Properties of the quartet distance are explored fully in Steel (1993). As quartet distances of 1 can only be accomplished for small trees (five or fewer leaves), it is perhaps more appropriate to consider whether or not trees are more dissimilar than a pair of random trees, whose distance will be, on average, \( \frac{2}{3} \).

### 0.1 Minimum quartet distance

When there are six or more tips in a bifurcating tree, some quartets are necessarily shared between trees. Consider the tree:

```r
tree_a <- ape::read.tree(text="((1, 2), (3, (4, 5)));")
```

![Tree A](attachment:image.png)

The only trees with no quartets in common with Tree A are symmetric with

```r
tree_b <- ape::read.tree(text="((1, 5), (3, (2, 4)));")
```

![Tree B](attachment:image.png)

Now create Tree C by adding a 6th tip as a sister to tip 3 on Tree A.

```r
tree_c <- ape::read.tree(text="((1, 2), ((3, 6), (4, 5)));")
```

![Tree C](attachment:image.png)
There's nowhere to add tip 6 to Tree B without creating a quartet that exists in Tree C.

0.2 Quartet distance in a pair of random trees

On average, \( \frac{1}{3} \) of the quartets resolved in a pair of random trees will match. This is because there are three quartets involving any set of four tips, each of which is equally likely to occur on a truly random tree.

The below code calculates the mean proportion of matching quartets for random trees with 4 to 20 tips, and the corresponding standard deviation.

```r
round(vapply(4:20, function (n_tip) {
  trees <- lapply(logical(56), function (X)
    ape::rtree(n_tip, tip.label=seq_len(n_tip), br=NULL))
```

2
results <- QuartetStatus(trees)[1, ] / choose(n_tip, 4)
c(mean(results[-1]), sd(results[-1]))
}, double(2), 3)

## [1,] 0.333 0.333 0.333 0.333 0.333 0.333 0.333 0.333 0.333 0.333 0.333
## [2,] 0.516 0.516 0.516 0.516 0.516 0.516 0.516 0.516 0.516 0.516 0.516
## [1,] 0.333 0.333 0.333 0.333 0.333 0.333
## [2,] 0.516 0.516 0.516 0.516 0.516 0.516

References