Package ‘Pade’

August 29, 2016

Type Package
Title Padé Approximant Coefficients
Version 0.1-4
Date 2015-07-29
Description Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order.
License GPL (>= 2) | BSD_2_clause + file LICENSE
Imports utils
Suggests testthat
URL https://bitbucket.org/aadler/pade
BugReports https://bitbucket.org/aadler/pade/issues
Encoding UTF-8
NeedsCompilation no
Author Avraham Adler [aut, cph, cre]
Maintainer Avraham Adler <Avraham.Adler@gmail.com>
Repository CRAN
Date/Publication 2015-07-29 20:48:44

R topics documented:

Pade-package ................................................................. 2
Pade ................................................................. 2

Index 4
Description

Given a vector of Taylor series coefficients of sufficient length as input, the function returns the numerator and denominator coefficients for the Padé approximant of appropriate order.

Author(s)

Maintainer: Avraham Adler <Avraham.Adler@gmail.com>

Usage

Pade(L, M, A)

Arguments

L Order of Padé numerator
M Order of Padé denominator
A vector of Taylor series coefficients, starting at \( x^0 \)

Details

As the Taylor series expansion is the “best” polynomial approximation to a function, the Padé approximants are the “best” rational function approximations to the original function. The Padé approximant often has a wider radius of convergence than the corresponding Taylor series, and can even converge where the Taylor series does not, which makes it very suitable for computer-based numerical analysis.

The \([L/M]\) Padé approximant to a Taylor series \( A(x) \) is the quotient

\[
\frac{P_L(x)}{Q_M(x)}
\]

where \( P_L(x) \) is of order \( L \) and \( Q_M(x) \) is of order \( M \). In this case:

\[
A(x) - \frac{P_L(x)}{Q_M(x)} = \mathcal{O}(x^{L+M+1})
\]
When \( q_0 \) is defined to be 1, there is a unique solution to the system of linear equations which can be used to calculate the coefficients.

The function accepts a vector \( A \) of length \( T + 1 \), composed of the \( a_n \) of the of truncated Taylor series

\[
A(x) = \sum_{j=0}^{T} a_j x^j
\]

and returns a list of two elements, \( P_x \) and \( Q_x \), the Padé numerator and denominator coefficients respectively, as long as \( L + M \leq T \).

**Value**

Pade returns a list with two entries:

- \( P_x \): Coefficients of the numerator polynomial starting at \( x^0 \).
- \( Q_x \): Coefficients of the denominator polynomial starting at \( x^0 \).

**Author(s)**

Avraham Adler <avraham.adler@gmail.com>

**References**


**See Also**

This package provides similar functionality to the `pade` function in the `pracma` package. However, it does not allow computation of coefficients beyond the supplied Taylor coefficients and it expects its input and provides its output in ascending, instead of descending, order.

**Examples**

```r
A <- 1 / factorial(seq_len(11) - 1) ## Taylor sequence for e^x up to x^[10] around x_0 = 0
Z <- Pade(5, 5, A)
print(Z) ## Padé approximant of order [5 / 5]
X <- .01
Actual <- exp(X)
## Test value
print(Actual, digits = 16)
Estimate <- sum(Z[1] * X ^ (seq_along(Z[1]) - 1)) / sum(Z[2] * X ^ (seq_along(Z[2]) - 1))
print(Estimate, digits = 16) ## Approximant value
all.equal(Actual, Estimate)
```
Index

* Topic NumericalMathematics
  Pade, 2
  Pade-package, 2
* Topic package
  Pade-package, 2

Pade, 2
pade, 3
Pade-package, 2