Package ‘PolynomF’

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as.character.polynom

Polynomial coercion to character

Description

Produce a text representation of a polynomial object

Usage

## S3 method for class 'polynom'
as.character(x, variable = "x", decreasing = FALSE, ...)

Arguments

x The polynomial object in question
variable Character string: what variable name should be used?
decreasing Logical: in decreasing powers or increasing powers?
... Additional arguments (ignored as yet)

Value

A character string representation of the polynomial
as.function.polynom

Examples

p <- poly_from_zeros(-2:3)
as.character(p, "z", FALSE)
as.character(p, "z", TRUE)
parse(text = as.character(p, "z", TRUE))[[1]]

description

PolynomF objects ARE functions, but this coercion method creates from a polynomial object a pure function with the coefficients fully exposed in the code and which evaluates the polynomial more efficiently.

Usage

## S3 method for class 'polynom'
as.function(x, variable = "x", ...)

## S3 method for class 'polylist'
as.function(x, ...)

Arguments

x
A polynomial object

variable
Character string: what variable name should be used?

... Additional arguments

Value

An explicit R function evaluating the polynomial

Examples

p <- poly_from_zeros(-2:3)
p
as.function(p)
c.polynom

Concatenation of polynomial objects into lists

Description

Concatenation of polynomial objects into lists

Usage

## S3 method for class 'polynom'
c(..., recursive = FALSE)

## S3 method for class 'polylist'
c(..., recursive = FALSE)

Arguments

... Polynomial or polylist objects
recursive Logical, should the concatenation flatten all component lists?

Value

A polylist object with all arguments included

change_origin

Change origin of a polynomial

Description

Given a polynomial \( P(x) \) and a new origin \( o \), find the polynomial \( Q(x) = P(x + o) \). I.e. \( Q(0) = P(o) \)

Usage

change_origin(p, o, ...)

## Default S3 method:
change_origin(p, o, ...)

## S3 method for class 'polynom'
change_origin(p, o, ...)

## S3 method for class 'polylist'
change_origin(p, o, ...)
**Arguments**

- **p** A polynom or polylist object
- **o** A single numeric quantity specifying the new x-origin
- **...** currently not used

**Value**

A polynom or polylist object with x measured from the new origin

---

**Description**

Extract polynomial coefficients

**Usage**

```r
## S3 method for class 'polynom'
coef(object, ...)

## S3 method for class 'polylist'
coef(object, ...)
```

**Arguments**

- **object** A polynomial object or list thereof
- **...** Ignored

**Value**

A numeric vector of coefficients

**Examples**

```r
p <- polynomial(1:3)*polynomial(5:1)
coef(p)
```
Derivative Polynomial Calculus

Description

Find the derivative or indefinite integral of a polynomial object, or list thereof.

Usage

```r
## S3 method for class 'polynom'
deriv(expr, ...)

integral(expr, ...)

## Default S3 method:
integral(expr, ...)

## S3 method for class 'polynom'
integral(expr, limits = NULL, ...)

## S3 method for class 'polylist'
deriv(expr, ...)

## S3 method for class 'polylist'
integral(expr, ...)
```

Arguments

- `expr`: A polynomial object, or list thereof
- `...`: Unused as yet
- `limits`: Real limits of a definite integral

Value

A coefficient vector, or list thereof

Examples

```r
p <- poly_from_roots(-2:3)
p
deriv(p)
integral(p)
```
**GCD**

*Greatest common divisor*

**Description**

Find a monic polynomial of maximal degree that divides each of a set of polynomials exactly.

**Usage**

GCD(...)

greatest_common_divisor(...)

```r
## S3 method for class 'polynom'
GCD(...)
```

```r
## S3 method for class 'polylist'
GCD(...)
```

**Arguments**

...  
A list of polynomials or polylist objects

**Value**

A polynomial giving the greatest common divisor, as defined above.

**Examples**

```r
p <- poly_calc(0:5)
r <- poly_calc(1:6)
greatest_common_divisor(p, r)
solve(greatest_common_divisor(p, r))
lowest_common_multiple(p, r)
solve(lowest_common_multiple(p, r))
```

---

**GroupGenerics**

*Summary and Math methods for polynomials*

**Description**

These provide methods for the generic function `Summary` and `Math` for polynomial and polylist objects. For `Summary` only `sum` and `prod` members are implemented.
**LCM**

**Usage**

```r
## S3 method for class 'polynom'
Summary(..., na.rm = FALSE)

## S3 method for class 'polylist'
Summary(..., na.rm = FALSE)

## S3 method for class 'polynom'
Math(x, ...)

## S3 method for class 'polylist'
Math(x, ...)
```

**Arguments**

- `...`: Additional arguments
- `na.rm`: Logical: should missing values be removed?
- `x`: a "polynom" or "polylist" objects.

**Value**

The result of the group generic operation

**Examples**

```r
lis <- as_polylist(lapply(-2:3, function(x) polynomial() - x))
prod(lis)
sum(lis)
solve(prod(lis))
solve(sum(lis))
```

---

**LCM**

**Lowest Common Multiple**

**Description**

For a list of polynomials, find the lowest degree monic polynomial into which each divides exactly

**Usage**

```r
LCM(...)  # same as lowest_common_multiple(...)
```

```r
## S3 method for class 'polynom'
LCM(...)  # same as lowest_common_multiple(...)

## S3 method for class 'polylist'
LCM(...)  # same as lowest_common_multiple(...)
```
Arguments

... A list of polynomials or polylist objects

Value

A polynomial giving the lowest common multiple

Examples

```r
p <- poly_calc(0:5)
r <- poly_calc(1:6)
greatest_common_divisor(p, r)
solve(greatest_common_divisor(p, r))
lowest_common_multiple(p, r)
solve(lowest_common_multiple(p, r))
```

---

**Ops.polynom**

**Polyomial arithmetic**

Description

Group generic function to implement arithmetic operations on polynomial objects

Usage

```r
## S3 method for class 'polynom'
Ops(e1, e2)

## S3 method for class 'polylist'
Ops(e1, e2)
```

Arguments

e1, e2 A numeric vector of a polynomial object. At least one of e1 or e2 must be an object of class "polynom" or "polylist".

Value

A polynomial or polylist object representing the result of the operation.

Examples

```r
x <- polynomial()
(p <- (x-1)^5 - 1)
(p1 <- (p + 1)/(x - 1)^2 - 1)
for(i in 0:10) cat(coef((x+1)^i), "\n")
```
plot.polylist  
Plot method for polynomials

Description
Plot methods for polynom or polylist objects

Usage

```r
## S3 method for class 'polylist'
plot(
  x,
  xlim = 0:1,
  ylim = range(Px),
  type = "l",
  xlab = "x",
  ylab = "P(x)",
  ..., 
  col = seq_along(x),
  lty = if (length(col) == 1) seq_along(x) else "solid",
  len = 1000,
  legend = FALSE
)

## S3 method for class 'polynom'
plot(
  x,
  xlim = 0:1,
  ylim = range(Px),
  type = "l",
  xlab = "x",
  ylab = "p(x)",
  ..., 
  len = 1000,
  limits = pu[1:2]
)

## S3 method for class 'polynom'
lines(x, ..., len = 1000, limits = pu[1:2])

## S3 method for class 'polynom'
points(x, ..., len = 100, limits = pu[1:2])

## S3 method for class 'polylist'
lines(
  x,
  ..., 
)```

```
### Arguments

- **x** A `polynom` or `polylist` object to be plotted
- **xlim, ylim** as for `graphics::plot`
- **type** as for `graphics::plot`
- **xlab, ylab** as for `graphics::plot`
- **...** additional arguments passed on to methods
- **col, lty** Colour(s) and line type(s) as for `graphics::plot`
- **len** positive integer defining the point or curve resolution
- **legend** logical: for "polylist" objects, should a legend be drawn alongside the main plot?
- **limits** x-limits for the polynomial, default: the entire plot. For polylist objects this may be a two column matrix.

### Value

Nothing of interest, invisibly

### Examples

```r
p <- poly_from_zeros((-3):4)
plot(p)
lines(deriv(p), col = "red")
```

---

**polynom**  
*Polynomial construction*

**Description**

Functions to construct polynomial objects and check class membership
Usage

polynom(a = c(0, 1), ..., eps = 0)

polynomial(a = c(0, 1), ..., eps = 0)

as_polynom(a)

is_polynom(a)

polylist(...)

is_polylist(x)

as_polylist(x)

Arguments

a A polynom object, or a numeric vector of coefficients (in "power series" order) or a vector object which can be coerced to one.

... Additional arguments, currently ignored.

eps A small non-negative tolerance to check for zero components.

x An object of class "polylist", at least potentially.

Value

A polynomial object.

Examples

(s <- polynomial())
(p <- polynomial(c(1, 5, 4, 1)/11))
oldPar <- par(mar = c(5,5,2,2)+0.1)
plot(p, xlim = 0:1, ylim = 0:1, type = "n", bty="n",
     xlab = "s", ylab = expression(P(n)(s)))
lines(s, limits = 0:1)

P <- p
for(j in 1:7) {
  lines(P, col = j+1, limits = 0:1)
  P <- p(P)
}
lines(P, limits = 0:1, col = 9)
(r <- Re(solve((p-s)/(1-s))))
arrows(r, p(r), r, par("usr")[3], lwd = 0.5,
       length = 0.125, angle = 15)
text(r, 0.025, paste("r =", format(r, digits = 3)))
leg <- sapply(0:8, function(x) bquote(P((x))(s)))
legend("topleft", legend = as.expression(leg),
       lty = "solid", col = 1:9, bty = "n", ncol=3)
par(oldPar)
rm(leg, oldPar, p, P, r, s, j)
**poly_calc**

**Lagrange interpolation polynomial**

**Description**

Calculate the Lagrange interpolation polynomial, or list of polynomials, given a set of (x, y) points to fit.

**Usage**

```r
poly_calc(x, y, tol = sqrt(.Machine$double.eps), lab = dimnames(y)[[2]])
poly_from_zeros(...)
poly_from_roots(...)
poly_from_roots(...)
poly_from_values(x, y, tol = sqrt(.Machine$double.eps), lab = dimnames(y)[[2]])
```

**Arguments**

- `x`: A numeric vector of x-points at which the y-values are specified.
- `y`: Either a numeric vector of the same length as `x` or a numeric matrix with rows matching the length of `x`. If `y` is missing (not specified) then a polynomial with zero at `x` is returned.
- `tol`: A numeric tolerance for duplicated x values.
- `lab`: A character string vector of names for the list result when `y` is a matrix.
- `...`: A list of specified zeros (for subsidiary functions).

**Value**

An interpolation polynomial, or list of interpolating polynomials.

**Examples**

```r
(p <- poly_calc(0:5)) # same as poly_from_zeros(0:5)
(p <- poly_calc(0:5, exp(0:5)))
plot(p)
curve(exp, add = TRUE, col = "red")
```
**poly_orth**

*Simpl orthogonal polynomials*

**Description**

Generate a list of polynomials up to a specified degree, orthogonal with respect to the natural inner product on a discrete, finite set of x-values with equal weights.

**Usage**

```
poly_orth(x, degree = length(unique(x)) - 1, norm = TRUE)
```

**Arguments**

- `x` A numeric vector
- `degree` The desired maximum degree
- `norm` Logical: should polynomials be normalised to length one?

**Value**

A list of orthogonal polynomials as a polylist object

**Examples**

```
x <- c(0:3, 5)
P <- poly_orth(x)
plot(P, lty = "solid")
PF <- as.function(P)
zap(crossprod(PF(x)))
```

---

**poly_orth_general**

*General Orthogonal Polynomials*

**Description**

Generate sets of polynomials orthogonal with respect to a general inner product. The inner product is specified by an R function of (at least) two polynomial arguments.
poly_orth_general

Usage

poly_orth_general(inner_product, degree, norm = FALSE, ...)

Hermite(p, q = p)

Legendre(p, q = p)

ChebyshevT(p, q = p)

ChebyshevU(p, q = p)

Jacobi(p, q = p, alpha = -0.5, beta = alpha)

Discrete(p, q = p, x, w = function(x, ...) 1, ...)

Arguments

inner_product  An R function of two "polynom" arguments with the second polynomial having a default value equal to the first. Additional arguments may be specified. See examples

degree  A non-negative integer specifying the maximum degree

norm  Logical: should the polynomials be normalized?

...  additional arguments passed on to the inner product function

p, q  Polynomials

alpha, beta  Family parameters for the Jacobi polynomials

x  numeric vector defining discrete orthogonal polynomials

w  a weight function for discrete orthogonal polynomials

Details

Discrete orthogonal polynomials, equally or unequally weighted, are included as special cases. See the Discrete inner product function.

Computations are done using the recurrence relation with computed coefficients. If the algebraic expressions for these recurrence relation coefficients are known the computation can be made much more efficient.

Value

A "polylist" object containing the orthogonal set

Examples

(P0 <- poly_orth(0:5, norm = FALSE))
(P1 <- poly_orth_general(Discrete, degree = 5, x = 0:5, norm = FALSE))
sapply(P0-P1, function(x) max(abs(coef(x))))  ## visual check for equality
(P0 <- poly_orth_general(Legendre, 5))

### should be same as P0, up to roundoff
(P1 <- poly_orth_general(Jacobi, 5, alpha = 0, beta = 0))

### check
sapply(P0-P1, function(x) max(abs(coef(x)))))

---

**predict.polynom**  
*Evaluate a polynomial*

---

**Description**

Evaluate a polynomial, or polylist object components.

**Usage**

```
## S3 method for class 'polynom'
predict(object, newdata, ...)
```

```
## S3 method for class 'polylist'
predict(object, newdata, ...)
```

**Arguments**

- `object`: A polynomial or polylist object
- `newdata`: A target object at which to evaluate.
- `...`: Not used

**Value**

If `newdata` is a numeric vector, a numeric vector of results. If `newdata` is a polynomial, then the composition is returned as a polynomial, or polylist object.

---

**print.polylist**  
*Print method for polynomial objects*

---

**Description**

Print method for polynomial objects

**Usage**

```
## S3 method for class 'polylist'
print(x, ...)
```

**Arguments**

- `x`: A polynomial object or list thereof
- `...`: Additional arguments passed on to methods
**print.polynom**

**Value**

The original object, invisibly.

---

**print.polynom**  
*Print method for polynomial objects*

**Description**

Standard method for printing polynomial objects

**Usage**

```r
## S3 method for class 'polynom'
print(x, variable = "x", digits = getOption("digits"), decreasing = FALSE, ...)
```

**Arguments**

- `x`: A polynomial object
- `variable`: Character string: what variable name should be given?
- `digits`: Integer: how many decimal digits to use?
- `decreasing`: Logical: in descending powers, or ascending?
- `...`: Additional arguments

**Value**

The original object `x`, invisibly

---

**rep.polylist**  
*Component repetition*

**Description**

Repeat components of a polylist object

**Usage**

```r
## S3 method for class 'polylist'
rep(x, times, ...)
```

```r
## S3 method for class 'polynom'
rep(x, times, ...)
```
solve.polynom

Arguments

x A single polynom or polylist object
times, ... As for the base package function rep.

Value

The resulting polylist object.

Description

Solve polynomial equations, a(x) = b(x), or alternatively find the zeros of the polynomial a(x) - b(x)

Usage

## S3 method for class 'polynom'
solve(a, b, ...)

## S3 method for class 'polylist'
solve(a, b, ...)

Arguments

a, b Polynomials for the LHS and RHS respectively
... Currently unused

Value

A vector of roots, usually complex

Examples

p <- poly_calc(0:5)
solve(p)
solve(p, 1)
summary.polynom

Polynomial summary

Description

Provide a succinct summary of the critical points of a polynomial, or list thereof

Usage

```r
## S3 method for class 'polynom'
summary(object, ...)

## S3 method for class 'polylist'
summary(object, ...)

## S3 method for class 'summary.polynom'
print(x, ...)
```

Arguments

- `object, x` A polynomial or polylist object
- `...` Currently unused

Value

A list giving the zeros, stationary points and points of inflexion of the polynomial(s)

Examples

```r
p <- poly_calc(0:5)
summary(p)
```

tangent

Tangent lines

Description

Find the tangent line to a polynomial at one or more x-points

Usage

```r
tangent(p, x0)
```

Arguments

- `p` A polynomial object
- `x0` A numeric vector of values at which the tangent line(s) are required
unique.polylist

**Value**

A linear polynomial giving the tangent line, or a list of such polynomials

**Examples**

```r
p <- poly_from_zeros(c(0, 0:5, 4))
plot(p, xlab = expression(italic(x)), ylab = expression(italic(P(x))),
    main = parse(text = paste("italic(P(x) ==",
                         as.character(p, decreasing = TRUE),"")))
x0 <- solve(deriv(p))  ## stationary points
lines(tangent(p, x0), col = "dark green", lty = "solid",
       limits = cbind(x0-1/4, x0+1/4))
points(x0, p(x0), col = "dark green")

x0 <- solve(deriv(deriv(p)))  ## points of inflexion
lines(tangent(p, x0), col = "red", lty = "solid", lwd = 2,
       limits = cbind(x0-1/4, x0+1/4))
points(x0, p(x0), col = "red")
legend("bottomleft", c("Stationary points", "Points of inflexion"),
        pch = 19, col = c("dark green", "red"), lty = "solid",
        cex = 0.7, bg = "beige", box.lwd = 0.25)
```

---

**unique.polylist**

*Unique components*

**Description**

Remove duplicated polynomials in a polylist object

**Usage**

```r
## S3 method for class 'polylist'
unique(x, incomparables = FALSE, ...)
```

**Arguments**

- **x**  
  A polylist object

- **incomparables**  
  Logical: as for the base function unique

- **...**  
  As for the base function unique

**Value**

A polylist object with no duplicated components
zap

*zap*  
*Remove minuscule coefficients*

**Description**

A convenience function for setting polynomial coefficients likely to be entirely round-off error to zero. The decision is relegated to the function `base::zapsmall`, to which this is a front-end.

**Usage**

```r
zap(x, digits = getOption("digits"))
```

```r
## Default S3 method:
zap(x, digits = getOption("digits"))
```

```r
## S3 method for class 'polynom'
zap(x, digits = getOption("digits"))
```

```r
## S3 method for class 'polylist'
zap(x, digits = getOption("digits"))
```

```r
## S3 method for class 'list'
zap(x, digits = getOption("digits"))
```

**Arguments**

- **x**: A polynomial or polylist object
- **digits**: As for `base::zapsmall`

**Value**

A polynomial or polylist object with minuscule coefficients set to zero.

**Examples**

```r
(P <- poly_orth(-2:2, norm = FALSE))
zap(35*P)
```
Description
Extract components of a list of polynomials

Usage
```r
## S3 method for class 'polylist'
x[i]
```

Arguments
- `x`: A polylist object
- `i`: An index vector of any congruent form

Value
A polylist object of the components
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