Package ‘Pade’

December 9, 2019

Type Package
Title Padé Approximant Coefficients
Version 1.0.0
Date 2019-12-08
Description Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order.
License GPL (>= 2) | BSD_2_clause + file LICENSE
Imports utils
Suggests testthat
URL https://github.com/aadler/Pade
BugReports https://github.com/aadler/Pade/issues
Encoding UTF-8
NeedsCompilation no
Author Avraham Adler [aut, cph, cre] (<https://orcid.org/0000-0002-3039-0703>)
Maintainer Avraham Adler <Avraham.Adler@gmail.com>
Repository CRAN
Date/Publication 2019-12-09 01:00:02 UTC

R topics documented:

  Pade-package .................................................. 2
  Pade ......................................................... 2

Index 4
Description

Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order.

Author(s)

Maintainer: Avraham Adler <Avraham.Adler@gmail.com>

Usage

Pade(L, M, A)

Arguments

L  Order of Padé numerator
M  Order of Padé denominator
A  vector of Taylor series coefficients, starting at $x^0$

Details

As the Taylor series expansion is the “best” polynomial approximation to a function, the Padé approximants are the “best” rational function approximations to the original function. The Padé approximant often has a wider radius of convergence than the corresponding Taylor series, and can even converge where the Taylor series does not, which makes it very suitable for computer-based numerical analysis.

The $[L/M]$ Padé approximant to a Taylor series $A(x)$ is the quotient

$$\frac{P_L(x)}{Q_M(x)}$$

where where $P_L(x)$ is of order $L$ and $Q_M(x)$ is of order $M$. In this case:

$$A(x) - \frac{P_L(x)}{Q_M(x)} = O(x^{L+M+1})$$
When $q_0$ is defined to be 1, there is a unique solution to the system of linear equations which can be used to calculate the coefficients.

The function accepts a vector $A$ of length $T + 1$, composed of the $a_n$ of the truncated Taylor series

$$A(x) = \sum_{j=0}^{T} a_j x^j$$

and returns a list of two elements, $P_x$ and $Q_x$, the Padé numerator and denominator coefficients respectively, as long as $L + M \leq T$.

Value

Pade returns a list with two entries:

$P_x$ Coefficients of the numerator polynomial starting at $x^0$.

$Q_x$ Coefficients of the denominator polynomial starting at $x^0$.

Author(s)

Avraham Adler <Avraham.Adler@gmail.com>

References


See Also

This package provides similar functionality to the pade function in the pracma package. However, it does not allow computation of coefficients beyond the supplied Taylor coefficients and it expects its input and provides its output in ascending, instead of descending, order.

Examples

```r
A <- 1 / factorial(seq_len(11) - 1) ## Taylor sequence for $e^x$ up to $x^{10}$ around $x_0 = 0$
Z <- Pade(5, 5, A)
print(Z) ## Padé approximant of order [5 / 5]
x <- -.01 ## Test value
Actual <- exp(x) ## Proper value
print(Actual, digits = 16)
Estimate <- sum(Z[[1]] * x ^ (seq_along(Z[[1]]) - 1)) / sum(Z[[2]] * x ^ (seq_along(Z[[2]]) - 1))
print(Estimate, digits = 16) ## Approximant value
all.equal(Actual, Estimate)
```
Index

*Topic **NumericalMathematics**
  Pade, 2
  Pade-package, 2
*Topic **package**
  Pade-package, 2

Pade, 2
pade, 3
Pade-package, 2