Package ‘HypergeoMat’

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Type Package
Title Hypergeometric Function of a Matrix Argument
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Description Evaluates the hypergeometric functions of a matrix argument, which appear in random matrix theory. This is an implementation of Koev & Edelman's algorithm (2006) <doi:10.1090/S0025-5718-06-01824-2>.
License GPL-3
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Description

Evaluates the type one Bessel function of Herz.

Usage

\[
\text{BesselA}(m, x, \nu)
\]

Arguments

- \(m\): truncation weight of the summation, a positive integer
- \(x\): either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix
- \(\nu\): the order parameter, real or complex number with \(\text{Re}(\nu) \geq 1\)

Value

A real or complex number.

Note

This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

References


Examples

```r
# for a scalar x, the relation with the Bessel J-function:
t <- 2
nu <- 3
besselJ(t, nu)
BesselA(m=15, t^2/4, nu) * (t/2)^nu
```

```
# it also holds for a complex variable:
if(require("Bessel")) {
t <- 1 + 2i
Bessel::BesselJ(t, nu)
BesselA(m=15, t^2/4, nu) * (t/2)^nu
}
```
Evaluates a truncated hypergeometric function of a matrix argument.

```
Usage

hypergeomPFQ(m, a, b, x, alpha = 2)
```

**Arguments**

- **m**: truncation weight of the summation, a positive integer
- **a**: the "upper" parameters, a numeric or complex vector, possibly empty (or NULL)
- **b**: the "lower" parameters, a numeric or complex vector, possibly empty (or NULL)
- **x**: either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix
- **alpha**: the alpha parameter, a positive number

**Details**

This is an implementation of Koev & Edelman's algorithm (see the reference). This algorithm is split into two parts: the case of a scalar matrix (multiple of an identity matrix) and the general case. The case of a scalar matrix is much faster (try e.g. `x = c(1,1,1)` vs `x = c(1,1,0.999)`).

**Value**

A real or a complex number.

**Note**

The hypergeometric function of a matrix argument is usually defined for a symmetric real matrix or a Hermitian complex matrix.

**References**

Examples

# a scalar x example, the Gauss hypergeometric function
hypergeomPFQ(m = 10, a = c(1,2), b = c(3), x = 0.2)
gsl::hyperg_2F1(1, 2, 3, 0.2)
# 0F0 is the exponential of the trace
X <- toeplitz(c(3,2,1))/10
hypergeomPFQ(m = 10, a = NULL, b = NULL, x = X)
exp(sum(diag(X)))
# 1F0 is det(I-X)^(-a)
X <- toeplitz(c(3,2,1))/100
hypergeomPFQ(m = 10, a = 3, b = NULL, x = X)
det(diag(3)-X)^(-3)
# Herz's relation for 1F1
hypergeomPFQ(m = 10, a = 2, b = 3, x = X)
exp(sum(diag(X))) * hypergeomPFQ(m = 10, a = 3-2, b = 3, x = -X)
# Herz's relation for 2F1
hypergeomPFQ(10, a = c(1,2), b = 3, x = X)
det(diag(3)-X)^(-2) *
    hypergeomPFQ(10, a = c(3-1,2), b = 3, -X %*% solve(diag(3)-X))
**Description**

Evaluates the incomplete Beta function of a matrix argument.

**Usage**

\[ \text{IncBeta}(m, a, b, x) \]

**Arguments**

- **m**: truncation weight of the summation, a positive integer
- **a, b**: real or complex parameters with Re(a)>(p-1)/2 and Re(b)>(p-1)/2, where p is the dimension (the order of the matrix)
- **x**: either a real positive symmetric matrix or a complex positive Hermitian matrix "smaller" than the identity matrix (i.e. I-x is positive), or a numeric or complex vector, the eigenvalues of the matrix

**Value**

A real or a complex number.

**Note**

The eigenvalues of a real symmetric matrix or a complex Hermitian matrix are always real numbers, and moreover they are positive under the constraints on x. However we allow to input a numeric or complex vector x because the definition of the function makes sense for such a x.

**References**


**Examples**

```r
# for a scalar x, this is the incomplete Beta function:
a <- 2; b <- 3
x <- 0.75
IncBeta(m = 15, a, b, x)
gsl::beta_inc(a, b, x)
 pbeta(x, a, b)
```
IncGamma

Incomplete Gamma function of a matrix argument

Description

Evaluates the incomplete Gamma function of a matrix argument.

Usage

IncGamma(m, a, x)

Arguments

m  truncation weight of the summation, a positive integer
a  real or complex parameter with Re(a)>(p-1)/2, where p is the dimension (the order of the matrix)
x  either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix

Value

A real or complex number.

Note

This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

References


Examples

# for a scalar x, this is the incomplete Gamma function:
a <- 2
x <- 1.5
IncGamma(m = 15, a, x)
gsl::gamma_inc_P(a, x)
pgamma(x, shape = a, rate = 1)
**mvbeta**  
*Multivariate Beta function (of complex variable)*

**Description**

The multivariate Beta function (mvbeta) and its logarithm (lmvbeta).

**Usage**

\[
\text{lmvbeta}(a, b, p) \\
\text{mvbeta}(a, b, p)
\]

**Arguments**

- **a, b**: real or complex numbers with \( \text{Re}(a) > 0 \) and \( \text{Re}(b) > 0 \)
- **p**: a positive integer, the dimension

**Value**

A real or a complex number.

**Examples**

\[
a <- 5; b <- 4; p <- 3 \\
\text{mvbeta}(a, b, p) \\
\text{mvgamma}(a, p) \times \text{mvgamma}(b, p) / \text{mvgamma}(a+b, p)
\]

---

**mvgamma**  
*Multivariate Gamma function (of complex variable)*

**Description**

The multivariate Gamma function (mvgamma) and its logarithm (lmvgamma).

**Usage**

\[
\text{lmvgamma}(x, p) \\
\text{mvgamma}(x, p)
\]

**Arguments**

- **x**: a real or a complex number; \( \text{Re}(x) > 0 \) for \( \text{lmvgamma} \) and \( x \) must not be a negative integer for \( \text{mvgamma} \)
- **p**: a positive integer, the dimension
**Value**

A real or a complex number.

**Examples**

```r
x <- 5
mvgamma(x, p = 2)
sqrt(pi)*gamma(x)*gamma(x-1/2)
```
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