Package ‘PolynomF’

June 18, 2019

Type Package
Title Polynomials in R
Description Implements univariate polynomial operations in R, including polynomial arithmetic, finding zeros, plotting, and some operations on lists of polynomials.
Version 2.0-2
Date 2019-06-18
NeedsCompilation yes
Imports stats, Rcpp, methods
Depends R (>= 3.0.0), graphics, grDevices
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License GPL-2
Encoding UTF-8
LinkingTo Rcpp
RoxygenNote 6.1.1
Suggests knitr, rmarkdown
VignetteBuilder knitr
Repository CRAN
Date/Publication 2019-06-18 06:50:25 UTC

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as.character.polynom  

Polynomial coercion to character

Description

Produce a text representation of a polynomial object

Usage

## S3 method for class 'polynom'
as.character(x, variable = "x", decreasing = FALSE,
...)

Arguments

x  
The polynomial object in question
variable  
Character string: what variable name should be used?
decreasing  
Logical: in decreasing powers or increasing powers?
...  
Additional arguments (ignored as yet)

Value

A character string representation of the polynomial
as.function.polynom

Examples

```r
p <- poly_from_zeros(-2:3)
as.character(p, "z", FALSE)
as.character(p, "z", TRUE)
parse(text = as.character(p, "z", TRUE))[[1]]
```

---

as.function.polynom  Coercion to function

Description

PolynomF objects ARE functions, but this coercion method creates from a polynomial object a pure function with the coefficients fully exposed in the code and which evaluates the polynomial more efficiently.

Usage

```r
## S3 method for class 'polynom'
as.function(x, variable = "x", ...)

## S3 method for class 'polylist'
as.function(x, ...)
```

Arguments

- `x`  
  A polynomial object
- `variable`  
  Character string: what variable name should be used?
- `...`  
  Additional arguments

Value

An explicit R function evaluating the polynomial

Examples

```r
p <- poly_from_zeros(-2:3)
p
as.function(p)
```
c.polynom

Concatenation of polynomial objects into lists

Description

Concatenation of polynomial objects into lists

Usage

## S3 method for class 'polynom'
c(..., recursive = FALSE)

## S3 method for class 'polylist'
c(..., recursive = FALSE)

Arguments

... Polynomial or polylist objects
recursive Logical, should the concatenation flatten all component lists?

Value

A polylist object with all arguments included

---

change_origin

Change origin of a polynomial

Description

Given a polynomial \( P(x) \) and a new origin \( o \), find the polynomial \( Q(x) = P(x + o) \). I.e. \( Q(0) = P(o) \)

Usage

change_origin(p, o, ...)

## Default S3 method:
change_origin(p, o, ...)

## S3 method for class 'polynom'
change_origin(p, o, ...)

## S3 method for class 'polylist'
change_origin(p, o, ...)
Arguments

p  A polynom or polylist object
o  A single numeric quantity specifying the new x-origin
... currently not used

Value

A polynom or polylist object with x measured from the new origin

---

coef.polynom  

Polynomial coefficients

Description

Extract polynomial coefficients

Usage

```r
## S3 method for class 'polynom'
coef(object, ...)

## S3 method for class 'polylist'
coef(object, ...)
```

Arguments

object  A polynomial object or list thereof
...  Ignored

Value

A numeric vector of coefficients

Examples

```r
p <- polynomial(1:3)*polynomial(5:1)
coef(p)
```
Defunct functions

Description

These functions have been removed from the package to allow for systematic nomenclature. Appropriate drop-in replacements are given if the function is called.

Usage

as.polynom(a)
is.polynom(a)
is.polylist(x)
as.polylist(x)
change.origin(p, o, ...)
poly.calc(...)
poly.from.zeros(...)
poly.from.values(...)

Arguments

a, p, x  A numeric vector or polynom object
o  A numeric vector of length one.
...  Additional arguments of appropriate class

Value

An error is triggered and appropriate error message is returned

Polynomial Calculus

Description

Find the derivative or indefinite integral of a polynomial object, or list thereof.
GCD

Usage

```r
## S3 method for class 'polynom'
deriv(expr, ...)

integral(expr, ...)

## Default S3 method:
integral(expr, ...)

## S3 method for class 'polynom'
integral(expr, limits = NULL, ...)

## S3 method for class 'polynomial'
deriv(expr, ...)

## S3 method for class 'polynomial'
integral(expr, ...)
```

Arguments

- `expr` A polynomial object, or list thereof
- `...` Unused as yet
- `limits` Real limits of a definite integral

Value

A coefficient vector, or list thereof

Examples

```r
p <- poly_from_roots(-2:3)
p
deriv(p)
integral(p)
```

Description

Find a monic polynomial of maximal degree that divides each of a set of polynomials exactly

GCD  Greatest common divisor
Usage

GCD(...)


greatest_common_divisor(...)

## S3 method for class 'polynom'
GCD(...)

## S3 method for class 'polylist'
GCD(...)

Arguments

... A list of polynomials or polylist objects

Value

A polynomial giving the greatest common divisor, as defined above

Examples

```r
p <- poly_calc(0:5)
r <- poly_calc(1:6)
greatest_common_divisor(p, r)
solve(greatest_common_divisor(p, r))
lowest_common_multiple(p, r)
solve(lowest_common_multiple(p, r))
```

---

Description

These provide methods for the generic function `Summary` and `Math` for polynomial and polylist objects. For `Summary` only `sum` and `prod` members are implemented

Usage

## S3 method for class 'polynom'
Summary(..., na.rm = FALSE)

## S3 method for class 'polylist'
Summary(..., na.rm = FALSE)

## S3 method for class 'polynom'
Math(x, ...)

## S3 method for class 'polylist'
Math(x, ...)
Arguments

... Additional arguments
na.rm Logical: should missing values be removed?
x a "polynom" or "polylist" objects.

Value

The result of the group generic operation

Examples

lis <- as_polylist(lapply(-2:3, function(x) polynomial() - x))
prod(lis)
sum(lis)
solve(prod(lis))
solve(sum(lis))

Description

For a list of polynomials, find the lowest degree monic polynomial into which each divides exactly

Usage

LCM(...)

lowest_common_multiple(...)

## S3 method for class 'polynom'
LCM(...)

## S3 method for class 'polylist'
LCM(...)

Arguments

... A list of polynomials or polylist objects

Value

A polynomial giving the lowest common multiple
Examples

```r
p <- poly_calc(0:5)
r <- poly_calc(1:6)
greatest_common_divisor(p, r)
solve(greatest_common_divisor(p, r))
lowest_common_multiple(p, r)
solve(lowest_common_multiple(p, r))
```

Description

Group generic function to implement arithmetic operations on polynomial objects

Usage

```r
## S3 method for class 'polynom'
Ops(e1, e2)
## S3 method for class 'polylist'
Ops(e1, e2)
```

Arguments

e1, e2

A numeric vector of a polynomial object. At least one of e1 or e2 must be an object of class "polynom" or "polylist".

Value

A polynomial or polylist object representing the result of the operation.

Examples

```r
x <- polynomial()
(p <- (x-1)^5 - 1)
(p1 <- (p + 1)/(x - 1)^2 - 1)
for(i in 0:10) cat(coef((x+1)^i), "\n")
```
**plot.polylist**

Plot method for polynomials

**Description**

Plot methods for polynom or polylist objects

**Usage**

```r
## S3 method for class 'polylist'
plot(x, xlim = 0:1, ylim = range(Px), type = "l",
     xlab = "x", ylab = "P(x)", ..., col = seq_along(x), lty = if
     (length(col) == 1) seq_along(x) else "solid", len = 1000,
     legend = FALSE)

## S3 method for class 'polynom'
plot(x, xlim = 0:1, ylim = range(Px), type = "l",
     xlab = "x", ylab = "p(x)", ..., len = 1000, limits = pu[1:2])

## S3 method for class 'polynom'
lines(x, ..., len = 1000, limits = pu[1:2])

## S3 method for class 'polynom'
points(x, ..., len = 100, limits = pu[1:2])

## S3 method for class 'polylist'
lines(x, ..., len = 1000, limits = pu[1:2],
     col = seq_along(x), lty = if (length(col) == 1) seq_along(x) else
     "solid")

## S3 method for class 'polylist'
points(x, ..., len = 100)
```

**Arguments**

- `x` A polynom or polylist object to be plotted
- `xlim, ylim` as for graphics::plot
- `type` as for graphics::plot
- `xlab, ylab` as for graphics::plot
- `...` additional arguments passed on to methods
- `col, lty` Colour(s) and line type(s) as for graphics::plot
- `len` positive integer defining the point or curve resolution
- `legend` logical: for "polylist" objects, should a legend be drawn alongside the main plot?
- `limits` x-limits for the polynomial, default: the entire plot. For polylist objects this may be a two column matrix.
Value

Nothing of interest, invisibly

Examples

```r
p <- poly_from_zeros((-3):4)
plot(p)
lines(deriv(p), col = "red")
```

---

**polynom**  
*Polynomial construction*

Description

Functions to construct polynomial objects and check class membership

Usage

```r
polynom(a = c(0, 1), ..., eps = 0)
polynomial(a = c(0, 1), ..., eps = 0)
as_polynom(a)
is_polynom(a)
polylist(...)
is_polylist(x)
as_polylist(x)
```

Arguments

- `a`  
  A polynom object, or a numeric vector of coefficients (in "power series" order) or a vector object which can be coerced to one.

- `...`  
  Additional arguments, currently ignored.

- `eps`  
  A small non-negative tolerance to check for zero components.

- `x`  
  An object of class "polylist", at least potentially.

Value

A polynomial object.
Examples

(s <- polynomial())
(p <- polynomial(c(1, 5, 4, 1)/11))
oldPar <- par(mar = c(5,5,2,2)+0.1)
plot(p, xlim = 0:1, ylim = 0:1, type = "n", bty="n",
     xlab = "s", ylab = expression((P^1,(s))))
lines(s, limits = 0:1)
P <- p
for(j in 1:7) {
    lines(P, col = j+1, limits = 0:1)
    P <- p(P)
}
lines(P, limits = 0:1, col = 9)
(r <- Re(solve((p-s)/(1-s))))
arrows(r, p(r), r, par("usr")[3], lwd = 0.5,
       length = 0.125, angle = 15)
text(r, 0.025, paste("r =", format(r, digits = 3)))
leg <- sapply(0:8, function(x) bquote((P^1,(x)))(s))
legend("topleft", legend = as.expression(leg),
       lty = "solid", col = 1:9, bty = "n", ncol=3)
par(oldPar)
rm(leg, oldPar, p, P, r, s, j)

---

**poly_calc**

Lagrange interpolation polynomial

**Description**

Calculate the Lagrange interpolation polynomial, or list of polynomials, given a set of (x, y) points to fit

**Usage**

```r
poly_calc(x, y, tol = sqrt(.Machine$double.eps),
          lab = dimnames(y)[[2]])
poly_from_zeros(...)
poly_from_roots(...)
poly_from_values(x, y, tol = sqrt(.Machine$double.eps),
                  lab = dimnames(y)[[2]])
```
poly_orth

Arguments

x A numeric vector of x-points at which the y-values are specified.

y Either a numeric vector of the same length as x or a numeric matrix with rows matching the length of x. If y is missing (not specified) then a polynomial with zero at x is returned.

tol A numeric tolerance for duplicated x values.

lab A character string vector of names for the list result when y is a matrix.

... A list of specified zeros (for subsidiary functions)

Value

An interpolation polynomial, or list of interpolating polynomials.

Examples

(p <- poly_calc(0:5)) # same as poly_from_zeros(0:5)
(p <- poly_calc(0:5, exp(0:5)))
plot(p)
curve(exp, add = TRUE, col = "red")

Description

Generate a list of polynomials up to a specified degree, orthogonal with respect to the natural inner product on a discrete, finite set of x-values with equal weights.

Usage

poly_orth(x, degree = length(unique(x)) - 1, norm = TRUE)
poly.orth(...)
Examples

```r
x <- c(0:3, 5)
P <- poly_orth(x)
plot(P, lty = "solid")
Pf <- as.function(P)
zap(crossprod(Pf(x)))
```

Description

Generate sets of polynomials orthogonal with respect to a general inner product. The inner product is specified by an R function of (at least) two polynomial arguments.

Usage

```r
poly_orth_general(inner_product, degree, norm = FALSE, ...)
```

Hermite(p, q = p)

Legendre(p, q = p)

ChebyshevT(p, q = p)

ChebyshevU(p, q = p)

Jacobi(p, q = p, alpha = -0.5, beta = alpha)

Discrete(p, q = p, x, w = function(x, ...) 1, ...)

Arguments

- `inner_product`: An R function of two "polynom" arguments with the second polynomial having a default value equal to the first. Additional arguments may be specified. See examples
- `degree`: A non-negative integer specifying the maximum degree
- `norm`: Logical: should the polynomials be normalized?
- `...`: additional arguments passed on to the inner product function
- `p, q`: Polynomials
- `alpha, beta`: Family parameters for the Jacobi polynomials
- `x`: numeric vector defining discrete orthogonal polynomials
- `w`: a weight function for discrete orthogonal polynomials
Details

Discrete orthogonal polynomials, equally or unequally weighted, are included as special cases. See the Discrete inner product function.

Computations are done using the recurrence relation with computed coefficients. If the algebraic expressions for these recurrence relation coefficients are known the computation can be made much more efficient.

Value

A "polylist" object containing the orthogonal set

Examples

(P0 <- polyOrth(0:5, norm = FALSE))
(P1 <- polyOrthGeneral(Discrete, degree = 5, x = 0:5, norm = FALSE))
sapply(P0-P1, function(x) max(abs(coef(x)))) ## visual check for equality
(P0 <- polyOrthGeneral(Legendre, 5))
## should be same as P0, up to roundoff
(P1 <- polyOrthGeneral(Jacobi, 5, alpha = 0, beta = 0))
## check
sapply(P0-P1, function(x) max(abs(coef(x))))

predict.polynom

Evaluate a polynomial

Description

Evaluate a polynomial, or polylist object components.

Usage

## S3 method for class 'polynom'
predict(object, newdata, ...)

## S3 method for class 'polylist'
predict(object, newdata, ...)

Arguments

object A polynomial or polylist object
newdata A target object at which to evaluate.
... Not used

Value

If newdata is a numeric vector, a numeric vector of results. If newdata is a polynomial, then the composition is returned as a polynomial, or polylist object.
print.polylist  
*Print method for polynomial objects*

**Description**

Print method for polynomial objects

**Usage**

```r
## S3 method for class 'polylist'
print(x, ...)
```

**Arguments**

- `x` A polynomial object or list thereof
- `...` Additional arguments passed on to methods

**Value**

The original object, invisibly.

---

print.polynom  
*Print method for polynomial objects*

**Description**

Standard method for printing polynomial objects

**Usage**

```r
## S3 method for class 'polynom'
print(x, variable = "x",
      digits = getOption("digits"), decreasing = FALSE, ...)
```

**Arguments**

- `x` A polynomial object
- `variable` Character string: what variable name should be given?
- `digits` Integer: how many decimal digits to use?
- `decreasing` Logical: in descending powers, or ascending?
- `...` Additional arguments

**Value**

The original object `x`, invisibly
**rep.polylist**  \hspace{1cm} \textit{Component repetition}

**Description**

Repeat components of a polylist object

**Usage**

```r
## S3 method for class 'polylist'
rep(x, times, ...)

## S3 method for class 'polynom'
rep(x, times, ...)
```

**Arguments**

- `x`  
  A single polynom or polylist object  
- `times, ...`  
  As for the base package function `rep`.

**Value**

The resulting polylist object.

---

**solve.polynom**  \hspace{1cm} \textit{Find Polynomial Zeros}

**Description**

Solve polynomial equations, \( a(x) = b(x) \), or alternatively find the zeros of the polynomial \( a(x) - b(x) \)

**Usage**

```r
## S3 method for class 'polynom'
solve(a, b, ...)  

## S3 method for class 'polylist'
solve(a, b, ...)
```

**Arguments**

- `a, b`  
  Polynomials for the LHS and RHS respectively
- `...`  
  Currently unused
summary.polynom

Value

A vector of roots, usually complex

Examples

```r
p <- poly_calc(0:5)
solve(p)
solve(p, 1)
```

summary.polynom  Polynomial summary

Description

Provide a succinct summary of the critical points of a polynomial, or list thereof

Usage

```r
## S3 method for class 'polynom'
summary(object, ...)

## S3 method for class 'polylist'
summary(object, ...)

## S3 method for class 'summary.polynom'
print(x, ...)
```

Arguments

- `object, x` A polynomial or polylist object
- `...` Currently unused

Value

A list giving the zeros, stationary points and points of inflexion of the polynomial(s)

Examples

```r
p <- poly_calc(0:5)
summary(p)
```
tangent

Tangent lines

Description

Find the tangent line to a polynomial at one or more x-points

Usage

tangent(p, x0)

Arguments

p
A polynomial object

x0
A numeric vector of values at which the tangent line(s) are required

Value

A linear polynomial giving the tangent line, or a list of such polynomials

Examples

p <- poly_from_zeros(c(0, 0.5, 4))
plot(p, xlab = expression(italic(x)), ylab = expression(italic(P(x))),
main = parse(text = paste("italic(P(x) ==",
as.character(p, decreasing = TRUE), "\n")))))
x0 <- solve(deriv(p))  # stationary points
lines(tangent(p, x0), col = "dark green", lty = "solid",
limits = cbind(x0-1/4, x0+1/4))
p(x0, col = "dark green")

x0 <- solve(deriv(deriv(p)))  # points of inflexion
lines(tangent(p, x0), col = "red", lty = "solid", lwd = 2,
limits = cbind(x0-1/4, x0+1/4))
p(x0, col = "red")
legend("bottomleft", c("Stationary points", "Points of inflexion"),
pch = 19, col = c("dark green", "red"), lty = "solid",
cex = 0.7, bg = "beige", box.lwd = 0.25)
unique.polylist  Unique components

Description
Remove duplicated polynomials in a polylist object

Usage
```r
## S3 method for class 'polylist'
unique(x, incomparables = FALSE, ...)
```

Arguments
- `x`: A polylist object
- `incomparables`: Logical: as for the base function `unique`
- `...`: As for the base function `unique`

Value
A polylist object with no duplicated components

zap  Remove minuscule coefficients

Description
A convenience function for setting polynomial coefficients likely to be entirely round-off error to zero. The decision is relegated to the function `base::zapsmall`, to which this is a front-end.

Usage
```r
zap(x, digits =getOption("digits"))
```

## Default S3 method:
```r
zap(x, digits =getOption("digits"))
```

## S3 method for class 'polynom'
```r
zap(x, digits =getOption("digits"))
```

## S3 method for class 'polylist'
```r
zap(x, digits =getOption("digits"))
```

## S3 method for class 'list'
```r
zap(x, digits =getOption("digits"))
```
Arguments

- x: A polynomial or polylist object
- digits: As for base::zapsmall
- ...: Passed on to base::zapsmall

Value

A polynomial or polylist object with minuscule coefficients set to zero.

Examples

```r
(P <- polyOrth(-2:2, norm = FALSE))
zap(35*P)
```

Description

Extract components of a list of polynomials

Usage

```r
## S3 method for class 'polylist'
x[i]
```

Arguments

- x: A polylist object
- i: An index vector of any congruent form

Value

A polylist object of the components
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