounds and p-value

In this section it is explained how the critical value bounds and p-values are obtained.

- If \( \text{exact} = \text{FALSE} \), then the asymptotic (\( T = 1000 \)) critical value bounds and p-value are provided.
- Only the asymptotic critical value bounds and p-values, and only for \( k \leq 10 \) are precalculated, everything else has to be computed.
- Precalculated critical value bounds and p-values were simulated using \( \text{set.seed}(2020) \) and \( R = 70000 \).
- Precalculated critical value bounds exist only for \( \alpha \) being one of the 0.005, 0.01, 0.025, 0.05, 0.075, 0.1, 0.15 or 0.2, everything else has to be computed.
- If \( \alpha \) is one of the 0.1, 0.05, 0.025 or 0.01 (and \( \text{exact} = \text{FALSE} \) and \( k \leq 10 \)), \text{PSS2001parameters} shows the critical value bounds presented in Pesaran et al. (2001) (less precise).

Cases

According to Pesaran et al. (2001), we distinguish the long-run relationship (cointegrating equation) (and thus the bounds-test and the Restricted ECMs) between 5 different cases. These differ in terms of whether the 'intercept' and/or the 'trend' are restricted to participate in the long-run relationship or they are unrestricted and so they participate in the short-run relationship.

**Case 1:**
- No intercept and no trend.
- case inputs: 1 or "n" where "n" stands for none.

**Case 2:**
- Restricted intercept and no trend.
- case inputs: 2 or "rc" where "rc" stands for restricted constant.

**Case 3:**
- Unrestricted intercept and no trend.
- case inputs: 3 or "uc" where "uc" stands for unrestricted constant.

**Case 4:**
- Unrestricted intercept and restricted trend.
- case inputs: 4 or "ucrt" where "ucrt" stands for unrestricted constant and restricted trend.

**Case 5:**
- Unrestricted intercept and unrestricted trend.
- case inputs: 5 or "ucut" where "ucut" stands for unrestricted constant and unrestricted trend.

Note that you can’t restrict (or leave unrestricted) a parameter that doesn’t exist in the input model. For example, you can’t compute \( \text{recm(object,case=3)} \) if the object is an ARDL (or UECM) model with no intercept. The same way, you can’t compute \( \text{bounds_f_test(object, case=5)} \) if the object is an ARDL (or UECM) model with no linear trend.

References

Author(s)
Kleanthis Natsiopoulos, <klnatsio@gmail.com>

See Also
bounds_f_test ardl uecm

Examples

data(denmark)
## How to use cases under different models (regarding deterministic terms)
## Construct an ARDL(3,1,3,2) model with different deterministic terms -
# Without constant
ardl_3132_n <- ardl(LRM ~ LRY + IBO + IDE -1, data = denmark, order = c(3,1,3,2))
# With constant
ardl_3132_c <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
# With constant and trend
ardl_3132_ct <- ardl(LRM ~ LRY + IBO + IDE + trend(LRM), data = denmark, order = c(3,1,3,2))
## t-bounds test for no level relationship (no cointegration) ----------
# For the model without a constant
bounds_t_test(ardl_3132_n, case = 1)
# or
bounds_t_test(ardl_3132_n, case = "n")
# For the model with a constant
# Including the constant term in the short-run relationship (unrestricted constant)
bounds_t_test(ardl_3132_c, case = "uc")
# or
bounds_t_test(ardl_3132_c, case = 3)
# For the model with constant and trend
# Including the constant term and the trend in the short-run relationship
# (unrestricted constant and unrestricted trend)
bounds_t_test(ardl_3132_ct, case = "ucut")
# or
bounds_t_test(ardl_3132_ct, case = 5)
## Note that you can't use bounds t-test for cases 2 and 4, or use a wrong model
# For example, the following tests will produce an error:
## Not run:
bounds_t_test(ardl_3132_n, case = 2)
bounds_t_test(ardl_3132_c, case = 4)
bounds_t_test(ardl_3132_ct, case = 3)
## End(Not run)

## Asymptotic p-value and critical value bounds (assuming T = 1000) ----

# Include critical value bounds for a certain level of significance

# t-statistic is larger than the I(1) bound (for a=0.05) as expected (p-value < 0.05)
```
btt <- bounds_t_test(ardl.3132.c, case = 3, alpha = 0.05)
btt
btt$tab
```

# Traditional but less precise critical value bounds, as presented in Pesaran et al. (2001)
```
btt$PSS2001parameters
```

# t-statistic doesn't exceed the I(1) bound (for a=0.005) as p-value is greater than 0.005
```
bounds_t_test(ardl.3132.c, case = 3, alpha = 0.005)
```

## Exact sample size p-value and critical value bounds -----------------

# Setting a seed is suggested to allow the replication of results
# 'R' can be increased for more accurate results

# t-statistic is smaller than the I(1) bound (for a=0.01) as expected (p-value > 0.01)
# Note that the exact sample p-value (0.009874) is very different than the asymptotic (0.005538)
# It can take more than 90 seconds
## Not run:
```
set.seed(2020)
bounds_t_test(ardl.3132.c, case = 3, alpha = 0.01, exact = TRUE)
```
## End(Not run)

coint_eq

### Cointegrating equation (long-run level relationship)

#### Description

Creates the cointegrating equation (long-run level relationship) providing an 'ardl', 'uecm' or 'recm' model.

#### Usage

```
coint_eq(object, case)
```

## S3 method for class 'recm'
coint_eq(object, ...)

## Default S3 method:
coint_eq(object, case)
coint_eq

Arguments

- **object**: An object of class 'ardl', 'uecm' or 'recm'.
- **case**: An integer from 1-5 or a character string specifying whether the 'intercept' and/or the 'trend' have to participate in the long-run level relationship (cointegrating equation) (see section 'Cases' below). If the input object is of class 'recm', case is not needed as the model is already under a certain case.

... Currently unused argument.

Value

coint_eq returns a numeric vector containing the cointegrating equation.

Cases

According to Pesaran et al. (2001), we distinguish the long-run relationship (cointegrating equation) (and thus the bounds-test and the Restricted ECMs) between 5 different cases. These differ in terms of whether the 'intercept' and/or the 'trend' are restricted to participate in the long-run relationship or they are unrestricted and so they participate in the short-run relationship.

**Case 1**: • No intercept and no trend.
  - case inputs: 1 or "n" where "n" stands for none.

**Case 2**: • Restricted intercept and no trend.
  - case inputs: 2 or "rc" where "rc" stands for restricted constant.

**Case 3**: • Unrestricted intercept and no trend.
  - case inputs: 3 or "uc" where "uc" stands for unrestricted constant.

**Case 4**: • Unrestricted intercept and restricted trend.
  - case inputs: 4 or "ucrt" where "ucrt" stands for unrestricted constant and restricted trend.

**Case 5**: • Unrestricted intercept and unrestricted trend.
  - case inputs: 5 or "ucut" where "ucut" stands for unrestricted constant and unrestricted trend.

Note that you can’t restrict (or leave unrestricted) a parameter that doesn’t exist in the input model. For example, you can’t compute recm(object, case=3) if the object is an ARDL (or UECM) model with no intercept. The same way, you can’t compute bounds_f_test(object, case=5) if the object is an ARDL (or UECM) model with no linear trend.

References


Author(s)

Kleanthis Natsiopoulos, <klnatsio@gmail.com>

See Also

plot_lr ardl uecm recm bounds_f_test bounds_t_test
Examples

data(denmark)
library(zoo) # for cbind.zoo()

## Estimate the Cointegrating Equation of an ARDL(3,1,3,2) model -------

# From an ARDL model (under case 2, restricted constant)
ar_dl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
ce2_ar_dl <- coint_eq(ar_dl_3132, case = 2)

# From an UECM (under case 2, restricted constant)
uecm_3132 <- uecm(ar_dl_3132)
ce2_uecm <- coint_eq(uecm_3132, case = 2)

# From a RECM (under case 2, restricted constant)
# Notice that if a RECM has already been estimated under a certain case,
# the 'coint_eq()' can't be under different case, so no 'case' argument needed.
recm_3132 <- recm(uecm_3132, case = 2)
# The RECM is already under case 2, so the 'case' argument is no needed
ce2_recm <- coint_eq(recm_3132)

identical(ce2_ar_dl, ce2_uecm, ce2_recm)

## Check for a degenerate level relationship ---------------------------

# The bounds F-test under both cases reject the Null Hypothesis of no level relationship.
bounds_f_test(ar_dl_3132, case = 2)
bounds_f_test(ar_dl_3132, case = 3)

# The bounds t-test also rejects the NULL Hypothesis of no level relationship.
bounds_t_test(ar_dl_3132, case = 3)

# But when the constant enters the long-run equation (case 3)
# this becomes a degenerate relationship.
ce3_ar_dl <- coint_eq(ar_dl_3132, case = 3)

plot_lr(ar_dl_3132, coint_eq = ce2_ar_dl, show.legend = TRUE)
plot_lr(ar_dl_3132, coint_eq = ce3_ar_dl, show.legend = TRUE)
plot_lr(ar_dl_3132, coint_eq = ce3_ar_dl, facets = TRUE, show.legend = TRUE)

---

The Danish data on money income prices and interest rates

Description

This data set contains the series used by S. Johansen and K. Juselius for estimating a money demand function of Denmark.
multipliers

Usage
denmark

Format

LRM logarithm of real money, M2
LRY logarithm of real income
LPY logarithm of price deflator
IBO bond rate
IDE bank deposit rate

Details
An object of class "zooreg" "zoo".

Source
http://web.math.ku.dk/~sjo/data/danish_data.html

References

multipliers multipliers

Description
multipliers is a generic function used to estimate short-run (impact), delay, interim and long-run (total) multipliers, accompanied by their corresponding standard errors, t-statistics and p-values.

Usage
multipliers(object, type = "lr", vcov_matrix = NULL, se = FALSE)

## S3 method for class 'ardl'
multipliers(object, type = "lr", vcov_matrix = NULL, se = FALSE)

## S3 method for class 'uecm'
multipliers(object, type = "lr", vcov_matrix = NULL, se = FALSE)
Arguments

object  An object of class 'ardl' or 'uecm'.
type  A character string describing the type of multipliers. Use "lr" for long-run (total) multipliers (default), "sr" or 0 for short-run (impact) multipliers or an integer between 1 and 200 for delay and interim multipliers.
vcov_matrix  The estimated covariance matrix of the random variable that the transformation function uses to estimate the standard errors (and so the t-statistics and p-values) of the multipliers. The default is vcov(object) (when vcov_matrix = NULL), but other estimations of the covariance matrix of the regression’s estimated coefficients can also be used (e.g., using vcovHC or vcovHAC).
se  A logical indicating whether you want standard errors for delay multipliers to be provided. The default is FALSE. Note that this parameter does not refer to the standard errors for the long-run multipliers, which are always calculated. IMPORTANT: Calculating standard errors for long periods of delays may cause your computer to run out of memory and terminate your R session, losing important unsaved work. As a rule of thumb, try not to exceed type = 19 when se = TRUE.

Details

The function invokes two different methods, one for objects of class 'ardl' and one for objects of class 'uecm'. This is because of the different (but equivalent) transformation functions that are used for each class/model ('ardl' and 'uecm') to estimate the multipliers.
type = 0 is equivalent to type = "sr".

Note that the interim multipliers are the cumulative sum of the delays, and that the sum of the interim multipliers (for long enough periods) and thus a distant enough interim multiplier match the long-run multipliers.
The delay (interim) multiplier can be interpreted as the effect in period t+s, of an instant (sustained) shock in period t.
The delta method is used for approximating the standard errors (and thus the t-statistics and p-values) of the estimated long-run and delay multipliers.

Value

multipliers returns (for long and short run multipliers) a data.frame containing the independent variables (including possibly existing intercept or trend and excluding the fixed variables) and their corresponding standard errors, t-statistics and p-values. For delay and interim multipliers it returns a list with a data.frame for each variable, containing the delay and interim multipliers for each period.

Mathematical Formula

**Short-Run Multipliers:**

As derived from an ARDL:

\[
\frac{\partial y_t}{\partial x_{j,t}} = b_{j,0} \quad j \in \{1, \ldots, k\}
\]
multipliers

As derived from an Unrestricted ECM:

\[
\frac{\partial y_t}{\partial x_j,t} = \omega_j \quad j \in \{1, \ldots, k\}
\]

Constant and Linear Trend:

\[c_0\]
\[c_1\]

Delay & Interim Multipliers:

As derived from an ARDL:

\[
Delay_{x_j,s} = \frac{\partial y_{t+s}}{\partial x_j,t} = b_{j,s} + \sum_{i=1}^{\min\{p,s\}} b_{y,i} \frac{\partial y_{t+i}}{\partial x_j,t} \quad b_{j,s} = 0 \quad \forall \ s > q
\]

\[
Interim_{x_j,s} = \sum_{i=0}^{s} Delay_{x_j,s}
\]

Constant and Linear Trend:

\[
Delay_{\text{intercept},s} = c_0 + \sum_{i=1}^{\min\{p,s\}} b_{y,i} Delay_{\text{intercept},s-i} \quad c_0 = 0 \quad \forall \ s \neq 0
\]

\[
Interim_{\text{intercept},s} = \sum_{i=0}^{s} Delay_{\text{intercept},s}
\]

\[
Delay_{\text{trend},s} = c_1 + \sum_{i=1}^{\min\{p,s\}} b_{y,i} Delay_{\text{trend},s-i} \quad c_1 = 0 \quad \forall \ s \neq 0
\]

\[
Interim_{\text{trend},s} = \sum_{i=0}^{s} Delay_{\text{trend},s}
\]

Long-Run Multipliers:

As derived from an ARDL:

\[
\frac{\partial y_{t+\infty}}{\partial x_j,t} = \theta_j = \frac{\sum_{l=0}^{q_j} b_{j,l}}{1 - \sum_{i=1}^{p} b_{y,i}} \quad j \in \{1, \ldots, k\}
\]

Constant and Linear Trend:
\[ \mu = \frac{c_0}{1 - \sum_{i=1}^{p} \theta_{x,i}} \]

\[ \delta = \frac{c_1}{1 - \sum_{i=1}^{p} \theta_{x,i}} \]

As derived from an Unrestricted ECM:

\[ \frac{\partial y_{t+\infty}}{\partial x_{j,t}} = \theta_j = \frac{\pi_j}{-\pi_y} \quad j \in \{1, \ldots, k\} \]

Constant and Linear Trend:

\[ \mu = \frac{c_0}{-\pi_y} \]

\[ \delta = \frac{c_1}{-\pi_y} \]

Author(s)

Kleanthis Natsiopoulos, <klnatsio@gmail.com>

See Also

adr1, uecm, plot_delay

Examples

data(denmark)

## Estimate the long-run multipliers of an ARDL(3,1,3,2) model --------

# From an ARDL model
ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
mult_ardl <- multipliers(ardl_3132)
mult_ardl

# From an UECM
uecm_3132 <- uecm(ardl_3132)
mult_uecm <- multipliers(uecm_3132)
mult_uecm

all.equal(mult_uecm, mult_ardl)

## Estimate the short-run multipliers of an ARDL(3,1,3,2) model --------

mult_sr <- multipliers(uecm_3132, type = "sr")
mult_0 <- multipliers(uecm_3132, type = 0)
all.equal(mult_sr, mult_0)
## Estimate the delay & interim multipliers of an ARDL(3,1,3,2) model --

```r
mult_lr <- multipliers(uecm_3132, type = "lr")
mult_inter80 <- multipliers(uecm_3132, type = 80)

mult_lr
sum(mult_inter80$'(Intercept)'$Delay)
mult_inter80$'(Intercept)'$Interim[nrow(mult_inter80$'(Intercept)')]  
sum(mult_inter80$LRY$Delay) 
mult_inter80$LRY$Interim[nrow(mult_inter80$LRY)]]
sum(mult_inter80$IBO$Delay) 
mult_inter80$IBO$Interim[nrow(mult_inter80$IBO)]]
sum(mult_inter80$IDE$Delay) 
mult_inter80$IDE$Interim[nrow(mult_inter80$IDE)]] 
plot(mult_inter80$LRY$Delay, type='l')
plot(mult_inter80$LRY$Interim, type='l')
mult_inter12 <- multipliers(uecm_3132, type = 12, se = TRUE) 
plot_delay(mult_inter12, interval = 0.95)
```

---

**NT2022**  
*The UK earnings equation data from Natsiopoulos and Tzeremes (2022)*

---

### Description

This data set contains the series used by Natsiopoulos and Tzeremes (2022) for re-estimating the UK earnings equation. The clean format of the data retrieved from the Data Archive of Natsiopoulos and Tzeremes (2022).

### Usage

NT2022

### Format


- **time** time variable
- **w** real wage
- **Prod** labor productivity
- **UR** unemployment rate
- **Wedge** wedge effect
- **Union** union power
- **D7475** income policies 1974:Q1-1975:Q4
- **D7579** income policies 1975:Q1-1979:Q4
- **UnionR** union membership
Details

An object of class "zooreg" "zoo".

Source

http://qed.econ.queensu.ca/jae/datasets/natsiopoulos001/

References


plot_delay

Create plots for the delay multipliers

Description

Creates plots for the delay multipliers and their uncertainty intervals based on their estimated standard errors. This is a basic ggplot with a few customizable parameters.

Usage

plot_delay(
  multipliers,
  facets_ncol = 2,
  interval = FALSE,
  interval_color = "blue",
  show.legend = FALSE,
  xlab = "Period",
  ylab = "Delay",
  ...
)

Arguments

multipliers A list returned from multipliers, in which type is a positive integer to return delay multipliers.
facets_ncol If a positive integer, it indicates the number of the columns in the facet. If FALSE, each plot is created separately. The default is 2.
interval If FALSE (default), no uncertainty intervals are drawn. If a positive integer, the intervals are this number times the standard error. If a number between 0 and 1 (e.g. 0.95), the equivalent confidence interval is drawn (e.g. 95 the Gaussian distribution.
interval_color The color of the uncertainty intervals. Default is "blue".
show.legend A logical indicating whether the interval legend is shown. Default is FALSE.
plot_lr

Create plot for the long-run (cointegrating) equation

Description

Creates a plot for the long-run relationship in comparison with the dependent variable, and the fitted values of the model. This is a basic \texttt{ggplot} with a few customizable parameters.

\begin{verbatim}
plot_lr

    xlab, ylab    Names displayed at the x and y axes respectively. Default is "Period" and "Delay" respectively.

    ...    Currently unused argument.

Value

plot_delay returns a number of \texttt{ggplot} objects.

Author(s)

Kleanthis Natsiopoulos, <klnatsio@gmail.com>

See Also

\texttt{multipliers}

Examples

ardl_3132 <- ardl(\texttt{LRM} \texttt{\sim} \texttt{LRY} \texttt{+ IBO} \texttt{+ IDE}, data = \texttt{denmark}, order = \texttt{c(3,1,3,2)})
delay_mult <- multipliers(ardl_3132, type = 12, se = \texttt{TRUE})

## Simply plot the delay multipliers -----------------------------------
plot_delay(delay_mult)

## Rearrange them ------------------------------------------------------
plot_delay(delay_mult, facets_ncol = 1)

## Add 1 standard deviation uncertainty intervals ----------------------
plot_delay(delay_mult, interval = 1)

## Add 95\% confidence intervals, change color and add legend -----------
plot_delay(delay_mult, interval = 0.95, interval_color = "darkgrey",
           show.legend = \texttt{TRUE})
\end{verbatim}
plot_lr

Usage

plot_lr(
  object,
  coint_eq,
  facets = FALSE,
  show_fitted = FALSE,
  show.legend = FALSE,
  xlab = "Time",
  ...
)

Arguments

  object          An object of class `class` 'ardl'.
  coint_eq       The objected returned from `coint_eq`.
  facets         A logical indicating whether the long-run relationship appears in a separate plot. Default is FALSE.
  show_fitted    A logical indicating whether the fitted values are shown. Default is FALSE.
  show.legend    A logical indicating whether the legend is shown. Default is FALSE.
  xlab           Name displayed at the x axis. Default is "Time".
  ...            Currently unused argument.

Value

plot_lr returns a `ggplot` object.

Author(s)

Kleanthis Natsiopoulos, <klnatsio@gmail.com>

See Also

coint_eq

Examples

  ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
  ce2 <- coint_eq(ardl_3132, case = 2)

  plot_lr(ardl_3132, coint_eq = ce2)

  ## Compare fitted values and place long-run relationship separately ----

  ce3 <- coint_eq(ardl_3132, case = 3)
  plot_lr(ardl_3132, coint_eq = ce3, facets = TRUE, show_fitted = TRUE,
           show.legend = TRUE)
Description

This data set contains the series used by Pesaran et al. (2001) for estimating the UK earnings equation. The clean format of the data retrieved from the Data Archive of Natsiopoulos and Tzeremes (2022).

Usage

PSS2001

Format


- w real wage
- Prod labor productivity
- UR unemployment rate
- Wedge wedge effect
- Union union power
- D7475 income policies 1974:Q1-1975:Q4
- D7579 income policies 1975:Q1-1979:Q4

Details

An object of class "zooreg" "zoo".

Source


References


**recm**  
*Restricted ECM regression*

**Description**

Creates the Restricted Error Correction Model (RECM). This is the conditional RECM, which is the RECM of the underlying ARDL.

**Usage**

```
recm(object, case)
```

**Arguments**

- **object**: An object of class ‘ardl’ or ‘uecm’.
- **case**: An integer from 1-5 or a character string specifying whether the ‘intercept’ and/or the ‘trend’ have to participate in the short-run or the long-run relationship (cointegrating equation) (see section ‘Cases’ below).

**Details**

Note that the statistical significance of ‘ect’ in a RECM should not be tested using the corresponding t-statistic (or the p-value) because it doesn’t follow a standard t-distribution. Instead, the `bounds_t_test` should be used.

**Value**

`recm` returns an object of class c("dynlm", "lm", "recm"). In addition, attributes ‘order’, ‘data’, ‘parsed_formula’ and ‘full_formula’ are provided.

**Mathematical Formula**

The formula of a Restricted ECM conditional to an ARDL($p, q_1, \ldots, q_k$) is:

\[
\Delta y_t = c_0 + c_1 t + \sum_{i=1}^{p-1} \psi_{yi,i} \Delta y_{t-i} + \sum_{j=1}^{k} \sum_{l=1}^{q_j-1} \psi_{yj,l} \Delta x_{j,t-l} + \sum_{j=1}^{k} \omega_j \Delta x_{j,t} + \pi_y ECT_t + \epsilon_t
\]

\[
\psi_{yj,t} = 0 \quad \forall \quad q_j = 1, \psi_{yj,t} = \omega_j = 0 \quad \forall \quad q_j = 0
\]

**Under Case 1:**

- \( c_0 = c_1 = 0 \)
- \( ECT = y_{t-1} - (\sum_{j=1}^{k} \theta_j x_{j,t-1}) \)

**Under Case 2:**

- \( c_0 = c_1 = 0 \)
- \( ECT = y_{t-1} - (\mu + \sum_{j=1}^{k} \theta_j x_{j,t-1}) \)

**Under Case 3:**

- \( c_1 = 0 \)
- \( ECT = y_{t-1} - (\sum_{j=1}^{k} \theta_j x_{j,t-1}) \)

**Under Case 4:**

- \( c_1 = 0 \)
$ECT = y_{t-1} - (\delta(t-1) + \sum_{j=1}^{k} \theta_j x_{j,t-1})$

Under Case 5:

$ECT = y_{t-1} - (\sum_{j=1}^{k} \theta_j x_{j,t-1})$

In all cases, $x_{j,t-1}$ in $ECT$ is replaced by $x_{j,t}$ \( \forall q_j = 0 \)

Cases

According to Pesaran et al. (2001), we distinguish the long-run relationship (cointegrating equation) (and thus the bounds-test and the Restricted ECMs) between 5 different cases. These differ in terms of whether the 'intercept' and/or the 'trend' are restricted to participate in the long-run relationship or they are unrestricted and so they participate in the short-run relationship.

Case 1:
- No intercept and no trend.
  - case inputs: 1 or "n" where "n" stands for none.

Case 2:
- Restricted intercept and no trend.
  - case inputs: 2 or "rc" where "rc" stands for restricted constant.

Case 3:
- Unrestricted intercept and no trend.
  - case inputs: 3 or "uc" where "uc" stands for unrestricted constant.

Case 4:
- Unrestricted intercept and restricted trend.
  - case inputs: 4 or "ucrt" where "ucrt" stands for unrestricted constant and restricted trend.

Case 5:
- Unrestricted intercept and unrestricted trend.
  - case inputs: 5 or "ucut" where "ucut" stands for unrestricted constant and unrestricted trend.

Note that you can’t restrict (or leave unrestricted) a parameter that doesn’t exist in the input model. For example, you can’t compute recm(object,case=3) if the object is an ARDL (or UECM) model with no intercept. The same way, you can’t compute bounds_f_test(object, case=5) if the object is an ARDL (or UECM) model with no linear trend.

References


Author(s)

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See Also

ardl uecm
Examples

```r
data(denmark)

## Estimate the RECM, conditional to it's underlying ARDL(3,1,3,2) -----

# Indirectly from an ARDL
ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
recm_3132 <- recm(ardl_3132, case = 2)

# Indirectly from an UECM
uecm_3132 <- uecm(ardl_3132)
recm_3132_ <- recm(uecm_3132, case = 2)
identical(recm_3132, recm_3132_)
summary(recm_3132)

## Error Correction Term (ect) & Speed of Adjustment -------------------

# The coefficient of the ect,
# shows the Speed of Adjustment towards equilibrium.
# Note that this can be also be obtained from an UECM,
# through the coefficient of the term L(y, 1) (where y is the dependent variable).
tail(recm_3132$coefficients, 1)
uecm_3132$coefficients[2]
```

to_lm  

Convert dynlm model (`ardl`, `uecm`, `recm`) to `lm` model

Description

Takes a `dynlm` model of class `ardl`, `uecm` or `recm` and converts it into an `lm` model. This can help using the model as a regular `lm` model with functions that are not compatible with `dynlm` models such as the `predict` function to forecast.

Usage

```r
to_lm(object, ...)
```

Arguments

- `object` An object of class `ardl`, `uecm` or `recm`.
- `...` Currently unused argument.

Value

`to_lm` returns an object of class `"lm"`.

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See Also

ardl, uecm, recm

Examples

```r
## Convert ARDL into lm ---------------------------------------------
ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
ardl_3132_lm <- to_lm(ardl_3132)
summary(ardl_3132)$coefficients
summary(ardl_3132_lm)$coefficients

## Convert UECM into lm ---------------------------------------------
uecm_3132 <- uecm(ardl_3132)
uecm_3132_lm <- to_lm(uecm_3132)
summary(uecm_3132)$coefficients
summary(uecm_3132_lm)$coefficients

## Convert RECM into lm ---------------------------------------------
recm_3132 <- recm(ardl_3132, case = 2)
recm_3132_lm <- to_lm(recm_3132)
summary(recm_3132)$coefficients
summary(recm_3132_lm)$coefficients

## Use the lm model to forecast ------------------------------------
# Forecast using the in-sample data
in_sample_data <- ardl_3132$model
head(in_sample_data)
predicted_values <- predict(ardl_3132_lm, newdata = in_sample_data)

# The predicted values are expected to be the same as the fitted values
ardl_3132$fitted.values
predicted_values

# Convert to ts class for the plot
predicted_values <- ts(predicted_values, start = c(1974,4), frequency=4)
plot(denmark$LRM, lwd=4) #The input dependent variable
lines(ardl_3132$fitted.values, lwd=4, col="blue") #The fitted values
lines(predicted_values, lty=2, lwd=2, col="red") #The predicted values
```

---

**uecm**

*Unrestricted ECM regression*

**Description**

*uecm* is a generic function used to construct Unrestricted Error Correction Models (UECM). The function invokes two different methods. The default method works exactly like *ardl*. The other
method requires an object of \texttt{class} 'ardl'. Both methods create the conditional UECM, which is the UECM of the underlying ARDL.

Usage

\begin{verbatim}
uecm(...)  
  ## S3 method for class 'ardl'
  uecm(object, ...)

  ## Default S3 method:
  uecm(formula, data, order, start = NULL, end = NULL, ...)
\end{verbatim}

Arguments

\begin{itemize}
\item \texttt{...} Additional arguments to be passed to the low level regression fitting functions.
\item \texttt{object} An object of \texttt{class} 'ardl'.
\item \texttt{formula} A "formula" describing the linear model. Details for model specification are given under 'Details'.
\item \texttt{data} A time series object (e.g., "ts", "zoo" or "zooreg") or a data frame containing the variables in the model. In the case of a data frame, it is coerced into a \texttt{ts} object with \texttt{start = 1}, \texttt{end = nrow(data)} and \texttt{frequency = 1}. If not found in data, the variables are NOT taken from any environment.
\item \texttt{order} A specification of the order of the underlying ARDL model (e.g., for the UECM of an ARDL(1,0,2) model it should be \texttt{order = c(1,0,2)}). A numeric vector of the same length as the total number of variables (excluding the fixed ones, see 'Details'). It should only contain positive integers or 0. An integer could be provided if all variables are of the same order.
\item \texttt{start} Start of the time period which should be used for fitting the model.
\item \texttt{end} End of the time period which should be used for fitting the model.
\end{itemize}

Details

The \texttt{formula} should contain only variables that exist in the data provided through \texttt{data} plus some additional functions supported by \texttt{dynlm} (i.e., \texttt{trend()}).

You can also specify fixed variables that are not supposed to be lagged (e.g. dummies etc.) simply by placing them after \texttt{|}. For example, \texttt{y ~ x1 + x2 | z1 + z2} where \texttt{z1} and \texttt{z2} are the fixed variables and should not be considered in \texttt{order}. Note that the \texttt{|} notion should not be confused with the same notion in \texttt{dynlm} where it introduces instrumental variables.

Value

\texttt{uecm} returns an object of \texttt{class} \texttt{c}"dynlm", "lm", "uecm"). In addition, attributes 'order', 'data', 'parsed_formula' and 'full_formula' are provided.
Mathematical Formula

The formula of an Unrestricted ECM conditional to an ARDL\((p, q_1, \ldots, q_k)\) is:

\[
\Delta y_t = c_0 + c_1 t + \pi_y y_{t-1} + \sum_{j=1}^{k} \pi_j x_{j,t-1} + \sum_{i=1}^{p-1} \psi_{y,i} \Delta y_{t-i} + \sum_{j=1}^{k} \sum_{l=1}^{q_j-1} \psi_{j,l} \Delta x_{j,t-l} + \sum_{j=1}^{k} \omega_j \Delta x_{j,t} + \epsilon_t
\]

\[
\psi_{j,l} = 0 \quad \forall \quad q_j \leq 1, \quad \psi_{y,i} = 0 \quad \text{if} \quad p = 1
\]

In addition, \(x_{j,t-1}\) and \(\Delta x_{j,t}\) cancel out becoming \(x_{j,t} \quad \forall \quad q_j = 0\)

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See Also

ardl recm

Examples

data(denmark)

## Estimate the UECM, conditional to its underlying ARDL(3,1,3,2) -----

# Indirectly
ardl_3132 <- ardl(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
uecm_3132 <- uecm(ardl_3132)

# Directly
uecm_3132_ <- uecm(LRM ~ LRY + IBO + IDE, data = denmark, order = c(3,1,3,2))
identical(uecm_3132, uecm_3132_)
summary(uecm_3132)
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