Package ‘ML2Pvae’

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Type Package

Title Variational Autoencoder Models for IRT Parameter Estimation

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Maintainer Geoffrey Converse <converseg@gmail.com>

Description Based on the work of Curi, Converse, Hajewski, and Oliveira (2019) <doi:10.1109/IJCNN.2019.8852333>. This package provides easy-to-use functions which create a variational autoencoder (VAE) to be used for parameter estimation in Item Response Theory (IRT) - namely the Multidimensional Logistic 2-Parameter (ML2P) model. To use a neural network as such, nontrivial modifications to the architecture must be made, such as restricting the nonzero weights in the decoder according to some binary matrix Q. The functions in this package allow for straight-forward construction, training, and evaluation so that minimal knowledge of ‘tensorflow’ or ‘keras’ is required.

Note The developer version of ‘keras’ should be used, rather than the CRAN version. The latter will cause tests to fail on an initial run, but work on subsequent tries. To avoid this, use devtools::install_github("rstudio/keras"). The user also must have an installation of 'Python 3'.

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Encoding UTF-8

LazyData true

Imports keras (>= 2.3.0), reticulate (>= 1.0), tensorflow (>= 2.2.0),
tfprobability (>= 0.11.0)

RoxygenNote 7.1.1

Suggests knitr, rmarkdown, testthat, R.rsp

VignetteBuilder R.rsp

Depends R (>= 3.6)

URL https://converseg.github.io

Display a message upon loading package

**Description**

Display a message upon loading package

**Usage**

```r
.onLoad(libnam, pkgname)
```
**build_hidden_encoder**  
*Build the encoder for a VAE*

---

**Arguments**

libnam  
the library name

pkgname  
the package name

---

**Description**

Build the encoder for a VAE

**Usage**

```r
build_hidden_encoder(
  input_size,
  layers,
  activations = rep("sigmoid", length(layers))
)
```

**Arguments**

input_size  
an integer representing the number of items

layers  
a list of integers giving the size of each hidden layer

activations  
a list of strings, the same length as layers

**Value**

two tensors: the input layer to the VAE and the last hidden layer of the encoder

---

**build_vae_correlated**  
*Build a VAE that fits to a normal, full covariance N(m,S) latent distribution*

---

**Description**

Build a VAE that fits to a normal, full covariance N(m,S) latent distribution
Usage

```r
build_vae_correlated(
  num_items,
  num_skills,
  Q_matrix,
  mean_vector = rep(0, num_skills),
  covariance_matrix = diag(num_skills),
  model_type = 2,
  enc_hid_arch = c(ceiling((num_items + num_skills)/2)),
  hid_enc_activations = rep("sigmoid", length(enc_hid_arch)),
  output_activation = "sigmoid",
  kl_weight = 1,
  learning_rate = 0.001
)
```

Arguments

- `num_items`: an integer giving the number of items on the assessment; also the number of nodes in the input/output layers of the VAE.
- `num_skills`: an integer giving the number of skills being evaluated; also the dimensionality of the distribution learned by the VAE.
- `Q_matrix`: a binary, `num_skills` by `num_items` matrix relating the assessment items with skills.
- `mean_vector`: a vector of length `num_skills` specifying the mean of each latent trait; the default of `rep(0,num_skills)` should almost always be used.
- `covariance_matrix`: a symmetric, positive definite, `num_skills` by `num_skills` matrix giving the covariance of the latent traits.
- `model_type`: either 1 or 2, specifying a 1 parameter (1PL) or 2 parameter (2PL) model; if 1PL, then all decoder weights are fixed to be equal to one.
- `enc_hid_arch`: a vector detailing the size of hidden layers in the encoder; the number of hidden layers is determined by the length of this vector.
- `hid_enc_activations`: a vector specifying the activation function in each hidden layer in the encoder; must be the same length as `enc_hid_arch`.
- `output_activation`: a string specifying the activation function in the output of the decoder; the ML2P model always used 'sigmoid'.
- `kl_weight`: an optional weight for the KL divergence term in the loss function.
- `learning_rate`: an optional parameter for the adam optimizer.

Value

returns three keras models: the encoder, decoder, and vae.
Examples

```r
Q <- matrix(c(1,0,1,1,0,1,1,0), nrow = 2, ncol = 4)
cov <- matrix(c(.7,.3,.3,1), nrow = 2, ncol = 2)
models <- build_vae_correlated(4, 2, Q,
  mean_vector = c(-0.5, 0),
  covariance_matrix = cov,
  enc_hid_arch = c(6, 3),
  hid_enc_activation = c('sigmoid', 'relu'),
  output_activation = 'tanh',
  kl_weight = 0.1)
vae <- models[[3]]
```

---

**build_vae_independent**  
*Build a VAE that fits to a standard N(0,I) latent distribution with independent latent traits*

Description

Build a VAE that fits to a standard N(0,I) latent distribution with independent latent traits

Usage

```r
build_vae_independent(
  num_items,
  num_skills,
  Q_matrix,
  model_type = 2,
  enc_hid_arch = c(ceiling((num_items + num_skills)/2)),
  hid_enc_activations = rep("sigmoid", length(enc_hid_arch)),
  output_activation = "sigmoid",
  kl_weight = 1,
  learning_rate = 0.001
)
```

Arguments

- **num_items**: an integer giving the number of items on the assessment; also the number of nodes in the input/output layers of the VAE
- **num_skills**: an integer giving the number of skills being evaluated; also the dimensionality of the distribution learned by the VAE
- **Q_matrix**: a binary, `num_skills` by `num_items` matrix relating the assessment items with skills
- **model_type**: either 1 or 2, specifying a 1 parameter (1PL) or 2 parameter (2PL) model; if 1PL, then all decoder weights are fixed to be equal to one
- **enc_hid_arch**: a vector detailing the size of hidden layers in the encoder; the number of hidden layers is determined by the length of this vector
hid_enc_activations
  a vector specifying the activation function in each hidden layer in the encoder;
  must be the same length as enc_hid_arch
output_activation
  a string specifying the activation function in the output of the decoder; the ML2P
  model always uses 'sigmoid'
kl_weight
  an optional weight for the KL divergence term in the loss function
learning_rate
  an optional parameter for the adam optimizer

Value

returns three keras models: the encoder, decoder, and vae.

Examples

Q <- matrix(c(1,0,1,1,0,1,1,0), nrow = 2, ncol = 4)
models <- build_vae_independent(4, 2, Q,
  enc_hid_arch = c(6, 3), hid_enc_activation = c('sigmoid', 'relu'),
  output_activation = 'tanh', kl_weight = 0.1)
models <- build_vae_independent(4, 2, Q)
vae <- models[[3]]
diff_true

Simulated difficulty parameters

Description
Difficulty parameters for an exam with 30 items.

Usage
diff_true

Format
A data frame with 30 rows and one column. Each entry corresponds to the true value of a particular difficulty parameter.

Source
Each entry is sampled uniformly from [-3, 3].

disc_true

Simulated discrimination parameters

Description
Difficulty parameters for an exam of 30 items assessing 3 latent abilities.

Usage
disc_true

Format
A data frame with 3 rows and 30 columns. Entry \([k, i]\) represents the discrimination parameter between item \(i\) and ability \(k\).

Source
Each entry is sampled uniformly from \([0.25, 1.75]\). If an entry in q_matrix.rda is 0, then so is the corresponding entry in disc_true.rda.
get_ability_parameter_estimates

*Feed forward response sets through the encoder, which outputs student ability estimates*

**Description**

Feed forward response sets through the encoder, which outputs student ability estimates

**Usage**

```r
getAbilityParameterEstimates(encoder, responses)
```

**Arguments**

- `encoder`: a trained keras model; should be the encoder returned from either `build_vae_independent()` or `build_vae_correlated`
- `responses`: a `num_students` by `num_items` matrix of binary responses, as used in training

**Value**

a list where the first entry contains student ability estimates and the second entry holds the variance (or covariance matrix) of those estimates

**Examples**

```r
data <- matrix(c(1,1,0,0,1,0,1,1,0,1,1,0), nrow = 3, ncol = 4)
Q <- matrix(c(1,0,1,1,0,1,1,0), nrow = 2, ncol = 4)
models <- build_vae_independent(4, 2, Q, model_type = 2)
encoder <- models[[1]]
ability_parameter_estimates_variances <- get_ability_parameter_estimates(encoder, data)
student_ability_est <- ability_parameter_estimates_variances[,1]
```

get_item_parameter_estimates

*Get trainable variables from the decoder, which serve as item parameter estimates.*

**Description**

Get trainable variables from the decoder, which serve as item parameter estimates.

**Usage**

```r
getItemParameterEstimates(decoder, model_type = 2)
```
ML2Pvae

Arguments

- decoder: a trained keras model; can either be the decoder or vae returned from build_vae_independent() or build_vae_correlated
- model_type: either 1 or 2, specifying a 1 parameter (1PL) or 2 parameter (2PL) model; if 1PL, then only the difficulty parameter estimates (output layer bias) will be returned; if 2PL, then the discrimination parameter estimates (output layer weights) will also be returned

Value

a list which contains item parameter estimates; the length of this list is equal to model_type - the first entry in the list holds the difficulty parameter estimates, and the second entry (if 2PL) contains discrimination parameter estimates

Examples

```r
Q <- matrix(c(1,0,1,1,0,1,1,0), nrow = 2, ncol = 4)
models <- build_vae_independent(4, 2, Q, model_type = 2)
decoder <- models[[2]]
item_parameter_estimates <- get_item_parameter_estimates(decoder, model_type = 2)
difficulty_est <- item_parameter_estimates[[1]]
discrimination_est <- item_parameter_estimates[[2]]
```

ML2Pvae: A package for creating a VAE whose decoder recovers the parameters of the ML2P model. The encoder can be used to predict the latent skills based on assessment scores.

Description

The ML2Pvae package includes functions which build a VAE with the desired architecture, and fits the latent skills to either a standard normal (independent) distribution, or a multivariate normal distribution with a full covariance matrix. Based on the work "Interpretable Variational Autoencoders for Cognitive Models" by Curi, M., Converse, G., Hajewski, J., and Oliveira, S. Found in International Joint Conference on Neural Networks, 2019.
q_1pl_constraint

A custom kernel constraint function that forces nonzero weights to be equal to one, so the VAE will estimate the 1-parameter logistic model. Nonzero weights are determined by the Q matrix.

Usage

q_1pl_constraint(Q)

Arguments

Q  

a binary matrix of size num_skills by num_items

Value

returns a function whose parameters match keras kernel constraint format

q_constraint

A custom kernel constraint function that restricts weights between the learned distribution and output. Nonzero weights are determined by the Q matrix.

Usage

q_constraint(Q)

Arguments

Q  

a binary matrix of size num_skills by num_items

Value

returns a function whose parameters match keras kernel constraint format
Description
The Q-matrix determines the relation between items and abilities.

Usage
q_matrix

Format
A data frame with 3 rows and 30 columns. If entry \([k, i] = 1\), then item \(i\) requires skill \(k\).

Source
Generated by sampling each entry from \(\text{Bernoulli}(0.35)\), but ensures each item assess at least one latent ability.

Description
Simulated response sets for 5000 students on an exam with 30 items.

Usage
responses

Format
A data frame with 30 columns and 5000 rows. Entry \([j, i] = 1\) if student \(j\) answers item \(i\) correctly, and 0 otherwise.

Source
Generated by sampling from the probability of student success on a given item according to the ML2P model. Model parameters can be found in \text{diff_true.rda}, \text{disc_true.rda}, and \text{theta_true.rda}.
**sampling_correlated**

*A reparameterization in order to sample from the learned multivariate normal distribution of the VAE*

---

**Description**

A reparameterization in order to sample from the learned multivariate normal distribution of the VAE

**Usage**

`sampling_correlated(arg)`

**Arguments**

- `arg` a layer of tensors representing the mean and log cholesky transform of the covariance matrix

---

**sampling_independent**

*A reparameterization in order to sample from the learned standard normal distribution of the VAE*

---

**Description**

A reparameterization in order to sample from the learned standard normal distribution of the VAE

**Usage**

`sampling_independent(arg)`

**Arguments**

- `arg` a layer of tensors representing the mean and variance
theta_true  Simulated ability parameters

Description
Three correlated ability parameters for 5000 students.

Usage
theta_true

Format
A data frame with 5000 rows and 3 columns. Each row represents a particular student’s three latent abilities.

Source
Generated by sampling from a 3-dimensional multivariate Gaussian distribution with mean 0 and covariance matrix correlation_matrix.rda.

train_model  Trains a VAE or autoencoder model. This acts as a wrapper for keras::fit().

Description
Trains a VAE or autoencoder model. This acts as a wrapper for keras::fit().

Usage
train_model(
  model,
  train_data,
  num_epochs = 10,
  batch_size = 1,
  validation_split = 0.15,
  shuffle = FALSE,
  verbose = 1
)
**Arguments**

- **model**
  The keras model to be trained; this should be the VAE returned from `build_vae_independent()` or `build_vae_correlated`

- **train_data**
  Training data; this should be a binary `num_students` by `num_items` matrix of student responses to an assessment

- **num_epochs**
  Number of epochs to train for

- **batch_size**
  Batch size for mini-batch stochastic gradient descent; default is 1, detailing pure SGD; if a larger batch size is used (e.g. 32), then a larger number of epochs should be set (e.g. 50)

- **validation_split**
  Split percentage to use as validation data

- **shuffle**
  Whether or not to shuffle data

- **verbose**
  Verbosity levels; 0 = silent; 1 = progress bar and epoch message; 2 = epoch message

**Value**

A list containing training history; this holds the loss from each epoch which can be plotted

**Examples**

```r
data <- matrix(c(1,1,0,0,1,0,1,1,0,1,1,0), nrow = 3, ncol = 4)
Q <- matrix(c(1,0,1,1,0,1,1,0), nrow = 2, ncol = 4)
models <- build_vae_independent(4, 2, Q)
vaes <- models[[3]]
history <- train_model(vaes, data, num_epochs = 3, validation_split = 0, verbose = 0)
plot(history)
```

---

**vae_loss_correlated**

*A custom loss function for a VAE learning a multivariate normal distribution with a full covariance matrix*

**Description**

A custom loss function for a VAE learning a multivariate normal distribution with a full covariance matrix

**Usage**

```r
vae_loss_correlated(
  encoder,
  inv_skill_cov,
  det_skill_cov,
  skill_mean,
)```
vae_loss_independent

kls_weight,
rec_dim
)

Arguments

the encoder model of the VAE, used to obtain z_mean and z_log_cholesky from inputs
inv_skill_cov a constant tensor matrix of the inverse of the covariance matrix being learned
det_skill_cov a constant tensor scalar representing the determinant of the covariance matrix being learned
skill_mean a constant tensor vector representing the means of the latent skills being learned
kl_weight weight for the KL divergence term
rec_dim the number of nodes in the input/output of the VAE

Value

returns a function whose parameters match keras loss format

Description

A custom loss function for a VAE learning a standard normal distribution

Usage

vae_loss_independent(encoder, kl_weight, rec_dim)

Arguments

the encoder model of the VAE, used to obtain z_mean and z_log_var from inputs
kl_weight weight for the KL divergence term
rec_dim the number of nodes in the input/output of the VAE

Value

returns a function whose parameters match keras loss format
validate_inputs

Give error messages for invalid inputs in exported functions.

Description

Give error messages for invalid inputs in exported functions.

Usage

validate_inputs(
  num_items,  
  num_skills,  
  Q_matrix,  
  model_type = 2,  
  mean_vector = rep(0, num_skills),  
  covariance_matrix = diag(num_skills),  
  enc_hid_arch = c(ceiling((num_items + num_skills)/2)),  
  hid_enc_activations = rep("sigmoid", length(enc_hid_arch)),  
  output_activation = "sigmoid",  
  kl_weight = 1,  
  learning_rate = 0.001
)

Arguments

num_items  the number of items on the assessment; also the number of nodes in the input/output layers of the VAE
num_skills  the number of skills being evaluated; also the size of the distribution learned by the VAE
Q_matrix  a binary, num_skills by num_items matrix relating the assessment items with skills
model_type  either 1 or 2, specifying a 1 parameter (1PL) or 2 parameter (2PL) model
mean_vector  a vector of length num_skills specifying the mean of each latent trait
covariance_matrix  a symmetric, positive definite, num_skills by num_skills, matrix giving the covariance of the latent traits
enc_hid_arch  a vector detailing the number an size of hidden layers in the encoder
hid_enc_activations  a vector specifying the activation function in each hidden layer in the encoder; must be the same length as enc_hid_arch
output_activation  a string specifying the activation function in the output of the decoder; the ML2P model alsways used 'sigmoid'
kl_weight  an optional weight for the KL divergence term in the loss function
learning_rate  an optional parameter for the adam optimizer
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