Package ‘HypergeoMat’

Type Package
Title Hypergeometric Function of a Matrix Argument
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Author Stéphane Laurent
Maintainer Stéphane Laurent <laurent_step@outlook.fr>
Description Evaluates the hypergeometric functions of a matrix argument, which appear in random matrix theory. This is an implementation of Koev & Edelman's algorithm (2006) <doi:10.1090/S0025-5718-06-01824-2>.
License GPL-3
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**Description**

Evaluates the type one Bessel function of Herz.

**Usage**

```r
BesselA(m, x, nu)
```

**Arguments**

- `m`: truncation weight of the summation, a positive integer
- `x`: either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix
- `nu`: the order parameter, real or complex number with \( \text{Re}(\nu) > -1 \)

**Value**

A real or complex number.

**Note**

This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

**References**

hypergeomPFQ

Examples

# for a scalar x, the relation with the Bessel J-function:
t <- 2
nu <- 3
besselJ(t, nu)
BesselA(m=15, t^2/4, nu) * (t/2)^nu
# it also holds for a complex variable:
t <- 1 + 2i
Bessel::BesselJ(t, nu)
BesselA(m=15, t^2/4, nu) * (t/2)^nu

hypergeomPFQ

Hypergeometric function of a matrix argument

Description

Evaluates a truncated hypergeometric function of a matrix argument.

Usage

hypergeomPFQ(m, a, b, x, alpha = 2)

Arguments

m         truncation weight of the summation, a positive integer
a         the "upper" parameters, a numeric or complex vector, possibly empty (or NULL)
b         the "lower" parameters, a numeric or complex vector, possibly empty (or NULL)
x         either a real or complex square matrix, or a numeric or complex vector, the
eigenvalues of the matrix
alpha      the alpha parameter, a positive number

Details

This is an implementation of Koev & Edelman's algorithm (see the reference). This algorithm is
split into two parts: the case of a scalar matrix (multiple of an identity matrix) and the general case.
The case of a scalar matrix is much faster (try e.g. x = c(1, 1, 1) vs x = c(1, 1, 0.999)).

Value

A real or a complex number.

Note

The hypergeometric function of a matrix argument is usually defined for a symmetric real matrix or
a Hermitian complex matrix.
IncBeta

Incomplete Beta function of a matrix argument

Description
Evaluates the incomplete Beta function of a matrix argument.

Usage
IncBeta(m, a, b, x)

Arguments
m truncation weight of the summation, a positive integer
a, b real or complex parameters with Re(a)>(p-1)/2, Re(b)>(p-1)/2, where p is the dimension (the order of the matrix)
x either a real positive symmetric matrix or a complex positive Hermitian matrix "smaller" than the identity matrix (i.e. I-x is positive), or a numeric or complex vector, the eigenvalues of the matrix

Value
A real or a complex number.

References

Examples
# a scalar x example, the Gauss hypergeometric function
hypergeomPFQ(m = 10, a = c(1,2), b = c(3), x = 0.2)
gsl::hyperg_2F1(1, 2, 3, 0.2)
# 0F0 is the exponential of the trace
X <- toeplitz(c(3,2,1))/10
hypergeomPFQ(m = 10, a = NULL, b = NULL, x = X)
exp(sum(diag(X)))
# 1F0 is det(I-X)^(-a)
X <- toeplitz(c(3,2,1))/100
hypergeomPFQ(m = 10, a = 3, b = NULL, x = X)
det(diag(3)-X)^(-3)
# Herz' s relation for 1F1
hypergeomPFQ(m = 10, a = 2, b = 3, x = X)
exp(sum(diag(X)))*hypergeomPFQ(m = 10, a = 3-2, b = 3, x = -X)
# Herz' s relation for 2F1
hypergeomPFQ(10, a = c(1,2), b = 3, x = X)
det(diag(3)-X)^(-2) *
hypergeomPFQ(10, a = c(3-1,2), b = 3, -X %% solve(diag(3)-X))
Note
The eigenvalues of a real symmetric matrix or a complex Hermitian matrix are always real numbers, and moreover they are positive under the constraints on x. However we allow to input a numeric or complex vector x because the definition of the function makes sense for such a x.

References

Examples
# for a scalar x, this is the incomplete Beta function:
a <- 2; b <- 3
x <- 0.75
IncBeta(m = 15, a, b, x)
gsl::beta_inc(a, b, x)
pbeta(x, a, b)

IncGamma
Incomplete Gamma function of a matrix argument

Description
Evaluates the incomplete Gamma function of a matrix argument.

Usage
IncGamma(m, a, x)

Arguments
m truncation weight of the summation, a positive integer
a real or complex parameter with Re(a)>((p-1)/2, where p is the dimension (the order of the matrix)
x either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix

Value
A real or complex number.

Note
This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

References
Examples

# for a scalar x, this is the incomplete Gamma function:
\[
a <- 2 \\
x <- 1.5 \\
IncGamma(m = 15, a, x) \\
gsl::gamma_inc_P(a, x) \\
pgamma(x, shape = a, rate = 1)
\]

mvbeta

Multivariate Beta function (of complex variable)

Description

The multivariate Beta function (mvbeta) and its logarithm (lmvbeta).

Usage

\[
\text{lmvbeta}(a, b, p) \\
mvbeta(a, b, p)
\]

Arguments

\[
a, b \quad \text{real or complex numbers with } \Re(a)>0, \Re(b)>0 \\
p \quad \text{a positive integer, the dimension}
\]

Value

A real or a complex number.

Examples

\[
a <- 5; \ b <- 4; \ p <- 3 \\
mvbeta(a, b, p) \\
mvgamma(a, p) * mvgamma(b, p) / mvgamma(a+b, p)
\]
**Description**

The multivariate Gamma function (\texttt{mvgamma}) and its logarithm (\texttt{lmvgamma}).

**Usage**

\[
\texttt{lmvgamma}(x, p) \\
\texttt{mvgamma}(x, p)
\]

**Arguments**

\begin{itemize}
  \item \texttt{x} \hspace{1cm} a real or a complex number; \text{Re}(x)>0 \text{ for } \texttt{lmvgamma} \text{ and } x \text{ must not be a negative integer for } \texttt{mvgamma}
  \item \texttt{p} \hspace{1cm} a positive integer, the dimension
\end{itemize}

**Value**

A real or a complex number.

**Examples**

\[
\begin{align*}
  x & \leftarrow 5 \\
  \texttt{mvgamma}(x, p = 2) & \hspace{1cm} \sqrt{\pi} \cdot \text{gamma}(x) \cdot \text{gamma}(x-1/2)
\end{align*}
\]
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