Package ‘Pareto’

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Version 1.1.5
Description Utilities for the Pareto and piecewise Pareto distribution that are useful for reinsurance pricing. In particular, the package provides a non-trivial algorithm that can be used to match the expected losses of a tower of reinsurance layers with a layer-independent collective risk model. The theoretical background of the matching algorithm and most other methods are described in Ulrich Riegel (2018) <doi:10.1007/s13385-018-0177-3>.
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dPareto ......................................................... 2
dPiecewisePareto ........................................... 3
Example1_AP .................................................. 4
Example1_EL .................................................. 4
Density of the Pareto Distribution

Description

Calculates the density function of the Pareto distribution

Usage

dPareto(x, t, alpha, truncation = NULL)

Arguments

x
Numeric. The function evaluates the density at x.

t
Numeric. Threshold of the Pareto distribution.

alpha
Numeric. Pareto alpha.

truncation
Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
**dPiecewisePareto**

**Value**

Density function of the Pareto distribution with parameters \(t\) and \(\alpha\) evaluated at \(x\).

**Examples**

```r
x <- 0:10 * 1000
dPareto(x, 1000, 2)
dPareto(x, 1000, 2, truncation = 5000)
```

---

**dPiecewisePareto**  
*Density of the Piecewise Pareto Distribution*

**Description**

Calculates the density function of the piecewise Pareto distribution.

**Usage**

```r
dPiecewisePareto(x, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

- **x**: Numeric. The function evaluates the density at \(x\).
- **t**: Numeric vector. Thresholds of the piecewise Pareto distribution.
- **alpha**: Numeric vector. \(\alpha[i]\) is the Pareto alpha in excess of \(t[i]\).
- **truncation**: Numeric. If truncation is not NULL and truncation > \(t\), then the Pareto distribution is truncated at truncation.
- **truncation_type**: Character. If \(\text{truncation_type} = \"wd\"\) then the whole distribution is truncated. If \(\text{truncation_type} = \"lp\"\) then a truncated Pareto is used for the last piece.

**Value**

Density function of the piecewise Pareto distribution with parameter vectors \(t\) and \(\alpha\) evaluated at \(x\).

**Examples**

```r
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
x <- 0:10 * 1000
dPiecewisePareto(x, t, alpha)
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
dPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```
Example 1: Attachment Points

**Description**
Example data: Attachment Points

**Usage**
Example 1 AP

**Format**
An object of class numeric of length 5.

---

Example 1: Expected Losses

**Description**
Example data: Expected Losses

**Usage**
Example 1 EL

**Format**
An object of class numeric of length 5.

---

Local Pareto Alpha

**Description**
Calculates the local Pareto alpha of the normal, lognormal and gamma distribution

**Usage**
Local_Pareto_Alpha(x, distribution, ...)

**Arguments**

- `x` Numeric. Vector of thresholds at which the local Pareto alpha is calculated.
- `distribution` Character. Arguments for the selected distribution
  - 'lnorm' for lognormal distribution (arguments: meanlog, sdlog)
  - 'norm' for normal distribution (arguments: mean, sd)
  - 'gamma' for gamma distribution (arguments: shape, rate, scale)

**Value**

Local Pareto alpha of the selected distribution at `x`

**References**


**Examples**

```r
x <- 1:10
Local_Pareto_Alpha(x, "norm", mean = 1, sd = 5)

x <- 1:10 * 1000000
Local_Pareto_Alpha(x, "lnorm", meanlog = 1, sdlog = 5)
```

---

**Pareto_CDF**  
*Distribution Function of the Pareto Distribution*

**Description**

Calculates the cumulative distribution function of a Pareto distribution. This function is deprecated. Use `pPareto` instead.

**Usage**

`Pareto_CDF(x, t, alpha, truncation = NULL)`

**Arguments**

- `x` Numeric. The function evaluates the CDF at `x`.
- `t` Numeric. Threshold of the Pareto distribution.
- `alpha` Numeric. Pareto alpha.
- `truncation` Numeric. If truncation is not `NULL` and `truncation > t`, then the Pareto distribution is truncated at `truncation`.
Value

Distribution function of the Pareto distribution with parameters \( t \) and \( \alpha \) evaluated at \( x \)

Examples

```r
x <- 0:10 * 1000
pPareto(x, 1000, 2)
pPareto(x, 1000, 2, truncation = 5000)
```

Description

Uses a Pareto distribution to derive the expected loss of a layer from the expected loss of another layer.

Usage

```r
Pareto_Extrapolation(
  Cover_1,
  AttachmentPoint_1,
  Cover_2,
  AttachmentPoint_2,
  alpha,
  ExpLoss_1 = NULL,
  truncation = NULL
)
```

Arguments

- **Cover_1**: Numeric. Cover of the layer from which we extrapolate. Use \( \text{Inf} \) for unlimited layers.
- **AttachmentPoint_1**: Numeric. Attachment point of the layer from which we extrapolate.
- **Cover_2**: Numeric. Cover of the layer to which we extrapolate. Use \( \text{Inf} \) for unlimited layers.
- **AttachmentPoint_2**: Numeric. Attachment point of the layer to which we extrapolate.
- **alpha**: Numeric. Pareto alpha used for the extrapolation.
- **ExpLoss_1**: Numeric. Expected loss of the layer from which we extrapolate. If NULL (default) then the function provides only the ratio between the expected losses of the layers.
- **truncation**: Numeric. If truncation is not NULL and \( \text{truncation} > \text{AttachmentPoint}_1 \), then the Pareto distribution is truncated at \( \text{truncation} \).
**Value**

The expected loss of the layer \( \text{Cover}_2 \times \text{AttachmentPoint}_2 \) given that \( \text{Cover}_1 \times \text{AttachmentPoint}_1 \) has expected loss \( \text{ExpLoss}_1 \) and assuming a (truncated) Pareto distribution with parameters \( t \) and \( \alpha \). If missing then \( \text{ExpLoss}_1 = 1 \) is assumed.

**References**


**Examples**

- `Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, ExpLoss_1 = 100)`
- `Pareto_Extrapolation(1000, 1000, 2000, 2000, 2) * 100`
- `Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, truncation = 5000, ExpLoss_1 = 100)`
- `Pareto_Extrapolation(1000, 1000, 2000, 2000, 2, truncation = 5000) * 100`

---

**Pareto_Find_Alpha_btw_FQs**

*Pareto Alpha Between Two Frequencies*

**Description**

Finds the Pareto alpha between two excess frequencies

**Usage**

```r
Pareto_Find_Alpha_btw_FQs(
  Threshold_1,
  Frequency_1,
  Threshold_2,
  Frequency_2,
  max_alpha = 100,
  tolerance = 1e-10,
  truncation = NULL
)
```

**Arguments**

- **Threshold_1**  Numeric. Threshold 1
- **Frequency_1**  Numeric. Expected frequency in excess of **Threshold_1**
- **Threshold_2**  Numeric. Threshold 2
- **Frequency_2**  Numeric. Expected frequency in excess of **Threshold_2**
- **max_alpha**  Numeric. Upper limit for the alpha that is returned.
- **tolerance**  Numeric. Accuracy of the result.
- **truncation**  Numeric. If **truncation** is not **NULL** then the Pareto distribution is truncated at **truncation**.
Pareto_Find_Alpha_btw_FQ_Layer

Value
The Pareto alpha between the expected number of claims Frequency_1 excess Threshold_1 and the expected number of claims Frequency_2 excess Threshold_2.

References

Examples
Pareto_Find_Alpha_btw_FQs(1000, 1, 2000, 0.5)
Pareto_Find_Alpha_btw_FQs(1000, 1, 2000, 0.5, truncation = 5000)

Description
Finds the Pareto alpha between an excess frequency and the expected loss of a layer.

Usage
Pareto_Find_Alpha_btw_FQ_Layer(
  Threshold,
  Frequency,
  Cover,
  AttachmentPoint,
  ExpLoss,
  max_alpha = 100,
  tolerance = 1e-10,
  truncation = NULL
)

Arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold</td>
<td>Numeric. Threshold</td>
</tr>
<tr>
<td>Frequency</td>
<td>Numeric. Expected frequency in excess of Threshold</td>
</tr>
<tr>
<td>Cover</td>
<td>Numeric. Cover of the second layer.</td>
</tr>
<tr>
<td>AttachmentPoint</td>
<td>Numeric. Attachment point of the layer.</td>
</tr>
<tr>
<td>ExpLoss</td>
<td>Numeric. Expected loss of the layer.</td>
</tr>
<tr>
<td>max_alpha</td>
<td>Numeric. Upper limit for the alpha that is returned.</td>
</tr>
<tr>
<td>tolerance</td>
<td>Numeric. Accuracy of the result.</td>
</tr>
<tr>
<td>truncation</td>
<td>Numeric. If truncation is not NULL then the Pareto distribution is truncated at truncation.</td>
</tr>
</tbody>
</table>
**Value**

The Pareto alpha between the expected number of claims Frequency excess Threshold and the layer Cover xs AttachmentPoint with expected loss ExpLoss

**References**


**Examples**

Pareto_Find_Alpha_btw_FQ_Layer(1000, 1, 1000, 1000, 500)
Pareto_Find_Alpha_btw_FQ_Layer(1000, 1, 1000, 1000, 500, truncation = 5000)

---

**Pareto_Alpha_Between_Two_Layers**

**Pareto Alpha Between Two Layers**

**Description**

Finds the Pareto alpha between two layers

**Usage**

Pareto_Find_Alpha_btw_Layers(
    Cover_1,
    AttachmentPoint_1,
    ExpLoss_1,
    Cover_2,
    AttachmentPoint_2,
    ExpLoss_2,
    max_alpha = 100,
    tolerance = 1e-10,
    truncation = NULL
)

**Arguments**

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cover_1</td>
<td>Numeric. Cover of the first layer.</td>
</tr>
<tr>
<td>AttachmentPoint_1</td>
<td>Numeric. Attachment point of the first layer.</td>
</tr>
<tr>
<td>ExpLoss_1</td>
<td>Numeric. Expected loss of the first layer.</td>
</tr>
<tr>
<td>Cover_2</td>
<td>Numeric. Cover of the second layer.</td>
</tr>
<tr>
<td>AttachmentPoint_2</td>
<td>Numeric. Attachment point of the second layer.</td>
</tr>
</tbody>
</table>
Pareto_Layer_Mean

ExpLoss_2        Numeric. Expected loss of the second layer.
max_alpha        Numeric. Upper limit for the alpha that is returned.
tolerance        Numeric. Accuracy of the result.
truncation       Numeric. If truncation is not NULL then the Pareto distribution is truncated at truncation.

Value

The Pareto alpha between the layer Cover_1 xs AttachmentPoint_1 with expected loss ExpLoss_1 and the layer Cover_2 xs AttachmentPoint_2 with expected loss ExpLoss_2

References


Examples

Pareto_Find_Alpha_btw_Layers(100, 100, 100, 200, 200, 50)
Pareto_Find_Alpha_btw_Layers(100, 100, 100, 200, 200, 50, truncation = 500)

---

Pareto_Layer_Mean

Layer Mean of the Pareto Distribution

Description

Calculates the expected loss of a Pareto distribution in a reinsurance layer

Usage

Pareto_Layer_Mean(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)

Arguments

Cover        Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
AttachmentPoint        Numeric. Attachment point of the reinsurance layer.
alpha        Numeric. Pareto alpha.
t        Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <= Attachment Point is used.
truncation        Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
Value

The expected loss of the (truncated) Pareto distribution with parameters \( t \) and \( \alpha \) in the layer \( \text{Cover} \times \text{AttachmentPoint} \)

Examples

```
Pareto_Layer_Mean(4000, 1000, 2)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 1000)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 5000)
Pareto_Layer_Mean(4000, 1000, alpha = 2, t = 1000, truncation = 5000)
Pareto_Layer_Mean(9000, 1000, alpha = 2, t = 1000, truncation = 5000)
```

---

**Pareto_Layer_SM**  
*Second Layer Moment of the Pareto Distribution*

Description

Calculates the second moment of a Pareto distribution in a reinsurance layer

Usage

```r
Pareto_Layer_SM(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)
```

Arguments

- **Cover**: Numeric. Cover of the reinsurance layer. Use `Inf` for unlimited layers.
- **AttachmentPoint**: Numeric. Attachment point of the reinsurance layer.
- **alpha**: Numeric. Pareto alpha.
- **t**: Numeric. Threshold of the Pareto distribution. If \( t \) is `NULL` (default) then \( t \) <-Attachment Point is used
- **truncation**: Numeric. If truncation is not `NULL` and truncation > \( t \), then the Pareto distribution is truncated at truncation.

Value

The second moment of the (truncated) Pareto distribution with parameters \( t \) and \( \alpha \) in the layer \( \text{Cover} \times \text{AttachmentPoint} \)

Examples

```
Pareto_Layer_SM(4000, 1000, 2)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 1000)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 5000)
Pareto_Layer_SM(4000, 1000, alpha = 2, t = 1000, truncation = 5000)
Pareto_Layer_SM(9000, 1000, alpha = 2, t = 1000, truncation = 5000)
```
**Pareto_Layer_Var**  
*Layer Variance of the Pareto Distribution*

**Description**
Calculates the variance of a Pareto distribution in a reinsurance layer

**Usage**
Pareto_Layer_Var(Cover, AttachmentPoint, alpha, t = NULL, truncation = NULL)

**Arguments**
- **Cover**: Numeric. Cover of the reinsurance layer. Use Inf for unlimited layers.
- **AttachmentPoint**: Numeric. Attachment point of the reinsurance layer.
- **alpha**: Numeric. Pareto alpha.
- **t**: Numeric. Threshold of the Pareto distribution. If t is NULL (default) then t <- Attachment Point is used.
- **truncation**: Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.

**Value**
The variance of the(truncated) Pareto distribution with parameters t and alpha in the layer Cover xs AttachmentPoint

**Examples**
Pareto_Layer_Var(4000, 1000, 2)  
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 1000)  
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 5000)  
Pareto_Layer_Var(4000, 1000, alpha = 2, t = 1000, truncation = 5000)  
Pareto_Layer_Var(9000, 1000, alpha = 2, t = 1000, truncation = 5000)

---

**Pareto_ML_Estimator_Alpha**  
*Maximum Likelihood Estimation of the Pareto Alpha*

**Description**
Calculates the maximum likelihood estimator of the parameter alpha of a Pareto distribution
Usage

Pareto_ML_Estimator_Alpha(
  losses,
  t,
  truncation = NULL,
  tol = 1e-07,
  max_iterations = 1000,
  alpha_min = 0,
  alpha_max = Inf
)

Arguments

- **losses**: Numeric vector. Losses that are used for the ML estimation.
- **t**: Numeric or numeric vector. Threshold of the Pareto distribution. Alternatively, 
t can be a vector of same length as losses. In this case 
t[i] is the reporting threshold of losses[i].
- **truncation**: Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
- **tol**: Numeric. Desired accuracy (only relevant in the truncated case).
- **max_iterations**: Numeric. Maximum number of iteration in the case truncation < Inf (only relevant in the truncated case).
- **alpha_min**: Numeric. Deprecated.
- **alpha_max**: Numeric. Deprecated.

Value

Maximum likelihood estimator for the parameter alpha of a Pareto distribution with threshold t given the observations losses

Examples

```
losses <- rPareto(100, 1000, 2)
Pareto_ML_Estimator_Alpha(losses, 1000)
losses <- rPareto(100, 1000, 2, truncation = 2000)
Pareto_ML_Estimator_Alpha(losses, 1000)
Pareto_ML_Estimator_Alpha(losses, 1000, truncation = 2000)

t <- rPareto(10000, 100, 2)
alpha <- 2
losses <- rPareto(10000, t, alpha)
Pareto_ML_Estimator_Alpha(losses, t)
losses <- rPareto(10000, t, alpha, truncation = 2 * max(t))
Pareto_ML_Estimator_Alpha(losses, t, truncation = 2 * max(t))
```
### Pareto_PDF

**Density of the Pareto Distribution**

**Description**

Calculates the density function of the Pareto distribution. This function is deprecated. Use `dPareto` instead.

**Usage**

```r
Pareto_PDF(x, t, alpha, truncation = NULL)
```

**Arguments**

- `x`: Numeric. The function evaluates the density at `x`
- `t`: Numeric. Threshold of the Pareto distribution.
- `truncation`: Numeric. If `truncation` is not `NULL` and `truncation > t`, then the Pareto distribution is truncated at `truncation`.

**Value**

Density function of the Pareto distribution with parameters `t` and `alpha` evaluated at `x`

**Examples**

```r
x <- 0:10 * 1000
dPareto(x, 1000, 2)
dPareto(x, 1000, 2, truncation = 5000)
```

### PiecewisePareto_CDF

**Distribution Function of the Piecewise Pareto Distribution**

**Description**

Calculates the cumulative distribution function of a Piecewise Pareto Distribution. This function is deprecated. Use `pPiecewisePareto` instead.

**Usage**

```r
PiecewisePareto_CDF(x, t, alpha, truncation = NULL, truncation_type = "lp")
```
Arguments

- **x**: Numeric. The function evaluates the CDF at x.
- **t**: Numeric vector. Thresholds of the piecewise Pareto distribution.
- **alpha**: Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
- **truncation**: Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.
- **truncation_type**: Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

Value

Distribution function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at x

References


Examples

```r
# t <- c(1000, 2000, 3000)
# alpha <- c(1, 1.5, 2)
# x <- 0:10 * 1000
# pPiecewisePareto(x, t, alpha)
# pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "lp")
# pPiecewisePareto(x, t, alpha, truncation = 5000, truncation_type = "wd")
```

PiecewisePareto_Layer_Mean

*Layer Mean of the Piecewise Pareto Distribution*

Description

Calculates the expected loss of a piecewise Pareto distribution in a reinsurance layer

Usage

```r
PiecewisePareto_Layer_Mean(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)
```
Arguments

Cover Numeric. Cover of the reinsurance layer.
AttachmentPoint Numeric. Attachment point of the reinsurance layer.
t Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
truncation_type Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

Value

The expected loss of the (truncated) piecewise Pareto distribution with parameter vectors t and alpha in the layer Cover xs AttachmentPoint

References


Examples

t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_Mean(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")

Description

Calculates the second moment of a piecewise Pareto distribution in a reinsurance layer
Usage

PiecewisePareto_Layer_SM(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)

Arguments

Cover  Numeric. Cover of the reinsurance layer.
AttachmentPoint
  Numeric. Attachment point of the reinsurance layer.
t  Numeric vector. Thresholds of the piecewise Pareto distribution.
alpha  Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
truncation  Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
truncation_type  Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

Value

The second moment of the (truncated) piecewise Pareto distribution with parameter vectors t and alpha in the layer Cover xs AttachmentPoint

Examples

t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_SM(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")

PiecewisePareto_Layer_Var

Layer Variance of the Piecewise Pareto Distribution

Description

Calculate the variance of a piecewise Pareto distribution in a reinsurance layer
Usage

PiecewisePareto_Layer_Var(
  Cover,
  AttachmentPoint,
  t,
  alpha,
  truncation = NULL,
  truncation_type = "lp"
)

Arguments

- **Cover**: Numeric. Cover of the reinsurance layer.
- **AttachmentPoint**: Numeric. Attachment point of the reinsurance layer.
- **t**: Numeric vector. Thresholds of the piecewise Pareto distribution.
- **alpha**: Numeric vector. *alpha[i]* is the Pareto alpha in excess of *t[i]*.
- **truncation**: Numeric. If truncation is not NULL and truncation > *t*, then the Pareto distribution is truncated at truncation.
- **truncation_type**: Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

Value

The variance of the (truncated) piecewise Pareto distribution with parameter vectors *t* and *alpha* in the layer *Cover xs AttachmentPoint*

Examples

```r
t <- c(1000, 2000, 3000)
alPHA <- c(1, 1.5, 2)
PiecewisePareto_Layer_Var(4000, 1000, t, alpha)
PiecewisePareto_Layer_SM(4000, 1000, t, alpha) - PiecewisePareto_Layer_Mean(4000, 1000, t, alpha)^2
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000)
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000, truncation_type = "lp")
PiecewisePareto_Layer_Var(4000, 1000, t, alpha, truncation = 5000, truncation_type = "wd")
```

---

**PiecewisePareto_Match_Layer_Losses**

*Match a Tower of Expected Layers Losses*

Description

Matches the expected losses of a tower of reinsurance layers using a piecewise Pareto severity
Usage

PiecewisePareto_Match_Layer_Losses(
    Attachment_Points,
    Expected_Layer_Losses,
    Unlimited_Layers = FALSE,
    Frequencies = NULL,
    FQ_at_lowest_AttPt = NULL,
    FQ_at_highest_AttPt = NULL,
    TotalLoss_Frequencies = NULL,
    minimize_ratios = TRUE,
    Use_unlimited_Layer_for_FQ = TRUE,
    truncation = NULL,
    truncation_type = "lp",
    tolerance = 1e-10,
    alpha_max = 100,
    merge_tolerance = 1e-06,
    RoL_tolerance = 1e-06
)

Arguments

Attachment_Points
    Numeric vector. Vector containing the attachment points of consecutive layers
    in increasing order

Expected_Layer_Losses
    Numeric vector. Vector containing the expected losses of layers xs the attach-
    ment points.

Unlimited_Layers
    Logical. If TRUE, then Expected_Layer_Losses[i] contains the expected loss
    of Inf xs Attachment_Points[i]. If FALSE then Expected_Layer_Losses[i]
    contains the expected loss of the layer Attachment_Points[i+1] xs Attachment_Points[i]

Frequencies
    Numeric vector. Expected frequencies excess the attachment points. If NULL
    then the function calculates frequencies.

FQ_at_lowest_AttPt
    Numerical. Expected frequency excess Attachment_Points[1]

FQ_at_highest_AttPt
    Numerical. Expected frequency excess Attachment_Points[k]

TotalLoss_Frequencies
    Numeric vector. TotalLoss_Frequencies[i] is the frequency of total losses to
    layer i (i.e. Attachment_Points[i+1] - Attachment_Points[i]) xs Attachment_Points[i].
    TotalLoss_Frequencies[i] is the frequency for losses larger than or equal
    to Attachment_Points[i+1], whereas Frequencies[i] is the frequency of
    losses larger than Attachment_Points[i]. TotalLoss_Frequencies[i] > Frequencies[i+1]
    means that there is a point mass of the severity at Attachment_Points[i+1].

minimize_ratios
    Logical. If TRUE then ratios between alphas are minimized.
Use_unlimited_Layer_for_FQ
   Logical. Only relevant if no frequency is provided for the highest attachment point by the user. If TRUE then the frequency is calculated using the Pareto alpha between the last two layers.

truncation
   Numeric. If truncation is not NULL and truncation > max(Attachment_Points), then the last Pareto piece is truncated at truncation (truncation_type = "lp").

truncation_type
   Character. Currently only truncation_type = "lp" supported. A truncated Pareto is used for the last piece.

tolerance
   Numeric. Numerical tolerance.

alpha_max
   Numerical. Maximum alpha to be used for the matching.

merge_tolerance
   Numerical. Consecutive Pareto pieces are merged if the alphas deviate by less than merge_tolerance.

RoL_tolerance
   Numerical. Consecutive layers are merged if RoL decreases less than factor \(1 - \text{RoL_tolerance}\).

Value

A list containing the following objects:

- \(t\) Numeric vector. Vector containing the thresholds for the piecewise Pareto distribution
- \(\alpha\) Numeric vector. Vector containing the Pareto alphas of the piecewise Pareto distribution
- Status Character. Information on whether the fit was successful
- \(FQ\) Numerical. Frequency in excess of the lowest threshold of the piecewise Pareto distribution

References


Examples

\[
\begin{align*}
\text{AP} & \leftarrow \text{Example1_AP} \\
\text{EL} & \leftarrow \text{Example1_EL} \\
\text{PiecewisePareto_Match_Layer_Losses}(\text{AP}, \text{EL}) \\
\text{EL_unlimited} & \leftarrow \text{rev(cumsum(rev(Example1_EL)))} \\
\text{PiecewisePareto_Match_Layer_Losses}(\text{AP}, \text{EL_unlimited}, \text{Unlimited_Layers} = \text{TRUE}) \\
\text{PiecewisePareto_Match_Layer_Losses}(\text{AP}, \text{EL}, \text{FQ_at_lowest_AttPt} = 0.5) \\
\text{Example1_FQ} & \leftarrow \text{c}(0.3, 0.15, 0.08, 0.02, 0.005) \\
\text{PiecewisePareto_Match_Layer_Losses}(\text{AP}, \text{EL}, \text{Frequencies} = \text{Example1_FQ})
\end{align*}
\]
Maximum Likelihood Estimation of the Alphas of the Piecewise Pareto Distribution

Description
Calculates the maximum likelihood estimator of the parameter vector alpha of a piecewise Pareto distribution

Usage
PiecewisePareto_ML_Estimator_Alpha(
  losses,
  t,
  truncation = NULL,
  truncation_type = "lp",
  tol = 1e-07,
  max_iterations = 1000
)

Arguments
losses Numeric vector. Losses that are used for the ML estimation.
t Numeric vector. Thresholds of the piecewise Pareto distribution.
truncation Numeric. If truncation is not NULL and truncation > max(t), then the distribution is truncated at truncation.
truncation_type Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.
tol Numeric. Desired accuracy (only relevant in the truncated case).
max_iterations Numeric. Maximum number of iteration in the case truncation < Inf (only relevant in the truncated case).

Value
Maximum likelihood estimator for the parameter alpha of a Pareto distribution with threshold t given the observations losses

Examples
losses <- rPiecewisePareto(10000, t = c(100,200,300), alpha = c(1,2,3))
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300))
losses <- rPiecewisePareto(10000, t = c(100,200,300), alpha = c(1,2,3),
  truncation = 500, truncation_type = "lp")
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300))
PiecewisePareto_ML_Estimator_Alpha(losses, c(100,200,300),
```r
truncation = 500, truncation_type = "lp")
losses <- rPiecewisePareto(10000, t = c(100, 200, 300), alpha = c(1, 2, 3),
                         truncation = 500, truncation_type = "wd")
PiecewisePareto_ML_Estimator_Alpha(losses, c(100, 200, 300))
PiecewisePareto_ML_Estimator_Alpha(losses, c(100, 200, 300),
                         truncation = 500, truncation_type = "wd")
```
**pPareto**

*Distribution Function of the Pareto Distribution*

**Description**

Calculates the cumulative distribution function of a Pareto distribution

**Usage**

```r
pPareto(x, t, alpha, truncation = NULL)
```

**Arguments**

- `x` Numeric. The function evaluates the CDF at `x`.
- `t` Numeric. Threshold of the Pareto distribution.
- `alpha` Numeric. Pareto alpha.
- `truncation` Numeric. If `truncation` is not `NULL` and `truncation > t`, then the Pareto distribution is truncated at `truncation`.

**Value**

Distribution function of the Pareto distribution with parameters `t` and `alpha` evaluated at `x`.

**Examples**

```r
x <- 0:10 * 1000
pPareto(x, 1000, 2)
pPareto(x, 1000, 2, truncation = 5000)
```

---

**pPiecewisePareto**

*Distribution Function of the Piecewise Pareto Distribution*

**Description**

Calculates the cumulative distribution function of a Piecewise Pareto Distribution

**Usage**

```r
pPiecewisePareto(x, t, alpha, truncation = NULL, truncation_type = "lp")
```
Arguments

- **x**: Numeric. The function evaluates the CDF at x.
- **t**: Numeric vector. Thresholds of the piecewise Pareto distribution.
- **alpha**: Numeric vector. alpha[i] is the Pareto alpha in excess of t[i].
- **truncation**: Numeric. If truncation is not NULL and truncation > t, then the distribution is truncated at truncation.
- **truncation_type**: Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

Value

Distribution function of the piecewise Pareto distribution with parameter vectors t and alpha evaluated at x

References


Examples

```r
t <- c(1000, 2000, 3000)
algebra <- c(1, 1.5, 2)
x <- 0:10 * 1000
pPiecewisePareto(x, t, algebra)
pPiecewisePareto(x, t, algebra, truncation = 5000, truncation_type = "lp")
pPiecewisePareto(x, t, algebra, truncation = 5000, truncation_type = "wd")
```

qPareto

Quantile Function of the Pareto Distribution

Description

Calculates the quantile function of a Pareto distribution

Usage

```r
qPareto(p, t, alpha, truncation = NULL)
```

Arguments

- **p**: Numeric. The function evaluates the inverse CDF at p.
- **t**: Numeric. Threshold of the piecewise Pareto distribution.
- **alpha**: Numeric. Pareto alpha.
- **truncation**: Numeric. If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation.
qPiecewisePareto

**Value**

Quantile function of the Pareto distribution with parameters `t` and `alpha`, evaluated at `p`

**Examples**

```r
p <- 0:10 * 0.1
tPareto(p, 1000, 2)
tPareto(p, 1000, 2, truncation = 5000)
```

---

**qPiecewisePareto**  
*Quantile Function of the Piecewise Pareto Distribution*

**Description**

Calculates the quantile function of a piecewise Pareto distribution

**Usage**

```r
tPiecewisePareto(p, t, alpha, truncation = NULL, truncation_type = "lp")
```

**Arguments**

- `p`: Numeric. The function evaluates the quantile function at `p`.
- `t`: Numeric vector. Thresholds of the piecewise Pareto distribution.
- `alpha`: Numeric vector. `alpha[i]` is the Pareto alpha in excess of `t[i]`.
- `truncation`: Numeric. If `truncation` is not `NULL` and `truncation > t`, then the distribution is truncated at `truncation`.
- `truncation_type`: Character. If `truncation_type = "wd"` then the whole distribution is truncated. If `truncation_type = "lp"` then a truncated Pareto is used for the last piece.

**Value**

Quantile function of the piecewise Pareto distribution with parameter vectors `t` and `alpha` evaluated at `p`

**Examples**

```r
t <- c(1000, 2000, 3000)
alpaha <- c(1, 1.5, 2)
p <- 0:10 * 0.1
tPiecewisePareto(p, t, alpha)
tPiecewisePareto(p, t, alpha, truncation = 5000, truncation_type = "lp")
tPiecewisePareto(p, t, alpha, truncation = 5000, truncation_type = "wd")
```
rPareto

Simulation of the Pareto Distribution

Description

Generates random deviates of a Pareto distribution

Usage

rPareto(n, t, alpha, truncation = NULL)

Arguments

n Numeric. Number of observations.
t Numeric vector. Thresholds of the Pareto distributions
alpha Numeric vector. Pareto alphas of the Pareto distributions.
truncation If truncation is not NULL and truncation > t, then the Pareto distribution is truncated at truncation (resampled Pareto)

Value

A vector of n samples from the (truncated) Pareto distribution with parameters t and alpha

Examples

rPareto(100, 1000, 2)
rPareto(100, 1000, 2, truncation = 2000)
rPareto(5, t = c(1, 10, 100, 1000, 10000), alpha = c(1,2,4,8,16))

rPiecewisePareto

Simulation of the Piecewise Pareto Distribution

Description

Generates random deviates of a piecewise Pareto distribution

Usage

rPiecewisePareto(  
n, 
t, 
alpha, 
truncation = NULL, 
truncation_type = "lp", 
scale_pieces = NULL 
)
Arguments

- **n**
  Numeric. Number of simulations

- **t**
  Numeric vector. Thresholds of the piecewise Pareto distribution.

- **alpha**
  Numeric vector. \( \alpha[i] \) is the Pareto alpha in excess of \( t[i] \).

- **truncation**
  Numeric. If truncation is not NULL and truncation > \( t \), then the distribution is truncated at truncation.

- **truncation_type**
  Character. If truncation_type = "wd" then the whole distribution is truncated. If truncation_type = "lp" then a truncated Pareto is used for the last piece.

- **scale_pieces**
  Numeric vector. If not NULL then the density of the \( i \)-th Pareto piece (on the interval \((t[i],t[i+1])\)) is scaled with the factor \( \text{const} \times \text{scale_pieces}[i] \) (where \( \text{const} \) is a normalization constant)

Value

A vector of \( n \) samples from the (truncated) piecewise Pareto distribution with parameter vectors \( t \) and \( \alpha \)

Examples

```
t <- c(1000, 2000, 3000)
alpha <- c(1, 1.5, 2)
rPiecewisePareto(100, t, alpha)
rPiecewisePareto(100, t, alpha, truncation = 5000)
rPiecewisePareto(100, t, alpha, truncation = 5000, truncation_type = "lp")
rPiecewisePareto(100, t, alpha, truncation = 5000, truncation_type = "wd")
```
Index

*Topic datasets
  Example1_AP, 4
  Example1_EL, 4

dPareto, 2
dPiecewisePareto, 3

Example1_AP, 4
Example1_EL, 4

Local_Pareto_Alpha, 4

Pareto_CDF, 5
Pareto_Extrapolation, 6
Pareto_Find_Alpha_btw_FQ_Layer, 8
Pareto_Find_Alpha_btw_FQs, 7
Pareto_Find_Alpha_btw_Layers, 9
Pareto_Layer_Mean, 10
Pareto_Layer_SM, 11
Pareto_Layer_Var, 12
Pareto_ML_Estimator_Alpha, 12
Pareto_PDF, 14
PiecewisePareto_CDF, 14
PiecewisePareto_Layer_Mean, 15
PiecewisePareto_Layer_SM, 16
PiecewisePareto_Layer_Var, 17
PiecewisePareto_Match_Layer_Losses, 18
PiecewisePareto_ML_Estimator_Alpha, 21
PiecewisePareto_PDF, 22
pPareto, 23
pPiecewisePareto, 23

gPareto, 24
gPiecewisePareto, 25

rPareto, 26
rPiecewisePareto, 26