Package ‘BlakerCI’

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BlakerCI-package ........................................... 2
binom.blaker.acc ......................................... 2
binom.blaker.limits ...................................... 5
binom.blaker.VHadj.acc ................................. 7
binom.blaker.VHadj.limits ............................... 9
BlakerCI-internal ......................................... 12
poisson.blaker.acc ....................................... 13
poisson.blaker.limits ................................... 15

Index 17
BlakerCI-package

**Blaker's binomial and Poisson confidence limits**

Description

Fast and accurate calculation of Blaker's binomial and Poisson confidence limits.

Details

```
Package: BlakerCI
Type: Package
Version: 1.0-6
Date: 2019-04-29
License: GPL-3
LazyLoad: yes
```

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Author(s)

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Examples

```
binom.blaker.limits(3,10) # [1] 0.08726443 0.61941066
poisson.blaker.limits(3)  # [1] 0.8176914 8.5597971
```

Description

Calculates values of the acceptability function for the binomial distribution (see function acceptbin in Blaker (2000)) in a sequence of points (for, e.g., plotting purposes). The acceptability function may optionally be “unimodalized”, i.e. replaced with the smallest greater or equal unimodal function.
Usage

binom.blaker.acc(x, n, p, type = c("orig", "unimod"),
               acc.tol = 1e-10, ...)

Arguments

x number of successes.
n number of trials.
p vector (length 1 allowed) of hypothesized binomial parameters (between 0 and 1). In case of more than one point, an increasing sequence required.
type for type = "orig", original acceptability function calculated.
   For type = "unimod", smallest unimodal function greater or equal to the acceptability function calculated instead.
acc.tol numerical tolerance (relevant only for type = "unimod").
... additional arguments to be passed to binom.blaker.acc.single.p; in fact, just maxiter (see BlakerCI-internal).

Details

For type = "orig", essentially the same is calculated as – for single points – by acceptbin function from Blaker (2000).

Single values of the “unimodalized” acceptability function (for type = "unimod") are computed by an iterative numerical algorithm implemented in internal function binom.blaker.acc.single.p. The function cited is called just once in each of the intervals where the acceptability function is continuous (namely in the leftmost one of those points of p that fall into the interval when dealing with points below x/n, and the rightmost one when above x/n). The rest is done by function cummax. This is considerably faster than calling binom.blaker.acc.single.p for every point of p. Note that applying cummax directly to a vector of unmodified acceptability values is even faster and provides a unimodal output; it may, nevertheless, lack accuracy (see Examples).

Value

Vector of acceptability values (with or without unimodalization) in points of p.

Note

Inspired by M.P. Fay (2010), mentioning “unavoidable inconsistencies” between tests with non-unimodal acceptability functions and confidence intervals derived from them. When the acceptability functions are unimodalized and the test modified accordingly (i.e. p-values slightly increased in some cases), a perfectly matching test-CI pair is obtained.

Author(s)

Jan Klaschka <klaschka@cs.cas.cz>
References

(Corrigena: Canadian Journal of Statistics 29: 681.)


Examples

```r
p <- seq(0,1,length=1001)
acc <- binom.blaker.acc(3,10,p)
acc1 <- binom.blaker.acc(3,10,p,type="unimod")
## The two functions look the same at first glance.
plot(p,acc,type="l")
lines(p,acc1,col="red")
legend(x=.7,y=.8,c("orig","unimod"),col=c("black","red"),lwd=1)

## There is, nevertheless, a difference.
plot(p,acc1-acc,type="l")

## Focussing on the difference about p=0.4:
p <- seq(.395,.405,length=1001)
acc <- binom.blaker.acc(3,10,p)
acc1 <- binom.blaker.acc(3,10,p,type="unimod")
plot(p,acc,type="l",ylim=c(.749,.7495))
lines(p,acc1,col="red")
legend(x=.402,y=.7494,c("orig","unimod"),col=c("black","red"),lwd=1)

## Difference between type="unimod" and mere applying
## cummax to values obtained via type="orig":
p <- seq(0,1,length=1001)
x <- 59
n <- 355
## Upper confidence limit (at 0.95 level) is slightly above 0.209:
binom.blaker.limits(x,n) ## [1] 0.1300807 0.2090089
## Unmodified acceptability value fall below 0.05 at p = .209
## left to the limit (so that the null hypothesis p = .209
## would be rejected despite the fact that p lies within
## the confidence interval):
acc <- binom.blaker.acc(59,355,p)
rbind(p,acc)[,2:11]
## [,1] 0.2060000 0.2080000 0.2100000
## [,2] 0.06006867 0.05759836 0.04999082 0.04330283
## ## Modified acceptability is above 0.05 at p = 0.05 (so that
## ## hypothesis p = 0.05 is not rejected by the modified test):
acc1 <- binom.blaker.acc(59,355,p,type="unimod")
rbind(p,acc1)[,2:11]
## [,1] 0.2060000 0.2070000 0.2080000 0.2090000 0.2100000
## [,2] 0.06006867 0.05759836 0.05014189 0.04999082 0.04330283
```
Blaker’s binomial confidence limits

Description

Fast and accurate calculation of Blaker’s binomial confidence limits.

Usage

binom.blaker.limits(x, n, level = 0.95, tol = 1e-10, ...)

Arguments

x  number of successes.
n  number of trials.
ivel  confidence level.
tol  numerical tolerance.
...  additional arguments to be passed to binom.blaker.lower.limit; in fact, just maxiter (see BlakerCI-internal).

Details

Note that the Blaker’s $(1 - \alpha)$ confidence interval is the convex hull of the set $C$ of those points where the acceptability function (Blaker (2000)) exceeds level $\alpha$. The original numerical algorithm from Blaker (2000) is prone, when $C$ is a union of disjoint intervals, to skipping a short interval and finding inaccurate over-liberal confidence limits.

Function binom.blaker.limits is, by contrast, immune from such failures and yields always as its result the whole confidence interval (Klaschka (2010)).

Value

Length 2 vector – the lower and upper confidence limits.
Note

Package exactci by M. P. Fay includes another algorithm that calculates Blaker’s binomial confidence limits (see user-level function binom.exact and internal function exactbinomCI). It is more sophisticated than the original Blaker’s one, but considerably slower and sometimes less accurate than that of binom.blaker.limits.

Earlier 2010 versions of the algorithm of binom.blaker.limits were designed independently of (though already existing) M.P. Fay’s packages exact2x2 and exactci. Some later modifications, however, have been inspired by Fay’s programs.

Lecoutre & Poitevineau (2014) designed another algorithm for the calculation of the Blaker’s confidence limits. Despite more abstract theoretical background and broader scope (not confined to the binomial distribution), it is closely analogous to that of binom.blaker.limits.

Author(s)

Jan Klaschka <klaschka@cs.cas.cz>

References

(Corrigenda: Canadian Journal of Statistics 29: 681.)


See Also

exactci:binom.exact One of the options yields Blaker’s limits. The algorithm is more sophisticated than the original Blaker’s one.
propCIs:blakerci Implementation of the original algorithm from Blaker (2000).
binGroup:binBlaker Another implementation of the same algorithm.

Examples

binom.blaker.limits(3,10) # [1] 0.08726443 0.61941066

## Example of a failure of the original algorithm:
## Requires PropCIs package.
## Tolerance 1e-4 - default in the Blaker's paper.
## Not run:
blakerci(29,99,conf.level=0.95,tolerance=1e-4) # [1] 0.2096386 0.3923087
## The correct upper limit should be 0.3929...,
Blaker's binomial acceptability function with Vos-Hudson adjustment.

**Description**

Calculates values of the Vos-Hudson adjusted acceptability function in a sequence of points (for, e.g., plotting purposes). The adjusted acceptability function may optionally be “unimodalized”, i.e. replaced with the smallest greater or equal unimodal function.

**Usage**

```r
binom.blaker.VHadj.acc(x, n, p, type = c("orig", "unimod"),
                 acc.tol = 1e-10, nmax=n+1000,int.pts=1e-12, ...)
```

**Arguments**

- `x` number of successes.
- `n` number of trials.
- `p` vector (length 1 allowed) of hypothesized binomial parameters (between 0 and 1). In case of more than one point, an increasing sequence required.
type for type = "orig", Vos-Hudson adjustment applied to original acceptability function. For type = "unimod", smallest unimodal function greater or equal to the adjusted acceptability function.

acc.tol numerical tolerance (relevant only for type = "unimod").

nmax Pairs \((y, m)\) of number of trials and number of successes are allowed to contribute to the Vos-Hudson adjustment for only \(m\) up to \(n_{max}\). Warning is returned when greater numbers of trials are suspected to have influence.

int.eps Maximum expected error of machine representation of integers calculated from reals via multiplication and division. (Used in order to round numbers correctly if they happen to be integer, e.g. \(\text{ceiling}(xx - \text{int.eps})\) is calculated instead of \(\text{ceiling}(xx)\).)

... additional arguments to be passed to \texttt{binom.blaker.acc.single.p}; in fact, just \texttt{maxiter} (see \texttt{BlakerCI-internal}).

Details

The relationship between the adjusted acceptability function and the adjusted confidence intervals (see \texttt{binom.blaker.VHadj.limits}) is the same as between the unadjusted acceptability function and confidence interval (see \texttt{binom.blaker.acc}, \texttt{binom.blaker.limits}): The confidence interval is the convex hull of the set of those points where the function exceeds \(1 - \text{confidence level}\).

Value

Vector of Vos-Hudson adjusted acceptability values (with or without unimodalization) in points of \(p\).

Warning

(1) Comparing output of the function with that of \texttt{binom.blaker.acc} cannot answer positively the question whether the unadjusted and adjusted functions are identical on an interval (but, up to the numerical accuracy, in the points of \(p\) only).

(2) The Warning section of the \texttt{binom.blaker.VHadj.limits} documentation is relevant here, as well.

Author(s)

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Examples

```r
p <- seq(0.1, length=10001)
acc.adj <- binom.blaker.VHadj.acc(6,13,p)
acc <- binom.blaker.acc(6,13,p)

plot(p,acc.adj,type="l",col="red",ylab="acceptability"
 ,main=paste("Vos-Hudson adjustment of acceptability function"
 ,"for 6 successes in 13 trials"
 ,sep="\n")
)```
The above adjustment results from the fact that, though
15 > 13 and 7/15 > 6/13, the acceptability function
for 7 successes in 15 trials is greater that that for 6 successes
in 13 trials on a short interval:

```
acc.7.15 <- binom.blaker.acc(7,15,p)
plot(p,acc,type="1",ylab="acceptability"
 ,main=paste("A detail of acceptability functions"
 ,"for 7 success in 15 trials"
 ,sep="\n")
 ,ylim=c(0.02,0.09)
)
lines(p,acc,type="1")
legend(x=.210,y=.08,c("unadjusted","adjustment"),col=c("black","red"),lwd=1)
```

```
# The adjustment shifts the point where the 0.05 level is exceeded,
# i.e. the Blaker's lower 0.95 confidence limit, from 0.2158 to 0.2150.
# (Compare with Examples in binom.blaker.VHadj.limits section.)
```
Description

Blaker’s binomial confidence limits adjusted so that logical inconsistencies criticized by Vos and Hudson (2008) are avoided.

Usage

```r
binom.blaker.VHadj.limits(x, n, level = 0.95, tol = 1e-10, ...)
```

Arguments

- `x` number of successes.
- `n` number of trials.
- `level` confidence level.
- `tol` numerical tolerance.
- `...` additional arguments to be passed to `binom.blaker.VHadj.lower.limit`: `maxiter`, `nmax`, `int.eps` (see `BlakerCI-internal`).

Value

Length 2 vector – the lower and upper (adjusted) confidence limits.

Warning

The stopping rule used is not fully justified:

The Clopper-Pearson $1 - \alpha$ confidence bounds for $x$ successes in $n$ trials may be expressed as $\text{qbeta}(\alpha/2, x, n-x+1)$ and $\text{qbeta}(1-\alpha/2, x+1, n-x)$, and can be generalized this way to real (i.e. not only integer) values of $x$.

The stopping rule used in `binom.blaker.VHadj.limits` relies on the hypothesis that the generalized lower (upper) Clopper-Pearson confidence bounds grow (decrease) whenever the number of trials grows, and the proportion of successes grows (decreases) or remains unchanged (with obvious exceptions in extremes).

Though I firmly trust the hypothesis, I can prove it, so far, just for integer numbers of successes (i.e. for “ordinary” Clopper-Pearson confidence bounds, not the generalized ones), and lack a general proof. Should the hypothesis be invalid, the stopping rule implemented in `binom.blaker.VHadj.limits` would be incorrect, and the process of modifying the Blaker’s confidence bounds could be incomplete in some cases.

Note

Vos & Hudson (2008) gave examples of mutually contradictory inferences yielded by some binomial tests and confidence intervals, including the Blaker’s confidence interval. Their objections may be interpreted as follows: When the number of trials is increased so that the success proportion increases (decreases) or remains the same, the lower (upper) confidence limit at the same confidence level should not decrease (increase).

The adjustment implemented in `binom.blaker.VHadj.limits` replaces the lower (upper) Blaker’s
confidence limit for \( x \) successes in \( n \) trials with the infimum (supremum) of the Blaker's lower (upper) confidence limits over such pairs \( y, m \) that \( m \) is not less than \( n \), and \( y/m \) is not less (greater) than \( x/n \).

Note that Lecoure & Poitevineau (2014), referring to the criticism by Vos & Hudson, proposed a modification of the Blaker’s confidence limits. Their adjustment, however, eliminates only a subset of “discrepancies” treated by `binom.blaker.VHadj.limits`, namely nonmonotonicities of upper (lower) Blaker’s confidence bounds in the number of trials when the number of successes (failures) remains the same.

Author(s)

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References


Examples

```r
binom.blaker.VHadj.limits(6,13) # [1] 0.2150187 0.7395922

## Note that the lower limit differs from the
## unadjusted version:
binom.blaker.limits(6,13) # [1] 0.2158050 0.7395922

## The (unadjusted) lower limit was replaced with the
## Blaker’s lower limit (both unadjusted and adjusted)
## assigned to 7 successes in 15 trials:
binom.blaker.limits(7,15) # [1] 0.2150187 0.7096627
binom.blaker.VHadj.limits(7,15) # [1] 0.2150187 0.7096627

## The adjustment avoids a contradiction between
## inferences corresponding to
## 6 successes in 13 trials, and 7 successess in 15 trials:
## Though the latter situation means a higher succes proportion
## in a higher number of trials, it is assigned a smaller
## (unadjusted) Blaker’s 95% lower confidence limit.
```
BlakerCI-internal

Internal functions, not expected to be called by the user.

Description

For binomial distribution: Calculation of the lower Blaker’s confidence limit as defined by Blaker (binom.blaker.lower.limit), or with so called Vos-Hudson adjustment (binom.blaker.VHadj.lower.limit); a single acceptability value, optionally “unimodalized” (binom.blaker.acc.single.p).

For Poisson distribution: Calculation of the lower and upper Blaker’s confidence limits (poisson.blaker.lower.limit, poisson.blaker.upper.limit); a single acceptability value, optionally “unimodalized” (poisson.blaker.acc.single.p).

Usage

binom.blaker.lower.limit(x, n, level, tol = 1e-10, maxiter=100)
binom.blaker.VHadj.lower.limit(x,n,level,tol=1e-10,maxiter=100, nmax=n+1000,int.eps=1e-10)
binom.blaker.acc.single.p(x, n, p, type = "orig", acc.tol = 1e-10, output = "acc", maxiter=100)
poisson.blaker.lower.limit(x, level, tol = 1e-10, maxiter=100)
poisson.blaker.upper.limit(x, level, tol = 1e-10, maxiter=100)
poisson.blaker.acc.single.p(x, p, type = "orig", acc.tol = 1e-10, output = "acc", maxiter=100)

Arguments

x  number of successes (binomial case), or events (Poisson case).

n  number of trials.

level  confidence level.

tol  numerical tolerance (for the confidence limit).

p  point (binomial or Poisson parameter value) where to calculate the acceptability.

type  "orig", or "unimod" – either unmodified, or unimodalized acceptability (see binom.blaker.acc, poisson.blaker.acc).

acc.tol  numerical tolerance (for the acceptability values when type = "unimod").

output  the acceptability value output (output = "acc", the default) or, instead, an auxiliary integer-valued parameter q1, used for testing whether points belong to the same continuous segment of the acceptability function (output = "q1"), or both (output = "both").

maxiter  Maximum number of interval halving iterations during the search for a confidence limit (binom.blaker.lower.limit, poisson.blaker.lower.limit, poisson.blaker.upper.limit), or a discontinuity point of the acceptability function (binom.blaker.acc.single.p, or poisson.blaker.acc.single.p with type = "unimod"). When the required accuracy is not reached in maxiter steps – typically when too small tol or acc.tol exceeds capabilities of machine arithmetic – last step’s result is returned with warning.
Pairs \((y, m)\) of number of trials and number of successes are allowed to contribute to the Vos-Hudson adjustment for only \(m\) up to \(n_{\text{max}}\). Warning is returned when greater numbers of trials are suspect to have influence.

Maximum expected error of machine representation of integers calculated via multiplication and division from reals. (Used in order to round numbers correctly if they happen to be integer, e.g. \(\text{ceiling}(xx - \text{int.} \ \text{eps})\) is calculated instead of \(\text{ceiling}(xx)\).)

For \(\text{binom.blaker.lower.limit}\) and \(\text{binom.blaker.VHadj.lower.limit}\), a single number – the lower confidence limit.

For \(\text{binom.blaker.acc.single.p}\) – depending on the output parameter – a single acceptability value, or a single auxiliary integer, or both.

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Calculates values of the acceptability function for the Poisson distribution (see Blaker (2000)) in a sequence of points (for, e.g., plotting purposes). The acceptability function may optionally be “unimodalized”, i.e. replaced with the smallest greater or equal unimodal function.

**Usage**

```r
poisson.blaker.acc(x, p, type = c("orig", "unimod"), 
acc.tol = 1e-10, ...)
```

**Arguments**

- \(x\) number of events.
- \(p\) vector (length 1 allowed) of hypothesized Poisson parameters. In case of more than one point, an increasing sequence required.
- \(\text{type}\) for \(\text{type} = "orig"\), original acceptability function calculated.
  For \(\text{type} = "unimod"\), smallest unimodal function greater or equal to the acceptability function calculated instead.
- \(\text{acc.tol}\) numerical tolerance (relevant only for \(\text{type} = "unimod"\)).
... additional arguments to be passed to `poisson.blaker.acc.single.p`; in fact, just `maxiter` (see BlakerCI-internal).

Details

Single values of the “unimodalized” acceptability function (for `type = "unimod"`) are computed by an iterative numerical algorithm implemented in internal function `poisson.blaker.acc.single.p`. The function cited is called just once in each of the intervals where the acceptability function is continuous (namely in the leftmost one of those points of `p` that fall into the interval when dealing with points below `x`, and the rightmost one when above `x`). The rest is done by function `cummax`. This is considerably faster than calling `poisson.blaker.acc.single.p` for every point of `p`. Note that applying `cummax` directly to a vector of unmodified acceptability values is even faster and provides a unimodal output; it may, nevertheless, lack accuracy.

Value

Vector of acceptability values (with or without unimodalization) in points of `p`.

Note

Inspired by M.P. Fay (2010), mentioning “unavoidable inconsistencies” between tests with non-unimodal acceptability functions and confidence intervals derived from them. When the acceptability functions are unimodalized and the test modified accordingly (i.e. p-values slightly increased in some cases), a perfectly matching test-CI pair is obtained.

Author(s)

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References


Examples

```r
p <- seq(0,10,length=100)
acc <- poisson.blaker.acc(3,p)
acc1 <- poisson.blaker.acc(3,p,type="unimod")
plot(p,acc,type="l")
lines(p,acc1,col="red")
legend(x=7,y=8,c("orig","unimod"),col=c("black","red"),lwd=1)
```

```r
## The two lines -- the unimodalized and original acceptabilities --
## look almost the same but some small differences are slightly
## visible.
```
```r
## They can be seen better this way:
plot(p, acc1 - acc, type = "l")

## Focussing on one of them:
p <- seq(5.05, 5.6, length = 100)
acc <- poisson.blaker.acc(3, p)
acc1 <- poisson.blaker.acc(3, p, type = "unimod")
plot(p, acc, type = "l", ylim = c(.391, .396))
lines(p, acc1, col = "red")
legend(x = 5.4, y = .395, c("orig", "unimod"), col = c("black", "red"), lwd = 1)
```

---

**poisson.blaker.limits**  
*Blaker's Poisson confidence limits*

**Description**

Fast and accurate calculation of Blaker's Poisson confidence limits.

**Usage**

```r
poisson.blaker.limits(x, level = 0.95, tol = 1e-10, ...)
```

**Arguments**

- `x`  
  number of events.

- `level`  
  confidence level.

- `tol`  
  numerical tolerance.

- `...`  
  additional arguments to be passed to `poisson.blaker.lower.limit`; in fact, just `maxiter` (see `BlakerCI-internal`).

**Details**

Note that the Blaker's $(1 - \alpha)$ confidence interval is the convex hull of the set $C$ of those points where the acceptability function (Blaker (2000)) exceeds level $\alpha$. When $C$ is not connected, the algorithm is, analogously to `binom.blaker.limits` (see its details), immune from leaving out short intervals and making thus the confidence intervals over-liberal.

**Value**

Length 2 vector – the lower and upper confidence limits.

**Note**

Package `exactci` by M. P. Fay includes another algorithm that calculates Blaker's Poisson confidence limits (see user-level function `poisson.exact` and internal function `exactpoissonCI`).

Lecoutre & Poitevineau (2014) designed another algorithm for the calculation of the Blaker's confidence limits. It is closely analogous to that of `poisson.blaker.limits`. 
Author(s)

Jan Klaschka <klaschka@cs.cas.cz>

References


See Also

exactci:poisson.exact One of the options yields Blaker’s limits.

Examples

poisson.blaker.limits(3) # [1] 0.8176914 8.5597971
Index

*Topic htest
  binom.blaker.acc, 2
  binom.blaker.limits, 5
  binom.blaker.VHadj.acc, 7
  binom.blaker.VHadj.limits, 9
  BlakerCI-package, 2
  poisson.blaker.acc, 13
  poisson.blaker.limits, 15

*Topic package
  BlakerCI-package, 2

  binom.blaker.acc, 2, 8, 12, 13
  binom.blaker.acc.single.p, 3
  binom.blaker.acc.single.p (BlakerCI-internal), 12
  binom.blaker.limits, 5, 8, 13
  binom.blaker.lower.limit (BlakerCI-internal), 12
  binom.blaker.VHadj.acc, 7, 13
  binom.blaker.VHadj.limits, 8, 9, 13
  binom.blaker.VHadj.lower.limit (BlakerCI-internal), 12
  BlakerCI (BlakerCI-package), 2
  BlakerCI-internal, 12
  BlakerCI-package, 2

  poisson.blaker.acc, 12, 13, 13
  poisson.blaker.acc.single.p, 14
  poisson.blaker.acc.single.p (BlakerCI-internal), 12
  poisson.blaker.limits, 13, 15
  poisson.blaker.lower.limit (BlakerCI-internal), 12
  poisson.blaker.upper.limit (BlakerCI-internal), 12