Standard Errors for Risk and Performance Estimators in PerformanceAnalytics

Anthony Christidis and Doug Martin

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Abstract

The PerformanceAnalytics package allows users to perform risk analysis of financial instruments or portfolios. In general, the package requires returns data and allows users to compute risk estimators (including the standard deviation, semi-standard deviation, value-at-risk and expected shortfall), and performance estimators (such as the Sharpe ratio, Sortino ratio, and expected shortfall ratio). However, users have no way of assessing the statistical accuracy of the estimates. A new method for computing accurate standard errors of risk and performance measures for serially correlated or uncorrelated returns has been developed using a sophisticated method based on the spectral density of the influence-function (IF) transformed returns, and has been implemented in the RPESE package. This capability has been integrated in the PerformanceAnalytics package, and this vignette provides basic instruction on how to use the standard errors capability of the PerformanceAnalytics package.

1 Introduction

The current finance industry practice in reporting risk and performance estimates for individual assets and portfolios seldom includes reporting estimate standard errors (SEs). For this reason, consumers of such reports have no way of assessing the statistical accuracy of the estimates. As a leading example, SE’s are seldom reported for Sharpe ratios, and consequently one cannot tell whether or not two Sharpe ratios for two different portfolio products are significantly different. Chen and Martin [2018], henceforth (CM), have developed a method to compute accurate standard errors for risk and performance estimators that properly reflects the impacts of: (1) returns that are serially correlated as well as when
they are uncorrelated, and (2) fat-tailed and skewed non-normality of returns distributions.
The risk and performance standard errors computing package RPESE was developed to implement this new CM methodology, and this capability has been integrated in the highly popular PerformanceAnalytics package that is frequently used in the quantitative finance community.

The current release of PerformanceAnalytics supports computing SEs for the six risk estimators described in Table 1, and the eight performance estimators described in Table 2. The Name column in each of those two tables contains the name of the R function in the RPESE package, For each of the names in the Name column of the two tables, there is a corresponding R function in PerformanceAnalytics, except for LMP1 and LPM2 in Table 1 for which there is a single function with an optional argument to choose between these two risk estimators, and for SorR.µ and SorR.c in Table 2 for which there is a single function with an optional argument to choose between these two performance estimators. The names of the PerformanceAnalytics functions that correspond to the names in Tables 1 and 2 are provided in Tables 3 and 4 of Section 3.2.

The PerformanceAnalytics package internally computes the point estimates in Tables 1 and 2, and uses the RPESE package to compute the standard errors of the point estimates.

<table>
<thead>
<tr>
<th>Name</th>
<th>Estimator Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>Sample standard deviation</td>
</tr>
<tr>
<td>SemiSD</td>
<td>Semi-standard deviation</td>
</tr>
<tr>
<td>LPM1</td>
<td>Lower partial moment of order 1</td>
</tr>
<tr>
<td>LPM2</td>
<td>Lower partial moment of order 2</td>
</tr>
<tr>
<td>ES</td>
<td>Expected shortfall with tail probability $\alpha$</td>
</tr>
<tr>
<td>VaR</td>
<td>Value-at-risk with tail probability $\alpha$</td>
</tr>
</tbody>
</table>

Table 1: Risk Estimator Names and Descriptions
## The New Method of Computing Standard Errors of Risk and Performance Estimators

The essence of the new method is as follows. Given a risk or performance estimators, such as the Sharpe ratio, the time series of returns used to compute the estimate are transformed using the influence function (IF) of the estimator. For an introduction to influence functions for risk and performance estimators, and derivations of the influence functions of the estimators in Tables 1 and 2, see the paper Zhang et al. [2019]. In CM, it is shown that a risk or performance estimator can be represented as a sample mean of the time series of IF transformed returns.

It is a well-known result that an appropriately standardized (with respect to sample size) sum of a stationary time series has a variance that is approximated by the spectral density of the time series at zero frequency, with the approximation becoming exact as the sample size tends to infinity. Using this result, computing the standard error of a risk or performance estimator reduces to estimating the spectral density at zero frequency of a standardized sum of the influence-function transformed returns.

CM developed an effective method of doing so based on first computing the periodogram of the IF transformed returns, and then using a regularized generalized linear model (GLM) method for Exponential and Gamma distributions, to fit a polynomial to the periodogram values. The regularization method used is an elastic net (EN) penalty that is well-known in the machine learning community, and encourages sparsity of coefficients. The intercept of
such GLM fitting provides an estimate of the spectral density at zero frequency, and hence a risk or performance estimator standard error.

The interested reader can find further details in the vignette of the RPESE package available at CRAN and in the CM paper.

2 Packages Involved in the Computation of Standard Errors

In order to compute the standard errors of the point estimates in PerformanceAnalytics, the RPESE package must be installed by the user. However, note that if a user does not have the RPESE package downloaded, then the PerformanceAnalytics package will still work with all its many capabilities other than standard error computation. In the source code of PerformanceAnalytics, the RPESE package is only suggested. If a user attempts to compute standard errors without the required package, an error will be returned with the required instructions to download RPESE.

On a side note, the overall structure of the packages involved in the computation of standard errors for risk and performance estimators in PerformanceAnalytics are depicted in Figure 1. As the figure indicates, RPESE makes use of the following two new packages:

- **RPEIF** (Risk and Performance Estimators Influence Functions)
- **RPEGLMEN** (Risk and Performance Estimators Generalized Linear Model fitting with Elastic Net, for Exponential and Gamma distributions)

The purpose of RPEIF is to provide the analytic formulas of influence functions in support of computing the IF transformed returns for the risk and performance estimators. For each risk and performance estimator in Tables 1 and 2, the RPEGLMEN package fits an EN regularized GLM polynomial fit to the periodogram of the time series of IF-transformed returns, using a GLM for Exponential distributions or Gamma distributions.
If a user of PerformanceAnalytics wishes to compute standard errors of risk and performance estimators, only the RPESE package is needed. The other two packages, RPEIF and RPEGLMEN will be installed automatically if RPESE is installed (they are imported packages by RPESE).

3 How to Compute Standard Errors in PerformanceAnalytics

In the following sections, we show how to use the functions in PerformanceAnalytics to compute standard errors of risk and performance estimators using the edhec time series of monthly hedge fund returns contained in the PerformanceAnalytics package.

3.1 Installing and Loading RPESE and Loading an Examples Data Set

As previously mentioned, to use the standard errors computation capability in PerformanceAnalytics, the RPESE package must be installed. The RPESE package can be installed form CRAN as follows:
install.packages("RPESE")

We will use the `xts` data set `edhec` of hedge fund returns, contained in `PerformanceAnalytics`, in demonstrating the functionality of `PerformanceAnalytics`. The following code loads the `edhec` data, confirms the object’s class, lists the names of the hedge funds, and displays the range of dates of the data.

```r
library(PerformanceAnalytics)
data(edhec, package='PerformanceAnalytics')
class(edhec)

## [1] "xts" "zoo"

names(edhec)

## [1] "Convertible Arbitrage" "CTA Global"  
## [3] "Distressed Securities" "Emerging Markets"  
## [5] "Equity Market Neutral" "Event Driven"  
## [7] "Fixed Income Arbitrage" "Global Macro"  
## [9] "Long/Short Equity" "Merger Arbitrage"  
## [11] "Relative Value" "Short Selling"  
## [13] "Funds of Funds"

library(xts) # Need this for the next line and later use of plot.zoo
range(index(edhec))


Since the hedge fund names are too long for convenient display, the following code is used to create shorter two or three letter names:

```r
names(edhec) <- c("CA", "CTA", "DIS", "EM", "EMN", "ED", "FIA",  
"GM", "LS", "MA", "RV", "SS", "FOF")
```
### 3.2 PerformanceAnalytics Functions for Computing Standard Errors

The names of the PerformanceAnalytics for computing standard errors are provide in the second column, with the names of the corresponding RPESE functions in the first column.

<table>
<thead>
<tr>
<th>Name</th>
<th>PerformanceAnalytics Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SD$</td>
<td>StdDev</td>
</tr>
<tr>
<td>SemiSD</td>
<td>SemiSD</td>
</tr>
<tr>
<td>LPM1</td>
<td>lpm with argument n=1</td>
</tr>
<tr>
<td>LPM2</td>
<td>lpm with argument n=2</td>
</tr>
<tr>
<td>ES</td>
<td>ES</td>
</tr>
<tr>
<td>VaR</td>
<td>VaR</td>
</tr>
</tbody>
</table>

Table 3: Risk Estimator Names and Descriptions

<table>
<thead>
<tr>
<th>Name</th>
<th>PerformanceAnalytics Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Mean.arithmetic</td>
</tr>
<tr>
<td>SR</td>
<td>SharpeRatio with argument FUN=&quot;StdDev&quot;</td>
</tr>
<tr>
<td>SorR.$\mu$</td>
<td>Sortino with argument threshold=&quot;mean&quot;</td>
</tr>
<tr>
<td>SorR.$c$</td>
<td>Sortino with argument threshold=&quot;const&quot;</td>
</tr>
<tr>
<td>ESratio</td>
<td>SharpeRatio with argument FUN=&quot;ES&quot;</td>
</tr>
<tr>
<td>VaRratio</td>
<td>SharpeRatio with argument FUN=&quot;VaR&quot;</td>
</tr>
<tr>
<td>RachevRatio</td>
<td>RachevRatio</td>
</tr>
<tr>
<td>OmegaRatio</td>
<td>Omega</td>
</tr>
</tbody>
</table>

Table 4: Performance Estimator Names and Descriptions
3.3 Basic Functionality

The arguments of risk and performance estimator functions in PerformanceAnalytics are different for different functions, and sometimes quite extensive. For example, below we show the arguments of the StdDev and ES (expected shortfall) functions using the args function:

```r
args(StdDev)
## function (R, ..., clean = c("none", "boudt", "geltner", "locScaleRob"),
## portfolio_method = c("single", "component"), weights = NULL,
## mu = NULL, sigma = NULL, use = "everything", method = c("pearson",
## "kendall", "spearman"), SE = FALSE, SE.control = NULL)
## NULL
```

```r
args(ES)
## function (R = NULL, p = 0.95, ..., method = c("modified", "gaussian",
## "historical"), clean = c("none", "boudt", "geltner", "locScaleRob"),
## portfolio_method = c("single", "component"), weights = NULL,
## mu = NULL, sigma = NULL, m3 = NULL, m4 = NULL, invert = TRUE,
## operational = TRUE, SE = FALSE, SE.control = NULL)
## NULL
```

However, for computing standard errors of the risk and performance estimators of Tables 3 and 4 in PerformanceAnalytics, only two arguments are two arguments besides the data set name are needed, namely the arguments SE and SE.control, and often one only uses the SE argument by setting SE=TRUE. This works for example when you want to compute the standard error of an expected shortfall estimate for the convertible arbitrage (CA) hedge fund, as follows.

```r
# Expected shortfall SE computation for a single hedge fund
ESout.CA <- t(ES(edhec$CA, SE=TRUE))
ESout.CA
```

###

<table>
<thead>
<tr>
<th></th>
<th>ES</th>
<th>IFiid</th>
<th>IFcor</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>0.03655</td>
<td>0.008973782</td>
<td>0.01580664</td>
</tr>
</tbody>
</table>

8
The first column above contains the ES estimate for CA hedge fund, and the second and third column contain the standard errors for two different methods, "IFiid" and "IFcor". These are just two of the following possible choices, which can be set as demonstrated in Section 3.4:

- "IFiid": This results in an influence function (IF) method based computation of a standard error assuming i.i.d. returns
- "IFcor": This is the basic IF method computation of a standard error that takes into account serial correlation in the returns
- "IFcorAdapt": This IF based method adaptively interpolates between IFcor and IFcorPW, and can sometimes result in better accounting for serial correlation in the returns than with either IFcor or IFcorPW alone
- "IFcorPW": This IF based method uses pre-whitening of the IF transformed returns and is useful when serial correlation is large
- "BOOTiid": This choice results in computing a bootstrap standard error assuming i.i.d. returns
- "BOOTcor": This choice uses a block bootstrap method to compute a standard error that takes into account serial correlation of returns.

The two default choices of methods are:

- "IFiid" and "IFcor" for risk estimators, and for performance estimators when returns serial correlation are known to be small
- "IFiid" and "IFcorAdapt" for performance estimators when returns correlations are unknown and may be large

The value of including IFiid, along with IFcor and IFcorAdapt is that it allows the user to see whether or not serial correlation results in a difference in the standard error that assumes i.i.d. returns and the standard error that takes into account serial correlation. If there is no serial correlation there will not be much difference, but if there is serial correlation the difference can be considerable. For example, in the above case of computing SEs for the CA hedge fund, the IFcor standard error is a considerable 76% larger than that of the IFiid standard error.
The **BOOTiid** and **BOOTcor** methods are provided for users who want to see how these bootstrap methods of computing standard errors compare with the IF based methods. Our experience to date indicates that **BOOTiid** generally agrees quite well with **IFiid**, but that **BOOTcor** is not as consistent in giving values similar to those of **IFcor**.

The risk and performance estimator functions allow you to return the standard errors for more than one asset or portfolio, e.g. a portfolio of assets, at the same time. For example, the following code results in computing standard errors for all thirteen of the edhec hedge funds.

```r
# Expected shortfall SE computation for all hedge funds in data set
ESout <- t(ES(edhec, SE=TRUE))
ESout
```

```

<table>
<thead>
<tr>
<th></th>
<th>ES</th>
<th>IFiid</th>
<th>IFcor</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>0.03655000</td>
<td>0.008973782</td>
<td>0.015801898</td>
</tr>
<tr>
<td>CTA</td>
<td>0.04126429</td>
<td>0.003459795</td>
<td>0.003433112</td>
</tr>
<tr>
<td>DIS</td>
<td>0.03669286</td>
<td>0.006728856</td>
<td>0.010249305</td>
</tr>
<tr>
<td>EM</td>
<td>0.07236429</td>
<td>0.013914916</td>
<td>0.016417564</td>
</tr>
<tr>
<td>EMN</td>
<td>0.01687857</td>
<td>0.004262266</td>
<td>0.004570448</td>
</tr>
<tr>
<td>ED</td>
<td>0.03856429</td>
<td>0.006000260</td>
<td>0.007780399</td>
</tr>
<tr>
<td>FIA</td>
<td>0.02825714</td>
<td>0.007971074</td>
<td>0.013084620</td>
</tr>
<tr>
<td>GM</td>
<td>0.02062857</td>
<td>0.002098174</td>
<td>0.002104368</td>
</tr>
<tr>
<td>LS</td>
<td>0.04220714</td>
<td>0.005082402</td>
<td>0.007472049</td>
</tr>
<tr>
<td>MA</td>
<td>0.01914286</td>
<td>0.003892194</td>
<td>0.004335635</td>
</tr>
<tr>
<td>RV</td>
<td>0.02465000</td>
<td>0.004975177</td>
<td>0.009391730</td>
</tr>
<tr>
<td>SS</td>
<td>0.09682143</td>
<td>0.009600362</td>
<td>0.010901638</td>
</tr>
<tr>
<td>FOF</td>
<td>0.03320714</td>
<td>0.005092672</td>
<td>0.007234076</td>
</tr>
</tbody>
</table>

```

Help files for the functions in the `PerformanceAnalytics` are available as usual. For functions in Tables 3 and 4 that were already in `PerformanceAnalytics` (PA) prior to the current release, the help file will be almost the same as before, with the only difference being that you will see the addtional arguments `SE = FALSE` and `SE.control = NULL`. You can confirm this claim, for example, for the ES function with the following code.

```r
?ES
csv(ES)
```

PA functions for computing SEs that are new with this release, will of course also have the optional arguments `SE = FALSE` and `SE.control = NULL`.
3.4 Controlling Standard Errors Computation Using the \texttt{RPESE.control} Function

The argument \texttt{SE.control} in the \texttt{PA} risk and performance estimator function in Tables 3 and 4 is used to control the parameters in the computation of standard errors, namely: the standard error methods used, as listed in Section 3.3, along with the outlier cleaning option, the model fitting method, the frequency decimation or truncation option, and the adaptive correlation standard errors tuning parameters described in this Section.

The \texttt{SE.control} argument should be set using the \texttt{RPESE.control} function, whose arguments are:

\begin{verbatim}
args(RPESE.control)
## function (estimator = c("Mean", "SD", "VaR", "ES", "SR", "SoR",
##   "ESratio", "VaRratio", "SoR", "LPM", "OmegaRatio", "SemiSD",
##   "RachevRatio")[[1]], se.method = NULL, cleanOutliers = NULL,
##   fitting.method = NULL, freq.include = NULL, freq.par = NULL,
##   a = NULL, b = NULL)
## NULL
\end{verbatim}

Here is what the \texttt{RPESE.control} arguments do:

- "estimator": This argument takes one of the options "Mean", "SD", "VaR", "ES", "SR", "SoR", "ESratio", "VaRratio", "LPM", "OmegaRatio", "SemiSD", "RachevRatio" and sets the other arguments to the function with the default for the measure used. If this argument is ignored, then the default used is "Mean" as indicated in the documentation. If this argument is used and the other arguments listed below are also used, then the default from the measure is overwritten by the user set option. See example code for demonstration.

- "se.method": This argument controls which of the standard errors methods listed in Section 3.3 are used.

- "cleanOutliers": Argument \texttt{TRUE} or \texttt{FALSE} (default) to determine if the returns data outlier should be shrunk.

- "fitting.method": Distribution used in \texttt{RPEGLMEN} fit method. Should be one of "Exponential" (default) or "Gamma".
- "freq.include": DFT frequencies inclusion criteria. Must be one of "All" (default), "Decimate" or "Truncate" choices. If the argument "freq.include" is set to "Decimate" or "Truncate", a value of 0.5 is used for the "freq.par" argument: every second frequency is used in the decimation case, and only the first half of the frequencies are used in the truncation case.

- "freq.par": Percentage of the frequency used if "freq.include" is "Decimate" or "Truncate." Default is 0.5.

- "a" and "b": Adaptive parameters for the standard errors computation if when "IFcorAdapt" is chosen.

For example, for the control parameters for the expected shortfall, we can use the simple default:

```r
ES.control <- RPESE.control(estimator="ES")
ES.control
```

Note that the return value is a list with the control parameters, and we can change individual components of the list, as for example:

```r
ES.control$cleanOutliers <- TRUE
```

If we want to enforce some parameters directly by the function call, we can use the relevant arguments of `RPESE.control` directly.

```r
ES.control <- RPESE.control(estimator="ES", cleanOutliers=TRUE, freq.include="Decimate")
ES.control
```

This the above `ES.control` is to be used for the `SE.control` argument if `SE=TRUE`.

```r
# Expected shortfall SE computation with control parameters
ESout <- t(ES(edhec, SE=TRUE, SE.control=ES.control))
ESout
```
## ES IFiid IFcor
## CA 0.03655000 0.008973782 0.003723893
## CTA 0.04126429 0.003459795 0.003532085
## DIS 0.03669286 0.006728856 0.006105000
## EM 0.07236429 0.013914916 0.007730818
## EMN 0.01687857 0.004262266 0.001670694
## ED 0.03856429 0.006000260 0.003387294
## FIA 0.02825714 0.007971074 0.001991072
## GM 0.02062857 0.002098174 0.002253347
## LS 0.04220714 0.005082402 0.006085118
## MA 0.01914286 0.003892194 0.002280315
## RV 0.02465000 0.004975177 0.003088431
## SS 0.09682143 0.009600362 0.011187394
## FOF 0.03320714 0.005092672 0.003038685

# We can use RPESE.control directly in the function call
ESout <- t(ES(edhec,
            SE=TRUE,
            SE.control=RPESE.control(estimator="ES",
                                   se.method=c("IFiid",
                                                "IFcor",
                                                "BOOTiid",
                                                "BOOTcor"))))
ESout

## ES IFiid IFcor BOOTiid BOOTcor
## CA 0.03655000 0.008973782 0.003723893 0.015803015 0.008884762 0.013433128
## CTA 0.04126429 0.003459795 0.003532085 0.003433112 0.003243576 0.003474148
## DIS 0.03669286 0.006728856 0.006105000 0.010245235 0.007232031 0.007567787
## EM 0.07236429 0.013914916 0.007730818 0.016426163 0.007232031 0.007567787
## EMN 0.01687857 0.004262266 0.001670694 0.004570448 0.004004225 0.005120100
## ED 0.03856429 0.006000260 0.003387294 0.013084620 0.005482296 0.005301204
## FIA 0.02825714 0.007971074 0.001991072 0.016426163 0.007232031 0.007567787
## GM 0.02062857 0.002098174 0.002253347 0.002105179 0.001898005 0.001852979
## LS 0.04220714 0.005082402 0.006085118 0.004570448 0.004004225 0.005120100
## MA 0.01914286 0.003892194 0.002280315 0.004335635 0.003725457 0.003211319
## RV 0.02465000 0.004975177 0.003088431 0.009393838 0.004787178 0.007092247
3.5 Outlier Cleaning

There is also an outlier cleaning functionality in the RPEIF package that is fully described in Section 7 of CM, and is available in RPESE. Here we illustrate the use of the outlier cleaning facility in terms of the influence function transformed returns for the sample mean estimator. It is shown in Section 2 of CM that the influence function for the sample mean estimator is \( IF(r; \mu) = r - \mu \). Thus the IF transformed returns time series, computed with the function \( \text{IF.mean} \) is just \( r_t - \mu \), where \( \mu \) is replaced by the sample mean in the SE computation implementation. The function \( \text{IF.mean} \) is made accessible by loading the package RPEIF. The following code produces Figure 2, which illustrate the effect of outlier cleaning relative to no outlier cleaning for the FIA hedge fund returns.

```r
library(RPEIF)
IFout <- IF.mean(returns = edhec[,"FIA"], cleanOutliers = F, IFprint = T)
IFout.clean <- IF.mean(returns = edhec[,"FIA"], cleanOutliers = T, IFprint = T)
par(mfrow = c(2,1))
ylim = c(-.1,.035)
plot.zoo(IFout,type = "b", ylab = expression(paste("Returns - \( \mu \))), main = "FIA Returns", pch = 20, lwd = .8, cex = .9, ylim = ylim)
plot.zoo(IFout.clean,type = "b", ylab = expression(paste("Returns - \( \mu \))), main = "FIA Outlier Cleaned Returns", pch = 20, lwd = .8, cex = .9, ylim = ylim)
par(mfrow = c(1,1))
```
You can use the following code to compare expected shortfall SE’s without and with outlier cleaning.

```r
# IFcor SE results with outliers present and with outliers cleaned
ESout <- t(ES(edhec, SE=TRUE, SE.control=RPESE.control(estimator="ES")))
ESout.clean <- t(ES(edhec, SE=TRUE, SE.control=RPESE.control(estimator="ES", cleanOutliers=T)))
clean.compare <- data.frame(ESout[,3], ESout.clean[,3])
names(clean.compare) <- c("With Outliers", "Outliers Cleaned")
row.names(clean.compare) <- names(edhec)
round(clean.compare,3)
```

Figure 2: FIA Returns Versus Outlier Cleaned FIA Returns (with sample mean subtracted)
It is not surprising that the SE’s of the expected shortfall are smaller with outlier cleaning than with the outliers in the returns, as outliers generally inflate estimator variability.

### 3.6 Remark for Conflicting Arguments for Standard Errors Computation

There are some arguments in some of the PerformanceAnalytics functions that will be assumed to hold a certain value when standard errors are computed. For example, recall the arguments of the ES function.

```
args(ES)
```

```
## function (R = NULL, p = 0.95, ..., method = c("modified", "gaussian", "historical"), clean = c("none", "boudt", "geltner", "locScaleRob"), portfolio_method = c("single", "component"), weights = NULL, mu = NULL, sigma = NULL, m3 = NULL, m4 = NULL, invert = TRUE, operational = TRUE, SE = FALSE, SE.control = NULL)
## NULL
```

The following ES arguments will be overwritten if SE=TRUE and the arguments do not contain the values below.

- "method": This argument will be enforced to be "historical".
• "portfolio_method": This argument will be enforced to be "single".

• "invert": This argument will be enforced to be "FALSE".

• "clean": This argument will be enforced to be match the argument in the "SE.control" argument.

References
