

# Package ‘CUB’

October 12, 2017

**Title** A Class of Mixture Models for Ordinal Data

**Version** 1.1.1

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**Description** For ordinal rating data, estimate and test models within the family of CUB models and their extensions (where CUB stands for Combination of a discrete Uniform and a shifted Binomial distributions). Simulation routines, plotting facilities and fitting measures are also provided.

**Depends** R (>= 2.15.2), Formula

**License** GPL-2 | GPL-3

**Encoding** UTF-8

**Imports** methods

**LazyData** true

**RoxygenNote** 5.0.1

**NeedsCompilation** no

**Suggests** knitr

**VignetteBuilder** knitr

**Repository** CRAN

**Date/Publication** 2017-10-11 22:51:42 UTC

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betabinomial	<i>Beta-Binomial probabilities of ordinal responses, with feeling and overdispersion parameters for each observation</i>
--------------	--

---

### Description

Compute the Beta-Binomial probabilities of ordinal responses, given feeling and overdispersion parameters for each observation.

### Usage

```
betabinomial(m,ordinal,csivett,phivett)
```

### Arguments

m	Number of ordinal categories
ordinal	Vector of ordinal responses. Missing values are not allowed: they should be preliminarily deleted or imputed
csivett	Vector of feeling parameters of the Beta-Binomial distribution for given ordinal responses
phivett	Vector of overdispersion parameters of the Beta-Binomial distribution for given ordinal responses

### Details

The Beta-Binomial distribution is the Binomial distribution in which the probability of success at each trial is random and follows the Beta distribution. It is frequently used in Bayesian statistics, empirical Bayes methods and classical statistics as an overdispersed binomial distribution.

### Value

A vector of the same length as ordinal, containing the Beta-Binomial probabilities of each observation, for the corresponding feeling and overdispersion parameters.

## References

- Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
- Piccolo D. (2015). Inferential issues for CUBE models with covariates. *Communications in Statistics - Theory and Methods*, **44**(23), 771–786.

## See Also

[betar](#), [betabinomialcsi](#)

## Examples

```
data(relgoods)
m<-10
ordinal<-relgoods$Tv
age<-2014-relgoods$BirthYear
no_na<-na.omit(cbind(ordinal,age))
ordinal<-no_na[,1]; age<-no_na[,2]
lage<-log(age)-mean(log(age))
gama<-c(-0.6, -0.3)
csivett<-logis(lage,gama)
alpha<-c(-2.3,0.92);
ZZ<-cbind(1,lage)
phivett<-exp(ZZ**alpha)
pr<-betabinomial(m,ordinal,csivett,phivett)
plot(density(pr))
```

---

betabinomialcsi	<i>Beta-Binomial probabilities of ordinal responses, given feeling parameter for each observation</i>
-----------------	---

---

## Description

Compute the Beta-Binomial probabilities of given ordinal responses, with feeling parameter specified for each observation, and with the same overdispersion parameter for all the responses.

## Usage

```
betabinomialcsi(m,ordinal,csivett,phi)
```

## Arguments

m	Number of ordinal categories
ordinal	Vector of ordinal responses. Missing values are not allowed: they should be preliminarily deleted or imputed
csivett	Vector of feeling parameters of the Beta-Binomial distribution for given ordinal responses
phi	Overdispersion parameter of the Beta-Binomial distribution

**Value**

A vector of the same length as ordinal: each entry is the Beta-Binomial probability for the given observation for the corresponding feeling and overdispersion parameters.

**References**

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

Piccolo D. (2015). Inferential issues for CUBE models with covariates. *Communications in Statistics - Theory and Methods*, **44**(23), 771–786.

**See Also**

[betar](#), [betabinomial](#)

**Examples**

```
data(relgoods)
m<-10
ordinal<-relgoods$Tv
age<-2014-relgoods$BirthYear
no_na<-na.omit(cbind(ordinal,age))
ordinal<-no_na[,1]; age<-no_na[,2]
lage<-log(age)-mean(log(age))
gama<-c(-0.61,-0.31)
phi<-0.16
csivett<-logis(lage,gama)
pr<-betabinomialcsi(m,ordinal,csivett,phi)
plot(density(pr))
```

---

betar

*Beta-Binomial distribution*

---

**Description**

Return the Beta-Binomial distribution with parameters  $m$ ,  $csi$  and  $phi$ .

**Usage**

```
betar(m,csi,phi)
```

**Arguments**

m	Number of ordinal categories
csi	Feeling parameter of the Beta-Binomial distribution
phi	Overdispersion parameter of the Beta-Binomial distribution

**Value**

The vector of length  $m$  of the Beta-Binomial distribution.

**References**

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

**See Also**

[betabinomial](#)

**Examples**

```
m<-9
csi<-0.8
phi<-0.2
pr<-betar(m,csi,phi)
plot(1:m,pr,type="h", main="Beta-Binomial distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

BIC.GEM

*S3 BIC method for class "GEM"*

---

**Description**

S3 BIC method for objects of class [GEM](#).

**Usage**

```
## S3 method for class 'GEM'
BIC(object, ...)
```

**Arguments**

object	An object of class "GEM"
...	Other arguments

**Value**

BIC index for the fitted model.

**See Also**

[logLik](#), [GEM](#)

---

bitcsi	<i>Shifted Binomial probabilities of ordinal responses</i>
--------	--

---

### Description

Compute the shifted Binomial probabilities of ordinal responses.

### Usage

```
bitcsi(m, ordinal, csi)
```

### Arguments

m	Number of ordinal categories
ordinal	Vector of ordinal responses
csi	Feeling parameter of the shifted Binomial distribution

### Value

A vector of the same length as `ordinal`, where each entry is the shifted Binomial probability of the corresponding observation.

### References

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables, *Quaderni di Statistica*, **5**, 85–104

### See Also

[probcub00](#), [probcubp0](#), [probcub0q](#)

### Examples

```
data(univer)
m<-7
csi<-0.7
ordinal<-univer$informat
pr<-bitcsi(m,ordinal,csi)
```

---

`bitgama`*Shifted Binomial distribution with covariates*

---

**Description**

Return the shifted Binomial probabilities of ordinal responses where the feeling component is explained by covariates via a logistic link.

**Usage**

```
bitgama(m, ordinal, W, gama)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>ordinal</code>	Vector of ordinal responses
<code>W</code>	Matrix of covariates for the feeling component
<code>gama</code>	Vector of parameters for the feeling component, with length equal to $\text{NCOL}(W)+1$ to account for an intercept term (first entry of <code>gama</code> )

**Value**

A vector of the same length as `ordinal`, where each entry is the shifted Binomial probability for the corresponding observation and feeling value.

**See Also**

[logis](#), [probcub0q](#), [probcubpq](#)

**Examples**

```
n<-100
m<-7
W<-sample(c(0,1),n,replace=TRUE)
gama<-c(0.2,-0.2)
csivett<-logis(W,gama)
ordinal<-rbinom(n,m-1,csivett)+1
pr<-bitgama(m,ordinal,W,gama)
```



---

chi2cub	<i>Pearson <math>X^2</math> statistic</i>
---------	---

---

**Description**

Compute the  $X^2$  statistic of Pearson for CUB models with one or two discrete covariates for the feeling component.

**Usage**

```
chi2cub(m,ordinal,W,pai,gama)
```

**Arguments**

m	Number of ordinal categories
ordinal	Vector of ordinal responses
W	Matrix of covariates for the feeling component
pai	Uncertainty parameter
gama	Vector of parameters for the feeling component, with length equal to NCOL(W)+1 to account for an intercept term (first entry of gama)

**Details**

No missing value should be present neither for ordinal nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

**Value**

A list with the following components:

df	Degrees of freedom
chi2	Value of the Pearson fitting measure
dev	Deviance indicator

**References**

Tutz, G. (2012). *Regression for Categorical Data*, Cambridge University Press, Cambridge

**Examples**

```
data(univer)
m<-7
pai<-0.3
gama<-c(0.1,0.7)
ordinal<-univer$informat; W<-univer$gender;
pearson<-chi2cub(m,ordinal,W,pai,gama)
degfree<-pearson$df
statvalue<-pearson$chi2
deviance<-pearson$dev
```

---

coef.GEM	<i>S3 Method: coef for class "GEM"</i>
----------	--

---

### Description

S3 method: coef for objects of class [GEM](#).

### Usage

```
## S3 method for class 'GEM'
coef(object, ...)
```

### Arguments

object	An object of class <a href="#">GEM</a>
...	Other arguments

### Details

Returns estimated values of coefficients of the fitted model

### Value

ML estimates of parameters of the fitted GEM model.

### See Also

[GEM](#), [summary](#)

---

cormat	<i>Correlation matrix for estimated model</i>
--------	---

---

### Description

Compute parameter correlation matrix for estimated model as returned by an object of class "GEM".

### Usage

```
cormat(object,digits=options()$digits)
```

### Arguments

object	An object of class "GEM"
digits	Number of significant digits to be printed. Default is options()\$digits

**Value**

Parameters correlation matrix for fitted GEM models.

**See Also**

GEM, vcov

---

cubevisual

*Plot an estimated CUBE model*

---

**Description**

Plotting facility for the CUBE estimation of ordinal responses.

**Usage**

```
cubevisual(ordinal,csiplot=FALSE,paipplot=FALSE,...)
```

**Arguments**

ordinal	Vector of ordinal responses
csiplot	Logical: should $\xi$ or $1 - \xi$ be the $y$ coordinate
paipplot	Logical: should $\pi$ or $1 - \pi$ be the $x$ coordinate
...	Additional arguments to be passed to <code>plot()</code> and <code>text()</code> . Optionally, the number $m$ of ordinal categories may be passed: this is recommended if some category has zero frequency.

**Details**

It represents an estimated CUBE model as a point in the parameter space with the overdispersion being labeled.

**Value**

For a CUBE model fitted to `ordinal`, by default it returns a plot of the estimated  $(1 - \pi, 1 - \xi)$  as a point in the parameter space, labeled with the estimated overdispersion  $\phi$ . Depending on `csiplot` and `paipplot` and on desired output,  $x$  and  $y$  coordinates may be set to  $\pi$  and  $\xi$ , respectively.

**Examples**

```
data(univer)
ordinal<-univer$global
cubevisual(ordinal,xlim=c(0,0.5),main="Global Satisfaction",
  ylim=c(0.5,1),cex=0.8,digits=3,col="red")
```

---

cubshevisual

*Plot an estimated CUB model with shelter*


---

**Description**

Plotting facility for the CUB estimation of ordinal responses when a shelter effect is included

**Usage**

```
cubshevisual(ordinal,shelter,csiplot=FALSE,paiplot=FALSE,...)
```

**Arguments**

ordinal	Vector of ordinal responses
shelter	Category corresponding to the shelter choice
csiplot	Logical: should $\xi$ or $1 - \xi$ be the $y$ coordinate
paiplot	Logical: should $\pi$ or $1 - \pi$ be the $x$ coordinate
...	Additional arguments to be passed to <code>plot()</code> and <code>text()</code> . Optionally, the number $m$ of ordinal categories may be passed: this is recommended if some category has zero frequency.

**Details**

It represents an estimated CUB model with shelter effect as a point in the parameter space with shelter estimate indicated as label.

**Value**

For a CUB model with shelter fitted to `ordinal`, by default it returns a plot of the estimated  $(1 - \pi, 1 - \xi)$  as a point in the parameter space, labeled with the estimated shelter parameter  $\delta$ . Depending on `csiplot` and `paiplot` and on desired output,  $x$  and  $y$  coordinates may be set to  $\pi$  and  $\xi$ , respectively.

**See Also**

[cubvisual](#), [multicub](#)

**Examples**

```
data(univer)
ordinal<-univer$global
cubshevisual(ordinal,shelter=7,digits=3,col="blue",main="Global Satisfaction")
```

---

cubvisual	<i>Plot an estimated CUB model</i>
-----------	------------------------------------

---

### Description

Plotting facility for the CUB estimation of ordinal responses.

### Usage

```
cubvisual(ordinal,csiplot=FALSE,paiplot=FALSE,...)
```

### Arguments

ordinal	Vector of ordinal responses
csiplot	Logical: should $\xi$ or $1 - \xi$ be the $y$ coordinate
paiplot	Logical: should $\pi$ or $1 - \pi$ be the $x$ coordinate
...	Additional arguments to be passed to <code>plot()</code> and <code>text()</code> . Optionally, the number $m$ of ordinal categories may be passed: this is recommended if some category has zero frequency.

### Details

It represents an estimated CUB model as a point in the parameter space with some useful options.

### Value

For a CUB model fit to `ordinal`, by default it returns a plot of the estimated  $(1 - \pi, 1 - \xi)$  as a point in the parameter space. Depending on `csiplot` and `paiplot` and on desired output,  $x$  and  $y$  coordinates may be set to  $\pi$  and  $\xi$ , respectively.

### Examples

```
data(univer)
ordinal<-univer$global
cubvisual(ordinal,xlim=c(0,0.5),ylim=c(0.5,1),cex=0.8,main="Global Satisfaction")
```

---

CUB\_package

*CUB package*

---

## Description

The analysis of human perceptions is often carried out by resorting to questionnaires, where respondents are asked to express ratings about the items being evaluated. The standard goal of the statistical framework proposed for this kind of data (e.g. cumulative models) is to explicitly characterize the respondents' perceptions about a latent trait, by taking into account, at the same time, the ordinal categorical scale of measurement of the involved statistical variables.

The new class of models starts from a particular assumption about the unconscious mechanism leading individuals' responses to choose an ordinal category on a rating scale. The basic idea derives from the awareness that two latent components move the psychological process of selection among discrete alternatives: attractiveness towards the item and uncertainty in the response. Both components of models concern the stochastic mechanism in term of feeling, which is an internal/personal movement of the subject towards the item, and uncertainty pertaining to the final choice.

Thus, on the basis of experimental data and statistical motivations, the response distribution is modelled as the convex Combination of a discrete Uniform and a shifted Binomial random variable (denoted as CUB model) whose parameters may be consistently estimated and validated by maximum likelihood inference. In addition, subjects' and objects' covariates can be included in the model in order to assess how the characteristics of the respondents may affect the ordinal score.

CUB models have been firstly introduced by Piccolo (2003) and implemented on real datasets concerning ratings and rankings by D'Elia and Piccolo (2005), Iannario and Piccolo (2012).

The CUB package allows the user to estimate and test CUB models and their extensions by using maximum likelihood methods. The package covers the main models of the class of Generalized Mixture Models with uncertainty (GEM - Iannario and Piccolo (2016a)), a comprehensive framework for modelling ordinal data. The accompanying vignettes supplies the user with detailed usage instructions and examples.

Acknowledgements: The Authors are grateful to Maria Antonietta Del Ferraro, Francesco Miranda and Giuseppe Porpora for their preliminary support in the implementation of the first version of the package.

## Details

Package:	CUB
Type:	Package
Version:	1.1.1
Date:	2017-10-11
License:	GPL-2   GPL-3

## Author(s)

Maria Iannario, Domenico Piccolo, Rosaria Simone

**Source**

<http://www.labstat.it/home/research/resources/cub-data-sets-2/>

**References**

- D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78
- Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables, *Quaderni di Statistica*, **5**, 85–104
- D'Elia A. and Piccolo D. (2005). A mixture model for preferences data analysis, *Computational Statistics & Data Analysis*, **49**, 917–937
- Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
- Iannario M. and Piccolo D. (2016a). A comprehensive framework for regression models of ordinal data. *Metron*, **74**(2), 233–252.
- Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

---

deltaprob

*Mean difference of a discrete random variable*

---

**Description**

Compute the Gini mean difference of a discrete distribution

**Usage**

deltaprob(prob)

**Arguments**

prob                      Vector of the probability distribution

**Value**

Numeric value of the Gini mean difference of the input probability distribution, computed according to the de Finetti-Paciello formulation.

**Examples**

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
deltaprob(prob)
```

---

dissim	<i>Normalized dissimilarity measure</i>
--------	---

---

**Description**

Compute the normalized dissimilarity measure between observed relative frequencies and estimated (theoretical) probabilities of a discrete distribution.

**Usage**

```
dissim(proba,probb)
```

**Arguments**

proba	Vector of observed relative frequencies
probb	Vector of estimated (theoretical) probabilities

**Value**

Numeric value of the dissimilarity index, assessing the distance to a perfect fit.

**Examples**

```
proba<-c(0.01,0.03,0.08,0.07,0.27,0.37,0.17)
probb<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
dissim(proba,probb)
```

---

ellecub	<i>Log-likelihood function of a CUB model without covariates</i>
---------	--

---

**Description**

Compute the log-likelihood function of a CUB model without covariates fitting ordinal responses, possibly with subjects' specific parameters.

**Usage**

```
ellecub(m,ordinal,assepai,assecsi)
```

**Arguments**

m	Number of ordinal categories
ordinal	Vector of ordinal responses
assepai	Vector of uncertainty parameters for given observations (with the same length as ordinal)
assecsi	Vector of feeling parameters for given observations (with the same length as ordinal)



**See Also**[loglikCUB](#)**Examples**

```

m<-7
n0<-230
n1<-270
bet<-c(-1.5,1.2)
gama<-c(0.5,-1.2)
pai0<-logis(0,bet); csi0<-logis(0,gama)
pai1<-logis(1,bet); csi1<-logis(1,gama)
ordinal0<-simcub(n0,m,pai0,csi0)
ordinal1<-simcub(n1,m,pai1,csi1)
ordinal<-c(ordinal0,ordinal1)
assepai<-c(rep(pai0,n0),rep(pai1,n1))
assecsi<-c(rep(csi0,n0),rep(csi1,n1))
lli<-ellecub(m,ordinal,assepai,assecsi)

```

expcub00

*Expectation of CUB distributions***Description**

Compute the expectation of a CUB model without covariates.

**Usage**

```
expcub00(m, pai, csi)
```

**Arguments**

m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter

**References**

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

**See Also**[varcub00](#), [expcube](#), [varcube](#)

## Examples

```
m<-10
pai<-0.3
csi<-0.7
meancub<-expcub00(m,pai,csi)
```

---

expcube

*Expectation of CUBE models*

---

## Description

Compute the expectation of a CUBE model without covariates.

## Usage

```
expcube(m,pai,csi,phi)
```

## Arguments

m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter
phi	Overdispersion parameter

## References

Iannario M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786  
Iannario, M. (2015). Detecting latent components in ordinal data with overdispersion by means of a mixture distribution, *Quality & Quantity*, **49**, 977–987

## See Also

[varcube](#), [varcub00](#), [expcub00](#)

## Examples

```
m<-10
pai<-0.1
csi<-0.7
phi<-0.2
meancube<-expcube(m,pai,csi,phi)
```

---

fitted.GEM	<i>S3 method "fitted" for class "GEM"</i>
------------	---

---

**Description**

S3 method fitted for objects of class [GEM](#).

**Usage**

```
## S3 method for class 'GEM'
fitted(object, ...)
```

**Arguments**

object	An object of class <a href="#">GEM</a>
...	Other arguments

**Details**

Returns the fitted probability distribution for GEM models with no covariates. If only one dichotomous covariate is included in the model to explain some components, it returns the fitted probability distribution for each profile.

**See Also**

[GEM](#)

**Examples**

```
fitcub<-GEM(Formula(global~0|freqserv|0),family="cub",data=univer)
fitted(fitcub,digits=4)
```

---

GEM	<i>Main function for GEM models</i>
-----	-------------------------------------

---

**Description**

Main function to estimate and validate GEneralized Mixture models with uncertainty.

**Usage**

```
GEM(Formula,family=c("cub","cube","ihg","cush"),data,...)
```

**Arguments**

Formula	Object of class Formula. Response variable is the vector of ordinal observations - see Details.
family	Character string indicating which class of GEM models to fit.
data	an optional data frame (or object coercible by <code>as.data.frame</code> to a data frame) containing the variables in the model. If missing, the variables are taken from <code>environment(Formula)</code> .
...	Additional arguments to be passed for the specification of the model. See details and examples.

**Details**

It is the main function for GEM models estimation, calling for the corresponding function for the specified subclass. The number of categories  $m$  is internally retrieved but it is advisable to pass it as an argument to the call if some category has zero frequency.

If `family="cub"`, then a CUB mixture model is fitted to the data to explain uncertainty, feeling and possible shelter effect by further passing the extra argument `shelter` for the corresponding category. Subjects' covariates can be included by specifying covariates matrices in the Formula as `ordinal~Y|W|X`, to explain uncertainty (Y), feeling (W) or shelter (X). Notice that covariates for shelter effect can be included only if specified for both feeling and uncertainty (GeCUB models).

If `family="cube"`, then a CUBE mixture model (Combination of Uniform and Beta-Binomial) is fitted to the data to explain uncertainty, feeling and overdispersion. Subjects' covariates can be also included to explain the feeling component or all the three components by specifying covariates matrices in the Formula as `ordinal~Y|W|Z` to explain uncertainty (Y), feeling (W) or overdispersion (Z). An extra logical argument `expinform` indicates whether or not to use the expected or the observed information matrix (default is FALSE).

If `family="ihg"`, then an IHG model is fitted to the data. IHG models (Inverse Hypergeometric) are nested into CUBE models (see the references below). The parameter  $\theta$  gives the probability of observing the first category and is therefore a direct measure of preference, attraction, pleasantness toward the investigated item. This is the reason why  $\theta$  is customarily referred to as the preference parameter of the IHG model. Covariates for the preference parameter  $\theta$  have to be specified in matrix form in the Formula as `ordinal~U`.

If `family="cush"`, then a CUSH model is fitted to the data (Combination of Uniform and SHelter effect). The category corresponding to the inflation should be passed via argument `shelter`. Covariates for the shelter parameter  $\delta$  are specified in matrix form Formula as `ordinal~X`.

Even if no covariate is included in the model for a given component, the corresponding model matrix needs always to be specified: in this case, it should be set to 0 (see examples below). Extra arguments include the maximum number of iterations (`maxiter`, default: `maxiter=500`) for the optimization algorithm and the required error tolerance (`toler`, default: `toler=1e-6`).

Standard methods: `logLik()`, `BIC()`, `vcov()`, `fitted()`, `coef()`, `print()`, `summary()` are implemented.

The optimization procedure is run via `optim()` when required. If the estimated variance-covariance matrix is not positive definite, the function returns a warning message and produces a matrix with NA entries.

**Value**

An object of the class "GEM" is a list containing the following elements:

estimates	Maximum likelihood estimates of parameters
loglik	Log-likelihood function at the final estimates
varmat	Variance-covariance matrix of final estimates
niter	Number of executed iterations
BIC	BIC index for the estimated model
ordinal	Vector of ordinal responses on which the model has been fitted
time	Processor time for execution
ellipsis	Retrieve the arguments passed to the call and extra arguments generated via the call
family	Character string indicating the sub-class of the fitted model
formula	Returns the Formula of the call for the fitted model
call	Returns the executed call

## References

- D’Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78
- D’Elia A. and Piccolo D. (2005). A mixture model for preferences data analysis, *Computational Statistics & Data Analysis*, **49**, 917–937
- Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
- Iannario M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
- Piccolo D. (2015). Inferential issues for CUBE models with covariates, *Communications in Statistics. Theory and Methods*, **44**(23), 771–786.
- Iannario M. (2015). Detecting latent components in ordinal data with overdispersion by means of a mixture distribution, *Quality & Quantity*, **49**, 977–987
- Iannario M. and Piccolo D. (2016a). A comprehensive framework for regression models of ordinal data. *Metron*, **74**(2), 233–252.
- Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

## See Also

[logLik](#), [coef](#), [BIC](#), [makeplot](#), [summary](#), [vcov](#), [fitted](#), [cormat](#)

## Examples

```
library(CUB)
## CUB models with no covariates
model<-GEM(Formula(Walking~0|0|0),family="cub",data=relgoods)
coef(model,digits=5)      # Estimated parameter vector (pai,csi)
logLik(model)             # Log-likelihood function at ML estimates
vcov(model,digits=4)     # Estimated Variance-Covariance matrix
cormat(model)            # Parameter Correlation matrix
```

```

fitted(model)          # Fitted probability distribution
makeplot(model)
#####
## CUB model with shelter effect
model<-GEM(Formula(officcho~0|0|0),family="cub",shelter=7,data=univer)
BICshe<-BIC(model,digits=4)
#####
## CUB model with covariate for all components - GeCUB
data(univer)
dichoage<-ifelse(univer$age<=25,0,1)
modelgecub<-GEM(Formula(officcho~gender|dichoage|freqserv),family="cub",shelter=7,data=univer)
BICgecub<-BIC(modelgecub)
#####
## CUB model with covariate for uncertainty
modelcovpai<-GEM(Formula(Parents~Smoking|0|0),family="cub",data=relgoods)
fitted(modelcovpai)
makeplot(modelcovpai)
#####
## CUB model with covariate for feeling
# all methods are available for this variable specification
modelcovcsi<-GEM(Formula(RelFriends~0|WalkAlone|0),family="cub",data=relgoods,maxiter=10)
fitted(modelcovcsi)
makeplot(modelcovcsi)
#####
## CUB model with covariates for both uncertainty and feeling components
data(univer)
lage<-log(univer$age)-mean(log(univer$age))
model<-GEM(Formula(global~gender|lage|0),family="cub",data=univer,maxiter=150)
param<-coef(model)
bet<-param[1:2]      # ML estimates of coefficients for uncertainty covariate: gender
gama<-param[3:4]    # ML estimates of coefficients for feeling covariate: lage
#####
## CUBE models with no covariates
model<-GEM(Formula(MeetRelatives~0|0|0),family="cube",starting=c(0.5,0.5,0.1),
  data=relgoods,expinform=TRUE)
coef(model,digits=4)  # Final ML estimates
logLik(model)        # Maximum value of the log-likelihood function
vcov(model)
print(model)
BIC(model)
fitted(model)
makeplot(model)
summary(model)
#####
## CUBE with covariate 'WalkAlone' only for the feeling component
modelcovcsi<-GEM(Formula(MeetRelatives~0|WalkAlone|0),family="cube",data=relgoods)
summary(modelcovcsi)
#####
## CUBE with covariates for all components
modelcov<-GEM(Formula(MeetRelatives~WalkAlone|WalkAlone|WalkAlone),family="cube",
  maxiter=50,toler=1e-2,data=relgoods)
BIC(modelcovcsi)
BIC(modelcov)

```

```
#####
## IHG models without covariates
model<-GEM(willingn~0,family="ihg",data=univer)
coef(model)          ## ML estimate of preference parameter theta
fitted(model)       ## fitted probabilities
makeplot(model)
#####
## IHG with covariates
modelcov<-GEM(willingn~freqserv,family="ihg",data=univer)
omega<-coef(modelcov)  ## ML estimates
maxlik<-logLik(modelcov) ##
makeplot(modelcov)
summary(modelcov)
#####
## CUSH models without covariate
model<-GEM(Dog~0,family="cush",shelter=1,data=relgoods)
delta<-coef(model)    # ML estimates of delta
maxlik<-logLik(model) # Log-likelihood at ML estimates
summary(model)
makeplot(model)
#####
## CUSH model with covariates
modelcov<-GEM(MusicInstr~Smoking,family="cush",shelter=1,data=relgoods)
omega<-coef(modelcov)
maxlik<-logLik(modelcov)
varmat<-vcov(modelcov)
summary(modelcov,digits=3)
fitted(modelcov,digits=3)
```

---

gini

*Normalized Gini heterogeneity index*


---

### Description

Compute the normalized Gini heterogeneity index for a given discrete probability distribution.

### Usage

```
gini(prob)
```

### Arguments

prob            Vector of probability distribution or relative frequencies

### See Also

[laakso](#)

### Examples

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
gini(prob)
```

---

inibest

*Preliminary estimators for CUB models without covariates*

---

### Description

Compute preliminary parameter estimates of a CUB model without covariates for given ordinal responses. These preliminary estimators are used within the package code to start the E-M algorithm.

### Usage

```
inibest(m, freq)
```

### Arguments

m	Number of ordinal categories
freq	Vector of the absolute frequencies of given ordinal responses

### Value

A vector  $(\pi, \xi)$  of the initial parameter estimates for a CUB model without covariates, given the absolute frequency distribution of ordinal responses

### References

Iannario M. (2009). A comparison of preliminary estimators in a class of ordinal data models, *Statistica & Applicazioni*, **VII**, 25–44

Iannario M. (2012). Preliminary estimators for a mixture model of ordinal data, *Advances in Data Analysis and Classification*, **6**, 163–184

### See Also

[inibestgama](#)

### Examples

```
m<-9
freq<-c(10,24,28,36,50,43,23,12,5)
estim<-inibest(m, freq)
pai<-estim[1]
csi<-estim[2]
```



---

inibestcube	<i>Naive estimates for CUBE models without covariates</i>
-------------	---

---

**Description**

Compute *naive* parameter estimates of a CUBE model without covariates for given ordinal responses. These preliminary estimators are used within the package code to start the E-M algorithm.

**Usage**

```
inibestcube(m, ordinal)
```

**Arguments**

m	Number of ordinal categories
ordinal	Vector of ordinal responses

**Value**

A vector  $(\pi, \xi, \phi)$  of parameter estimates of a CUBE model without covariates.

**See Also**

[inibestcubecov](#), [inibestcubecsi](#)

**Examples**

```
data(relgoods)
m<-10
ordinal<-relgoods$SocialNetwork
estim<-inibestcube(m,ordinal) # Preliminary estimates (pai,csi,phi)
```

---

inibestcubecov	<i>Preliminary parameter estimates for CUBE models with covariates</i>
----------------	--

---

**Description**

Compute preliminary parameter estimates for a CUBE model with covariates for all the three parameters. These estimates are set as initial values to start the E-M algorithm within maximum likelihood estimation.

**Usage**

```
inibestcubecov(m, ordinal, Y, W, Z)
```

**Arguments**

m	Number of ordinal categories
ordinal	Vector of ordinal responses
Y	Matrix of selected covariates to explain the uncertainty parameter
W	Matrix of selected covariates to explain the feeling parameter
Z	Matrix of selected covariates to explain the overdispersion parameter

**Value**

A vector (inibet, inigama, inialpha) of preliminary estimates of parameter vectors for  $\pi = \pi(\beta)$ ,  $\xi = \xi(\gamma)$ ,  $\phi = \phi(\alpha)$ , respectively, of a CUBE model with covariates for all the three parameters. In details, inibet, inigama and inialpha have length equal to  $\text{NCOL}(Y)+1$ ,  $\text{NCOL}(W)+1$  and  $\text{NCOL}(Z)+1$ , respectively, to account for an intercept term for each component.

**See Also**

[inibestcube](#), [inibestcubecsi](#), [inibestgama](#)

**Examples**

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Tv))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$year.12))
nacovphi<-which(is.na(relgoods$EducationDegree))
na<-union(union(naord,nacovpai),union(nacovcsi,nacovphi))
ordinal<-relgoods$Tv[-na]
Y<-relgoods$Gender[-na]
W<-relgoods$year.12[-na]
Z<-relgoods$EducationDegree[-na]
ini<-inibestcubecov(m,ordinal,Y,W,Z)
p<-NCOL(Y)
q<-NCOL(W)
inibet<-ini[1:(p+1)]           # Preliminary estimates for uncertainty
inigama<-ini[(p+2):(p+q+2)]   # Preliminary estimates for feeling
inialpha<-ini[(p+q+3):length(ini)] # Preliminary estimates for overdispersion
```

---

inibestcubecsi	<i>Preliminary estimates of parameters for CUBE models with covariates only for feeling</i>
----------------	---

---

**Description**

Compute preliminary parameter estimates of a CUBE model with covariates only for feeling, given ordinal responses. These estimates are set as initial values to start the corresponding E-M algorithm within the package.

**Usage**

```
inibestcubecsi(m,ordinal,W,starting,maxiter,toler)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>ordinal</code>	Vector of ordinal responses
<code>W</code>	Matrix of selected covariates to explain the feeling component
<code>starting</code>	Starting values for preliminary estimation of a CUBE without covariate
<code>maxiter</code>	Maximum number of iterations allowed for preliminary iterations
<code>toler</code>	Fixed error tolerance for final estimates for preliminary iterations

**Details**

Preliminary estimates for the uncertainty and the overdispersion parameters are computed by short runs of EM. As to the feeling component, it considers the nested CUB model with covariates and calls [inibestgama](#) to derive initial estimates for the coefficients of the selected covariates for feeling.

**Value**

A vector (`pai`, `gamaest`, `phi`), where `pai` is the initial estimate for the uncertainty parameter, `gamaest` is the vector of initial estimates for the feeling component (including an intercept term in the first entry), and `phi` is the initial estimate for the overdispersion parameter.

**See Also**

[inibestcube](#), [inibestcubecov](#), [inibestgama](#)

**Examples**

```
data(relgoods)
isnacov<-which(is.na(relgoods$Gender))
isnaord<-which(is.na(relgoods$Tv))
na<-union(isnacov,isnaord)
ordinal<-relgoods$Tv[-na]; W<-relgoods$Gender[-na]
m<-10
starting<-rep(0.1,3)
ini<-inibestcubecsi(m,ordinal,W,starting,maxiter=100,toler=1e-3)
nparam<-length(ini)
pai<-ini[1] # Preliminary estimates for uncertainty component
gamaest<-ini[2:(nparam-1)] # Preliminary estimates for coefficients of feeling covariates
phi<-ini[nparam] # Preliminary estimates for overdispersion component
```

---

inibestgama	<i>Preliminary parameter estimates of a CUB model with covariates for feeling</i>
-------------	---

---

### Description

Compute preliminary parameter estimates for the feeling component of a CUB model fitted to ordinal responses. These estimates are set as initial values for parameters to start the E-M algorithm.

### Usage

```
inibestgama(m, ordinal, W)
```

### Arguments

m	Number of ordinal categories
ordinal	Vector of ordinal responses
W	Matrix of selected covariates for explaining the feeling component

### Value

A vector of length equal to  $NCOL(W)+1$ , whose entries are the preliminary estimates of the parameters for the feeling component, including an intercept term as first entry.

### References

Iannario M. (2008). Selecting feeling covariates in rating surveys, *Rivista di Statistica Applicata*, **20**, 103–116

Iannario M. (2009). A comparison of preliminary estimators in a class of ordinal data models, *Statistica & Applicazioni*, **VII**, 25–44

Iannario M. (2012). Preliminary estimators for a mixture model of ordinal data, *Advances in Data Analysis and Classification*, **6**, 163–184

### See Also

[inibest](#), [inibestcubecsi](#)

### Examples

```
data(univer)
m<-7; ordinal<-univer$global; cov<-univer$diploma
ini<-inibestgama(m, ordinal, W=cov)
```

---

`inigrd`*Grid-based preliminary parameter estimates for CUB models*

---

**Description**

Compute the log-likelihood function of a CUB model with parameter vector  $(\pi, \xi)$  ranging in the Cartesian product between  $x$  and  $y$ , for a given absolute frequency distribution.

**Usage**

```
inigrd(m, freq, x, y)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>freq</code>	Vector of length $m$ of the absolute frequency distribution
<code>x</code>	A set of values to assign to the uncertainty parameter $\pi$
<code>y</code>	A set of values to assign to the feeling parameter $\xi$

**Value**

It returns the parameter vector corresponding to the maximum value of the log-likelihood for a CUB model without covariates for given frequencies.

**See Also**

[inibest](#)

**Examples**

```
m<-9
x<-c(0.1, 0.4, 0.6, 0.8)
y<-c(0.2, 0.5, 0.7)
freq<-c(10, 24, 28, 36, 50, 43, 23, 12, 5)
ini<-inigrd(m, freq, x, y)
pai<-ini[1]
csi<-ini[2]
```

---

iniihg	<i>Moment estimate for the preference parameter of the IHG distribution</i>
--------	---

---

### Description

Compute the moment estimate of the preference parameter of the IHG distribution. This preliminary estimate is set as initial value within the optimization procedure for an IHG model fitting the observed frequencies.

### Usage

```
iniihg(m, freq)
```

### Arguments

m	Number of ordinal categories
freq	Vector of the absolute frequency distribution of the categories

### Value

Moment estimator of the preference parameter  $\theta$ .

### References

D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78.

### See Also

[inibest](#), [inibestcube](#)

### Examples

```
m<-9
freq<-c(70,51,48,38,29,23,12,10,5)
initheta<-iniihg(m, freq)
```

---

laakso	<i>Normalized Laakso and Taagepera heterogeneity index</i>
--------	--

---

**Description**

Compute the normalized Laakso and Taagepera heterogeneity index for a given discrete probability distribution.

**Usage**

```
laakso(prob)
```

**Arguments**

prob	Vector of a probability or relative frequency distribution
------	--

**References**

Laakso, M. and Taagepera, R. (1989). Effective number of parties: a measure with application to West Europe, *Comparative Political Studies*, **12**, 3–27.

**See Also**

[gini](#)

**Examples**

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
laakso(prob)
```

---

logis	<i>The logistic transform</i>
-------	-------------------------------

---

**Description**

Create a matrix YY binding array Y with a vector of ones, placed as the first column of YY. It applies the logistic transform componentwise to the standard matrix multiplication between YY and param.

**Usage**

```
logis(Y,param)
```

**Arguments**

Y	A generic matrix or one dimensional array
param	Vector of coefficients, whose length is NCOL(Y) + 1 (to consider also an intercept term)

**Value**

Return a vector whose length is  $NROW(Y)$  and whose  $i$ -th component is the logistic function at the scalar product between the  $i$ -th row of  $YY$  and the vector `param`.

**Examples**

```
n<-50
Y<-sample(c(1,2,3),n,replace=TRUE)
param<-c(0.2,0.7)
logis(Y,param)
```

---

`logLik.GEM`*logLik S3 Method for class "GEM"*

---

**Description**

S3 method: `logLik()` for objects of class "GEM".

**Usage**

```
## S3 method for class 'GEM'
logLik(object, ...)
```

**Arguments**

<code>object</code>	An object of class "GEM"
<code>...</code>	Other arguments

**Value**

Log-likelihood at the final ML estimates for parameters of the fitted GEM model.

**See Also**

[loglikCUB](#), [loglikCUBE](#), [GEM](#), [loglikIHG](#), [loglikCUSH](#), [BIC](#)



---

loglikCUB	<i>Log-likelihood function for CUB models</i>
-----------	---

---

### Description

Compute the log-likelihood value of a CUB model fitting given data, with or without covariates to explain the feeling and uncertainty components, or for extended CUB models with shelter effect.

### Usage

```
loglikCUB(ordinal, m, param, Y=0, W=0, X=0, shelter=0)
```

### Arguments

ordinal	Vector of ordinal responses
m	Number of ordinal categories
param	Vector of parameters for the specified CUB model
Y	Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model)
W	Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model)
X	Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model)
shelter	Category corresponding to the shelter choice (default: no shelter effect is included in the model)

### Details

If no covariate is included in the model, then `param` should be given in the form  $(\pi, \xi)$ . More generally, it should have the form  $(\beta, \gamma)$  where, respectively,  $\beta$  and  $\gamma$  are the vectors of coefficients explaining the uncertainty and the feeling components, with length  $\text{NCOL}(Y)+1$  and  $\text{NCOL}(W)+1$  to account for an intercept term in the first entry. When shelter effect is considered, `param` corresponds to the first possible parameterization and hence should be given as  $(\text{pai1}, \text{pai2}, \text{csi})$ . No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

### See Also

[logLik](#)

## Examples

```

## Log-likelihood of a CUB model with no covariate
m<-9; n<-300
pai<-0.6; csi<-0.4
ordinal<-simcub(n,m,pai,csi)
param<-c(pai,csi)
loglikcub<-loglikCUB(ordinal,m,param)
#####
## Log-likelihood of a CUB model with covariate for uncertainty

data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]; Y<-relgoods$Gender[-na]
bbet<-c(-0.81,0.93); ccsi<-0.2
param<-c(bbet,ccsi)
loglikcubp0<-loglikCUB(ordinal,m,param,Y=Y)
#####
## Log-likelihood of a CUB model with covariate for feeling
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]; W<-relgoods$Gender[-na]
pai<-0.44; gama<-c(-0.91,-0.7)
param<-c(pai,gama)
loglikcub0q<-loglikCUB(ordinal,m,param,W=W)
#####
## Log-likelihood of a CUB model with covariates for both parameters
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Walking))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$Smoking))
na<-union(naord,union(nacovpai,nacovcsi))
ordinal<-relgoods$Walking[-na]
Y<-relgoods$Gender[-na]; W<-relgoods$Smoking[-na]
bet<-c(-0.45,-0.48); gama<-c(-0.55,-0.43)
param<-c(bet,gama)
loglikcubpq<-loglikCUB(ordinal,m,param,Y=Y,W=W)
#####
### Log-likelihood of a CUB model with shelter effect
m<-7; n<-400
pai<-0.7; csi<-0.16; delta<-0.15
shelter<-5
ordinal<-simcubshe(n,m,pai,csi,delta,shelter)
pai1<- pai*(1-delta); pai2<-1-pai1-delta
param<-c(pai1,pai2,csi)
loglik<-loglikCUB(ordinal,m,param,shelter=shelter)

```

```
#####
### Log-likelihood of a GeCUB
data(univer)
ordinal<-univer$officeho; Y<-W<-X<-univer$gender;
modelgecub<-GEM(Formula(ordinal~Y|W|X),family="cub",shelter=7,maxiter=100)
logLik(modelgecub)
param<-rep(0.1,6)
loglik<-loglikCUBE(ordinal,m=7,param=param,shelter=7,Y=Y,W=W,X=X)
```

---

loglikCUBE

*Log-likelihood function for CUBE models*


---

### Description

Compute the log-likelihood function for CUBE models. It is possible to include covariates in the model for explaining the feeling component or all the three parameters.

### Usage

```
loglikCUBE(ordinal,m,param,Y=0,W=0,Z=0)
```

### Arguments

ordinal	Vector of ordinal responses
m	Number of ordinal categories
param	Vector of parameters for the specified CUBE model
Y	Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model)
W	Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model)
Z	Matrix of selected covariates to explain the overdispersion component (default: no covariate is included in the model)

### Details

If no covariate is included in the model, then param has the form  $(\pi, \xi, \phi)$ . More generally, it has the form  $(\beta, \gamma, \alpha)$  where, respectively,  $\beta, \gamma, \alpha$  are the vectors of coefficients explaining the uncertainty, the feeling and the overdispersion components, with length  $\text{NCOL}(Y)+1$ ,  $\text{NCOL}(W)+1$ ,  $\text{NCOL}(Z)+1$  to account for an intercept term in the first entry. No missing value should be present neither for ordinal nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

### See Also

[logLik](#)

**Examples**

```
#### Log-likelihood of a CUBE model with no covariate
m<-7; n<-400
pai<-0.83; csi<-0.19; phi<-0.045
ordinal<-simcube(n,m,pai,csi,phi)
loglik<-loglikCUBE(ordinal,m,param=c(pai,csi,phi))
#####
#### Log-likelihood of a CUBE model with covariate for feeling
data(relgoods)
m<-10
nacov<-which(is.na(relgoods$BirthYear))
naord<-which(is.na(relgoods$Tv))
na<-union(nacov,naord)
age<-2014-relgoods$BirthYear[-na]
lage<-log(age)-mean(log(age))
ordinal<-relgoods$Tv[-na]; W<-lage
pai<-0.63; gama<-c(-0.61,-0.31); phi<-0.16
param<-c(pai,gama,phi)
loglik<-loglikCUBE(ordinal,m,param,W=W)
##### Log-likelihood of a CUBE model with covariates for all parameters
Y<-W<-Z<-lage
bet<-c(0.18, 1.03); gama<-c(-0.6, -0.3); alpha<-c(-2.3,0.92)
param<-c(bet,gama,alpha)
loglik<-loglikCUBE(ordinal,m,param,Y=Y,W=W,Z=Z)
```

---

loglikCUSH

*Log-likelihood function for CUSH models*


---

**Description**

Compute the log-likelihood function for CUSH models with or without covariates to explain the shelter effect.

**Usage**

```
loglikCUSH(ordinal,m,param,shelter,X=0)
```

**Arguments**

ordinal	Vector of ordinal responses
m	Number of ordinal categories
param	Vector of parameters for the specified CUSH model
shelter	Category corresponding to the shelter choice
X	Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model)

**Details**

If no covariate is included in the model, then `param` is the estimate of the shelter parameter (`delta`), otherwise `param` has length equal to  $NCOL(X) + 1$  to account for an intercept term (first entry). No missing value should be present neither for ordinal nor for  $X$ .

**See Also**

[GEM](#), [logLik](#)

**Examples**

```
## Log-likelihood of CUSH model without covariates
n<-300
m<-7
shelter<-2; delta<-0.4
ordinal<-simcush(n,m,delta,shelter)
loglik<-loglikCUSH(ordinal,m,param=delta,shelter)
#####
## Log-likelihood of CUSH model with covariates
data(relgoods)
m<-10
naord<-which(is.na(relgoods$SocialNetwork))
nacov<-which(is.na(relgoods$Gender))
na<-union(nacov,naord)
ordinal<-relgoods$SocialNetwork[-na]; cov<-relgoods$Gender[-na]
omega<-c(-2.29, 0.62)
loglikcov<-loglikCUSH(ordinal,m,param=omega,shelter=1,X=cov)
```

---

loglikIHG

*Log-likelihood function for IHG models*


---

**Description**

Compute the log-likelihood function for IHG models with or without covariates to explain the preference parameter.

**Usage**

```
loglikIHG(ordinal,m,param,U=0)
```

**Arguments**

<code>ordinal</code>	Vector of ordinal responses
<code>m</code>	Number of ordinal categories
<code>param</code>	Vector of parameters for the specified IHG model
<code>U</code>	Matrix of selected covariates to explain the preference parameter (default: no covariate is included in the model)

**Details**

If no covariate is included in the model, then `param` is the estimate of the preference parameter (*theta*), otherwise `param` has length equal to `NCOL(U) + 1` to account for an intercept term (first entry). No missing value should be present neither for `ordinal` nor for `U`.

**See Also**

[GEM](#), [logLik](#)

**Examples**

```
#### Log-likelihood of an IHG model with no covariate
m<-10; theta<-0.14; n<-300
ordinal<-simihg(n,m,theta)
loglik<-loglikIHG(ordinal,m,param=theta)
#####
#### Log-likelihood of a IHG model with covariate
data(relgoods)
m<-10
naord<-which(is.na(relgoods$HandWork))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$HandWork[-na]; U<-relgoods$Gender[-na]
nu<-c(-1.55,-0.11) # first entry: intercept term
loglik<-loglikIHG(ordinal,m,param=nu,U=U); loglik
```

---

logscore

*Logarithmic score*

---

**Description**

Compute the logarithmic score of a CUB model with covariates both for the uncertainty and the feeling parameters.

**Usage**

```
logscore(m,ordinal,Y,W,bet,gama)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>ordinal</code>	Vector of ordinal responses
<code>Y</code>	Matrix of covariates for explaining the uncertainty component
<code>W</code>	Matrix of covariates for explaining the feeling component
<code>bet</code>	Vector of parameters for the uncertainty component, with length <code>NCOL(Y)+1</code> to account for an intercept term (first entry of <code>bet</code> )
<code>gama</code>	Vector of parameters for the feeling component, with length <code>NCOL(W)+1</code> to account for an intercept term (first entry of <code>gama</code> )

**Details**

No missing value should be present neither for ordinal nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

**References**

Tutz, G. (2012). *Regression for Categorical Data*, Cambridge University Press, Cambridge

**Examples**

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Walking))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$Smoking))
na<-union(naord,union(nacovpai,nacovcsi))
ordinal<-relgoods$Walking[-na]
Y<-relgoods$Gender[-na]
W<-relgoods$Smoking[-na]
bet<-c(-0.45,-0.48)
gama<-c(-0.55,-0.43)
logscore(m,ordinal,Y=Y,W=W,bet,gama)
```

---

makeplot

*Plot facilities for GEM objects*


---

**Description**

Plot facilities for objects of class "GEM".

**Usage**

```
makeplot(object)
```

**Arguments**

object            An object of class "GEM"

**Details**

Returns a plot comparing fitted probabilities and observed relative frequencies for GEM models without covariates. If only one explanatory dichotomous variable is included in the model for one or all components, then the function returns a plot comparing the distributions of the responses conditioned to the value of the covariate.

**See Also**

[cubvisual](#), [cubevisual](#), [cubshevisual](#), [multicub](#), [multicube](#)

---

 multicub

*Joint plot of estimated CUB models in the parameter space*


---

**Description**

Return a plot of estimated CUB models represented as points in the parameter space.

**Usage**

```
multicub(listord,mvett,csiplot=FALSE,paipplot=FALSE,...)
```

**Arguments**

listord	A data matrix, data frame, or list of vectors of ordinal observations (for variables with different number of observations)
mvett	Vector of number of categories for ordinal variables in listord (optional: if missing, the number of categories is retrieved from data: it is advisable to specify it in case some category has zero frequency)
csiplot	Logical: should $\xi$ or $1 - \xi$ be the $y$ coordinate
paipplot	Logical: should $\pi$ or $1 - \pi$ be the $x$ coordinate
...	Additional arguments to be passed to <a href="#">plot</a> , <a href="#">text</a> , and <a href="#">GEM</a>

**Value**

Fit a CUB model to list elements, and then by default it returns a plot of the estimated  $(1 - \pi, 1 - \xi)$  as points in the parameter space. Depending on csiplot and paipplot and on desired output,  $x$  and  $y$  coordinates may be set to  $\pi$  and  $\xi$ , respectively.

**Examples**

```
data(univer)
listord<-univer[,8:12]
multicub(listord,colours=rep("red",5),cex=c(0.4,0.6,0.8,1,1.2),
  pch=c(1,2,3,4,5),xlim=c(0,0.4),ylim=c(0.75,1),pos=c(1,3,3,3,3))
#####
m1<-5; m2<-7; m3<-9
pai<-0.7;csi<-0.6
n1<-1000; n2<-500; n3<-1500
ord1<-simcub(n1,m1,pai,csi)
ord2<-simcub(n2,m2,pai,csi)
ord3<-simcub(n3,m3,pai,csi)
listord<-list(ord1,ord2,ord3)
multicub(listord,labels=c("m=5","m=7","m=9"),pos=c(3,1,4))
```



---

multicube	<i>Joint plot of estimated CUBE models in the parameter space</i>
-----------	---

---

### Description

Return a plot of estimated CUBE models represented as points in the parameter space, where the overdispersion is labeled.

### Usage

```
multicube(listord,mvett,csiplot=FALSE,paipplot=FALSE,...)
```

### Arguments

listord	A data matrix, data frame, or list of vectors of ordinal observations (for variables with different number of observations)
mvett	Vector of number of categories for ordinal variables in listord (optional: if missing, the number of categories is retrieved from data: it is advisable to specify it in case some category has zero frequency)
csiplot	Logical: should $\xi$ or $1 - \xi$ be the $y$ coordinate
paipplot	Logical: should $\pi$ or $1 - \pi$ be the $x$ coordinate
...	Additional arguments to be passed to <a href="#">plot</a> , <a href="#">text</a> , and <a href="#">GEM</a>

### Value

Fit a CUBE model to list elements, and then by default it returns a plot of the estimated  $(1 - \pi, 1 - \xi)$  as points in the parameter space, labeled with the estimated overdispersion. Depending on csiplot and paipplot and on desired output,  $x$  and  $y$  coordinates may be set to  $\pi$  and  $\xi$ , respectively.

### Examples

```
m1<-5; m2<-7; m3<-9
pai<-0.7;csi<-0.6;phi=0.1
n1<-1000; n2<-500; n3<-1500
ord1<-simcube(n1,m1,pai,csi,phi)
ord2<-simcube(n2,m2,pai,csi,phi)
ord3<-simcube(n3,m3,pai,csi,phi)
listord<-list(ord1,ord2,ord3)
multicube(listord,labels=c("m=5","m=7","m=9"),pos=c(3,1,4),expinform=TRUE)
```

---

plotloglikihg	<i>Plot of the log-likelihood function of the IHG distribution</i>
---------------	--

---

**Description**

Plot the log-likelihood function of an IHG model fitted to a given absolute frequency distribution, over the whole support of the preference parameter. It returns also the ML estimate.

**Usage**

```
plotloglikihg(m, freq)
```

**Arguments**

m	Number of ordinal categories
freq	Vector of the absolute frequency distribution

**See Also**

[loglikIHG](#)

**Examples**

```
m<-7
freq<-c(828,275,202,178,143,110,101)
max<-plotloglikihg(m, freq)
```

---

print.GEM	<i>S3 method: print for class "GEM"</i>
-----------	---

---

**Description**

S3 method print for objects of class [GEM](#).

**Usage**

```
## S3 method for class 'GEM'
print(x, ...)
```

**Arguments**

x	An object of class <a href="#">GEM</a>
...	Other arguments

**Value**

Brief summary results of the fitting procedure, including parameter estimates, their standard errors and the executed call.

---

`probbit`*Probability distribution of a shifted Binomial random variable*

---

**Description**

Return the shifted Binomial probability distribution.

**Usage**

```
probbit(m,csi)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>csi</code>	Feeling parameter

**Value**

The vector of the probability distribution of a shifted Binomial model.

**See Also**

[bitcsi](#), [probcub00](#)

**Examples**

```
m<-7
csi<-0.7
pr<-probbit(m,csi)
plot(1:m,pr,type="h",main="Shifted Binomial probability distribution",xlab="Categories")
points(1:m,pr,pch=19)
```

---

`probcub00`*Probability distribution of a CUB model without covariates*

---

**Description**

Compute the probability distribution of a CUB model without covariates.

**Usage**

```
probcub00(m,pai,csi)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>pai</code>	Uncertainty parameter
<code>csi</code>	Feeling parameter

**Value**

The vector of the probability distribution of a CUB model.

**References**

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

**See Also**

[bitcsi](#), [probcub0q](#), [probcubp0](#), [probcubpq](#)

**Examples**

```
m<-9
pai<-0.3
csi<-0.8
pr<-probcub00(m,pai,csi)
plot(1:m,pr,type="h",main="CUB probability distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

<code>probcub0q</code>	<i>Probability distribution of a CUB model with covariates for the feeling component</i>
------------------------	--

---

**Description**

Compute the probability distribution of a CUB model with covariates for the feeling component.

**Usage**

```
probcub0q(m,ordinal,W,pai,gama)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>ordinal</code>	Vector of ordinal responses
<code>W</code>	Matrix of covariates for explaining the feeling component $NCOL(Y)+1$ to include an intercept term in the model (first entry)

pai	Uncertainty parameter
gama	Vector of parameters for the feeling component, whose length equals $NCOL(W)+1$ to include an intercept term in the model (first entry)

### Value

A vector of the same length as `ordinal`, whose  $i$ -th component is the probability of the  $i$ -th observation according to a CUB distribution with the corresponding values of the covariates for the feeling component and coefficients specified in `gama`.

### References

- Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78
- Piccolo D. and D’Elia A. (2008). A new approach for modelling consumers’ preferences, *Food Quality and Preference*, **18**, 247–259
- Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

### See Also

[bitgama](#), [probcub00](#), [probcubp0](#), [probcubpq](#)

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-relgoods$Gender[-na]
pai<-0.44; gama<-c(-0.91,-0.7)
pr<-probcub0q(m,ordinal,W,pai,gama)
```

---

probcube

*Probability distribution of a CUBE model without covariates*

---

### Description

Compute the probability distribution of a CUBE model without covariates.

### Usage

```
probcube(m,pai,csi,phi)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>pai</code>	Uncertainty parameter
<code>csi</code>	Feeling parameter
<code>phi</code>	Overdispersion parameter

**Value**

The vector of the probability distribution of a CUBE model without covariates.

**References**

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

**See Also**

[betar](#), [betabinomial](#)

**Examples**

```
m<-9
pai<-0.3
csi<-0.8
phi<-0.1
pr<-probcube(m,pai,csi,phi)
plot(1:m,pr,type="h", main="CUBE probability distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

probcubp0

*Probability distribution of a CUB model with covariates for the uncertainty component*

---

**Description**

Compute the probability distribution of a CUB model with covariates for the uncertainty component.

**Usage**

```
probcubp0(m,ordinal,Y,bet,csi)
```

**Arguments**

m	Number of ordinal categories
ordinal	Vector of ordinal responses
Y	Matrix of covariates for explaining the uncertainty component
bet	Vector of parameters for the uncertainty component, whose length equals NCOL(Y) + 1 to include an intercept term in the model (first entry)
csi	Feeling parameter

**Value**

A vector of the same length as `ordinal`, whose *i*-th component is the probability of the *i*-th observation according to a CUB model with the corresponding values of the covariates for the uncertainty component and coefficients for the covariates specified in `bet`.

**References**

- Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78
- Piccolo D. and D’Elia A. (2008). A new approach for modelling consumers’ preferences, *Food Quality and Preference*, **18**, 247–259
- Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

**See Also**

[bitgama](#), [probcub00](#), [probcubpq](#), [probcub0q](#)

**Examples**

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
Y<-relgoods$Gender[-na]
bet<-c(-0.81,0.93); csi<-0.20
probi<-probcubp0(m,ordinal,Y,bet,csi)
```

---

probcubpq	<i>Probability distribution of a CUB model with covariates for both feeling and uncertainty</i>
-----------	---

---

### Description

Compute the probability distribution of a CUB model with covariates for both the feeling and the uncertainty components.

### Usage

```
probcubpq(m,ordinal,Y,W,bet,gama)
```

### Arguments

m	Number of ordinal categories
ordinal	Vector of ordinal responses
Y	Matrix of covariates for explaining the uncertainty component
W	Matrix of covariates for explaining the feeling component
bet	Vector of parameters for the uncertainty component, whose length equals $\text{NCOL}(Y) + 1$ to include an intercept term in the model (first entry)
gama	Vector of parameters for the feeling component, whose length equals $\text{NCOL}(W)+1$ to include an intercept term in the model (first entry)

### Value

A vector of the same length as `ordinal`, whose  $i$ -th component is the probability of the  $i$ -th rating according to a CUB distribution with given covariates for both uncertainty and feeling, and specified coefficients vectors `bet` and `gama`, respectively.

### References

Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78

Piccolo D. and D’Elia A. (2008). A new approach for modelling consumers’ preferences, *Food Quality and Preference*, **18**, 247–259

Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

### See Also

[bitgama](#), [probcub00](#), [probcubp0](#), [probcub0q](#)



**Examples**

```

data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-Y<-relgoods$Gender[-na]
gama<-c(-0.91,-0.7); bet<-c(-0.81,0.93)
probi<-probcubpq(m,ordinal,Y,W,bet,gama)

```

probcubshe1

*probcubshe1***Description**

Probability distribution of an extended CUB model with a shelter effect.

**Usage**

```
probcubshe1(m,pai1,pai2,csi,shelter)
```

**Arguments**

m	Number of ordinal categories
pai1	Mixing coefficient for the shifted Binomial component of the mixture distribution
pai2	Mixing coefficient for the discrete Uniform component of the mixture distribution
csi	Feeling parameter
shelter	Category corresponding to the shelter choice

**Details**

An extended CUB model is a mixture of three components: a shifted Binomial distribution with probability of success  $\xi$ , a discrete uniform distribution with support  $\{1, \dots, m\}$ , and a degenerate distribution with unit mass at the shelter category (*shelter*).

**Value**

The vector of the probability distribution of an extended CUB model with a shelter effect at the shelter category

**References**

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

**See Also**

[probcubshe2](#), [probcubshe3](#)

**Examples**

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
pr<-probcubshe1(m,pai1,pai2,csi,shelter)
plot(1:m,pr,type="h",main="Extended CUB probability distribution with shelter effect",
xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

probcubshe2

*probcubshe2*

---

**Description**

Probability distribution of a CUB model with explicit shelter effect

**Usage**

```
probcubshe2(m,pai,csi,delta,shelter)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>pai</code>	Uncertainty parameter
<code>csi</code>	Feeling parameter
<code>delta</code>	Shelter parameter
<code>shelter</code>	Category corresponding to the shelter choice

**Details**

A CUB model with explicit shelter effect is a mixture of two components: a CUB distribution with uncertainty parameter  $\pi$  and feeling parameter  $\xi$ , and a degenerate distribution with unit mass at the shelter category (`shelter`) with mixing coefficient specified by  $\delta$ .

**Value**

The vector of the probability distribution of a CUB model with explicit shelter effect.

**References**

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

**See Also**

[probcubshe1](#), [probcubshe3](#)

**Examples**

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
delta<-1-pai1-pai2
pai<-pai1/(1-delta)
pr2<-probcubshe2(m,pai,csi,delta,shelter)
plot(1:m,pr2,type="h", main="CUB probability distribution with
explicit shelter effect",xlab="Ordinal categories")
points(1:m,pr2,pch=19)
```

---

probcubshe3

*probcubshe3*

---

**Description**

Probability distribution of a CUB model with explicit shelter effect: satisficing interpretation

**Usage**

```
probcubshe3(m,lambda,eta,csi,shelter)
```

**Arguments**

m	Number of ordinal categories
lambda	Mixing coefficient for the shifted Binomial component
eta	Mixing coefficient for the mixture of the uncertainty component and the shelter effect
csi	Feeling parameter
shelter	Category corresponding to the shelter choice

**Details**

The "satisficing interpretation" provides a parametrization for CUB models with explicit shelter effect as a mixture of two components: a shifted Binomial distribution with feeling parameter  $\xi$  (meditated choice), and a mixture of a degenerate distribution with unit mass at the shelter category (shelter) and a discrete uniform distribution over  $m$  categories, with mixing coefficient specified by  $\eta$  (lazy selection of a category).

**Value**

The vector of the probability distribution of a CUB model with shelter effect.

**References**

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

**See Also**

[probcubshe1](#), [probcubshe2](#)

**Examples**

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
lambda<-pai1
eta<-1-pai2/(1-pai1)
pr3<-probcubshe3(m,lambda,eta,csi,shelter)
plot(1:m,pr3,type="h",main="CUB probability distribution with explicit
shelter effect",xlab="Ordinal categories")
points(1:m,pr3,pch=19)
```

---

 probcush

*Probability distribution of a CUSH model*


---

**Description**

Compute the probability distribution of a CUSH model without covariates, that is a mixture of a degenerate random variable with mass at the shelter category and the Uniform distribution.

**Usage**

```
probcush(m,delta,shelter)
```

**Arguments**

m	Number of ordinal categories
delta	Shelter parameter
shelter	Category corresponding to the shelter choice

**Value**

The vector of the probability distribution of a CUSH model without covariates.

**References**

- Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
- Capecchi S. and Iannario M. (2016). Gini heterogeneity index for detecting uncertainty in ordinal data surveys, *Metron*, **74**(2), 223–232

**Examples**

```
m<-10
shelter<-1
delta<-0.4
pr<-probcush(m,delta,shelter)
plot(1:m,pr,type="h",xlab="Number of categories")
points(1:m,pr,pch=19)
```

---

 probgecub

---

*Probability distribution of a GeCUB model*


---

**Description**

Compute the probability distribution of a GeCUB model, that is a CUB model with shelter effect with covariates specified for all component.

**Usage**

```
probgecub(ordinal, Y, W, X, bet, gama, omega, shelter)
```

**Arguments**

ordinal	Vector of ordinal responses
Y	Matrix of covariates for explaining the uncertainty component
W	Matrix of covariates for explaining the feeling component
X	Matrix of covariates for explaining the shelter effect
bet	Vector of parameters for the uncertainty component, whose length equals NCOL(Y)+1 to include an intercept term in the model (first entry)

gama	Vector of parameters for the feeling component, whose length equals $\text{NCOL}(W)+1$ to include an intercept term in the model (first entry)
omega	Vector of parameters for the shelter effect, whose length equals $\text{NCOL}(X)+1$ to include an intercept term in the model (first entry)
shelter	Category corresponding to the shelter choice

**Value**

A vector of the same length as `ordinal`, whose  $i$ -th component is the probability of the  $i$ -th observation according to a GeCUB model with the corresponding values of the covariates for all the components and coefficients specified in `bet`, `gama`, `omega`.

**References**

Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

---

 probihg

---

*Probability distribution of an IHG model*


---

**Description**

Compute the probability distribution of an IHG model (Inverse Hypergeometric) without covariates.

**Usage**

```
probihg(m, theta)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>theta</code>	Preference parameter

**Value**

The vector of the probability distribution of an IHG model.

**References**

D’Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78

**Examples**

```

m<-10
theta<-0.30
pr<-probihg(m,theta)
plot(1:m,pr,type="h",xlab="Ordinal categories")
points(1:m,pr,pch=19)

```

---

probihgcovn

*Probability distribution of an IHG model with covariates*


---

**Description**

Given a vector of  $n$  ratings over  $m$  categories, it returns a vector of length  $n$  whose  $i$ -th element is the probability of observing the  $i$ -th rating for the corresponding IHG model with parameter  $\theta_i$ , obtained via logistic link with covariates and coefficients.

**Usage**

```
probihgcovn(m,ordinal,U,nu)
```

**Arguments**

<code>m</code>	Number of ordinal categories
<code>ordinal</code>	Vector of ordinal responses
<code>U</code>	Matrix of selected covariates for explaining the preference parameter
<code>nu</code>	Vector of coefficients for covariates, whose length equals $\text{NCOL}(U)+1$ to include an intercept term in the model (first entry)

**Details**

The matrix  $U$  is expanded with a vector with entries equal to 1 in the first column to include an intercept term in the model.

**See Also**

[probihg](#)

**Examples**

```

n<-100
m<-7
theta<-0.30
ordinal<-simihg(n,m,theta)
U<-sample(c(0,1),n,replace=TRUE)
nu<-c(0.12,-0.5)
pr<-probihgcovn(m,ordinal,U,nu)

```

relgoods

*Relational goods and Leisure time dataset***Description**

Dataset consists of the results of a survey aimed at measuring the evaluation of people living in the metropolitan area of Naples, Italy, with respect to of relational goods and leisure time collected in December 2014. Every participant was asked to assess on a 10 point ordinal scale his/her personal score for several relational goods (for instance, time dedicated to friends and family) and to leisure time. In addition, the survey asked respondents to self-evaluate their level of happiness by marking a sign along a horizontal line of 110 millimeters according to their feeling, with the left-most extremity standing for "extremely unhappy", and the right-most extremity corresponding to the status "extremely happy".

**Usage**

```
data(relgoods)
```

**Format**

The description of subjects' covariates is the following:

ID An identification number

Gender A factor with levels: 0 = man, 1 = woman

BirthMonth A variable indicating the month of birth of the respondent

BirthYear A variable indicating the year of birth of the respondent

Family A factor variable indicating the number of members of the family

Year.12 A factor with levels: 1 = if there is any child aged less than 12 in the family, 0 = otherwise

EducationDegree A factor with levels: 1 = compulsory school, 2 = high school diploma, 3 = Graduated-Bachelor degree, 4 = Graduated-Master degree, 5 = Post graduated

MaritalStatus A factor with levels: 1 = Unmarried, 2 = Married/Cohabitee, 3 = Separated/Divorced, 4 = Widower

Residence A factor with levels: 1 = City of Naples, 2 = District of Naples, 3 = Others Campania, 4 = Others Italia, 5 = Foreign countries

Glasses A factor with levels: 1 = wearing glasses or contact lenses, 0 = otherwise

RightHand A factor with levels: 1 = right-handed, 0 = left-handed

Smoking A factor with levels: 1 = smoker, 0 = not smoker

WalkAlone A factor with levels: 1 = usually walking alone, 0 = usually walking in company

job A factor with levels: 1 = Not working, 2 = Retired, 3 = occasionally, 4 = fixed-term job, 5 = permanent job

PlaySport A factor with levels: 1 = Not playing any sport, 2 = Yes, individual sport, 3 = Yes, team sport

Pets A factor with levels: 1 = owning a pet, 0 = not owning any pet



1) Respondents were asked to evaluate the following items on a 10 point Likert scale, ranging from 1 = "never, at all" to 10 = "always, a lot":

WalkOut How often the respondent goes out for a walk

Parents How often respondent talks at least to one of his/her parents

MeetRelatives How often respondent meets his/her relatives

Association Frequency of involvement in volunteering or different kinds of associations/parties, etc

RelFriends Quality of respondent's relationships with friends

RelNeighbours Quality of the relationships with neighbors

NeedHelp Easiness in asking help whenever in need

Environment Level of comfort with the surrounding environment

Safety Level of safety in the streets

EndofMonth Family making ends meet

MeetFriend Number of times the respondent met his/her friends during the month preceding the interview

Physician Importance of the kindness/sympathy in the selection of respondent's physician

Happiness Each respondent was asked to mark a sign on a 110mm horizontal line according to his/her feeling of happiness (left endpoint corresponding to completely unhappy, right-most endpoint corresponding to extremely happy)

2) The same respondents were asked to score the activities for leisure time listed below, according to their involvement/degree of amusement, on a 10 point Likert scale ranging from 1 = "At all, nothing, never" to 10 = "Totally, extremely important, always":

Videogames

Reading

Cinema

Drawing

Shopping

Writing

Bicycle

Tv

StayWFriend Spending time with friends

Groups Taking part to associations, meetings, etc.

Walking

HandWork Hobby, gardening, sewing, etc.

Internet

Sport

SocialNetwork

Gym

Quiz Crosswords, sudoku, etc.  
MusicInstr Playing a musical instrument  
GoAroundCar Hanging out by car  
Dog Walking out the dog  
GoOutEat Go to restaurants/pubs

### Details

Period of data collection: December 2014  
Mode of collection: questionnaire  
Number of observations: 2459  
Number of subjects' covariates: 16  
Number of analyzed items: 34  
Warning: with a limited number of missing values

### Source

<http://www.labstat.it/home/wp-content/uploads/2015/09/relgoods.txt>

---

simcub

*Simulation routine for CUB models*

---

### Description

Generate  $n$  pseudo-random observations following the given CUB distribution.

### Usage

```
simcub(n,m,pai,csi)
```

### Arguments

n	Number of simulated observations
m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter

### See Also

[probcub00](#)

**Examples**

```
n<-300
m<-9
pai<-0.4
csi<-0.7
simulation<-simcub(n,m,pai,csi)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

simcube

*Simulation routine for CUBE models*

---

**Description**

Generate  $n$  pseudo-random observations following the given CUBE distribution.

**Usage**

```
simcube(n,m,pai,csi,phi)
```

**Arguments**

n	Number of simulated observations
m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter
phi	Overdispersion parameter

**See Also**

[probcube](#)

**Examples**

```
n<-300
m<-9
pai<-0.7
csi<-0.4
phi<-0.1
simulation<-simcube(n,m,pai,csi,phi)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

simcubshe                      *Simulation routine for CUB models with shelter effect*

---

**Description**

Generate  $n$  pseudo-random observations following the given CUB distribution with shelter effect.

**Usage**

```
simcubshe(n,m,pai,csi,delta,shelter)
```

**Arguments**

n	Number of simulated observations
m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter
delta	Shelter parameter
shelter	Category corresponding to the shelter choice

**See Also**

[probcubshe1](#), [probcubshe2](#), [probcubshe3](#)

**Examples**

```
n<-300
m<-9
pai<-0.7
csi<-0.3
delta<-0.2
shelter<-3
simulation<-simcubshe(n,m,pai,csi,delta,shelter)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

simcush                      *Simulation routine for CUSH models*

---

**Description**

Generate  $n$  pseudo-random observations following the distribution of a CUSH model without co-variates.

**Usage**

```
simcush(n,m,delta,shelter)
```

**Arguments**

n	Number of simulated observations
m	Number of ordinal categories
delta	Shelter parameter
shelter	Category corresponding to the shelter choice

**See Also**

[probcush](#)

**Examples**

```
n<-200
m<-7
delta<-0.3
shelter<-3
simulation<-simcush(n,m,delta,shelter)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

simihg

*Simulation routine for IHG models*

---

**Description**

Generate  $n$  pseudo-random observations following the given IHG distribution.

**Usage**

```
simihg(n,m,theta)
```

**Arguments**

n	Number of simulated observations
m	Number of ordinal categories
theta	Preference parameter

**See Also**

[probihg](#)

**Examples**

```
n<-300
m<-9
theta<-0.4
simulation<-simihg(n,m,theta)
plot(table(simulation),xlab="Number of categories",ylab="Frequencies")
```

---

summary.GEM

*S3 method: summary for class "GEM"*


---

### Description

S3 method summary for objects of class `GEM`.

### Usage

```
## S3 method for class 'GEM'
summary(object, correlation = FALSE, ...)
```

### Arguments

object	An object of class <code>GEM</code>
correlation	Logical: should the estimated correlation matrix be returned? Default is FALSE
...	Other arguments

### Value

Extended summary results of the fitting procedure, including parameter estimates, their standard errors and Wald statistics, maximized log-likelihood compared with that of the saturated model and of a Uniform sample. AIC, BIC and ICOMP indeces are also displayed for model selection. Execution time and number of executed iterations for the fitting procedure are also returned.

### Examples

```
model<-GEM(Formula(MeetRelatives~0|0|0),family="cube",data=relgoods)
summary(model,correlation=TRUE,digits=4)
```

---

univer

*Evaluation of the Orientation Services 2002*


---

### Description

A sample survey on students evaluation of the Orientation services was conducted across the 13 Faculties of University of Naples Federico II in five waves: participants were asked to express their ratings on a 7 point scale (1 = "very unsatisfied", 7 = "extremely satisfied"). Here dataset collected during 2002 is loaded.

### Usage

```
data(univer)
```

**Format**

The description of subjects' covariates is:

Faculty A factor variable, with levels ranging from 1 to 13 indicating the coding for the different university faculties

Freqserv A factor with levels: 0 = for not regular users, 1 = for regular users

Age Variable indicating the age of the respondent in years

Gender A factor with levels: 0 = man, 1 = woman

Diploma A factor with levels: 1 = classic studies, 2 = scientific studies, 3 = linguistic, 4 = Professional, 5 = Technical/Accountancy, 6 = others

Residence A factor with levels: 1 = city NA, 2 = district NA, 3 = others

ChangeFa A factor with levels: 1 = changed faculty, 2 = not changed faculty

Analyzed ordinal variables (Likert ordinal scale): 1 = "extremely unsatisfied", 2 = "very unsatisfied", 3 = "unsatisfied", 4 = "indifferent", 5 = "satisfied", 6 = "very satisfied", 7 = "extremely satisfied"

Informat Level of satisfaction about the collected information

Willingn Level of satisfaction about the willingness of the staff

Officeho Judgment about the Office hours

Competen Judgement about the competence of the staff

Global Global satisfaction

**Details**

Period of data collection: 2002

Mode of collection: questionnaire

Number of observations: 2179

Number of subjects' covariates: 7

Number of analyzed items: 5

**Source**

<http://www.labstat.it/home/wp-content/uploads/2016/12/univer2002.txt>

---

varcub00

*Variance of CUB models without covariates*

---

**Description**

Compute the variance of a CUB model without covariates.

**Usage**

`varcub00(m, pai, csi)`

**Arguments**

m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter

**References**

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

**See Also**

[expcub00](#), [probcub00](#)

**Examples**

```
m<-9
pai<-0.6
csi<-0.5
varcub<-varcub00(m,pai,csi)
```

---

varcube

*Variance of CUBE models without covariates*


---

**Description**

Compute the variance of a CUBE model without covariates.

**Usage**

```
varcube(m,pai,csi,phi)
```

**Arguments**

m	Number of ordinal categories
pai	Uncertainty parameter
csi	Feeling parameter
phi	Overdispersion parameter

**References**

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

**See Also**

[probcube](#), [expcube](#)



**Examples**

```

m<-7
pai<-0.8
csi<-0.2
phi<-0.05
varianceCUBE<-varcube(m,pai,csi,phi)

```

varmatCUB

*Variance-covariance matrix for CUB models***Description**

Compute the variance-covariance matrix of parameter estimates for CUB models with or without covariates for the feeling and the uncertainty parameter, and for extended CUB models with shelter effect.

**Usage**

```
varmatCUB(ordinal,m,param,Y=0,W=0,X=0,shelter=0)
```

**Arguments**

ordinal	Vector of ordinal responses
m	Number of ordinal categories
param	Vector of parameters for the specified CUB model
Y	Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model)
W	Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model)
X	Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model)
shelter	Category corresponding to the shelter choice (default: no shelter effect is included in the model)

**Details**

The function checks if the variance-covariance matrix is positive-definite: if not, it returns a warning message and produces a matrix with NA entries. No missing value should be present neither for ordinal nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## References

- Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78
- Iannario, M. (2012). Modelling shelter choices in ordinal data surveys. *Statistical Modelling and Applications*, **21**, 1–22
- Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

## See Also

[vcov](#), [cormat](#)

## Examples

```
data(univer)
m<-7
### CUB model with no covariate
pai<-0.87; csi<-0.17
param<-c(pai,csi)
varmat<-varmatCUB(univer$global,m,param)
#####
### and with covariates for feeling
data(univer)
m<-7
pai<-0.86; gama<-c(-1.94,-0.17)
param<-c(pai,gama)
ordinal<-univer$willingn; W<-univer$gender
varmat<-varmatCUB(ordinal,m,param,W)
#####
### CUB model with uncertainty covariates
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
Y<-relgoods$Gender[-na]
bet<-c(-0.81,0.93); csi<-0.20
varmat<-varmatCUB(ordinal,m,param=c(bet,csi),Y=Y)
#####
### and with covariates for both parameters
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-Y<-relgoods$Gender[-na]
gama<-c(-0.91,-0.7); bet<-c(-0.81,0.93)
varmat<-varmatCUB(ordinal,m,param=c(bet,gama),Y=Y,W=W)
#####
```

```

### Variance-covariance for a CUB model with shelter
m<-8; n<-300
pai1<-0.5; pai2<-0.3; csi<-0.4
shelter<-6
pr<-probcubshe1(m,pai1,pai2,csi,shelter)
ordinal<-sample(1:m,n,prob=pr,replace=TRUE)
param<-c(pai1,pai2,csi)
varmat<-varmatCUB(ordinal,m,param,shelter=shelter)

```

---

varmatCUBE

*Variance-covariance matrix for CUBE models*


---

### Description

Compute the variance-covariance matrix of parameter estimates for CUBE models when no covariate is specified, or when covariates are included for all the three parameters.

### Usage

```
varmatCUBE(ordinal,m,param,Y=0,W=0,Z=0,expinform=FALSE)
```

### Arguments

ordinal	Vector of ordinal responses
m	Number of ordinal categories
param	Vector of parameters for the specified CUBE model
Y	Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model)
W	Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model)
Z	Matrix of selected covariates to explain the overdispersion component (default: no covariate is included in the model)
expinform	Logical: if TRUE and no covariate is included in the model, the function returns the expected variance-covariance matrix (default is FALSE: the function returns the observed variance-covariance matrix)

### Details

The function checks if the variance-covariance matrix is positive-definite: if not, it returns a warning message and produces a matrix with NA entries. No missing value should be present neither for ordinal nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## References

- Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
- Piccolo D. (2015). Inferential issues for CUBE models with covariates, *Communications in Statistics. Theory and Methods*, **44**(23), 771–786.

## See Also

[vcov](#), [cormat](#)

## Examples

```
m<-7; n<-500
pai<-0.83; csi<-0.19; phi<-0.045
ordinal<-simcube(n,m,pai,csi,phi)
param<-c(pai,csi,phi)
varmat<-varmatCUBE(ordinal,m,param)
#####
### Including covariates

data(relgoods)
m<-10
naord<-which(is.na(relgoods$Tv))
nacov<-which(is.na(relgoods$BirthYear))
na<-union(naord,nacov)
age<-2014-relgoods$BirthYear[-na]
lage<-log(age)-mean(log(age))
Y<-W<-Z<-lage
ordinal<-relgoods$Tv[-na]
estbet<-c(0.18,1.03); estgama<-c(-0.6,-0.3); estalpha<-c(-2.3,0.92)
param<-c(estbet,estgama,estalpha)
varmat<-varmatCUBE(ordinal,m,param,Y=Y,W=W,Z=Z,exinform=TRUE)
```

---

vcov.GEM

*S3 method vcov() for class "GEM"*

---

## Description

S3 method: vcov for objects of class [GEM](#).

## Usage

```
## S3 method for class 'GEM'
vcov(object, ...)
```

**Arguments**

object	An object of class <a href="#">GEM</a>
...	Other arguments

**Value**

Variance-covariance matrix of the final ML estimates for parameters of the fitted GEM model. It returns the square of the estimated standard error for CUSH and IHG models with no covariates.

**See Also**

[varmatCUB](#), [varmatCUBE](#), [GEM](#)

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