Package 'RSpectra'

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Type Package

Title Solvers for Large Scale Eigenvalue and SVD Problems

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Description R interface to the 'Spectra' library
<http://yixuan.cos.name/spectra/> for large scale eigenvalue and SVD problems. It is typically used to compute a few eigenvalues/vectors of an n by n matrix, e.g., the k largest eigenvalues, which is usually more efficient than eigen() if k << n. This package provides the 'eigs()' function which does the similar job as in 'Matlab', 'Octave', 'Python SciPy' and 'Julia'. It also provides the 'svds()' function to calculate the largest k singular values and corresponding singular vectors of a real matrix. Matrices can be given in either dense or sparse form.

License MPL (>= 2)

URL https://github.com/yixuan/RSpectra

BugReports https://github.com/yixuan/RSpectra/issues

Depends R (>= 3.0.2)

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Suggests knitr

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R topics documented:

`eigs` *Find a Specified Number of Eigenvalues/vectors for Square Matrix*

Description

Given an n by n matrix A, function `eigs()` can calculate a limited number of eigenvalues and eigenvectors of A. Users can specify the selection criteria by argument `which`, e.g., choosing the k largest or smallest eigenvalues and the corresponding eigenvectors.

Currently `eigs()` supports matrices of the following classes:

`matrix`	The most commonly used matrix type, defined in **base** package.
`dgeMatrix`	General matrix, equivalent to `matrix`, defined in **Matrix** package.
`dgCMatrix`	Column oriented sparse matrix, defined in **Matrix** package.
`dgRMatrix`	Row oriented sparse matrix, defined in **Matrix** package.
`dsyMatrix`	Symmetrix matrix, defined in **Matrix** package.
`function`	Implicitly specify the matrix through a function that has the effect of calculating $f(x) = Ax$. See section **Funct**

`eigs_sym()` assumes the matrix is symmetric, and only the lower triangle (or upper triangle, which is controlled by the argument `lower`) is used for computation, which guarantees that the eigenvalues and eigenvectors are real, and in some cases reduces the workload. One exception is when `A` is a function, in which case the user is responsible for the symmetry of the operator.

`eigs_sym()` supports "matrix", "dgeMatrix", "dgCMatrix", "dgRMatrix" and "function" typed matrices.

Usage

```
eigs(A, k, which = "LM", sigma = NULL, opts = list(), ...)

## S3 method for class 'matrix'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ...)

## S3 method for class 'dgeMatrix'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ...)

## S3 method for class 'dgCMatrix'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ...)
```

```
## S3 method for class 'dgRMatrix'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ...)

## S3 method for class 'dsyMatrix'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ...)

## S3 method for class 'function'
eigs(A, k, which = "LM", sigma = NULL, opts = list(),
  ..., n = NULL, args = NULL)

eigs_sym(A, k, which = "LM", sigma = NULL, opts = list(),
   lower = TRUE, ...)

## S3 method for class 'function'
eigs_sym(A, k, which = "LM", sigma = NULL,
  opts = list(), lower = TRUE, ..., n = NULL, args = NULL)
```

Arguments

A	The matrix whose eigenvalues/vectors are to be computed. It can also be a function which receives a vector x and calculates Ax. See section **Function Interface** for details.
k	Number of eigenvalues requested.
which	Selection criteria. See **Details** below.
sigma	Shift parameter. See section **Shift-And-Invert Mode**.
opts	Control parameters related to the computing algorithm. See **Details** below.
n	Only used when A is a function, to specify the dimension of the implicit matrix. See section **Function Interface** for details.
args	Only used when A is a function. This argument will be passed to the A function when it is called. See section **Function Interface** for details.
lower	For symmetric matrices, should the lower triangle or upper triangle be used.
...	Arguments for specialized S3 function calls, for example lower, n and args.

Details

The which argument is a character string that specifies the type of eigenvalues to be computed. Possible values are:

"LM"	The k eigenvalues with largest magnitude. Here the magnitude means the Euclidean norm of complex numbers.
"SM"	The k eigenvalues with smallest magnitude.
"LR"	The k eigenvalues with largest real part.
"SR"	The k eigenvalues with smallest real part.
"LI"	The k eigenvalues with largest imaginary part.
"SI"	The k eigenvalues with smallest imaginary part.
"LA"	The k largest (algebraic) eigenvalues, considering any negative sign.

"SA" The k smallest (algebraic) eigenvalues, considering any negative sign.
"BE" Compute k eigenvalues, half from each end of the spectrum. When k is odd, compute more from the high and then fr

`eigs()` with matrix type "matrix", "dgeMatrix", "dgCMatrix" and "dgRMatrix" can use "LM", "SM", "LR", "SR", "LI" and "SI".

`eigs_sym()`, and `eigs()` with matrix type "dsyMatrix" can use "LM", "SM", "LA", "SA" and "BE".

The `opts` argument is a list that can supply any of the following parameters:

`ncv` Number of Lanzcos basis vectors to use. More vectors will result in faster convergence, but with greater memory use. For general matrix, `ncv` must satisfy $k + 2 \leq ncv \leq n$, and for symmetric matrix, the constraint is $k < ncv \leq n$. Default is `min(n, max(2*k+1, 20))`.

`tol` Precision parameter. Default is 1e-10.

`maxitr` Maximum number of iterations. Default is 1000.

`retvec` Whether to compute eigenvectors. If FALSE, only calculate and return eigenvalues.

Value

A list of converged eigenvalues and eigenvectors.

`values`	Computed eigenvalues.
`vectors`	Computed eigenvectors. `vectors[, j]` corresponds to `values[j]`.
`nconv`	Number of converged eigenvalues.
`niter`	Number of iterations used in the computation.
`nops`	Number of matrix operations used in the computation.

Shift-And-Invert Mode

The `sigma` argument is used in the shift-and-invert mode.

When `sigma` is not `NULL`, the selection criteria specified by argument `which` will apply to

$$\frac{1}{\lambda - \sigma}$$

where λ's are the eigenvalues of A. This mode is useful when user wants to find eigenvalues closest to a given number. For example, if $\sigma = 0$, then `which = "LM"` will select the largest values of $1/|\lambda|$, which turns out to select eigenvalues of A that have the smallest magnitude. The result of using `which = "LM", sigma = 0` will be the same as `which = "SM"`, but the former one is preferable in that `eigs()` is good at finding large eigenvalues rather than small ones. More explanation of the shift-and-invert mode can be found in the SciPy document, http://docs.scipy.org/doc/scipy/reference/tutorial/arpack.html.

Function Interface

The matrix A can be specified through a function with the definition

```
function(x, args)
{
    ## should return A %*% x
}
```

which receives a vector `x` as an argument and returns a vector of the same length. The function should have the effect of calculating Ax, and extra arguments can be passed in through the `args` parameter. In `eigs()`, user should also provide the dimension of the implicit matrix through the argument `n`.

Author(s)

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See Also

`eigen()`, `svd()`, `svds()`

Examples

```
library(Matrix)
n = 20
k = 5

## general matrices have complex eigenvalues
set.seed(111)
A1 = matrix(rnorm(n^2), n)  ## class "matrix"
A2 = Matrix(A1)             ## class "dgeMatrix"

eigs(A1, k)
eigs(A2, k, opts = list(retvec = FALSE))  ## eigenvalues only

## Sparse matrices
A1[sample(n^2, n^2 / 2)] = 0
A3 = as(A1, "dgCMatrix")
A4 = as(A1, "dgRMatrix")

eigs(A3, k)
eigs(A4, k)

## Function interface
f = function(x, args)
{
    as.numeric(args %*% x)
}
eigs(f, k, n = n, args = A3)
```

```
## Symmetric matrices have real eigenvalues
A5 = crossprod(A1)
eigs_sym(A5, k)

## Find the smallest (in absolute value) k eigenvalues of A5
eigs_sym(A5, k, which = "SM")

## Another way to do this: use the sigma argument
eigs_sym(A5, k, sigma = 0)

## The results should be the same,
## but the latter method is far more stable on large matrices
```

---

svds *Find the Largest k Singular Values/Vectors of a Matrix*

Description

Given an m by n matrix A, function `svds()` can find its largest k singular values and the corresponding singular vectors. It is also called the Truncated SVD or Partial SVD since it only calculates a subset of the whole singular triplets.

Currently `svds()` supports matrices of the following classes:

`matrix`	The most commonly used matrix type, defined in **base** package.
`dgeMatrix`	General matrix, equivalent to `matrix`, defined in **Matrix** package.
`dgCMatrix`	Column oriented sparse matrix, defined in **Matrix** package.
`dgRMatrix`	Row oriented sparse matrix, defined in **Matrix** package.
`dsyMatrix`	Symmetrix matrix, defined in **Matrix** package.
`function`	Implicitly specify the matrix through two functions that calculate $f(x) = Ax$ and $g(x) = A'x$. See section **Fun**

Note that when A is symmetric, SVD reduces to eigen decomposition, so you may consider using `eigs()` instead.

Usage

```
svds(A, k, nu = k, nv = k, opts = list(), ...)

## S3 method for class 'matrix'
svds(A, k, nu = k, nv = k, opts = list(), ...)

## S3 method for class 'dgeMatrix'
svds(A, k, nu = k, nv = k, opts = list(), ...)

## S3 method for class 'dgCMatrix'
svds(A, k, nu = k, nv = k, opts = list(), ...)
```

```
## S3 method for class 'dgRMatrix'
svds(A, k, nu = k, nv = k, opts = list(), ...)

## S3 method for class 'dsyMatrix'
svds(A, k, nu = k, nv = k, opts = list(), ...)

## S3 method for class 'function'
svds(A, k, nu = k, nv = k, opts = list(), ..., Atrans,
  dim, args = NULL)
```

Arguments

A	The matrix whose truncated SVD is to be computed.
k	Number of singular values requested.
nu	Number of left singular vectors to be computed. This must be between 0 and k.
nv	Number of right singular vectors to be computed. This must be between 0 and k.
opts	Control parameters related to the computing algorithm. See **Details** below.
Atrans	Only used when A is a function. A is a function that calculates the matrix multiplication Ax, and Atrans is a function that calculates the transpose multiplication $A'x$.
dim	Only used when A is a function, to specify the dimension of the implicit matrix. A vector of length two.
args	Only used when A is a function. This argument will be passed to the A and Atrans functions.
...	Arguments for specialized S3 function calls, for example Atrans, dim and args.

Details

The opts argument is a list that can supply any of the following parameters:

ncv Number of Lanzcos basis vectors to use. More vectors will result in faster convergence, but with greater memory use. ncv must be satisfy $k < ncv \leq p$ where p = min(m, n). Default is min(p, max(2*k+1, 20)).

tol Precision parameter. Default is 1e-10.

maxitr Maximum number of iterations. Default is 1000.

Value

A list with the following components:

d	A vector of the computed singular values.
u	An m by nu matrix whose columns contain the left singular vectors. If nu == 0, NULL will be returned.
v	An n by nv matrix whose columns contain the right singular vectors. If nv == 0, NULL will be returned.

nconv	Number of converged singular values.
niter	Number of iterations used.
nops	Number of matrix-vector multiplications used.

Function Interface

The matrix A can be specified through two functions with the following definitions

```
A <- function(x, args)
{
    ## should return A %*% x
}

Atrans <- function(x, args)
{
    ## should return t(A) %*% x
}
```

They receive a vector x as an argument and returns a vector of the proper dimension. These two functions should have the effect of calculating Ax and $A'x$ respectively, and extra arguments can be passed in through the args parameter. In svds(), user should also provide the dimension of the implicit matrix through the argument dim.

Author(s)

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See Also

eigen(), svd(), eigs().

Examples

```
m = 100
n = 20
k = 5
set.seed(111)
A = matrix(rnorm(m * n), m)

svds(A, k)
svds(t(A), k, nu = 0, nv = 3)

## Sparse matrices
library(Matrix)
A[sample(m * n, m * n / 2)] = 0
Asp1 = as(A, "dgCMatrix")
Asp2 = as(A, "dgRMatrix")

svds(Asp1, k)
svds(Asp2, k, nu = 0, nv = 0)
```

```
## Function interface
Af = function(x, args)
{
    as.numeric(args %*% x)
}

Atf = function(x, args)
{
    as.numeric(crossprod(args, x))
}

svds(Af, k, Atrans = Atf, dim = c(m, n), args = Asp1)
```

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