Robust Regression with Particle Swarm Optimisation and Differential Evolution

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1 Introduction

We provide a code example for a robust regression problem; for more details, please see Gilli et al. [2011]. (The vignette builds on the script comparisonLMS.R.)

2 Data and settings

We start by attaching the NMOF package and fixing a seed. We will use the function lqs from the MASS package [Venables and Ripley, 2002], so we attach that package as well.

```
> require("NMOF")
> require("MASS")
> set.seed(11223344)
```

We will use an artificial data set with n observations and p regressors, created with the function createData.

```
> createData <- function(n, p, constant = TRUE,
                         sigma = 2, oFrac = 0.1) {
      X <- array(rnorm(n * p), dim = c(n, p))
      if (constant)
          X[ ,1L] <- 1L
      b <- rnorm(p)
      y <- X %*% b + rnorm(n)*0.5
      nO <- ceiling(oFrac*n)
      when <- sample.int(n, nO)
      X[when, -1L] <- X[when, -1L] + rnorm(nO, sd = sigma)
      list(X = X, y = y, outliers = when)
  }
```

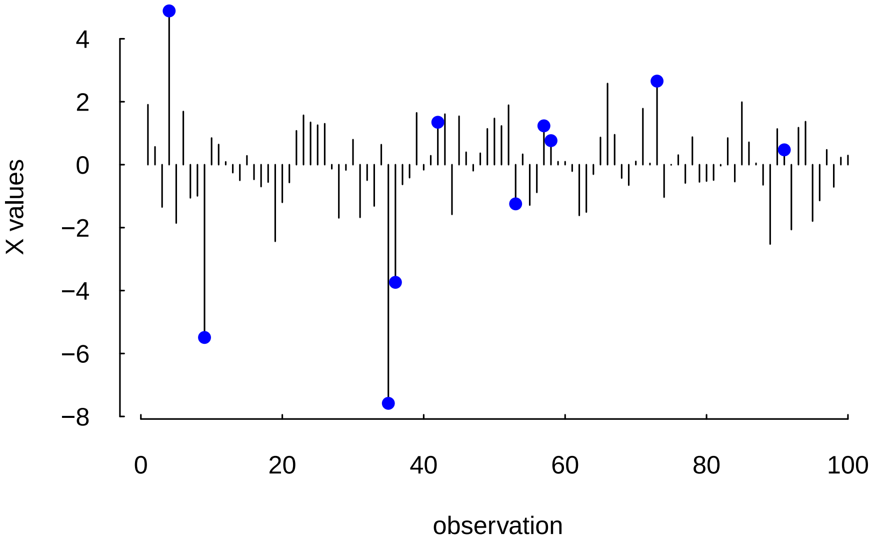
The function also takes arguments constant (logical: should the data-generating model contain a constant?); sigma (standard deviation of the outliers); and oFrac (fraction of outliers). The function evaluates to a list containing the regressors X, the regressand y and a list of the outliers.

We put X and y into the list Data. We also add the scalar h, which gives the order statistic of the squared residuals to be minimised. Note that we put as.vector(y) into Data so that the vector gets 'recycled' in the objective function.

```
> n <- 100L   ## number of observations
> p <- 10L    ## number of regressors
> constant <- TRUE; sigma <- 5; oFrac  <- 0.1
> h <- 75L    ## ... or use something like floor((n+1)/2)
> aux <- createData(n, p, constant, sigma, oFrac)
> X <- aux$X; y <- aux$y
> Data <- list(y = as.vector(y), X = X, h = h)
```

The outliers, added in blue, are often visible.

```
> par(bty = "n", las = 1, tck = 0.01, mar = c(4,4,1,1))
> plot(X[ ,2L], type = "h", ylab = "X values", xlab = "observation")
> lines(aux$outliers, X[aux$outliers ,2L], type = "p", pch = 21,
        col = "blue", bg = "blue")
```



Two example objective functions, Least Trimmed Squares (LTS) and Least Quantile of Squares (LQS). Note that they are identical except for their last line.

```
> OF <- function(param, Data) {
      X <- Data$X; y <- Data$y
      aux <- y - X %*% param
      aux <- aux * aux
      aux <- apply(aux, 2L, sort, partial = Data$h)
      colSums(aux[1:Data$h, ])  ## LTS
  }
> OF <- function(param, Data) {
      X <- Data$X; y <- Data$y
      aux <- y - X %*% param
      aux <- aux * aux
      aux <- apply(aux, 2L, sort, partial = Data$h)
      aux[Data$h, ]  ## LQS
  }
```

Both functions are vectorised. They work with a single solution (`param` would be a vector) or a whole population (`param` would be a matrix; each column would be one solution).

3 Using DE and PSO

We run DE and PSO. We compare the result with `lqs`.

```
> popsize <- 100L; generations <- 500L
> ps <- list(min = rep(-10,p),
             max = rep( 10,p),
             c1 = 0.9,
             c2 = 0.9,
             iner = 0.9,
             initV = 1,
             nP = popsize,
             nG = generations,
             maxV = 5,
             loopOF = FALSE,
             printBar = FALSE,
             printDetail = FALSE)
> de <- list(min = rep(-10,p),
             max = rep( 10,p),
             nP = popsize,
```

```
             nG = generations,
             F = 0.7,
             CR = 0.9,
             loopOF = FALSE,
             printBar = FALSE,
             printDetail = FALSE)
> system.time(solPS <- PSopt(OF = OF, algo = ps, Data = Data))
```

```
   user  system elapsed 
  1.080   0.005   1.085 
```

```
> system.time(solDE <- DEopt(OF = OF, algo = de, Data = Data))
```

```
   user  system elapsed 
  1.068   0.000   1.069 
```

```
> if (require("MASS", quietly = TRUE)) {
      system.time(test1 <- lqs(y ~ X[ ,-1L], adjust = TRUE,
                                nsamp = 100000L, method = "lqs",
                                quantile = h))
      res1 <- sort((y - X %*% as.matrix(coef(test1)))^2)[h]
  } else
      res1 <- NA
> res2 <- sort((y - X %*% as.matrix(solPS$xbest))^2)[h]
> res3 <- sort((y - X %*% as.matrix(solDE$xbest))^2)[h]
> cat("lqs:   ", res1, "\n",
      "PSopt: ", res2, "\n",
      "DEopt: ", res3, "\n", sep = "")
```

```
lqs:   0.38073
PSopt: 0.25895
DEopt: 0.28958
```

To demonstrate the advantage of a vectorised objective function, we can compare it with looping over the solutions. We first set `loopOF` to `TRUE`, so we actually loop over the solutions. (We also reduce the number of objective function evaluations since we do not care about the actual solution, only about speed of computation.)

```
> popsize <- 100L; generations <- 20L
> de$nP <- popsize; de$nG <- generations
> ps$nP <- popsize; ps$nG <- generations
> de$loopOF <- TRUE; ps$loopOF <- TRUE
> t1ps <- system.time(solPS <- PSopt(OF = OF, algo = ps, Data = Data))
> t1de <- system.time(solDE <- DEopt(OF = OF, algo = de, Data = Data))
```

To evaluate the objective function in one step, we `loopOF` to `FALSE`.

```
> de$loopOF <- FALSE; ps$loopOF <- FALSE
> t2ps <- system.time(solPS <- PSopt(OF = OF, algo = ps, Data = Data))
> t2de <- system.time(solDE <- DEopt(OF = OF, algo = de, Data = Data))
```

Speedup:

```
> t1ps[[3L]]/t2ps[[3L]]  ## PS
```

```
[1] 2.7209
```

```
> t1de[[3L]]/t2de[[3L]]  ## DE
```

```
[1] 2.7381
```

References

Manfred Gilli, Dietmar Maringer, and Enrico Schumann. *Numerical Methods and Optimization in Finance*. Elsevier, 2011.

William N. Venables and Brian D. Ripley. *Modern Applied Statistics with S*. Springer, 4th edition, 2002. URL `http://www.stats.ox.ac.uk/pub/MASS4`.