Package ‘FoReco’

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Description

An R package offering classical (bottom-up and top-down), and modern (optimal and heuristic combination) forecast reconciliation procedures for cross-sectional, temporal, and cross-temporal linearly constrained time series.

Details

The FoReco package is designed for forecast reconciliation, a post-forecasting process aimed to improve the accuracy of the base forecasts for a system of linearly constrained (e.g., hierarchical/grouped) time series. The main functions are:

- `htsrec()`: cross-sectional (contemporaneous) forecast reconciliation.
- `thfrec()`: forecast reconciliation for a single time series through temporal hierarchies.
- `lccrec()`: level conditional forecast reconciliation for genuine hierarchical/grouped time series.
- `tdrec()`: top-down (cross-sectional, temporal, cross-temporal) forecast reconciliation for genuine hierarchical/grouped time series.
- `ctbu()`: bottom-up cross-temporal forecast reconciliation.
- `tcsrec()`: heuristic first-temporal-then-cross-sectional cross-temporal forecast reconciliation.
- `cstrec()`: heuristic first-cross-sectional-then-temporal cross-temporal forecast reconciliation.
- `iterc()`: heuristic iterative cross-temporal forecast reconciliation.
- `octrec()`: optimal combination cross-temporal forecast reconciliation.

Author(s)

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References


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**agg_ts**

*Non-overlapping temporal aggregation of a time series*

**Description**

Non-overlapping temporal aggregation of a time series according to a specific aggregation order.

**Usage**

```r
agg_ts(agg_order, x, align = "end", rm_na = FALSE)
```

**Arguments**

- `agg_order`: Aggregation order to consider.
- `x`: Univariate time series: a vector or a `ts` object.
- `align`: Specifies whether the aggregates should be aligned with the start () or end of the series.
- `rm_na`: logical. Should missing values be removed?

**Value**

A vector or `ts` object

**See Also**

Other utilities: `Cmatrix()`, `FoReco2ts()`, `arrange_hres()`, `commat()`, `ctf_tools()`, `hts_tools()`, `lcmat()`, `oct_bounds()`, `residuals_matrix()`, `score_index()`, `shrink_estim()`, `thf_tools()`

**Examples**

```r
data(FoReco_data)
annual_ts <- agg_ts(12, FoReco_data$obs$k1[,1]) # == FoReco_data$obs$k12[,1]
```
arrange_hres
Re-arrange the multi-step residuals

Description
Re-arrange the multi-step residuals

Usage
arrange_hres(list_res)

Arguments
list_res a list of $H$ multi-step residuals. Each element of the list can be a vector (univariate time series) or a matrix (multivariate time series).

Details
Let $Z_t, t = 1, \ldots, T$, be a univariate time series. We can define the multi-step residuals such as

$$\hat{\epsilon}_{h,t} = Z_{t+h} - \hat{Z}_{t+h|t} \quad h \leq t \leq T - h$$

where $\hat{Z}_{t+h|t}$ is the $h$-step fitted value, calculated as the $h$-step ahead forecast given the time $t$.

Given the list of errors at different step ($[\hat{\epsilon}_{1,1}, \ldots, \hat{\epsilon}_{1,T}], \ldots, [\hat{\epsilon}_{H,1}, \ldots, \hat{\epsilon}_{H,T}]$) this function returns a $T$-vector with the residuals, organized in the following way:

$$[\hat{\epsilon}_{1,1} \hat{\epsilon}_{2,2} \ldots \hat{\epsilon}_{H,H} \hat{\epsilon}_{1,H+1} \ldots \hat{\epsilon}_{H,T-H}]'$$

Same idea can be apply for a multivariate time series.

Value
A vector or a matrix of multi-step residuals

See Also
Other utilities: Cmatrix(), FoReco2ts(), agg_ts(), commat(), ctf_tools(), hts_tools(), lcmat(), oct_bounds(), residuals_matrix(), score_index(), shrink_estim(), thf_tools()
Arguments

- **fit**: A list of \( n \) base forecast models. It is important to note that the models must have the `simulate()` function available and implemented as with the package `forecast`, with the following mandatory parameters: `object`, `innov`, `future`, and `nsim`.

- **boot_size**: The number of bootstrap replicates.

- **h**: Block size of the bootstrap, which is typically equivalent to the forecast horizon.

- **seed**: An integer seed.

Value

A list with two elements: the seed used to sample the errors and a 3-d array (\( \text{boot}\_\text{size} \times n \times h \))

References


See Also

Other bootstrap: `boot_ct()`, `boot_te()`

---

**boot_ct**

*Cross-temporal Joint Bootstrap*

Description

Joint block bootstrap for generating probabilistic base forecasts that take into account the correlation between different time series and temporal aggregation order at the same time (Girolimetto et al. 2023).

Usage

`boot_ct(fit, boot_size, m, h = 1, seed = NULL)`

Arguments

- **fit**: A list of \( n \) elements. Each element is a list with the \( (k^*+m) \) base forecast models ordered as `[lowest_freq' ... highest_freq']` of the cross-sectional series. It is important to note that the models must have the `simulate()` function available and implemented as with the package `forecast`, with the following mandatory parameters: `object`, `innov`, `future`, and `nsim`.

- **boot_size**: The number of bootstrap replicates.

- **m**: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \( m \)), or a subset of \( p \) factors of \( m \).

- **h**: Forecast horizon for the most temporally aggregated series.

- **seed**: An integer seed.
Value

A list with two elements: the seed used to sample the errors and a \((\text{boot\_size} \times h(k^* + m))\) matrix

References


See Also

Other bootstrap: `boot_cs()`, `boot_te()`

---

**boot_te**  
*Temporal Joint Bootstrap*

Description

Joint block bootstrap for generating probabilistic base forecasts that take into account the correlation between different temporal aggregation order (Girolimetto et al. 2023).

Usage

`boot_te(fit, boot_size, m, h = 1, seed = NULL)`

Arguments

- **fit**: A list of \((k^* + m)\) base forecast models ordered as \([\text{lowest\_freq} \ldots \text{highest\_freq}]\). It is important to note that the models must have the `simulate()` function available and implemented as with the package `forecast`, with the following mandatory parameters: `object`, `innov`, `future`, and `nsim`.
- **boot_size**: The number of bootstrap replicates.
- **m**: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).
- **h**: Forecast horizon for the most temporally aggregated series.
- **seed**: An integer seed.

Value

A list with two elements: the seed used to sample the errors and a \((\text{boot\_size} \times h(k^* + m))\) matrix

References


See Also

Other bootstrap: `boot_cs()`, `boot_ct()`
Cmatrix

Cross-sectional (contemporaneous) aggregation matrix

Description

This function allows the user to easily build the \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the \(n_b\) bottom level series into the \(n_a\) higher level ones. (Experimental version)

Usage

\[
\text{Cmatrix}(\text{formula, data, sep = "_", sparse = TRUE, top_label = "Total"})
\]

Arguments

- **formula**: Specification of the hierarchical structure: grouped hierarchies are specified using \(\sim g_1 \times g_2\) and nested hierarchies are specified using \(\sim \text{parent/child}\). Mixtures of the two formulations are also possible, like \(\sim g_1 \times (\text{grandparent/parent/child})\).
- **data**: A dataset in which each column contains the values of the variables in the formula and each row identifies a bottom level time series.
- **sep**: Character to separate the names of the aggregated series (default is "_").
- **sparse**: Option to return sparse matrix (default is TRUE).
- **top_label**: Label of the top level variable (default is "Total").

Value

A \((na \times nb)\) matrix.

See Also

Other utilities: FoReco2ts(), agg_ts(), arrange_hres(), commat(), ctf_tools(), hts_tools(), lcmat(), oct_bounds(), residuals_matrix(), score_index(), shrink_estim(), thf_tools()

Examples

```r
## Balanced hierarchy
# |--------|
# A B
# AA AB BA BB BC
# Names of the bottom level variables
data_bts <- data.frame(X1 = c("A", "A", "B", "B", "B"),
                       X2 = c("A", "B", "A", "B", "C"),
stringsAsFactors = FALSE)

# Cross-sectional aggregation matrix
C <- Cmatrix(~ X1 / X2, data_bts, sep = "")

## Unbalanced hierarchy (1)
# |--------|
# A B C
```

# |---| |--|--|
# AA AB BA BB BC
# Names of the bottom level variables
data_bts <- data.frame(X1 = c("A", "A", "B", "B", "B", "C"),
  X2 = c("A", "B", "A", "B", "C", NA),
  stringsAsFactors = FALSE)

## Unbalanced hierarchy (2)
## Cross-sectional aggregation matrix
C <- Cmatrix(~ X1 / X2, data_bts, sep = "")

## Grouped hierarchy
## Cross-sectional aggregation matrix
C <- Cmatrix(~ Y1 * (X1 / X2), data_bts, sep = "")

---

### commat

**Commutation matrix**

**Description**

This function returns the \((rc \times rc)\) commutation matrix \(P\) such that \(P \text{vec}(Y) = \text{vec}(Y')\),

where \(Y\) is a \((r \times c)\) matrix.

**Usage**

`commat(r, c)`
cstrec

Arguments

- `r` Number of rows of $Y$.
- `c` Number of columns of $Y$.

Value

A sparse $(rc \times rc)$ matrix, $P$.

References


See Also

Other utilities: `Cmatrix()`, `FoReco2ts()`, `agg_ts()`, `arrange_hres()`, `ctf_tools()`, `hts_tools()`, `lcmat()`, `oct_bounds()`, `residuals_matrix()`, `score_index()`, `shrink_estim()`, `thf_tools()`

Examples

```r
Y <- matrix(rnorm(30), 5, 6)
P <- commat(5, 6)
P %*% as.vector(Y) == as.vector(t(Y)) # check
```

cstrec

Heuristic first-cross-sectional-then-temporal cross-temporal forecast reconciliation

Description

Cross-temporal forecast reconciliation according to the heuristic procedure by Kourentzes and Athanasopoulos (2019), where the order of application of the two reconciliation steps (temporal-first-then-cross-sectional, as in the function `tcsrec()`), is inverted. The function `cstrec()` performs cross-sectional reconciliation (`htsrec()`), first, then temporal reconciliation (`thfrec()`), and finally applies the average of the projection matrices obtained in the second step to the one dimensional reconciled values obtained in the first step.

Usage

```r
cstrec(basef, hts_comb, thf_comb, res, ...)
```

Arguments

- `basef` $(n \times h(k^* + m))$ matrix of base forecasts to be reconciled, $\hat{Y}$; $n$ is the total number of variables, $m$ is the highest time frequency, $k^*$ is the sum of (a subset of) $(p-1)$ factors of $m$, excluding $m$, and $h$ is the forecast horizon for the lowest frequency time series. Each row identifies a time series, and the forecasts are ordered as `[lowest_freq' ... highest_freq']`.
- `hts_comb`, `thf_comb` Type of covariance matrix (respectively $(n \times n)$ and $(k^* + m) \times (k^* + m))$ to be used in the cross-sectional and temporal reconciliation. More details in `comb` param of `htsrec()` and `thfrec()`.
res \((n \times N(k^* + m))\) matrix containing the residuals at all the temporal frequencies ordered as \([\text{lowest}_\text{freq} \ldots \text{highest}_\text{freq}]\) (columns) for each variable (row), needed to estimate the covariance matrix when \(hts\_comb = \{"wls", \"shr\", \"sam\}" and/or \(hts\_comb = \{"wlsv", \"wlsh", \"acov", \"strar1", \"sar1", \"har1\", \"shr\", \"sam\}\). The rows must be in the same order as \(basef\).

... Any other options useful for \(htsrec()\) and \(thfrec()\), e.g. \(m\), \(C\) (or \(Ut\) and \(nb\)), \(nn\) (for non-negative reconciliation only at the first step), \(mse\), \(corpcor\), \(type\), \(sol\), \(settings\), \(W\), \(Omega\),...

Details

**Warning**, the two-step heuristic reconciliation allows considering non negativity constraints only in the first step. This means that non-negativity is not guaranteed in the final reconciled values.

Value

The function returns a list with two elements:

- \(\text{recf} \ (n \times h(k^* + m))\) reconciled forecasts matrix, \(\tilde{Y}\).
- \(M\) Matrix which transforms the uni-dimensional reconciled forecasts of step 1 (projection approach).

References


See Also

Other reconciliation procedures: \(ctbu()\), \(htsrec()\), \(iterec()\), \(lccrec()\), \(octrec()\), \(tcsrec()\), \(tdrec()\), \(thfrec()\)

Examples

data(FoReco_data)
obj <- cstrec(FoReco_data$base, m = 12, C = FoReco_data$C, 
hts_comb = "shr", thf_comb = "acov", res = FoReco_data$res)
Description

Cross temporal reconciled forecasts for all series at any temporal aggregation level are computed by appropriate summation of the high-frequency bottom base forecasts $\hat{b}_{i}$, $i = 1, ..., n_b$, according to a bottom-up procedure like what is currently done in both the cross-sectional and temporal frameworks.

Usage

ctbu(Bmat, m, C)

Arguments

Bmat \((n_b \times hm)\) matrix of high-frequency bottom time series base forecasts $(\hat{B}^{[1]})$. $h$ is the forecast horizon for the lowest frequency (most temporally aggregated) time series.

m Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, $m$), or a subset of the $p$ factors of $m$.

C \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level series into the higher level ones.

Details

Denoting by $\tilde{Y}$ the $(n \times h(k^{*} + m))$ matrix containing the bottom-up cross temporal reconciled forecasts, it is:

$$
\tilde{Y} = \begin{bmatrix}
    C\hat{B}^{[1]}K'_{1} \\
    \hat{B}^{[1]}K'_{1} & C\hat{B}^{[1]}
\end{bmatrix},
$$

where $C$ is the cross-sectional (contemporaneous) aggregation matrix, $K_1$ is the temporal aggregation matrix with $h = 1$, and $\hat{B}^{[1]}$ is the matrix containing the high-frequency bottom time series base forecasts. This expression is equivalent to $\text{vec}(\tilde{Y}') = \tilde{S}\text{vec}(\hat{Y}')$ for $h = 1$, where $\tilde{S}$ is the cross-temporal summing matrix for $\text{vec}(\hat{Y}')$, and $\hat{Y}$ is the $(n \times h(k^{*} + m))$ matrix containing all the base forecasts at any temporal aggregation order.

Value

The function returns a $(n \times h(k^{*} + m))$ matrix of bottom-up cross-temporally reconciled forecasts, $\tilde{Y}$.

References


See Also

Other reconciliation procedures: cstrec(), htsrec(), iterec(), lccrec(), octrec(), tcsrec(), tdrec(), thfrec()
Examples

data(FoReco_data)
# monthly base forecasts
hfbts <- t(FoReco2matrix(FoReco_data$base, m = 12)$k1[, -c(1:3), drop = FALSE])
obj <- ctbu(Bmat = hfbts, m = 12, C = FoReco_data$C)
rownames(obj) <- rownames(FoReco_data$base)

ctf_tools

Cross-temporal reconciliation tools

Description

Some useful tools for the cross-temporal forecast reconciliation of a linearly constrained (hierarchi-
cal/grouped) multiple time series.

Usage

ctf_tools(C, m, h = 1, Ut, nb, sparse = TRUE)

Arguments

C  \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level
series into the higher level ones.
m Highest available sampling frequency per seasonal cycle (max. order of tempo-
arial aggregation, \(m\)), or a subset of the \(p\) factors of \(m\).
h Forecast horizon for the lowest frequency (most temporally aggregated) time
series (default is 1).
Ut Zero constraints cross-sectional (contemporaneous) kernel matrix \((U'y = 0)\)
spanning the null space valid for the reconciled forecasts. It can be used instead
of parameter \(C\), but \(nb\) \((n = n_a + n_b)\) is needed if \(U' \neq \begin{bmatrix} I & -C \end{bmatrix}\). If the hierarchy
admits a structural representation, \(U'\) has dimension \((n_a \times n)\).
nb Number of bottom time series; if \(C\) is present, \(nb\) and \(Ut\) are not used.
sparse Option to return sparse object (default is \(\text{TRUE}\)).

Value

ctf list with:

Ht Full row-rank cross-temporal zero constraints (kernel) matrix coherent with \(y = \text{vec}(Y')\): \(H'y = 0\).
Hbrevet Complete, not full row-rank cross-temporal zero constraints (kernel) matrix co-
herent with \(y = \text{vec}(Y')\): \(\tilde{H}'y = 0\).
Hcheckt Full row-rank cross-temporal zero constraints (kernel) matrix coherent with \(\tilde{y}\)
(structural representation): \(\tilde{H}'\tilde{y} = 0\).
Ccheck Cross-temporal aggregation matrix \(\tilde{C}\) coherent with \(\tilde{y}\) (structural representa-
tion).
Scheck Cross-temporal summing matrix \(\tilde{S}\) coherent with \(\tilde{y}\) (structural representa-
tion).
Fmat Cross-temporal summing matrix \(\tilde{F}\) coherent with \(y = \text{vec}(Y')\).

hts list from hts_tools.

thf list from thf_tools.
FoReco-hts

See Also

Other utilities: Cmatrix(), FoReco2ts(), agg_ts(), arrange_hres(), commat(), hts_tools(), lcmat(), oct_bounds(), residuals_matrix(), score_index(), shrink_estim(), thf_tools()

Examples

# One level hierarchy (na = 1, nb = 2) with quarterly data
obj <- ctf_tools(C = matrix(c(1, 1), 1), m = 4)

---

FoReco-hts

Simple examples to compare FoReco and hts packages

Description

Two datasets of the hts package are used to show how to get the same results using FoReco. First, we consider the htseg1 dataset (a simulated three level hierarchy, with a total of 8 series, each of length 10). Then, we take the htseg2 dataset (a simulated four level hierarchy with a total of 17 series, each of length 16). htseg1 and htseg2 are objects of class hts in hts.

References


Examples

## Not run:
library(hts)
require(FoReco)

#### htseg1 ####
data <- allts(htseg1)
n <- NCOL(data)
nb <- NCOL(htseg1$bts)
a <- n-nb
C <- smatrix(htseg1)[1:na, ]

# List containing the base forecasts
# Forecast horizon: 10
base <- list()
for (i in 1:n) {
  base[[i]] <- forecast(auto.arima(data[, i]))
}

# Create the matrix of base forecasts
BASE <- NULL
for (i in 1:n) {
  BASE <- cbind(BASE, base[[i]]$mean)
}
colnames(BASE) <- colnames(data)

# Create the matrix of residuals
```r
res <- NULL
for (i in 1:n) {
    res <- cbind(res, base[[i]]$residuals)
}
colnames(res) <- colnames(data)

## Comparisons
# ols
# two commands in hts...
Y_hts_forecast <- forecast(htseg1, method = "comb", fmethod = "arima", weights = "ols")
Y_hts_ols <- combinef(BASE, nodes = get_nodes(htseg1), keep = "all")
# ...with the same results:
sum(abs(allts(Y_hts_forecast) - Y_hts_ols) > 1e-10)
Y_FoReco_ols <- htsrec(BASE, C = C, comb = "ols")$recf
sum(abs(Y_hts_ols - Y_FoReco_ols) > 1e-10)

# struc
w <- 1 / apply(smatrix(htseg1), 1, sum)
Y_hts_struc <- combinef(BASE, nodes = get_nodes(htseg1), weights = w, keep = "all")
Y_FoReco_struc <- htsrec(BASE, C = C, comb = "struc")$recf
sum(abs(Y_hts_struc - Y_FoReco_struc) > 1e-10)

# shr
Y_hts_shr <- MinT(BASE, nodes = get_nodes(htseg1), keep = "all",
                   covariance = "shr", residual = res)
Y_FoReco_shr <- htsrec(BASE, C = C, comb = "shr", res = res)$recf
sum(abs(Y_hts_shr - Y_FoReco_shr) > 1e-10)

# sam - hts error "MinT needs covariance matrix to be positive definite."
# The covariance matrix is ill-conditioned, hts considers it as non-invertible
Y_hts_sam <- MinT(BASE, nodes = get_nodes(htseg1), keep = "all",
                   covariance = "sam", residual = res)
Y_FoReco_sam <- htsrec(BASE, C = C, comb = "sam", res = res)$recf
sum(abs(Y_hts_sam - Y_FoReco_sam) > 1e-10)

####### htseg2 #######
data <- allts(htseg2)
n <- NCOL(data)
na <- n - NCOL(htseg2$bts)
C <- smatrix(htseg2)[1:na, ]

## Computation of the base forecasts
# using the auto.arima() function of the package forecast (loaded by hts)
# List containing the base forecasts
# Forecast horizon: 10
base <- list()
for (i in 1:n) {
    base[[i]] <- forecast(auto.arima(data[, i]))
}

# Create the matrix of base forecasts
BASE <- NULL
for (i in 1:n) {
    BASE <- cbind(BASE, base[[i]]$mean)
}
```
The dataset in the thief package is used to show how to get the same results with the FoReco package. In particular, we take the weekly data of Accident and Emergency demand in the UK, AEdemand, from 1 January 2011 to 31 December 2014.

References


Examples

```r
## Not run:
library(thief)
require(FoReco)
dataset <- window(AEdemand[, 12], start = c(2011, 1), end = c(2014, 52))
data <- tsaggregates(dataset)
# Base forecasts
base <- list()
for (i in 1:5) {
  base[[i]] <- forecast(auto.arima(data[[i]]))
}
```

base[[6]] <- forecast(auto.arima(data[[6]]), h = 2)
# Base forecasts vector
base_vec <- NULL
for (i in 6:1) {
    base_vec <- c(base_vec, base[[i]]$mean)
}
# Residual vector
res <- NULL
for (i in 6:1) {
    res <- c(res, base[[i]]$residuals)
}
# OLS
# two commands in thief...
obj_thief <- thief(dataset, m = 52, h = 2 * 52, comb = "ols", usemodel = "arima")
obj_thief <- tsaggregates(obj_thief$mean)
y_thief <- NULL
for (i in 6:1) {
    y_thief <- c(y_thief, obj_thief[[i]])
}  
obj_thief_ols <- reconcilethief(base, comb="ols")
y_thief_ols <- NULL
for (i in 6:1) {
    y_thief_ols <- c(y_thief_ols, obj_thief_ols[[i]]$mean)
}
# ...with the same results:
sum(abs(y_thief_ols - y_thief) > 1e-10)
y_FoReco_ols <- thfrec(base_vec, 52, comb = "ols")$recf
sum(abs(y_FoReco_ols - y_thief_ols) > 1e-10)
# STRUC
obj_thief_struc <- reconcilethief(base, comb="struc")
y_thief_struc <- NULL
for (i in 6:1) {
    y_thief_struc <- c(y_thief_struc, obj_thief_struc[[i]]$mean)
}  
y_FoReco_struc <- thfrec(base_vec, 52, comb = "struc")$recf
sum(abs(y_FoReco_struc - y_thief_struc) > 1e-10)
# BU
obj_thief_bu <- reconcilethief(base, comb="bu")
y_thief_bu <- NULL
for (i in 6:1) {
    y_thief_bu <- c(y_thief_bu, obj_thief_bu[[i]]$mean)
}  
y_FoReco_bu <- thfrec(base_vec, 52, comb = "bu")$recf
sum(abs(y_FoReco_bu - y_thief_bu) > 1e-10)
# SHR
obj_thief_shr <- reconcilethief(base, comb="shr")
y_thief_shr <- NULL
for (i in 6:1) {
    y_thief_shr <- c(y_thief_shr, obj_thief_shr[[i]]$mean)
}  
y_FoReco_shr <- thfrec(base_vec, 52, comb = "shr", res = res)$recf
sum(abs(y_FoReco_shr - y_thief_shr) > 1e-10)
FoReco2ts

## End(Not run)

<table>
<thead>
<tr>
<th>FoReco2ts</th>
<th>Reconciled forecasts matrix/vector to time-series class</th>
</tr>
</thead>
</table>

### Description

Function to transform the matrix/vector of FoReco forecasts input and output into a list of time series/matrix/vector objects.

### Usage

- `FoReco2ts(recf, m, ...)`
- `FoReco2matrix(recf, m)`

### Arguments

- `recf` \((h(k^* + m) \times 1)\) forecasts vector from `thfrec`, \((h \times n)\) forecasts matrix from `htsrec` or \((n \times h(k^* + m))\) forecasts matrix from `octrec`, `tcsrec`, `cstrec`, `iterec`, `ctbu`.
- `m` Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).
- `...` optional arguments to `ts` (i.e. starting date); frequency is required only for the cross-sectional case.

### Value

A list of class "ts" objects for `FoReco2ts` and a list of matrix/vector for `FoReco2matrix`

### See Also

Other utilities: `Cmatrix()`, `agg_ts()`, `arrange_hres()`, `commat()`, `ctf_tools()`, `hts_tools()`, `lcmat()`, `oct_bounds()`, `residuals_matrix()`, `score_index()`, `shrink_estim()`, `thf_tools()`

### Examples

```r
# Load data
data(FoReco_data)

# Cross-temporal framework
oct_recf <- octrec(FoReco_data$base, m = 12, C = FoReco_data$C,
                   comb = "bdshr", res = FoReco_data$res)$recf
ts_oct <- FoReco2ts(recf = oct_recf, m = 12, start = c(15, 1))
mat_oct <- FoReco2matrix(recf = oct_recf, m = 12)

# Cross-sectional framework
# monthly base forecasts
mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
# monthly residuals
mres <- FoReco2matrix(FoReco_data$res, m = 12)$k1
hts_recf <- htsrec(mbase, C = FoReco_data$C, comb = "shr", res = mres)$recf
ts_hts <- FoReco2ts(recf = hts_recf, start = c(15, 1), frequency = 12)
```
FoReco_data <- FoReco2matrix(recf = hts_recf)

# Temporal framework
# Top ts base forecasts ([lowest_freq ... highest_freq'])
topbase <- FoReco_data$base[, ]
# Top ts residuals ([lowest_freq' ... highest_freq'])
topres <- FoReco_data$res[,]

thf_recf <- thfrec(topbase, m = 12, comb = "acov", res = topres)$recf
ts_thf <- FoReco2ts(recf = thf_recf, m = 12, start = c(15, 1))
mat_thf <- FoReco2matrix(recf = thf_recf, m = 12)

FoReco_data

Forecast reconciliation for a simulated linearly constrained, genuine hierarchical multiple time series

Description
A two-level hierarchy with \( n = 8 \) monthly time series. In the cross-sectional framework, at any
time it is \( \text{Tot} = A + B + C, A = AA + AB \) and \( B = BA + BB \) (the bottom time series being
\( AA, AB, BA, BB \), and \( C \), it is \( n_b = 5 \)). The monthly observations are aggregated to their annual
\( (k = 12) \), semi-annual \( (k = 6) \), four-monthly \( (k = 4) \), quarterly \( (k = 3) \), and bi-monthly \( (k = 2) \)
counterparts. The monthly bottom time series are simulated from five different SARIMA models
(see Using the 'FoReco' package). There are 180 (15 years) monthly observations: the first 168
values (14 years) are used as training set, and the last 12 form the test set.

Usage
data(FoReco_data)

Format
An object of class "list":

- **base** \((8 \times 28)\) matrix of base forecasts. Each row identifies a time series and the forecasts are
  ordered as [lowest_freq’ ... highest_freq’].
- **test** \((8 \times 28)\) matrix of test set. Each row identifies a time series and the observed values are ordered
  as [lowest_freq’ ... highest_freq’].
- **res** \((8 \times 392)\) matrix of in-sample residuals. Each row identifies a time series and the in-sample
  residuals are ordered as [lowest_freq’ ... highest_freq’].
- **C** \((3 \times 5)\) cross-sectional (contemporaneous) aggregation matrix.
- **obs** List of the observations at any levels and temporal frequencies.

Examples
data(FoReco_data)

# Cross-sectional reconciliation for all temporal aggregation levels
# (annual, ..., bi-monthly, monthly)
K <- c(12, 6, 4, 3, 2, 1)
mbase <- FoReco2matrix(FoReco_data$base, m = 12)
mres <- FoReco2matrix(FoReco_data$res, m = 12)
hts_recf <- lapply(K, function(k){
htsrec(mbase[[paste0("k", k)]]), C = FoReco_data$C, comb = "shr",
    res = mres[[paste0("k", k)]], keep = "recf")
})
names(hts_recf) <- paste("k", K, sep="")

# Forecast reconciliation through temporal hierarchies for all time series
# comb = "acov"

n <- NROW(FoReco_data$base)
thf_recf <- matrix(NA, n, NCOL(FoReco_data$base))
dimnames(thf_recf) <- dimnames(FoReco_data$base)
for(i in 1:n){
    # ts base forecasts ([lowest_freq/quotesingle.Var... highest_freq/quotesingle.Var])
    tsbase <- FoReco_data$base[i, ]
    # ts residuals ([lowest_freq/quotesingle.Var... highest_freq/quotesingle.Var])
    tsres <- FoReco_data$res[i, ]
    thf_recf[i,] <- thfrec(tsbase, m = 12, comb = "acov",
                          res = tsres, keep = "recf")
}

# Iterative cross-temporal reconciliation
# Each iteration: t-acov + cs-shr
ite_recf <- iterec(FoReco_data$base, note=FALSE,
               m = 12, C = FoReco_data$C,
               thf_comb = "acov", hts_comb = "shr",
               res = FoReco_data$res, start_rec = "thf")$recf

# Heuristic first-cross-sectional-then-temporal cross-temporal reconciliation
# cs-shr + t-acov
cst_recf <- cstrec(FoReco_data$base, m = 12, C = FoReco_data$C,
               thf_comb = "acov", hts_comb = "shr",
               res = FoReco_data$res)$recf

# Heuristic first-temporal-then-cross-sectional cross-temporal reconciliation
# t-acov + cs-shr
tcs_recf <- tcsrec(FoReco_data$base, m = 12, C = FoReco_data$C,
               thf_comb = "acov", hts_comb = "shr",
               res = FoReco_data$res)$recf

# Optimal cross-temporal reconciliation
# comb = "bdshr"
oct_recf <- octrec(FoReco_data$base, m = 12, C = FoReco_data$C,
               comb = "bdshr", res = FoReco_data$res, keep = "recf")

---

**htsrec**

*Cross-sectional (contemporaneous) forecast reconciliation*

**Description**

Cross-sectional (contemporaneous) forecast reconciliation of a linearly constrained (e.g., hierarchi-
cal/grouped) multiple time series. The reconciled forecasts are calculated either through a projection
approach (Byron, 1978, see also van Erven and Cugliari, 2015, and Wickramasuriya et al., 2019),
or the equivalent structural approach by Hyndman et al. (2011). Moreover, the classic bottom-up
approach is available.
Usage

htsrec(basef, comb, C, res, Ut, nb, mse = TRUE, corpcor = FALSE, type = "M", sol = "direct", keep = "list", v = NULL, nn = FALSE, nn_type = "osqp", settings = list(), bounds = NULL, W = NULL)

Arguments

basef \((h \times n)\) matrix of base forecasts to be reconciled; \(h\) is the forecast horizon and \(n\) is the total number of time series.

comb Type of the reconciliation. Except for Bottom-up, each option corresponds to a specific \((n \times n)\) covariance matrix:

- bu (Bottom-up);
- ols (Identity);
- struc (Structural variances);
- wls (Series variances) - uses res;
- shr (Shrunk covariance matrix - MinT-shr) - uses res;
- sam (Sample covariance matrix - MinT-sam) - uses res;
- w use your personal matrix \(W\) in param \(W\).

C \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level series into the higher level ones.

res \((N \times n)\) in-sample residuals matrix needed when \(\text{comb} = \{\text{wls", "shr", "sam"}\}\). The columns must be in the same order as \text{basef}.

Ut Zero constraints cross-sectional (contemporaneous) kernel matrix \((U'y = 0)\) spanning the null space valid for the reconciled forecasts. It can be used instead of parameter \(C\), but \(nb (n = n_a + n_b)\) is needed if \(U' \neq [I - C]\). If the hierarchy admits a structural representation, \(U'\) has dimension \((n_a \times n)\).

nb Number of bottom time series; if \(C\) is present, \(nb\) and \(Ut\) are not used.

mse Logical value: TRUE (default) calculates the covariance matrix of the in-sample residuals (when necessary) according to the original \text{hts} and \text{thief} formulation: no mean correction, \(T\) as denominator.

corpcor Logical value: TRUE if \text{corpcor} (Schäfer et al., 2017) must be used to shrink the sample covariance matrix according to Schäfer and Strimmer (2005), otherwise the function uses the same implementation as package \text{hts}.

type Approach used to compute the reconciled forecasts: "M" for the projection approach with matrix \(M\) (default), or "S" for the structural approach with summing matrix \(S\).

sol Solution technique for the reconciliation problem: either "direct" (default) for the closed-form matrix solution, or "osqp" for the numerical solution (solving a linearly constrained quadratic program using \text{solve_osqp}).

keep Return a list object of the reconciled forecasts at all levels (if keep = "list") or only the reconciled forecasts matrix (if keep = "recf").

v vector index of the fixed base forecast \((\min(v) > 0 \text{ and } \max(v) < n)\).

nn Logical value: TRUE if non-negative reconciled forecasts are wished.

nn_type "osqp" (default), "KAnn" (only type == "M") or "sntz".

settings Settings for \text{osqp} (object \text{osqpSettings}). The default options are: verbose = FALSE, eps_abs = 1e-5, eps_rel = 1e-5, polish_refine_iter = 100 and polish = TRUE. For details, see the \text{osqp} documentation (Stellato et al., 2019).
bounds  
(n × 2) matrix containing the cross-sectional bounds: the first column is the lower bound, and the second column is the upper bound.

\( W \)
This option permits to directly enter the covariance matrix:
1. \( W \) must be a p.d. \((n \times n)\) matrix or a list of \( h \) matrix (one for each forecast horizon);
2. if \( \text{comb} \) is different from "w", \( W \) is not used.

Details
Let \( y \) be a \((n \times 1)\) vector of target point forecasts which are wished to satisfy the system of linearly independent constraints
\[
U' y = 0_{(r \times 1)},
\]
where \( U' \) is a \((r \times n)\) matrix, with \( \text{rank}(U') = r \leq n \), and \( 0_{(r \times 1)} \) is a \((r \times 1)\) null vector. Let \( \hat{y} \) be a \((n \times 1)\) vector of unbiased point forecasts, not fulfilling the linear constraints (i.e., \( U' \hat{y} \neq 0 \)).

We consider a regression-based reconciliation method assuming that \( \hat{y} \) is related to \( y \) by
\[
\hat{y} = y + \varepsilon,
\]
where \( \varepsilon \) is a \((n \times 1)\) vector of zero mean disturbances, with known p.d. covariance matrix \( W \).

The reconciled forecasts \( \tilde{y} \) are found by minimizing the generalized least squares (GLS) objective function
\[
(\hat{y} - y)' W^{-1} (\hat{y} - y), \quad \text{s.t.} \, U' y = 0.
\]
The solution is given by
\[
\tilde{y} = \hat{y} - W U (U' W U)^{-1} U' \hat{y} = M \hat{y},
\]
where \( M = I_n - W U (U' W U)^{-1} U' \) is a \((n \times n)\) projection matrix. This solution is used by \texttt{htsrec} when \( \text{type} = "M" \).

Denoting with \( d_{\hat{y}} = 0 - U' \hat{y} \) the \((r \times 1)\) vector containing the coherency errors of the base forecasts, we can re-state the solution as
\[
\tilde{y} = \hat{y} + W U (U' W U)^{-1} d_{\hat{y}},
\]
which makes it clear that the reconciliation formula simply adjusts the vector \( \hat{y} \) with a linear combination – according to the smoothing matrix \( L = W U (U' W U)^{-1} \) – of the coherency errors of the base forecasts.

In addition, if the error term \( \varepsilon \) is gaussian, the reconciliation error \( \tilde{\varepsilon} = \tilde{y} - y \) is a zero-mean gaussian vector with covariance matrix
\[
E (\tilde{y} - y) (\tilde{y} - y)' = W - W U (U' W U)^{-1} U' = M W.
\]

Hyndman et al. (2011, see also Wickramasuriya et al., 2019) propose an equivalent, alternative formulation as for the reconciled estimates, obtained by GLS estimation of the model
\[
\hat{y} = S \beta + \varepsilon,
\]
where \( S \) is the \textit{structural summation matrix} describing the aggregation relationships operating on \( y \), and \( \beta \) is a subset of \( y \) containing the target forecasts of the bottom level series, such that \( y = S \beta \).

Since the hypotheses on \( \varepsilon \) remain unchanged,
\[
\tilde{\beta} = (S' W^{-1} S)^{-1} S' W^{-1} \hat{y}.
\]
is the best linear unbiased estimate of $\beta$, and the whole reconciled forecasts vector is given by
\[
\tilde{\mathbf{y}} = \mathbf{S}\tilde{\beta} = \mathbf{SG}\hat{\mathbf{y}} .
\]
where $\mathbf{G} = (\mathbf{S}'\mathbf{W}^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{W}^{-1}$, and $\mathbf{M} = \mathbf{SG}$. This solution is used by htsrec when type = "S".

Bounds on the reconciled forecasts

The user may impose bounds on the reconciled forecasts. The parameter bounds permits to consider lower (a) and upper (b) bounds like $a \leq \tilde{y} \leq b$ such that:
\[
a_1 \leq \tilde{y}_1 \leq b_1 \\
\vdots \\
a_n \leq \tilde{y}_n \leq b_n
\Rightarrow \text{bounds} = [\mathbf{a} \mathbf{b}] = \begin{bmatrix} a_1 & b_1 \\ \vdots & \vdots \\ a_n & b_n \end{bmatrix} ,
\]
where $a_i \in [-\infty, +\infty]$ and $b_i \in [-\infty, +\infty]$. If $y_i$ is unbounded, the i-th row of bounds would be equal to $c(-\text{Inf}, +\text{Inf})$. Notice that if the bounds parameter is used, sol = "osqp" must be used. This is not true in the case of non-negativity constraints:

- sol = "direct": first the base forecasts are reconciled without non-negativity constraints, then, if negative reconciled values are present, the "osqp" solver is used;
- sol = "osqp": the base forecasts are reconciled using the "osqp" solver.

In this case it is not necessary to build a matrix containing the bounds, and it is sufficient to set nn = "TRUE".

Non-negative reconciled forecasts may be obtained by setting nn_type alternatively as:

- nn_type = "sntz" ("set-negative-to-zero")
- nn_type = "osqp" (Stellato et al., 2020)

Value

If the parameter keep is equal to "recf", then the function returns only the $(h \times n)$ reconciled forecasts matrix, otherwise (keep="all") it returns a list that mainly depends on what type of representation (type) and solution technique (sol) have been used:

- ref = $(h \times n)$ reconciled forecasts matrix, $\tilde{\mathbf{Y}}$.
- $\mathbf{W}$ Covariance matrix used for forecast reconciliation, $\mathbf{W}$.
- nn_check Number of negative values (if zero, there are no values below zero).
- rec_check Logical value: rec_check = TRUE when the constraints have been fulfilled.
- varf (type="direct") $(n \times 1)$ reconciled forecasts variance vector for $h = 1$, diag(MW).
- M (type="direct") Projection matrix, $\mathbf{M}$ (projection approach).
- G (type="S" and type="direct") Projection matrix, $\mathbf{G}$ (structural approach, $\mathbf{M} = \mathbf{SG}$).
- S (type="S" and type="direct") Cross-sectional summing matrix, $\mathbf{S}$.
- info (type="osqp") matrix with information in columns for each forecast horizon $h$ (rows): run time (run_time), number of iteration (iter), norm of primal residual (pri_res), status of osqp’s solution (status) and polish’s status (status_polish). It will also be returned with nn = TRUE if a solver (see nn_type) will be used.

Only if comb = "bu", the function returns ref, S and M.
References


See Also

Other reconciliation procedures: *cstrec()*,*ctbu()*, *iterec()*,*lccrec()*, *octrec()*,*tcsrec()*, *tdrec()*, *thfrec()*

Examples

data(FoReco_data)
# monthly base forecasts
mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
# monthly residuals
mres <- FoReco2matrix(FoReco_data$res, m = 12)$k1
obj <- htsrec(mbase, C = FoReco_data$C, comb = “shr”, res = mres)

# FoReco is able to work also with covariance matrix that are not equal
# across all the forecast horizon. For example, we can consider the
# normalized squared differences (see Di Fonzo and Marini, 2011) where
# Wh = diag(|yh|):
Wh <- lapply(split(mbase, row(mbase)), function(x) diag(abs(x)))
# Now we can introduce the list of the covariance matrix in htsrec through
# the parameter "W" and setting comb = "w".
objh <- htsrec(mbase, C = FoReco_data$C, W = Wh, comb = "w")

hts_tools
Cross-sectional reconciliation tools

Description
Some useful tools for the cross-sectional forecast reconciliation of a linearly constrained (e.g.,
hierarchical/grouped) multiple time series.

Usage
hts_tools(C, h = 1, Ut, nb, sparse = TRUE)

Arguments
- **C**: \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level
  series into the higher level ones.
- **h**: Forecast horizon (default is 1).
- **Ut**: Zero constraints cross-sectional (contemporaneous) kernel matrix \((U'y = 0)\)
  spanning the null space valid for the reconciled forecasts. It can be used instead
  of parameter \(C\), but \(nb\) is needed if \(U' \neq [I - C]\). If the hierarchy admits a
  structural representation, \(U'\) has dimension \((n_a \times n)\).
- **nb**: Number of bottom time series; if \(C\) is present, \(nb\) and \(Ut\) are not used.
- **sparse**: Option to return sparse matrices (default is TRUE).

Value
A list of five elements:
- **C**: \((n \times n_b)\) cross-sectional (contemporaneous) aggregation matrix.
- **S**: \((n \times n_b)\) cross-sectional (contemporaneous) summing matrix, \(S = [C \ I_{n_b}]\).
- **Ut**: \((n_a \times n)\) zero constraints cross-sectional (contemporaneous) kernel matrix. If
  the hierarchy admits a structural representation \(U' = [I - C]\.
- **n**: Number of variables \(n_a + n_b\).
- **na**: Number of upper level variables.
- **nb**: Number of bottom level variables.

See Also
Other utilities: Cmatrix(), FoReco2ts(), agg_ts(), arrange_hres(), commat(), ctf_tools(),
lcmat(), oct_bounds(), residuals_matrix(), score_index(), shrink_estim(), thf_tools()

Examples
# One level hierarchy (na = 1, nb = 2)
obj <- hts_tools(C = matrix(c(1, 1), 1), sparse = FALSE)
Iterative heuristic cross-temporal forecast reconciliation

Description

Iterative procedure which produces cross-temporally reconciled forecasts by alternating forecast reconciliation along one single dimension (either cross-sectional or temporal) at each iteration step. Each iteration consists in the first two steps of the heuristic procedure by Kourentzes and Athanasopoulos (2019), so the forecasts are reconciled by alternating cross-sectional (contemporaneous) reconciliation, and reconciliation through temporal hierarchies in a cyclic fashion. The choice of the dimension along which the first reconciliation step in each iteration is performed is up to the user (param start_rec), and there is no particular reason why one should perform the temporal reconciliation first, and the cross-sectional reconciliation then. The iterative procedure allows the user to get non-negative reconciled forecasts.

Usage

iterrec(basef, thf_comb, hts_comb, res, itmax = 100, tol = 1e-5, start_rec = "thf", norm = "inf", note = TRUE, plot = "mti", ...)

Arguments

basef \((n \times h(k^* + m))\) matrix of base forecasts to be reconciled, \(\hat{Y}\): \(n\) is the total number of variables, \(m\) is the highest time frequency, \(k^*\) is the sum of (a subset of) \((p - 1)\) factors of \(m\), excluding \(m\), and \(h\) is the forecast horizon for the lowest frequency time series. Each row identifies a time series, and the forecasts are ordered as \([\text{lowest_freq' ... highest_freq'}]\).

hts_comb, thf_comb
Type of covariance matrix (respectively \((n \times n)\) and \(((k^* + m) \times (k^* + m))\)) to be used in the cross-sectional and temporal reconciliation, see more in comb param of htsrec() and thfrec().

res \((n \times N(k^* + m))\) matrix containing the residuals at all the temporal frequencies ordered \([\text{lowest_freq'} ... \text{highest_freq'}]\) (columns) for each variable (row), needed to estimate the covariance matrix when hts_comb = \{"wls", "shr", "sam"\} and/or hts_comb = \{"wlsv", "wlsh", "acov", "strar1", "sar1", "har1", "shr", "sam"\}. The row must be in the same order as basef.

itmax Max number of iteration (100, default) (old version maxit).

tol Convergence tolerance (1e-5, default).

start_rec Dimension along with the first reconciliation step in each iteration is performed: it start from temporal reconciliation with "thf" (default), from cross-sectional with "hts" and it does both reconciliation with "auto".

norm Norm used to calculate the temporal and the cross-sectional incoherence. There are two alternatives: "inf" (max \(|x_1|, |x_2|, \ldots, \text{default}\)) or "one" (\(\sum |x_i|\)).

note If note = TRUE (default) the function writes some notes to the console, otherwise no note is produced (also no plot).

plot Some useful plots: "mti" (default) marginal trend inconsistencies, "pat" step by step inconsistency pattern for each iteration, "distf" distance forecasts iteration i and i-1, "all" all the plots.
... any other options useful for \texttt{htsrec()} and \texttt{thfrec()}, e.g. m, C (or \texttt{Ut} and \texttt{nb}), nn (for non negativity reconciliation only at first step), mse, corpca, type, sol, settings, W, Omega,...

**Details**

This reconciliation procedure can be seen as an extension of the well known iterative proportional fitting procedure (Deming and Stephan, 1940, Johnston and Pattie, 1993), also known as RAS method (Miller and Blair, 2009), to adjust the internal cell values of a two-dimensional matrix until they sum to some predetermined row and column totals. In that case the adjustment follows a proportional adjustment scheme, whereas in the cross-temporal reconciliation framework each adjustment step is made according to the penalty function associated to the single-dimension reconciliation procedure adopted.

Control status of iterative reconciliation:

-2 Temporal/Cross-sectional reconciliation does not work.
-1 Convergence not achieved (maximum iteration limit reached).
 0 Convergence achieved.
+1 Convergence achieved: incoherence has increased in the next iteration (at least one time).
+2 Convergence achieved: incoherence has increased in the next two or more iteration (at least one time).
+3 The forecasts are already reconciled.

**Value**

\texttt{iterrec} returns a list with:

- \texttt{recf} \((n \times h(k^* + m))\) reconciled forecasts matrix, \(\tilde{Y}\).
- \texttt{d_cs} Cross-sectional incoherence at each iteration.
- \texttt{d_te} Temporal incoherence at each iteration.
- \texttt{start_rec} Starting coherence dimension (thf or hts).
- \texttt{tol} Tolerance.
- \texttt{flag} Control code (see details).
- \texttt{time} Elapsed time.
- \texttt{dist} If \texttt{start_rec} = "auto", matrix of distances of the forecasts reconciled from the base.

**References**


lccrec


See Also

Other reconciliation procedures: cstrec(), ctbu(), htsrec(), lccrec(), octrec(), tcsrec(), tdrec(), thfreq()

Examples

data(FoReco_data)
obj <- iterec(FoReco_data$base, note = FALSE,
m = 12, C = FoReco_data$C, thf_comb = "acov",
hts_comb = "shr", res = FoReco_data$res, start_rec = "thf")

### lccrec

**Level conditional coherent forecast reconciliation for genuine hierarchical/grouped time series**

**Description**

Forecast reconciliation procedure built on and extending the original proposal by Hollyman et al. (2021). Level conditional coherent reconciled forecasts may be computed in cross-sectional, temporal, and cross-temporal frameworks. The reconciled forecasts are conditional to (i.e., constrained by) the base forecasts of a specific upper level of the hierarchy (exogenous constraints). The linear constraints linking the variables may be dealt with endogenously as well (Di Fonzo and Girolimetto, 2022). *Combined Conditional Coherent* (CCC) forecasts may be calculated as simple averages of LCC and bottom-up reconciled forecasts, with either endogenous or exogenous constraints.

**Usage**

lccrec(basef, m, C, nl, weights, bnaive = NULL, const = "exogenous",
CCC = TRUE, nn = FALSE, nn_type = "osqp", settings = list(), ...)

**Arguments**

- **basef**: matrix/vector of base forecasts to be reconciled: \((h \times n)\) matrix in the cross-sectional framework; \((h(k^* + m) \times 1)\) vector in the temporal framework; \((n \times h(k^* + m))\) matrix in the cross-temporal framework. \(n\) is the total number of variables, \(m\) is the highest time frequency, \(k^*\) is the sum of (a subset of) \((p - 1)\) factors of \(m\), excluding \(m\), and \(h\) is the forecast horizon.
m Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of the \(p\) factors of \(m\).

\(m\)

\(C\) \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level series into the higher level ones (or a list of matrices forming \(C = [C_1' \ C_2' \ ... \ C_L']\)), \(1, ..., L\) being the number of the cross-sectional upper levels.

\(C\)

\(n1\) \((L \times 1)\) vector containing the number of time series in each level of the hierarchy \((n1[1] = 1)\).

\(n1\)

\(weights\) covariance matrix or a vector (weights used in the reconciliation: either \((n_b \times 1)\) for exogenous or \((n \times 1)\) for endogenous constraints).

\(weights\)

\(bnaive\) matrix/vector of naive bts base forecasts (e.g., seasonal averages, as in Hollyman et al., 2021): \((h \times n_b)\) matrix in the cross-sectional framework; \((hm \times 1)\) vector in the temporal framework; \((n_b \times hm)\) matrix in the cross-temporal framework. Ignore it, if only basef are to be used as base forecasts.

\(bnaive\)

\(const\) exogenous (default) or endogenous constraints

\(const\)

\(CCC\) Option to return Combined Conditional Coherent reconciled forecasts (default is TRUE).

\(CCC\)

\(nn\) Logical value: TRUE if non-negative reconciled forecasts are wished.

\(nn\)

\(nn_type\) Non-negative method: "osqp" (default) or "sntz" (set-negative-to-zero, only if CCC = TRUE) with exogenous constraints (const = "exo"); "osqp" (default), "KAnn" (only type == "M") or "sntz" with endogenous constraints (const = "endo").

\(nn_type\)

\(settings\) Settings for osqp (object osqpSettings). The default options are: verbose = FALSE, eps_abs = 1e-5, eps_rel = 1e-5, polish_refine_iter = 100 and polish = TRUE. For details, see the osqp documentation (Stellato et al., 2019).

\(settings\)

\(...\) Additional functional arguments passed to htsrec of FoReco.

\(...\)

**Details**

**Cross-sectional hierarchies**

To be as simple as possible, we fix the forecast horizon equal to 1. Let the base forecasts be a vector \(\hat{y} = [\hat{a}' \ \hat{b}']\), where vector \(\hat{a}\) consists of the sub-vectors forming each of the upper \(L\) levels of the hierarchy/grouping:

\[
\hat{a} = \begin{bmatrix}
\hat{a}_1 \\
\hat{a}_2 \\
\vdots \\
\hat{a}_L
\end{bmatrix},
\]

where \(\hat{a}_l, l = 1, \ldots, L\), has dimension \((n_l \times 1)\) and \(\sum_{l=1}^L n_l = n_a\). Denote \(C_l\) the \((n_l \times n_b)\) matrix mapping the bts into the level-\(l\) uts, then the aggregation matrix \(C\) may be written as

\[
C = \begin{bmatrix}
C_1 \\
C_2 \\
\vdots \\
C_L
\end{bmatrix},
\]

where the generic matrix \(C_L\) is \((n_L \times n_b)\), \(l = 1, \ldots, L\). Given a generic level \(l\), the reconciled forecasts coherent with the base forecasts of that level are the solution to a quadratic minimization
problem with linear exogenous constraints (const = "exo"): 
\[
\tilde{b}_l = \arg\min_b \left( b - \hat{b} \right)' W_b^{-1} \left( b - \hat{b} \right) \quad \text{s.t.} \quad C_l b = \hat{\alpha}_l, \quad l = 1, \ldots, L,
\]
\[
\tilde{b}_l = \tilde{b} + W_b C_l' (C_l W_b C_l')^{-1} \left( \hat{\alpha}_l - C_l \hat{\tilde{b}} \right), \quad l = 1, \ldots, L,
\]
where \( W_b \) is a \((n_b \times n_b)\) p.d. matrix (in Hollyman et al., 2021, \( W_b \) is diagonal). If endogenous constraints (const = "endo") are considered, denote 
\[
\tilde{y}_l = \arg\min_{y_l} \left( y_l - \hat{y}_l \right)' W_l^{-1} \left( y_l - \hat{y}_l \right) \quad \text{s.t.} \quad U_l y_l = 0, \quad l = 1, \ldots, L,
\]
\[
\tilde{y}_l = \left( I_{n_b + n_t} - W_l U_l (U_l' W_l U_l)^{-1} U_l' \right) \hat{y}_l, \quad l = 1, \ldots, L,
\]
where \( W_l \) is a \((n_l + n_b \times n_l + n_b)\) p.d. matrix. Combined Conditional Coherent (CCC) forecasts are obtained by taking the simple average of the \( L \) level conditional, and of the bottom-up reconciled forecasts, respectively (Di Fonzo and Girolimetto, 2022):
\[
\tilde{y}_{\text{CCC}} = \frac{1}{L+1} \sum_{l=1}^{L+1} S \tilde{b}_l,
\]
with
\[
\tilde{b}_{L+1} = \hat{b}.
\]
Non-negative reconciled forecasts may be obtained by setting \( \text{nn_type} \) alternatively as:
- \( \text{nn_type} = "\text{osqp}" \) to apply non-negative constraints to each level:
- \( \text{nn_type} = "\text{sntz}" \) ("set-negative-to-zero") to apply non-negative constraints only to the CCC forecasts:

Temporal hierarchies

The extension to the case of temporal hierarchies is quite simple. Using the same notation as in \( \text{thfrec}() \), the following ‘equivalences’ hold between the symbols used for the level conditional cross-sectional reconciliation and the ones used in temporal reconciliation:
\[
L \equiv p - 1, \quad (n_a, n_b, n) \equiv (k^*, m, k^* + m),
\]
and
\[
C \equiv K, \quad C_1 \equiv K_1 = I_m, \quad C_2 \equiv K_2 = I_{m \times m}, \quad \ldots, \quad C_L \equiv K_{p-1} = I_{m \times 1} \otimes I_{k^*}, \quad S \equiv R.
\]
The description of the cross-temporal extension is currently under progress.

Value

The function returns the level reconciled forecasts in case of an elementary hierarchy with one level. Otherwise it returns a list with

- \( \text{recf} \): Level Conditional Coherent (CCC = FALSE) forecasts matrix/vector calculated as simple averages of upper level conditional reconciled forecasts, with either endogenous or exogenous constraints. If CCC = TRUE then it is the Combined Conditional Coherent matrix/vector, as weighted averages of LCC and bottom-up reconciled forecasts.
- \( \text{levrecf} \): list of level conditional reconciled forecasts (+ BU).
- \( \text{info} \) (type="osqp"): matrix with some useful indicators (columns) for each forecast horizon \( h \) (rows): run time (run_time), number of iteration, norm of primal residual (pri_res), status of osqp’s solution (status) and polish’s status (status_polish).
References


See Also

Other reconciliation procedures: `cstrec()`, `ctbu()`, `htsrec()`, `iterec()`, `octrec()`, `tcsrec()`, `tdrec()`, `thfrec()`

Examples

```r
data(FoReco_data)
### LCC and CCC CROSS-SECTIONAL FORECAST RECONCILIATION
# Cross sectional aggregation matrix
C <- rbind(FoReco_data$C, c(0,0,0,1))
# monthly base forecasts
mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
mbase <- mbase[, c("T", "A","B","C","AA","AB","BA","BB","C")]
# residuals
mres <- FoReco2matrix(FoReco_data$res, m = 12)$k1
mres <- mres[, c("T", "A","B","C","AA","AB","BA","BB","C")]
# covariance matrix of all the base forecasts, computed using the in-sample residuals
Wres <- cov(mres)
# covariance matrix of the bts base forecasts, computed using the in-sample residuals
Wb <- Wres[cbind("AA","AB", "BA", "BB", "C"),
           c("AA","AB", "BA", "BB", "C")]
# bts seasonal averages
obs_1 <- FoReco_data$obs$k1
bts_sm <- apply(obs_1, 2, function(x) tapply(x[1:168],rep(1:12, 14), mean))
bts_sm <- bts_sm[,c("AA", "AB", "BA", "BB", "C")]
## EXOGENOUS CONSTRAINTS AND DIAGONAL COVARIANCE MATRIX (Hollyman et al., 2021)
# Forecast reconciliation for an elementary hierarchy:
# 1 top-level series + 5 bottom-level series (Level 2 base forecasts are not considered).
# The input is given by the base forecasts of the top and bottom levels series,
# along with a vector of positive weights for the bts base forecasts
exo_EHd <- lccrec(basef = mbase[, c("T","AA","AB","BA","BB","C")],
                   weights = diag(Wb), bnaive = bts_sm)
# Level conditional reconciled forecasts
# recf/L1: Level 1 reconciled forecasts for the whole hierarchy
# L2: Middle-Out reconciled forecasts hinging on Level 2 conditional reconciled forecasts
# L3: Bottom-Up reconciled forecasts
exo_LCd <- lccrec(basef = mbase, C = C, nl = c(1,3), weights = diag(Wb),
                   weights = diag(Wb), bnaive = bts_sm)
# Combined Conditional Coherent (CCC) reconciled forecasts
# recf: CCC reconciled forecasts matrix
# L1: Level 1 conditional reconciled forecasts for the whole hierarchy
# L2: Middle-Out reconciled forecasts hinging on Level 2 conditional reconciled forecasts
# L3: Bottom-Up reconciled forecasts
```
```r
exo_CCCd <- lccrec(basef = mbase, C = C, nl = c(1,3), weights = diag(Wb))

## EXOGENOUS CONSTRAINTS AND FULL COVARIANCE MATRIX
# Simply substitute weights=diag(Wb) with weights=Wb
exo_EHF <- lccrec(basef = mbase[, c("T","AA","AB","BA","BB","C")], weights = Wb, bnaive = bts_sm)
exo_LCf <- lccrec(basef = mbase, C = C, nl = c(1,3), weights = Wb, CCC = FALSE, bnaive = bts_sm)
exo_CCCf <- lccrec(basef = mbase, C = C, nl = c(1,3), weights = Wb, bnaive = bts_sm)

## ENDOGENOUS CONSTRAINTS AND DIAGONAL COVARIANCE MATRIX
# parameters of function htsrec(), like "type" and "nn_type" may be used
W1 <- Wres[, c("T","AA","AB","BA","BB","C")]
endo_EHd <- lccrec(basef = mbase[, c("T","AA","AB","BA","BB","C")],
                  weights = diag(W1), const = "endo", CCC = FALSE)
endo_LCd <- lccrec(basef = mbase, C = C, nl = c(1,3), weights = diag(Wres),
                  const = "endo", CCC = FALSE)
endo_CCCd <- lccrec(basef = mbase, C = C, nl = c(1,3),
                    weights = diag(Wres), const = "endo")

## ENDOGENOUS CONSTRAINTS AND FULL COVARIANCE MATRIX
# Simply substitute weights=diag(Wres) with weights=Wres
endo_EHF <- lccrec(basef = mbase[, c("T","AA","AB","BA","BB","C")],
                   weights = W1, const = "endo")
exo_LCf <- lccrec(basef = mbase, C = C, nl = c(1,3),
                 weights = Wres, const = "endo", CCC = FALSE)
exo_CCCf <- lccrec(basef = mbase-40, C = C, nl = c(1,3),
                   weights = Wres, const = "endo")

## LCC and CCC TEMPORAL FORECAST RECONCILIATION
# top ts base forecasts ([lowest_freq ... highest_freq'])
topbase <- FoReco_data$base[,1,]
# top ts residuals ([lowest_freq ... highest_freq'])
topres <- FoReco_data$res[,1,]
Om_bt <- cov(matrix(topres[which(simplify2array(strsplit(names(topres), "_"))[1,]=="k1")], ncol = 12, byrow = TRUE))
t_exo_LCd <- lccrec(basef = topbase, m = 12, weights = diag(Om_bt), CCC = FALSE)
t_exo_CCCd <- lccrec(basef = topbase, m = 12, weights = diag(Om_bt))

## LCC and CCC CROSS-TEMPORAL FORECAST RECONCILIATION
idr <- which(simplify2array(strsplit(colnames(FoReco_data$res), "_"))[1,]=="k1")
bres <- do.call(cbind, lapply(1:5, function(x) matrix(bres[x,], nrow=14, byrow = TRUE)))
cbase <- FoReco_data$base[c("T", "A","B","C","AA","AB","BA","BB","C")]
c_exo_LCf <- lccrec(basef = cbase, m = 12, C = C, nl = c(1,3),
                    weights = diag(cov(bres)), CCC = FALSE)
c_exo_CCCf <- lccrec(basef = cbase, m = 12, C = C, nl = c(1,3),
                    weights = diag(cov(bres)), CCC = TRUE)
```
Description

When working with a general linearly constrained multiple \((n\text{-variate})\) time series \((x_t)\), getting a linear combination matrix \(\bar{C}\) is a critical step to obtain a structural-like representation such that, for \(t = 1, \ldots, T\),

\[
\bar{U}' = [I - \bar{C}] \Rightarrow y_t = P x_t = \begin{bmatrix} v_t \\ f_t \end{bmatrix} = \begin{bmatrix} \bar{C} \\ I \end{bmatrix} f_t = \bar{S} f_t,
\]

where \(\bar{U}'\) is the \((n_v \times n)\) full rank zero constraints matrix, \(\bar{S}\) is the \((n \times n_f)\) matrix analogous of the summing matrix \(S\) for a genuine hierarchical/grouped times series, \(\bar{C}\) is the \((n_v \times n_f)\) linear combination matrix such that \(v_t = \bar{C} f_t\), \(v_t\) is the \((n_v \times 1)\) vector of 'basic' variables, and \(f_t\) is the \((n_f \times 1)\) vector of 'free' variables (Di Fonzo and Girolimetto, 2022).

Usage

\texttt{lcmat(Gt, alg = "rref", tol = sqrt(.Machine$double.eps), verbose = FALSE, sparse = TRUE)}

Arguments

- \(Gt\) \((r \times n)\) coefficient matrix \((\Gamma')\) for a general linearly constrained multiple time series \((x_t)\) such that \(\Gamma' x_t = 0_{(r \times 1)}\).
- \(alg\) Technique used to transform \(\Gamma'\) in \(U' = [I - \bar{C}]\), such that \(U' y_t = 0_{(n_v \times 1)}\). Use "rref" for the Row Reduced Echelon Form through Gauss-Jordan elimination \((default)\), or "qr" for the (pivoting) QR decomposition (Strang, 2019).
- \(tol\) Tolerance for the "rref" or "qr" algorithm.
- \(verbose\) If TRUE, intermediate steps are printed \((default\ is \ FALSE)\).
- \(sparse\) Option to return a sparse \(\bar{C}\) matrix \((default\ is \ TRUE)\).

Details

Looking for an analogous of the summing matrix \(S\), say \(S = \begin{bmatrix} \bar{C} \\ I \end{bmatrix}\), the \texttt{lcmat} function transforms \(\Gamma'\) into \(U' = [I - \bar{C}]\), such that \(U' y_t = 0_{(n_v \times 1)}\). Consider the simple example of a linearly constrained multiple time series consisting of two hierarchies, each with distinct bottom time series, with a common top-level series \((X)\):

1) \(X = C + D\),
2) \(X = A + B\),
3) \(A = A1 + A2\).

The coefficient matrix \(\Gamma'\) of the linear system \(\Gamma' x_t = 0\) \((x_t = [X C D A B A1 A2])\) is

\[
\Gamma' = \begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & -1
\end{bmatrix}.
\]
The `lcmat` function returns

\[
\hat{C} = \begin{bmatrix}
0 & 1 & 1 & 1 \\
-1 & 1 & 1 & 1 \\
0 & 0 & -1 & -1
\end{bmatrix}.
\]

Then

\[
\hat{U}' = \begin{bmatrix}
1 & 0 & 0 & 0 & -1 & -1 & -1 \\
0 & 1 & 0 & 1 & -1 & -1 & -1 \\
0 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}, \quad \text{with} \quad \hat{U}'y_t = \hat{U}' \begin{bmatrix} v_t \\ f_t \end{bmatrix} = 0,
\]

where \( v_t = [X \ C \ A] \), and \( f_t = [D \ B \ A_1 \ A_2] \).

**Value**

A list with

- `Cbar` \((n_v \times n_f)\) linear combination matrix \( \hat{C} \)
- `pivot` \((n \times 1)\) vector of the column permutations s.t. \( P = I[\text{pivot}] \)

**References**


**See Also**

Other utilities: `Cmatrix()`, `FoReco2ts()`, `agg_ts()`, `arrange_hres()`, `commat()`, `ctf_tools()`, `hts_tools()`, `oct_bounds()`, `residuals_matrix()`, `score_index()`, `shrink_estim()`, `thf_tools()`

**Examples**

```r
Gt <- matrix(c(1,-1,-1,0,0,0,0, 1,0,0,-1,-1,0,0, 0,0,0,1,0,-1,-1), nrow = 3, byrow = TRUE)
Cbar <- lcmat(Gt = Gt)$Cbar
P <- diag(1, NCOL(Gt))[,lcmat(Gt = Gt)$pivot]
```

**octrec**

*Optimal combination cross-temporal forecast reconciliation*

**Description**

Optimal (in least squares sense) combination cross-temporal forecast reconciliation. The reconciled forecasts are calculated either through a projection approach (Byron, 1978), or the equivalent structural approach by Hyndman et al. (2011).

**Usage**

```r
octrec(basef, m, C, comb, res, Ut, nb, mse = TRUE, corpcor = FALSE, type = "M", sol = "direct", keep = "list", v = NULL, nn = FALSE, nn_type = "osqp", settings = list(), bounds = NULL, W = NULL, Omega = NULL)
```
Arguments

**basef**  
\((n \times h(k^* + m))\) matrix of base forecasts to be reconciled, \(\hat{Y}\); \(n\) is the total number of variables, \(m\) is the highest time frequency, \(k^*\) is the sum of (a subset of) \((p - 1)\) factors of \(m\), excluding \(m\), and \(h\) is the forecast horizon for the lowest frequency time series. Each row identifies a time series, and the forecasts are ordered as \([\text{lowest_freq'} ... \text{highest_freq'}]\).

**m**  
Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).

**C**  
\((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level series into the higher level ones.

**comb**  
Type of the reconciliation. It corresponds to a specific \((n(k^* + m) \times n(k^* + m))\) covariance matrix (Di Fonzo and Girolimetto 2023, Girolimetto et al. 2023), where \(k^*\) is the sum of (a subset of) \((p - 1)\) factors of \(m\) (\(m\) is not considered) and \(n\) is the number of variables:

- **ols** (Identity);
- **struc** (Cross-temporal structural variances);
- **cs_struc** (Cross-sectional structural variances and temporal independence);
- **t_struc** (Cross-sectional independence and temporal structural variances);
- **wls** (Hierarchy variances);
- **wlsv** (Series variances);
- **bds** (Shrunk cross-covariance matrix, cross-sectional framework);
- **bdsm** (Sample cross-covariance matrix, cross-sectional framework);
- **acov** (Series auto-covariance matrix);
- **Sshr** (Series shrunk cross-covariance matrix);
- **Ssam** (Series cross-covariance matrix);
- **shr** (Shrunk cross-covariance matrix);
- **sam** (Sample cross-covariance matrix);
- **hbshr** (Shrunk high frequency bottom time series cross-covariance matrix);
- **hbasm** (Sample high frequency bottom time series cross-covariance matrix);
- **bshr** (Shrunk bottom time series cross-covariance matrix);
- **bsam** (Sample bottom time series cross-covariance matrix);
- **hs** (Shrunk high frequency cross-covariance matrix);
- **hsam** (Sample high frequency cross-covariance matrix);
- **w** use your personal matrix \(W\) in param \(W\);
- **omega** use your personal matrix \(\Omega\) in param \(\Omega\).

**res**  
\((n \times N(k^* + m))\) matrix containing the residuals at all the frequency levels \([\text{lowest_freq'} ... \text{highest_freq'}]\) (columns) for each variable (row), needed to estimate the covariance matrix.

**Ut**  
Zero constraints cross-sectional (contemporaneous) kernel matrix \((U'y = 0)\) spanning the null space valid for the reconciled forecasts. It can be used instead of parameter \(C\), but \(nb\) \((n = n_a + n_b)\) is needed if \(U' \neq [I - C]\). If the hierarchy admits a structural representation, \(U'\) has dimension \((n_a \times n)\).

**nb**  
Number of bottom time series; if \(C\) is present, \(nb\) and \(Ut\) are not used.

**mse**  
Logical value: **TRUE (default)** calculates the covariance matrix of the in-sample residuals (when necessary) according to the original **hts** and **thief** formulation: no mean correction, \(T\) as denominator.
corpcor Logical value: TRUE if corpcor (Schäfer et al., 2017) must be used to shrink the sample covariance matrix according to Schäfer and Strimmer (2005), otherwise the function uses the same implementation as package hts.

type Approach used to compute the reconciled forecasts: "M" for the projection approach with matrix M (default), or "S" for the structural approach with summing matrix S.

sol Solution technique for the reconciliation problem: either "direct" (default) for the closed-form matrix solution, or "osqp" for the numerical solution (solving a linearly constrained quadratic program using solve_osqp).

keep Return a list object of the reconciled forecasts at all levels (if keep = "list") or only the reconciled forecasts matrix (if keep = "recf").

v vector index of the fixed base forecast (min(v) > 0 and max(v) < n(k* + m)).

nn Logical value: TRUE if non-negative reconciled forecasts are wished.

nn_type "osqp" (default), "KAnn" (only type == "M") or "sntz".

settings Settings for osqp (object osqpSettings). The default options are: verbose = FALSE, eps_abs = 1e-5, eps_rel = 1e-5, polish_refine_iter = 100 and polish = TRUE. For details, see the osqp documentation (Stellato et al., 2019).

bounds (n(k* + m) × 2) matrix of the bounds on the variables: the first column is the lower bound, and the second column is the upper bound.

W, Omega This option permits to directly enter the covariance matrix:
1. W must be a p.d. (n(k* + m) × n(k* + m)) matrix or a list of h matrix (one for each forecast horizon);
2. Omega must be a p.d. (n(k* + m) × n(k* + m)) matrix or a list of h matrix (one for each forecast horizon);
3. if comb is different from "w" or "omega", W or Omega is not used.

Details

Considering contemporaneous and temporal dimensions in the same framework requires to extend and adapt the notations used in htsrec and ththree. To do that, we define the matrix containing the base forecasts at any considered temporal frequency as

$$\hat{Y}_{n \times h(k^* + m)} = \begin{bmatrix} \hat{A}^{[m]} & \hat{A}^{[k_{p-1}]} & \cdots & \hat{A}^{[k]} & \hat{A}^{[1]} \\ \hat{B}^{[m]} & \hat{B}^{[k_{p-1}]} & \cdots & \hat{B}^{[k]} & \hat{B}^{[1]} \end{bmatrix} \quad k \in K,$$

where $K$ is a subset of $p$ factors of $m$ and $\hat{A}^{[k]}$ and $\hat{B}^{[k]}$ are the matrices containing the $k$-order temporal aggregates of the bts and uts, of dimension $(n_b \times hm/k)$ and $(n_a \times hm/k)$, respectively.

Let us consider the multivariate regression model

$$\hat{Y} = Y + E,$$

where the involved matrices have each dimension $[n \times (k^* + m)]$ and contain, respectively, the base ($\hat{Y}$) and the target forecasts ($Y$), and the coherency errors ($E$) for the $n$ component variables of the linearly constrained time series of interest. For each variable, $k^* + m$ base forecasts are available, pertaining to all aggregation levels of the temporal hierarchy for a complete cycle of high-frequency observation, $m$. Consider now two vectorized versions of model, by transforming the matrices either in original form:

$$\text{vec} \left( \hat{Y} \right) = \text{vec} \left( Y \right) + \varepsilon \quad \text{with} \quad \varepsilon = \text{vec} \left( E \right)$$
or in transposed form:

$$\text{vec} \left( \hat{Y}' \right) = \text{vec} \left( Y' \right) + \eta \text{ with } \eta = \text{vec} (E') .$$

Denote with $P$ the $\left[ n(k^* + m) \times n(k^* + m) \right]$ commutation matrix such that $P \text{vec}(Y) = \text{vec}(Y')$, $P \text{vec}(\hat{Y}) = \text{vec}(\hat{Y}')$ and $P \varepsilon = \eta$. Let $W = E[\varepsilon \varepsilon']$ be the covariance matrix of vector $\varepsilon$, and $\Omega = E[\eta \eta']$ the covariance matrix of vector $\eta$. Clearly, $W$ and $\Omega$ are different parameterizations of the same statistical object for which the following relationships hold:

$$\Omega = PWP' , \quad W = P' \Omega P .$$

In order to apply the general point forecast reconciliation according to the projection approach (type = "M") to a cross-temporal forecast reconciliation problem, we may consider either two vec-forms, e.g. if we follow the first:

$$\tilde{y} = \hat{y} - \Omega H (H' \Omega H)^{-1} H' \hat{y} = M \hat{y} ,$$

where $\hat{y} = \text{vec}(\hat{Y}')$ is the row vectorization of the base forecasts matrix $\hat{Y}$. The alternative equivalent solution (type = "S") (following the structural reconciliation approach by Hyndman et al., 2011) is

$$\tilde{y} = \tilde{S} \left( \tilde{S}' \Omega^{-1} \tilde{S} \right)^{-1} \tilde{S}' \Omega^{-1} \hat{y} = \tilde{S} G \hat{y} .$$

where $\tilde{S}$ is the cross-temporal summing matrix.

**Bounds on the reconciled forecasts**

When the reconciliation uses the optimization package osqp, the user may impose bounds on the reconciled forecasts. The parameter bounds permits to consider lower ($a$) and upper ($b$) bounds like $a \leq \tilde{y} \leq b$, where $\tilde{y} = \text{vec}(\tilde{Y}')$, such that:

$$a_1 \leq \tilde{y}_1 \leq b_1 \quad \text{...} \quad a_n(k^* + m) \leq \tilde{y}_n(k^* + m) \leq b_n(k^* + m) \Rightarrow \text{bounds} = \begin{bmatrix} a_1 & b_1 \\ \vdots & \vdots \\ a_n(k^* + m) & b_n(k^* + m) \end{bmatrix} ,$$

where $a_i \in [-\infty, +\infty]$ and $b_i \in [-\infty, +\infty]$. If $y_i$ is unbounded, the i-th row of bounds would be equal to c(-Inf, +Inf). Notice that if the bounds parameter is used, sol = "osqp" must be used. This is not true in the case of non-negativity constraints:

- sol = "direct": first the base forecasts are reconciled without non-negativity constraints, then, if negative reconciled values are present, the "osqp" solver is used;
- sol = "osqp": the base forecasts are reconciled using the "osqp" solver.

In this case it is not necessary to build a matrix containing the bounds, and it is sufficient to set nn = "TRUE".

Non-negative reconciled forecasts may be obtained by setting nn_type alternatively as:

- nn_type = "sntz" ("set-negative-to-zero")
- nn_type = "osqp" (Stellato et al., 2020)

**Value**

If the parameter keep is equal to "recf", then the function returns only the $(n \times h(k^* + m))$ reconciled forecasts matrix, otherwise (keep="all") it returns a list that mainly depends on what type of representation (type) and solution technique (sol) have been used:
octrec

(ref \( (n \times h(k^* + m)) \) reconciled forecasts matrix, \( \tilde{Y} \).

Omega
Covariance matrix used for reconciled forecasts (vec(\( \tilde{Y} \)) representation).

W
Covariance matrix used for reconciled forecasts (vec(\( \tilde{Y} \)) representation).

nn_check
Number of negative values (if zero, there are no values below zero).

rec_check
Logical value: rec_check = TRUE when the constraints have been fulfilled.

varf (type="direct")
\( (n \times (k^* + m)) \) reconciled forecasts variance matrix for \( h = 1 \), diag(MW).

M (type="direct")
Projection matrix (projection approach).

G (type="S" and type="direct")
Projection matrix (structural approach, \( M = SG \)).

S (type="S" and type="direct")
Cross-temporal summing matrix (vec(\( \tilde{Y} \)) representation).

info (type="osqp")
matrix with some useful indicators (columns) for each forecast horizon \( h \) (rows): run time (run_time), number of iteration, norm of primal residual (pri_res), status of osqp's solution (status) and polish's status (status_polish).

References


See Also

Other reconciliation procedures: cstrec(), ctbu(), htsrec(), iterec(), lccrec(), tcsrec(), tdrec(), thfrec()

Examples

data(FoReco_data)
obj <- octrec(FoReco_data$base, m = 12, C = FoReco_data$C,
comb = "bdshr", res = FoReco_data$res)
oct_bounds

Optimal cross-temporal bounds

Description
Function to export the constraints designed for the cross-sectional and/or temporal reconciled forecasts

Usage
oct_bounds(hts_bounds, thf_bounds, m, C, Ut)

Arguments
hts_bounds  \((n \times 2)\) matrix with cross-sectional bounds: the first column is the lower bound, and the second column is the upper bound.

thf_bounds  \(((k^* + m) \times 2)\) matrix with temporal bounds: the first column is the lower bound, and the second column is the upper bound.

m  Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).

C  \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the bottom level series into the higher level ones.

Ut  Zero constraints cross-sectional (contemporaneous) kernel matrix \((U' y = 0)\) spanning the null space valid for the reconciled forecasts. It can be used instead of parameter \(C\), but \(nb (n = n_a + n_b)\) is needed if \(U' \neq [I - C]\). If the hierarchy admits a structural representation, \(U'\) has dimension \((n_a \times n)\).

Value
A matrix with the cross-temporal bounds.

See Also
Other utilities: Cmatrix(), FoReco2ts(), agg_ts(), arrange_hres(), commat(), ctf_tools(), hts_tools(), lcmat(), residuals_matrix(), score_index(), shrink_estim(), thf_tools()

Examples
data(FoReco_data)
  # monthly base forecasts
  mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
  # monthly residuals
  mres <- FoReco2matrix(FoReco_data$res, m = 12)$k1

  # For example, in FoReco_data we want that BA > 78, and C > 50
  cs_bound <- matrix(c(rep(-Inf, 5), 78, -Inf, 50, rep(+Inf, 8)), ncol = 2)
  ## Cross-sectional reconciliation
  csobj <- htsrec(mbase, C = FoReco_data$C, comb = "shr", res = mres, bounds = cs_bound)

  # Extension of the constraints to the cross-temporal case
  ct_bound <- oct_bounds(hts_bounds = cs_bound, m = 12)
## Cross-temporal reconciliation

```r
obj <- octrec(FoReco_data$base, m = 12, C = FoReco_data$C, comb = "bdshr",
              res = FoReco_data$res, bounds = ct_bound)
```

### residuals_matrix

**Description**

Arrange temporal and cross-temporal residuals in a matrix form

**Usage**

```r
residuals_matrix(res, m)
```

**Arguments**

- `res`: \((n \times N(k^* + m))\) matrix or \((N(k^* + m))\) vector containing the residuals at all the temporal frequencies ordered \([\text{lowest_freq} \ldots \text{highest_freq}]\).
- `m`: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).

**Value**

\((N \times n(k^* + m))\) matrix (if the input res is a vector then \(n = 1\))

**See Also**

Other utilities: `Cmatrix()`, `FoReco2ts()`, `agg_ts()`, `arrange_hres()`, `commat()`, `ctf_tools()`, `hts_tools()`, `lcmat()`, `oct_bounds()`, `score_index()`, `shrink_estim()`, `thf_tools()`

**Examples**

```r
data(FoReco_data)
mat <- residuals_matrix(FoReco_data$res, m = 12)
```

### score_index

**Description**

Function to calculate the accuracy indices of the reconciled point forecasts of a cross-temporal (not only, see examples) system (more in Average relative accuracy indices). *(Experimental version)*

**Usage**

```r
score_index(recf, base, test, m, nb, nl, type = "mse", compact = TRUE)
```
Arguments

`recf` list of q (forecast origins) reconciled forecasts’ matrices \([n \times h(k^* + m)]\) in the cross-temporal case, \([h \times n]\) in the cross-sectional case, and vectors of length \([h(k^* \times m)]\) in the temporal framework.

`base` list of q (forecast origins) base forecasts’ matrices \([n \times h(k^* + m)]\) in the cross-temporal case, \([h \times n]\) in the cross-sectional case, and vectors of length \([h(k^* \times m)]\) in the temporal framework.

`test` list of q (forecast origins) test observations’ matrices \([n \times h(k^* + m)]\) in the cross-temporal case, \([h \times n]\) in the cross-sectional case, and vectors of length \([h(k^* \times m)]\) in the temporal framework.

`m` Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).

`nb` number of bottom time series in the cross-sectional framework.

`nl` \((L \times 1)\) vector containing the number of time series in each cross-sectional level of the hierarchy (\(nl[1] = 1\)).

`type` type of accuracy measure ("mse" Mean Square Error, "rmse" Root Mean Square Error or "mae" Mean Absolute Error).

`compact` if TRUE returns only the summary matrix.

Value

It returns a summary table called `Avg_mat` (if `compact` option is TRUE, default), otherwise it returns a list of six tables (more in Average relative accuracy indices).

References


See Also

Other utilities: `Cmatrix()`, `FoReco2ts()`, `agg_ts()`, `arrange_hres()`, `commat()`, `ctf_tools()`, `hts_tools()`, `lcmat()`, `oct_bounds()`, `residuals_matrix()`, `shrink_estim()`, `thf_tools()`

Examples

data(FoReco_data)

# Cross-temporal framework
oct_recf <- octrec(FoReco_data$base, m = 12, C = FoReco_data$C, comb = "bdshr", res = FoReco_data$res)$recf
oxct_score <- score_index(recf = oct_recf, base = FoReco_data$base, test = FoReco_data@test, m = 12, nb = 5)

# Cross-sectional framework
# monthly base forecasts
mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
# monthly test set
mtest <- FoReco2matrix(FoReco_data@test, m = 12)$k1
# monthly residuals
mres <- FoReco2matrix(FoReco_data$res, m = 12)$k1
# monthly reconciled forecasts
mrecf <- htsrec(mbase, C = FoReco_data$C, comb = "shr", res = mres)$recf
# score
hts_score <- score_index(recf = mrecf, base = mbase, test = mtest, nb = 5)

# Temporal framework
# top ts base forecasts ([lowest_freq' ... highest_freq']')
topbase <- FoReco_data$base[1, ]
# top ts residuals ([lowest_freq' ... highest_freq']')
topres <- FoReco_data$res[1, ]
# top ts test ([lowest_freq' ... highest_freq']')
toptest <- FoReco_data$test[1, ]
# top ts recf ([lowest_freq' ... highest_freq']')
toprecf <- thfrec(topbase, m = 12, comb = "acov", res = topres)$recf
# score
thf_score <- score_index(recf = toprecf, base = topbase, test = toptest, m = 12)

---

**shrink_estim**

**Shrinkage of the covariance matrix**

**Description**

Shrinkage of the covariance matrix according to Schäfer and Strimmer (2005).

**Usage**

`shrink_estim(x, minT = T)`

**Arguments**

- **x**: residual matrix
- **minT**: this param allows to calculate the covariance matrix according to the original hts formulation (TRUE) or according to the standard approach (FALSE).

**Value**

A list with two objects: the first ($scov$) is the shrunk covariance matrix and the second ($lambda$) is the shrinkage intensity coefficient.

**Author(s)**

This function is a modified version of the `shrink_estim()` hidden function of `hts`.

**References**


tcsrec  

Heuristic first-temporal-then-cross-sectional cross-temporal forecast reconciliation

Description

The cross-temporal forecast reconciliation procedure by Kourentzes and Athanasopoulos (2019) can be viewed as an ensemble forecasting procedure which exploits the simple averaging of different forecasts. First, for each time series the forecasts at any temporal aggregation order are reconciled using temporal hierarchies (thfrec()), then time-by-time cross-sectional reconciliation is performed (htsrec()). The projection matrices obtained at this step are then averaged and used to cross-sectionally reconcile the forecasts obtained at step 1, by this way fulfilling both cross-sectional and temporal constraints.

Usage

```r
tcsrec(basef, thf_comb, hts_comb, res, avg = "KA", ...)
```

Arguments

- **basef**  
  $(n \times h(k^* + m))$ matrix of base forecasts to be reconciled, $\hat{Y}$; $n$ is the total number of variables, $m$ is the highest time frequency, $k^*$ is the sum of (a subset of) $(p - 1)$ factors of $m$, excluding $m$, and $h$ is the forecast horizon for the lowest frequency time series. Each row identifies a time series, and the forecasts are ordered as [lowest_freq' ... highest_freq']'.

- **hts_comb, thf_comb**  
  Type of covariance matrix (respectively $(n \times n)$ and $((k^* + m) \times (k^* + m))$) to be used in the cross-sectional and temporal reconciliation. More details in comb param of htsrec() and thfrec().

- **res**  
  $(n \times N(k^* + m))$ matrix containing the residuals at all the temporal frequencies ordered [lowest_freq' ... highest_freq']' (columns) for each variable (row), needed to estimate the covariance matrix when hts_comb = {"wls", "shr", "sam"} and/or hts_comb = {"wlsv", "wlsh", "acov", "strar1", "sar1", "har1", "shr", "sam"}. The row must be in the same order as basef.

- **avg**  
  If avg = "KA" (default), the final projection matrix $M$ is the one proposed by Kourentzes and Athanasopoulos (2019), otherwise it is calculated as simple average of all the involved projection matrices at step 2 of the procedure (see Di Fonzo and Girolimetto, 2020).

- **...**  
  Any other options useful for htsrec() and thfrec(), e.g. m, C (or Ut and nb), nn (for non negativity reconciliation only at first step), mse, corpcor, type, sol, settings, W, Omega,...
Details

This function performs a two-step cross-temporal forecast reconciliation using the covariance matrices chosen by the user. If the combinations used by Kourentzes and Athanasopoulos (2019) are wished, thf_comb must be set equal to either "struc" or "wlsv", and hts_comb equal to either "shr" or "wls".

Warning. the two-step heuristic reconciliation allows considering non negativity constraints only in the first step. This means that non-negativity is not guaranteed in the final reconciled values.

Value

The function returns a list with two elements:

- \( \text{recf} \) \((n \times h(k^* + m))\) reconciled forecasts matrix, \( \bar{Y} \).
- \( M \) Matrix which transforms the uni-dimensional reconciled forecasts of step 1 (projection approach).

References


See Also

Other reconciliation procedures: cstrec(), ctbu(), htsrec(), iterec(), lccrec(), octrec(), tdrec(), thfrec()

Examples

data(FoReco_data)
obj <- tcsrec(FoReco_data$base, m = 12, C = FoReco_data$C,
              thf_comb = "acov", hts_comb = "shr", res = FoReco_data$res)
**tdrec**

*Top-down forecast reconciliation for genuine hierarchical/grouped time series*

**Description**

Top-down forecast reconciliation for genuine hierarchical/grouped time series, where the forecast of a ‘Total’ (top-level series, expected to be positive) is disaggregated according to a proportional scheme given by a vector of proportions (weights). Besides the fulfillment of any aggregation constraint, the top-down reconciled forecasts should respect two main properties:

- the top-level value remains unchanged;
- all the bottom time series reconciled forecasts are non-negative.

The top-down procedure is extended to deal with both temporal and cross-temporal cases. Since this is a post-forecasting function, the vector of weights must be given in input by the user, and is not calculated automatically (see Examples).

**Usage**

```r
tdrec(topf, C, m, weights)
```

**Arguments**

- `topf`: \((h \times 1)\) vector of the top-level base forecast to be disaggregated; \(h\) is the forecast horizon (for the lowest temporal aggregation order in temporal and cross-temporal cases).
- `C`: \((n_a \times n_b)\) cross-sectional (contemporaneous) matrix mapping the \(n_b\) bottom level series into the \(n_a\) higher level ones.
- `m`: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of the \(p\) factors of \(m\).
- `weights`: vector of weights to be used to disaggregate \(topf\): \((n_b \times h)\) matrix in the cross-sectional framework; \((m \times h)\) matrix in the temporal framework; \((n_b m \times h)\) matrix in the cross-temporal framework.

**Details**

Fix \(h = 1\), then

\[
\tilde{y} = Sw\hat{a}_1
\]

where \(\tilde{y}\) is the vector of reconciled forecasts, \(S\) is the summing matrix (whose pattern depends on which type of reconciliation is being performed), \(w\) is the vector of weights, and \(\hat{a}_1\) is the top-level value to be disaggregated.

**Value**

The function returns an \((h \times n)\) matrix of cross-sectionally reconciled forecasts, or an \((h(k^* + m) \times 1)\) vector of top-down temporally reconciled forecasts, or an \((n \times h(k^* + m))\) matrix of top-down cross-temporally reconciled forecasts.
References
Athanasopoulos, G., Ahmed, R.A., Hyndman, R.J. (2009), Hierarchical forecasts for Australian

See Also
Other reconciliation procedures: `cstrec()`, `ctbu()`, `htsrec()`, `iterc()`, `lccrec()`, `octrec()`,
`tcsrec()`, `thfrec()`

Examples
```r
data(FoReco_data)
### CROSS-SECTIONAL TOP-DOWN RECONCILIATION
# Cross sectional aggregation matrix
C <- FoReco_data$C
# monthly base forecasts
mbase <- FoReco2matrix(FoReco_data$base, m = 12)$k1
obs_1 <- FoReco_data$obs$k1
# average historical proportions
props <- colMeans(obs_1[1:168,-c(1:3)])/obs_1[1:168,1]
cs_td <- tdrec(topf = mbase[,1], C = C, weights = props)

### TEMPORAL TOP-DOWN RECONCILIATION
# top ts base forecasts ([lowest_freq' ... highest_freq']')
top_obs12 <- FoReco_data$obs$k12[1:14,1]
# average historical proportions
props <- colMeans(matrix(bts_obs1, ncol = 12, byrow = TRUE)/top_obs12)
topbase <- FoReco_data$base[,1]
t_td <- tdrec(topf = topbase, m = 12, weights = props)

### CROSS-TEMPORAL TOP-DOWN RECONCILIATION
top_obs <- FoReco_data$obs$k12[1:14,1]
bts_obs <- lapply(1:5, function(x) matrix(bts_obs[,x], nrow=14, byrow = TRUE))
bts_obs <- do.call(cbind, bts_obs)
# average historical proportions
props <- colMeans(bts_obs/top_obs)
ct_td <- tdrec(topf = topbase, m = 12, C = C, weights = props)
```

---

**thfrec**

*Forecast reconciliation through temporal hierarchies (temporal reconcile-
ciation)*

**Description**
Forecast reconciliation of one time series through temporal hierarchies (Athanasopoulos et al., 2017). The reconciled forecasts are calculated either through a projection approach (Byron, 1978), or the equivalent structural approach by Hyndman et al. (2011). Moreover, the classic bottom-up approach is available.
Usage

thfrec(basef, m, comb, res, mse = TRUE, corpcor = FALSE,
        type = "M", sol = "direct", keep = "list", v = NULL, nn = FALSE,
        nn_type = "osqp", settings = list(), bounds = NULL, Omega = NULL)

Arguments

basef \((h(k^* + m) \times 1)\) vector of base forecasts to be reconciled, containing the forecasts at all the needed temporal frequencies ordered as \([\text{lowest_freq}' ... \text{highest_freq}']\).

m Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of \(p\) factors of \(m\).

comb Type of the reconciliation. Except for bottom up, all other options correspond to a different \((k^* + m) \times (k^* + m)\) covariance matrix, \(k^*\) is the sum of \((p - 1)\) factors of \(m\) (excluding \(m\)):

- \(\text{bu}\) (Bottom-up);
- \(\text{ols}\) (Identity);
- \(\text{struc}\) (Structural variances);
- \(\text{wlsv}\) (Series variances);
- \(\text{wlsh}\) (Hierarchy variances);
- \(\text{acov}\) (Auto-covariance matrix);
- \(\text{strar1}\) (Structural Markov);
- \(\text{sar1}\) (Series Markov);
- \(\text{har1}\) (Hierarchy Markov);
- \(\text{shr}\) (Shrunk cross-covariance matrix);
- \(\text{sam}\) (Sample cross-covariance matrix);
- \(\text{omega}\) use your personal matrix Omega in param Omega.

res vector containing the in-sample residuals at all the temporal frequencies ordered as basef, i.e. \([\text{lowest_freq}' ... \text{highest_freq}']\), needed to estimate the covariance matrix when \(\text{comb} = \{\text{"wlsv"}, \text{"wlsh"}, \text{"acov"}, \text{"strar1"}, \text{"sar1"}, \text{"har1"}, \text{"shr"}, \text{"sam"}\}\).

mse Logical value: TRUE (default) calculates the covariance matrix of the in-sample residuals (when necessary) according to the original \texttt{hts} and \texttt{thief} formulation: no mean correction, \(T\) as denominator.

corpcor Logical value: TRUE if \texttt{corpcor} (Schäfer et al., 2017) must be used to shrink the sample covariance matrix according to Schäfer and Strimmer (2005), otherwise the function uses the same implementation as package \texttt{hts}.

type Approach used to compute the reconciled forecasts: "M" for the projection approach with matrix \(M\) (default), or "S" for the structural approach with temporal summing matrix \(R\).

sol Solution technique for the reconciliation problem: either "direct" (default) for the closed-form matrix solution, or "osqp" for the numerical solution (solving a linearly constrained quadratic program using \texttt{solve_osqp}).

keep Return a list object of the reconciled forecasts at all levels (if keep = "list") or only the reconciled forecasts matrix (if keep = "recf").

v vector index of the fixed base forecast \((\min(v) > 0 \text{ and } \max(v) < (k^* + m))\).

nn Logical value: TRUE if non-negative reconciled forecasts are wished.
nn_type "osqp" (default), "KAnn" (only type == "M") or "sntz".

settings Settings for osqp (object osqpSettings). The default options are: verbose = FALSE, eps_abs = 1e-5, eps_rel = 1e-5, polish_refine_iter = 100 and polish = TRUE. For details, see the osqp documentation (Stellato et al., 2019).

bounds \(((k^* + m) \times 2)\) matrix with temporal bounds: the first column is the lower bound, and the second column is the upper bound.

Omega This option permits to directly enter the covariance matrix:

1. Omega must be a p.d. \((k^* + m) \times (k^* + m)\) matrix or a list of \(h\) matrices (one for each forecast horizon);
2. If comb is different from "omega", Omega is not used.

Details

Let \(m\) be the highest available sampling frequency per seasonal cycle, and denote \(\mathcal{K} = \{k_m, k_{p-1}, \ldots, k_2, k_1\}\) the \(p\) factors of \(m\), in descending order, where \(k_p = m\), and \(k_1 = 1\). Define \(K\) the \((k^* \times m)\) temporal aggregation matrix converting the high-frequency observations into lower-frequency (temporally aggregated) ones:

\[
K = \begin{bmatrix}
1_m' \\
\frac{1_{m'}}{I_{p-1}} \otimes 1_{k_{p-1}}' \\
\vdots \\
\frac{1_{m'}}{I_{k_2}} \otimes 1_{k_2}'
\end{bmatrix}
\]

Denote \(R = \begin{bmatrix} K \\ I_m \end{bmatrix}\) the \([(k^* + m) \times m]\) temporal summing matrix, and \(Z' = [I_{k^*} - K]\) the zero constraints kernel matrix.

Suppose we have the \([(k^* + m) \times 1]\) vector \(\hat{y}\) of unbiased base forecasts for the \(p\) temporal aggregates of a single time series \(Y\) within a complete time cycle, i.e., at the forecast horizon \(h = 1\) for the lowest (most aggregated) time frequency. If the base forecasts have been independently obtained, generally they do not fulfill the temporal aggregation constraints, i.e. \(Z'y \neq 0_{(k^* \times 1)}\). By adapting the general point forecast reconciliation according to the projection approach (type = "M"), the vector of temporally reconciled forecasts is given by:

\[
\hat{y} = \hat{y} - \Omega Z (Z' \Omega Z)^{-1} Z' \hat{y},
\]

where \(\Omega\) is a \([(k^* + m) \times (k^* + m)]\) p.d. matrix, assumed known. The alternative equivalent solution (type = "S") following the structural reconciliation approach by Athanasopoulos et al. (2017) is given by:

\[
\hat{y} = R (R' \Omega^{-1} R)^{-1} R' \Omega^{-1} \hat{y}.
\]

Bounds on the reconciled forecasts

When the reconciliation makes use of the optimization package osqp, the user may impose bounds on the reconciled forecasts. The parameter bounds permits to consider lower (a) and upper (b) bounds like \(a \leq \hat{y} \leq b\) such that:

\[
a_1 \leq \hat{y}_1 \leq b_1 \\
\ldots \\
a_{(k^* + m)} \leq \hat{y}_{(k^* + m)} \leq b_{(k^* + m)}
\Rightarrow \text{bounds} = [a \ b] = \begin{bmatrix} a_1 & b_1 \\
\vdots \\
a_{(k^* + m)} & b_{(k^* + m)} \end{bmatrix},
\]

where \(a_i \in [-\infty, +\infty]\) and \(b_i \in [-\infty, +\infty]\). If \(y_i\) is unbounded, the \(i\)-th row of bounds would be equal to \(c(-\infty, +\infty)\). Notice that if the bounds parameter is used, sol = "osqp" must be used. This is not true in the case of non-negativity constraints:
• sol = "direct": first the base forecasts are reconciled without non-negativity constraints, then, if negative reconciled values are present, the "osqp" solver is used;
• sol = "osqp": the base forecasts are reconciled using the "osqp" solver.

In this case it is not necessary to build a matrix containing the bounds, and it is sufficient to set nn = "TRUE".

Non-negative reconciled forecasts may be obtained by setting nn_type alternatively as:
• nn_type = "sntz" ("set-negative-to-zero")
• nn_type = "osqp" (Stellato et al., 2020)

Value
If the parameter keep is equal to "recf", then the function returns only the \((h(k^* + m) \times 1)\) reconciled forecasts vector, otherwise (keep="all") it returns a list that mainly depends on what type of representation (type) and solution technique (sol) have been used:

- **recf** \((h(k^* + m) \times 1)\) reconciled forecasts vector, \(\tilde{y}\).
- **Omega** Covariance matrix used for forecast reconciliation, \(\Omega\).
- **nn_check** Number of negative values (if zero, there are no values below zero).
- **rec_check** Logical value: \(rec\_check = \text{TRUE}\) when the constraints have been fulfilled.
- **varf** \((type="direct")\) \((\times 1)\) reconciled forecasts variance vector for \(h = 1\), diag(MW).
- **M** \((type="direct")\) Projection matrix, \(M\) (projection approach).
- **G** \((type="S" and type="direct")\) Projection matrix, \(G\) (structural approach, \(M = RG\)).
- **S** \((type="S" and type="direct")\) Temporal summing matrix, \(R\).
- **info** \((type="osqp")\) matrix with information in columns for each forecast horizon \(h\) (rows): run time (run_time), number of iteration (iter), norm of primal residual (pri_res), status of osqp’s solution (status) and polish’s status (status_polish). It will also be returned with \(nn = \text{TRUE}\) if a solver (see nn_type) will be use.

Only if \(comb = "bu"\), the function returns recf, R and M.

References
See Also

Other reconciliation procedures: `cstrec()`, `ctbu()`, `htsrec()`, `iterc()`, `lccrec()`, `octrec()`, `tcsrec()`, `tdrec()`

Examples

```r
data(FoReco_data)
# top ts base forecasts ([lowest_freq' ... highest_freq']')
topbase <- FoReco_data$base[, 1]
# top ts residuals ([lowest_freq' ... highest_freq']')
topres <- FoReco_data$res[, 1]
obj <- thfrec(topbase, m = 12, comb = "acov", res = topres)
```

Description

Some useful tools for forecast reconciliation through temporal hierarchies.

Usage

```r
thf_tools(m, h = 1, sparse = TRUE)
```

Arguments

- `m`: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation, \(m\)), or a subset of the \(p\) factors of \(m\).
- `h`: Forecast horizon for the lowest frequency (most temporally aggregated) time series (default is 1).
- `sparse`: Option to return sparse object (default is TRUE).
Value

A list of seven elements:

- **K**: Temporal aggregation matrix.
- **R**: Temporal summing matrix.
- **Zt**: Zero constraints temporal kernel matrix, $Z_t^h Y' = 0_{[h \times (k^* \times n)]}$. 
- **kset**: Set of factors ($p$) of $m$ in descending order (from $m$ to 1), $\mathcal{K} = \{k_p, k_{p-1}, \ldots, k_2, k_1\}$, $k_p = m$, $k_1 = 1$.
- **m**: Highest available sampling frequency per seasonal cycle (max. order of temporal aggregation).
- **p**: Number of elements of kset, $\mathcal{K}$.
- **ks**: Sum of $p - 1$ factors of $m$ (out of $m$ itself), $k^*$.
- **kt**: Sum of all factors of $m$, $k_{tot} = k^* + m$.

See Also

Other utilities: Cmatrix(), FoReco2ts(), agg_ts(), arrange_hres(), commat(), ctf_tools(), hts_tools(), lcmat(), oct_bounds(), residuals_matrix(), score_index(), shrink_estim()

Examples

```r
# quarterly data
obj <- thf_tools(m = 4, sparse = FALSE)
```
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