Package ‘LSE’

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**Type**  Package

**Title**  Constrained Least Squares and Generalized QR Factorization

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**Description**  The solution of equality constrained least squares problem (LSE) is given through four analytics methods (Generalized QR Factorization, Lagrange Multipliers, Direct Elimination and Null Space method). We expose the orthogonal decomposition called Generalized QR Factorization (GQR) and also RQ factorization. Finally some codes for the solution of LSE applied in quaternions.

**URL**  https://github.com/sergio05acm/LSE

**Imports**  MASS, pracma

**License**  GPL-3

**Encoding**  UTF-8

**RoxygenNote**  7.1.1

**NeedsCompilation**  no

**Repository**  CRAN

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**R topics documented:**

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Antiquaternion

| Description | A quaternion matrix obtained by the Quaternion function, can be transformed into a matrix, which contains as many quaternions as rows obtained, each column represent an imaginary axis (1,i,j,k). |
| Usage | Antiquaternion(x) |
| Arguments | x | Quaternion matrix object. |
| Details | This function shows in each row a quaternion, and in each column an axis in the order a+bi+cj+dk. |
| Value | Numeric matrix. |
| Author(s) | Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com |
| See Also | See Also as Quaternion |
| Examples | Antiquaternion(Quaternion(1,0,1,0)) |
**Direct Elimination**

*Direct Elimination for LSE problem.*

**Description**

Direct Elimination allows to give an analytic solution for equality constrained least squares problem (LSE). Requires MASS and pracma library.

**Usage**

```
Dir_Elimination(A,C,b,d)
```

**Arguments**

- **A**: Design matrix, m rows and n columns.
- **C**: Constraint matrix, p rows and n columns.
- **b**: Response vector for A, Ax=b, m rows and 1 column.
- **d**: Response vector for C, Cx=d, p rows and 1 column.

**Details**

Direct Elimination method gives a numerical vector as the solution of a least squares problem (Ax=b), when impose some restrictions (additional information, extramuestral information or a priori information) that lead to another linear equality system (Cx=d). See significance constraint (x=0) or inclusion restriction (x+y=1), etc.

**Value**

Numerical vector for a LSE problem.

**Author(s)**

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

**References**


Examples
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)
Dir_Elimination(A,C,b,d)

GQR  Generalized QR Factorization

Description
This code provides a simultaneous orthogonal factorization for two matrices A and B. This code requires pracma library.

Usage
GQR(x,y)

Arguments
x  Numerical matrix with m rows and n columns.
y  Numerical matrix with p rows and n columns.

Details
Given two matrices, with the same number of rows, this algorithm provides a single factorization, such that A=QR and (Q^T)B=WS.

Value
Q  Orthogonal matrix for A
R  Trapezoidal matrix for A
W  Orthogonal matrix for (Q^T)B
S  Trapezoidal matrix for (Q^T)B

Author(s)
Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References
**Lagrange**

**Examples**

\[
\begin{align*}
A &= \text{matrix}(c(1,1,1,1,1,-1,1,1,1),4,3,\text{byrow}=\text{TRUE}) \\
C &= \text{matrix}(c(1,1,1,1,1,-1),2,3,\text{byrow}=\text{TRUE}) \\
\text{GQR}(t(A),t(C))
\end{align*}
\]

---

**Description**

Lagrange multipliers allows to give an analytical solution for equality constrained least squares problem (LSE).

**Usage**

\[
\text{Lagrange}(A, C, b, d)
\]

**Arguments**

- \(A\): Design matrix, \(m\) rows and \(n\) columns.
- \(C\): Constraint matrix, \(p\) rows and \(n\) columns.
- \(b\): Response vector for \(A\), \(Ax=b\), \(m\) rows and 1 column.
- \(d\): Response vector for \(C\), \(Cx=d\), \(p\) rows and 1 column.

**Details**

The Lagrange multipliers method gives a numerical vector as the solution of a least squares problem \((Ax=b)\) through unification the model and their restrictions in one function, the restrictions impose in the model (additional information, extramuestral information or a priori information) lead to another linear equality system \((Cx=d)\). See significance constraint \((x=0)\) or inclusion restriction \((x+y=1)\), etc.

**Value**

Numerical vector for a LSE problem.

**Author(s)**

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

**References**


Examples

```r
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)

Lagrange(A,C,b,d)
```

Description

Solve the equality constrained least squares problem for real value and for quaternions, also allows to use the Generalized QR factorization for two matrices simultaneously.

Author(s)

Sergio Andrés Cabrera Miranda <sergio05acm@gmail.com>

Usage

```r
LSE_GQR(A, C, b, d)
```

Arguments

- **A**: Design matrix, m rows and n columns.
- **C**: Constraint matrix, p rows and n columns.
- **b**: Response vector for A, Ax=b, m rows and 1 column.
- **d**: Response vector for C, Cx=d, p rows and 1 column.

Details

This algorithm provides the solution of the equality constrained least squares problem through Generalized QR factorization. This algorithm requires the same number of columns for matrices A and C.
**Nullspace**

**Value**
Numerical vector for a LSE problem.

**Author(s)**
Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

**References**

**Examples**
```r
A = matrix(c(1,2,3,4,5,6),3,2,byrow = TRUE)
C = matrix(c(1,1),1,2,byrow=TRUE)
b = matrix(c(7,1,3),3,1,byrow=TRUE)
d = matrix(c(1),1,1,byrow=TRUE)
LSE.GQR(A,C,b,d) #You can verify that x+y=1 satisfies the constraint.
```

---

**Nullspace**

**Nullspace method for LSE problem.**

**Description**
Null Space method allows to give an analytic solution for equality constrained least squares problem (LSE). Requires pracma library.

**Usage**
```r
Nullspace(A,C,b,d)
```

**Arguments**

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Design matrix, m rows and n columns.</td>
</tr>
<tr>
<td>C</td>
<td>Constraint matrix, p rows and n columns.</td>
</tr>
<tr>
<td>b</td>
<td>Response vector for A, Ax=b, m rows and 1 column.</td>
</tr>
<tr>
<td>d</td>
<td>Response vector for C, Cx=d, p rows and 1 column.</td>
</tr>
</tbody>
</table>

**Details**
Null Space method gives a numerical vector as the solution of a least squares problem (Ax=b), using an unconstrained problem equivalent to the LSE proposed, this method an be applied when impose some restrictions (additional information, extramuestral information or a priori information) that lead to another linear equality system (Cx=d). See significance constraint (x=0) or inclusion restriction (x+y=1), etc.
**Value**

Numerical vector for a LSE problem.

**Author(s)**

Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

**References**


**Examples**

```r
A = matrix(runif(50,-1,1),10,5)
C = matrix(runif(20,-1,1),4,5)
b = matrix(runif(10,-1,1),10,1)
d = matrix(runif(4,-1,1),4,1)

Nullspace(A,C,b,d)
```

---

<table>
<thead>
<tr>
<th>Quaternion</th>
<th>Quaternion transformation</th>
</tr>
</thead>
</table>

**Description**

A quaternion q=a+bi+cj+dk can be transformed into a real value matrix M(4x4).

**Usage**

```r
Quaternion(a,b,c,d)
```

**Arguments**

<table>
<thead>
<tr>
<th>a</th>
<th>Real value coefficient.</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Coefficient of the imaginary i-axis.</td>
</tr>
<tr>
<td>c</td>
<td>Coefficient of the imaginary j-axis.</td>
</tr>
<tr>
<td>d</td>
<td>Coefficient of the imaginary k-axis.</td>
</tr>
</tbody>
</table>

**Value**

Real value matrix to represent a quaternion.
Author(s)
Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

References

Examples
Quat = Quaternion(1,0,1,0)

rb = rbind(cbind(Quat,-Quat),
          cbind(Quat,Quat))

RQ Factorization of a matrix

Description
RQ factorization allows to develop an orthogonal transformation in a matrix through Householder reflections. Requires pracma package.

Usage
RQ(y)

Arguments
y Numeric matrix or vector.

Details
RQ factorization make a orthogonal transformation at the rows of the matrix, beginning in the last one, and finishing with the first one row.

Value
Q Orthogonal matrix for x
R Triangular matrix for x

Author(s)
Sergio Andrés Cabrera Miranda Statician sergio05acm@gmail.com

Examples
A = matrix(runif(12,0,5),4,3,byrow=TRUE)
RQ(A)
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