Package ‘Rmpfr’

April 11, 2021

Title R MPFR - Multiple Precision Floating-Point Reliable

Version 0.8-4

Date 2021-04-08

DateNote Previous CRAN version 0.8-3 on 2021-03-23

Type Package

Description Arithmetic (via S4 classes and methods) for arbitrary precision floating point numbers, including transcendental ("special") functions. To this end, the package interfaces to the 'LGPL' licensed 'MPFR' (Multiple Precision Floating-Point Reliable) Library which itself is based on the 'GMP' (GNU Multiple Precision) Library.

SystemRequirements gmp (>= 4.2.3), mpfr (>= 3.0.0), pdfcrop (part of TexLive) is required to rebuild the vignettes.

SystemRequirementsNote 'MPFR' (MP Floating-Point Reliable Library, https://www.mpfr.org/) and 'GMP' (GNU Multiple Precision library, https://gmplib.org/), see >> README.md

Depends gmp (>= 0.6-1), R (>= 3.5.0)

Imports stats, utils, methods

Suggests MASS, Bessel, polynom, sfsmisc (>= 1.0-20), Matrix

SuggestsNote MASS, polynom, sfsmisc: only for vignette; Matrix: test-tools

Enhances dfoptim, pracma

EnhancesNote mentioned in Rd xrefs

URL https://rmpfr.r-forge.r-project.org/

BugReports https://r-forge.r-project.org/tracker/?group_id=386

License GPL (>= 2)

Encoding UTF-8

NeedsCompilation yes
Author  Martin Maechler [aut, cre] (https://orcid.org/0000-0002-8685-9910),
  Richard M. Heiberger [ctb] (formatHex(), *Bin, *Dec),
  John C. Nash [ctb] (hjkMpfr()), origin of unirootR()),
  Hans W. Borchers [ctb] (optimizeR(*, "GoldenRatio"); origin of
  hjkMpfr())
Maintainer  Martin Maechler <maechler@stat.math.ethz.ch>
Repository  CRAN
Date/Publication  2021-04-11 15:40:02 UTC

R topics documented:

Rmpfr-package ......................................................... 3
array_or_vector-class ............................................. 6
asNumeric-methods ................................................. 7
atomicVector-class ................................................. 8
Bernoulli ............................................................. 8
Bessel_mpf ............................................................ 9
bind-methods .......................................................... 11
chooseMpf ............................................................. 12
factorialMpf .......................................................... 14
formatHex ............................................................. 15
formatMpf ............................................................. 18
gmp-conversions ...................................................... 21
hjkMpf ................................................................. 22
igamma ................................................................. 24
integrateR ............................................................ 26
is.whole ............................................................... 28
log1mexp ............................................................. 29
matmult ............................................................... 31
Mnumber-class ........................................................ 33
mpfr ................................................................. 34
mpfr-class ............................................................ 36
mpfr-distr-etc ....................................................... 42
mpfr-special-functions ............................................. 43
mpfr-utils ............................................................. 45
mpfr.util ............................................................. 49
mpfrArray ............................................................ 50
mpfrMatrix ........................................................... 52
mpfrMatrix-utils .................................................... 55
optimizeR ............................................................. 56
pbetaI ................................................................. 59
pmax ................................................................. 61
Rmpfr-workarounds .................................................. 62
roundMpf ............................................................. 63
sapplyMpf ............................................................ 63
seqMpf ............................................................... 64
str.mpf ............................................................... 65
Rmpfr-package

Description

Rmpfr provides S4 classes and methods for arithmetic including transcendental ("special") functions for arbitrary precision floating point numbers, here often called "mpfr - numbers". To this end, it interfaces to the LGPL'ed MPFR (Multiple Precision Floating-Point Reliable) Library which itself is based on the GMP (GNU Multiple Precision) Library.

Details

Package: Rmpfr
Title: R MPFR - Multiple Precision Floating-Point Reliable
Version: 0.8-4
Date: 2021-04-08
DateNote: Previous CRAN version 0.8-3 on 2021-03-23
Type: Package
Authors@R: c(person("Martin","Maechler",role = c("aut","cre"), email = "maechler@stat.math.ethz.ch", comment = "RMPFR - Multiple Precision Floating-Point Reliable"), person("Richard M.Heiberger",role = "ctb", email="hwborchers@googlemail.com", comment = "optimizeR(*, "GoldenRatio"); origin of hjkMpfr()") )
Description: Arithmetic (via S4 classes and methods) for arbitrary precision floating point numbers, including 
most special functions, and transcendental numbers.
SystemRequirements: gmp (>= 4.2.3), mpfr (>= 3.0.0), pdfcrop (part of TexLive) is required to rebuild the vignettes.
SystemRequirementsNote: ‘MPFR’ (MP Floating-Point Reliable Library, https://www.mpfr.org/) and ‘GMP’ (GNU Multiple Precision Library) are required.
Depends: gmp (>= 0.6-1), R (>= 3.5.0)
Imports: stats, utils, methods
Suggests: MASS, Bessel, polynom, sfsmisc (>= 1.0-20), Matrix
SuggestsNote: MASS, polynom, sfsmisc: only for vignette; Matrix: test-tools
Enhances: dfoptim, pracma
EnhancesNote: mentioned in Rd xrefs
URL: https://rmpfr.r-forge.r-project.org/
BugReports: https://r-forge.r-project.org/tracker/?group_id=386
License: GPL (>= 2)
Encoding: UTF-8
Author: Martin Maechler [aut, cre] (<https://orcid.org/0000-0002-8685-9910>), Richard M. Heiberger [ctb]
Maintainer: Martin Maechler <maechler@stat.math.ethz.ch>

Index of help topics:

 sumBinomMpfr Conversion Utilities gmp <-> Rmpfr
 unirootR Bernoulli Numbers in Arbitrary Precision
 Bessel_mpfr Bessel functions of Integer Order in multiple precisions
Mnumber-class  Class "Mnumber" and "mNumber" of "mpfr" and regular numbers and arrays from them

Rmpfr-package  R MPFR - Multiple Precision Floating-Point Reliable

array_or_vector-class  Auxiliary Class "array_or_vector"

asNumeric-methods  Methods for 'asNumeric(<mpfr>)'

atomicVector-class  Virtual Class "atomicVector" of Atomic Vectors

c.mpfr  MPFR Number Utilities

cbind  "mpfr" '...' - Methods for Functions cbind(), rbind()

chooseMpfr  Binomial Coefficients and Pochhammer Symbol aka Rising Factorial

determinant.mpfrMatrix  Functions for mpfrMatrix Objects

factorialMpfr  Factorial 'n!' in Arbitrary Precision

formatHex  Flexibly Format Numbers in Binary, Hex and Decimal Format

formatMpfr  Formatting MPFR (multiprecision) Numbers

getPrec  Rmpfr - Utilities for Precision Setting, Printing, etc

hjkMpfrr  Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)

igamma  Incomplete Gamma Function

integrateR  One-Dimensional Numerical Integration - in pure R

is.whole.mpfr  Whole ("Integer") Numbers

log1mexp  Compute f(a) = log(1 +/- exp(-a)) Numerically Optimally

matmult  (MPFR) Matrix (Vector) Multiplication

mpfr  Create "mpfr" Numbers (Objects)

mpfr-class  Class "mpfr" of Multiple Precision Floating Point Numbers

mpfrArray  Construct "mpfrArray" almost as by 'array()

mpfrMatrix-class  Classes "mpfrMatrix" and "mpfrArray"

optimizeR  High Precision One-Dimensional Optimization

outer  Base Functions etc, as an Rmpfr version

pbetaI  Accurate Incomplete Beta / Beta Probabilities For Integer Shapes

pmax  Parallel Maxima and Minima

pnorm  Distribution Functions etc (MPFR)

roundMpfr  Rounding to Binary bits, "mpfr-internally"

sapplyMpfr  Apply a Function over a "mpfr" Vector

seqMpfr  "mpfr" Sequence Generation

str.mpfr  Compactly Show STRucture of Rmpfr Number Object

sumBinomMpfr  (Alternating) Binomial Sums via Rmpfr

unirrootR  One Dimensional Root (Zero) Finding - in pure R

zeta  Special Mathematical Functions (MPFR)

Further information is available in the following vignettes:
The following (help pages) index does not really mention that we provide many methods for mathematical functions, including gamma, digamma, etc, namely, all of R’s (S4) Math group (with the only exception of trigamma), see the list in the examples. Additionally also pnorm, the “error function”, and more, see the list in zeta, and further note the first vignette (below).

**Partial index:**

- mpfr
  - Create "mpfr" Numbers (Objects)
- mpfrArray
  - Construct "mpfrArray" almost as by array()
- mpfr-class
  - Class "mpfr" of Multiple Precision Floating Point Numbers
- mpfrMatrix-class
  - Classes "mpfrMatrix" and "mpfrArray"
- Bernoulli
  - Bernoulli Numbers in Arbitrary Precision
- Bessel_mpfr
  - Bessel functions of Integer Order in multiple precisions
- c.mpfr
  - MPFR Number Utilities
- cbind
  - "mpfr"... - Methods for Functions cbind(), rbind()
- chooseMpfr
  - Binomial Coefficients and Pochhammer Symbol aka Rising Factorial
- factorialMpfr
  - Factorial 'n!' in Arbitrary Precision
- formatMpfr
  - Formatting MPFR (multiprecision) Numbers
- getPrec
  - Rmpfr - Utilities for Precision Setting, Printing, etc
- roundMpfr
  - Rounding to Binary bits, "mpfr-internally"
- seqMpfr
  - "mpfr" Sequence Generation
- sumBinomMpfr
  - (Alternating) Binomial Sums via Rmpfr
- zeta
  - Special Mathematical Functions (MPFR)
- integrateR
  - One-Dimensional Numerical Integration - in pure R
- unirootR
  - One Dimensional Root (Zero) Finding - in pure R
- optimizeR
  - High Precisione One-Dimensional Optimization
- hjkMpfr
  - Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)

Further information is available in the following vignettes:

- Rmpfr-pkg
  - Arbitrarily Accurate Computation with R: The 'Rmpfr' package (source, pdf)
- log1mexp-note
  - Accurately Computing log(1 - exp(.)) – Assessed by Rmpfr (source, pdf)

**Author(s)**

Martin Maechler
References

MPFR (MP Floating-Point Reliable Library), https://www.mpfr.org/
GMP (GNU Multiple Precision library), https://gmplib.org/
and see the vignettes mentioned above.

See Also

The R package gmp for big integer and rational numbers (bigrational) on which Rmpfr now depends.

Examples

```r
## Using "mpfr" numbers instead of regular numbers...
n1.25 <- mpfr(5, precBits = 256)/4
n1.25
```

```r
## and then "everything" just works with the desired chosen precision: hig
n1.25 ^ c(1:7, 20, 30) ## fully precise; compare with
print(1.25 ^ 30, digits=19)
```

```r
exp(n1.25)
```

```r
## Show all math functions which work with "MPFR" numbers (1 exception: trigamma)
getGroupMembers("Math")
```

```r
## We provide *many* arithmetic, special function, and other methods:
showMethods(classes = "mpfr")
showMethods(classes = "mpfrArray")
```

---

array_or_vector-class  Auxiliary Class "array\_or\_vector"

Description

"array_or_vector" is the class union of c("array", "matrix", "vector") and exists for its use in signatures of method definitions.

Details

Using "array_or_vector" instead of just "vector" in a signature makes an important difference: E.g., if we had setMethod(crossprod,c(x="mpfr",y="vector"),function(x,y) CPR(x,y)), a call crossprod(x,matrix(1:6,2,3)) would extend into a call of CPR(x,as(y,"vector")) such that CPR()'s second argument would simply be a vector instead of the desired 2 × 3 matrix.

Objects from the Class

A virtual Class: No objects may be created from it.
asNumeric-methods

Examples

showClass("array_or_vector")

Methods for asNumeric(<mpfr>)

Description

Methods for function asNumeric (in package gmp).

Usage

## S4 method for signature 'mpfrArray'
asNumeric(x)

Arguments

x a “number-like” object, here, a mpfr or typically mpfrArray one.

Value

an R object of type (typeof) "numeric", a matrix or array if x had non-NULL dimension dim().

Methods

signature(x = "mpfrArray") this method also dispatches for mpfrMatrix and returns a numeric array.
signature(x = "mpfr") for non-array/matrix, asNumeric(x) is basically the same as as.numeric(x).

Author(s)

Martin Maechler

See Also

our lower level (non-generic) toNum(). Further, asNumeric (package gmp), standard R’s as.numeric().

Examples

x <- (0:7)/8 # (exact)
X <- mpfr(x, 99)
stopifnot(identical(asNumeric(x), x), identical(asNumeric(X), x))

m <- matrix(1:6, 3,2)
(M <- mpfr(m, 99) / 5) ##-> "mpfrMatrix"
asNumeric(M) # numeric matrix
stopifnot(all.equal(asNumeric(M), m/5), identical(asNumeric(m), m))# remains matrix
Bernoulli

Bernoulli Numbers in Arbitrary Precision

**Description**

Computes the Bernoulli numbers in the desired (binary) precision. The computation happens via the `zeta` function and the formula

\[ B_k = -k \zeta(1 - k), \]

and hence the only non-zero odd Bernoulli number is \( B_1 = +1/2 \). (Another tradition defines it, equally sensibly, as \(-1/2\).)
Bernoulli

Usage

Bernoulli(k, precBits = 128)

Arguments

k non-negative integer vector
precBits the precision in bits desired.

Value

an mpfr class vector of the same length as k, with i-th component the k[i]-th Bernoulli number.

Author(s)

Martin Maechler

References

https://en.wikipedia.org/wiki/Bernoulli_number

See Also

zeta is used to compute them.

The next version of package gmp is to contain BernoulliQ(), providing exact Bernoulli numbers as big rationals (class "bigq").

Examples

Bernoulli(0:10)
plot(as.numeric(Bernoulli(0:15)), type = "h")

curve(-x*zeta(1-x), -.2, 15.03, n=300,
     main = expression(-x %.% zeta(1-x)))
legend("top", paste(c("even","odd "), "Bernoulli numbers"),
       pch=c(1,3), col=2, pt.cex=2, inset=1/64)
abline(h=0,v=0, lty=3, col="gray")
k <- 0:15; k[1] <- 1e-4
points(k, -k*zeta(1-k), col=2, cex=2, pch=1+2*(k%%2))

## They pretty much explode for larger k :
k2 <- 2*(1:120)
plot(k2, abs(as.numeric(Bernoulli(k2))), log = "y")
title("Bernoulli numbers exponential growth")

Bernoulli(10000)# - 9.0494239636 * 10^27677
Bessel functions of Integer Order in multiple precisions

Description

Bessel functions of integer orders, provided via arbitrary precision algorithms from the MPFR library.

Note that the computation can be very slow when \( n \) and \( x \) are large (and of similar magnitude).

Usage

\[
\begin{align*}
&\text{Ai}(x) \\
&j_0(x) \\
&j_1(x) \\
&jn(n, x, \text{rnd.mode} = \text{c}("N", "D", "U", "Z", "A")) \\
&y_0(x) \\
&y_1(x) \\
&yn(n, x, \text{rnd.mode} = \text{c}("N", "D", "U", "Z", "A"))
\end{align*}
\]

Arguments

- \( x \): a numeric or mpfr vector.
- \( n \): non-negative integer (vector).
- \( \text{rnd.mode} \): a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

Value

Computes multiple precision versions of the Bessel functions of integer order, \( J_n(x) \) and \( Y_n(x) \), and—when using MPFR library 3.0.0 or newer—also of the Airy function \( \text{Ai}(x) \). Note that currently \( \text{Ai}(x) \) is very slow to compute for large \( x \).

See Also

besselJ, and besselY compute the same bessel functions but for arbitrary real order and only precision of a bit more than ten digits.

Examples

\[
\begin{align*}
x &\leftarrow (0:100)/8 \ # (have\ exact\ binary\ representation) \\
\text{stopifnot}( &\text{all.equal(besselY(x, 0), bY0 <- y0(x))} \\
&\text{, all.equal(besselJ(x, 1), bJ1 <- j1(x))} \\
&\text{, all.equal(yn(0,x), bY0)} \\
&\text{, all.equal(jn(1,x), bJ1)} \\
\})
\end{align*}
\]

if(mpfrVersion() >= "3.0.0") \{ ## Ai() not available previously
```r
print( aix <- Ai(x) )
plot(x, aix, log="y", type="l", col=2)
stopifnot(
  all.equal(Ai(0), 1/(3^(2/3) * gamma(2/3))), # see https://dlmf.nist.gov/9.2.ii
  all.equal(Ai(100), mpfr("2.6344821520881844895505525695264981561e-291"), tol=1e-37)
)
two3rd <- 2/mpfr(3, 144)
print( all.equal(Ai(0), 1/(3^two3rd * gamma(two3rd)), tol=0) ) # 1.7e-40
if(Rmpfr:::doExtras()) { # slowish:
  system.time(ai1k <- Ai(1000)) # 1.4 sec (on 2017 lynne)
  stopifnot(all.equal(log10(ai1k),
                   -9157.031193409585185582, tol=1e-16))
}
```

### Description

`cbind` and `rbind` methods for signature ... (see `dotsMethods` are provided for class `Mnumber`, i.e., for binding numeric vectors and class "mpfr" vectors and matrices ("mpfrMatrix") together.

### Usage

```r
cbind(..., deparse.level = 1)
rbind(..., deparse.level = 1)
```

### Arguments

- `...` matrix-/vector-like `R` objects to be bound together, see the base documentation, `cbind`
- `deparse.level` integer determining under which circumstances column and row names are built from the actual arguments’ 'expression', see `cbind`

### Value

typically a ‘matrix-like’ object, here typically of class "mpfrMatrix".

### Methods

- `... = "Mnumber"` is used to (c)rbind multiprecision “numbers” (inheriting from class "mpfr") together, maybe combined with simple numeric vectors.
- `... = "ANY"` reverts to `cbind` and `rbind` from package `base`.

### Author(s)

Martin Maechler
chooseMpfr

See Also

`cbind2`, `cbind`, Documentation in base R’s methods package

Examples

```r
cbind(1, mpfr(6:3, 70)/7, 3:0)
```

```
chooseMpfr | Binomial Coefficients and Pochhammer Symbol aka Rising Factorial

Description

Compute binomial coefficients, `chooseMpfr(a, n)` being mathematically the same as `choose(a, n)`, but using high precision (MPFR) arithmetic.

`chooseMpfr.all(n)` means the vector `choose(n, 1:n)`, using enough bits for exact computation via MPFR. However, `chooseMpfr.all()` is now deprecated in favor of `chooseZ` from package gmp, as that is now vectorized.

`pochMpfr()` computes the Pochhammer symbol or “rising factorial”, also called the “Pochhammer function”, “Pochhammer polynomial”, “ascending factorial”, “rising sequential product” or “upper factorial”:

\[ x^{(n)} = x(x + 1)(x + 2)\cdots(x + n - 1) = \frac{(x + n - 1)!}{(x - 1)!} = \frac{\Gamma(x + n)}{\Gamma(x)}. \]

Usage

```r
chooseMpfr (a, n, rnd.mode = c("N","D","U","Z","A"))
chooseMpfr.all(n, precBits=NULL, k0=1, alternating=FALSE)
pochMpfr(a, n, rnd.mode = c("N","D","U","Z","A"))
```

Arguments

- `a`: a numeric or `mpfr` vector.
- `n`: an integer vector; if not of length one, `n` and `a` are recycled to the same length.
- `rnd.mode`: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see `mpfr`.
- `precBits`: integer or NULL for increasing the default precision of the result.
- `k0`: integer scalar
- `alternating`: logical, for `chooseMpfr.all()`, indicating if alternating sign coefficients should be returned, see below.
chooseMpfr

Value

For

chooseMpfr(), pochMpfr(): an mpfr vector of length max(length(a), length(n));
chooseMpfr.all(n, k0): a mpfr vector of length n-k0+1, of binomial coefficients $C_{n,m}$ or, if
alternating is true, $(-1)^m \cdot C_{n,m}$ for $m \in k0:n$.

Note

Currently this works via a (C level) for(i in 1:n)-loop which really slow for large n, say $10^6$, with
computational cost $O(n^2)$. In such cases, if you need high precision choose(a,n) (or Pochham-
mer(a,n)) for large n, preferably work with the corresponding factorial(mpfr(..)), or gamma(mpfr(..))
terms.

See Also

choose(n,m) (base R) computes the binomial coefficient $C_{n,m}$ which can also be expressed via
Pochhammer symbol as $C_{n,m} = (n - m + 1)^{\binom{m}{m}} / m!$.
chooseZ from package gmp; for now, factorialMpfr.
For (alternating) binomial sums, directly use sumBinomMpfr, as that is potentially more efficient.

Examples

pochMpfr(100, 4) == 100*101*102*103 # TRUE
a <- 100:110
pochMpfr(a, 10) # exact (but too high precision)
x <- mpfr(a, 70)# should be enough
(px <- pochMpfr(x, 10)) # the same as above (needing only 70 bits)
stopifnot(pochMpfr(a, 10) == px, px[1] == prod(mpfr(100:109, 100)))# used to fail
(cl <- chooseMpfr(1000:997, 60)) # -> automatic "correct" precision
stopifnot(all.equal(cl, choose(1000:997, 60), tolerance=1e-12))
## --- Experimenting & Checking
n.set <- c(1:10, 20, 50:55, 100:105, 200:203, 300:303, 500:503,
699:702, 999:1001)
if(!Rmpfr:::doExtras()) { ## speed up: smaller set
n. <- n.set[-(1:10)]
n.set <- c(1:10, n.[c(TRUE, diff(n.) > 1)])
}
C1 <- C2 <- numeric(length(n.set))
for(i.n in seq_along(n.set)) {
  cat(n <- n.set[i.n],":")
  C1[i.n] <- system.time(c.c <- chooseMpfr.all(n))[1]
  C2[i.n] <- system.time(c.2 <- chooseMpfr(n, 1:n))[1]
  stopifnot(is.whole(c.c), c.c == c.2,
            if(n > 60) TRUE else all.equal(c.c, choose(n, 1:n), tolerance = 1e-15))
  cat(" [OK]\n")
}
factorialMpfr

Factorial 'n!' in Arbitrary Precision

Description

Efficiently compute \( n! \) in arbitrary precision, using the MPFR-internal implementation. This is mathematically (but not numerically) the same as \( \Gamma(n + 1) \).

\texttt{factorialZ} (package \texttt{gmp}) should typically be used instead of \texttt{factorialMpfr()} nowadays. Hence, \texttt{factorialMpfr} now is somewhat deprecated.

Usage

\begin{verbatim}
factorialMpfr(n, precBits = max(2, ceiling(lgamma(n+1)/log(2))),
              rnd.mode = c("N","D","U","Z","A"))
\end{verbatim}
Arguments

- **n**: non-negative integer (vector).
- **precBits**: desired precision in bits ("binary digits"); the default sets the precision high enough for the result to be exact.
- **rnd.mode**: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see `mpfr`.

Value

A number of (S4) class `mpfr`.

See Also

- `factorial` and `gamma` in base R.
- `factorialZ` (package `gmp`), to replace `factorialMpfr`, see above.
- `chooseMpfr()` and `pochMpfr()` (on the same page).

Examples

```r
factorialMpfr(200)
```

```r
n <- 1000:1010
f1000 <- factorialMpfr(n)
stopifnot(1e-15 > abs(as.numeric(1 - lfactorial(n)/log(f1000))))
```

---

**formatHex**

**Flexibly Format Numbers in Binary, Hex and Decimal Format**

Description

Show numbers in binary, hex and decimal format. The resulting character-like objects can be back-transformed to "mpfr" numbers via `mpfr()`.

Usage

```r
formatHex(x, precBits = min(getPrec(x)), style = "+", expAlign = TRUE)
```

```r
formatBin(x, precBits = min(getPrec(x)), scientific = TRUE,
          left.pad = "_", right.pad = left.pad, style = "+", expAlign = TRUE)
```
formatDec(x, precBits = min(getPrec(x)), digits = decdigits, 
    nsmall = NULL, scientific = FALSE, style = "+", 
    decimalPointAlign = TRUE, ...)  

Arguments

x
    a numeric or mpfr R object.

precBits
    integer, the number of bits of precision, typically derived from x, see getPrec. 
    Numeric, i.e., double precision numbers have 53 bits. For more detail, see mpfr.

style
    a single character, to be used in sprintf's format (fmt), immediately after the " 
    sets a sign in the output, i.e., "+" or "-", where as style = " " may seem more 
    standard.

expAlign
    logical indicating if for scientific ("exponential") representations the exp- 
    onents should be aligned to the same width, i.e., zero-padded to the same number 
    of digits.

scientific
    logical indicating that formatBin should display the binary representation in 
    scientific notation (mpfr(3,5) is displayed as +0b1.1000p+1). When FALSE, 
    formatBin will display the binary representation in regular format shifted to 
    align binary points (mpfr(3,5) is displayed +0b11.000).

... additional optional arguments.

formatHex, formatBin: precBits is the only ... argument acted on. Other 
    ... arguments are ignored.

formatDec: precBits is acted on. Any argument accepted by format (except 
    nsmall) is acted on. Other ... arguments are ignored.

left.pad, right.pad
    characters (one-character strings) that will be used for left- and right-padding of 
    the formatted string when scientific=FALSE. Do not change these unless for 
    display-only purpose !

nsmall
    only used when scientific is false, then passed to format(). If NULL, the 
    default is computed from the range of the non-zero values of x.

digits
    integer; the number of decimal digits displayed is the larger of this argument 
    and the internally generated value that is a function of precBits. This is related 
    to but different than digits in format.

decimalPointAlign
    logical indicating if padding should be used to ensure that the resulting strings 
    align on the decimal point (" .").

Details

For the hexadecimal representation, when the precision is not larger than double precision, sprintf() 
    is used directly, otherwise formatMpfr() is used and post processed. For the binary representation, 
    the hexadecimal value is calculated and then edited by substitution of the binary representation of
the hex characters coded in the HextoBin vector. For binary with scientific=FALSE, the result of the scientific=TRUE version is edited to align binary points. For the decimal representation, the hexadecimal value is calculated with the specified precision and then sent to the format function for scientific=FALSE or to the sprintf function for scientific=TRUE.

Value

a character vector (or matrix) like x, say r, containing the formatted representation of x, with a class (unless left.pad or right.pad were not "_". In that case, formatHex() and formatBin() return class "Ncharacter"; for that, mpfr(.) has a method and will basically return x, i.e., work as inverse function.

Since Rmpfr version 0.6-2, the S3 class "Ncharacter" extends "character". (class(.) is now of length two and class(.)[2] is "character".). There are simple [ and print methods; modifying or setting dim works as well.

Author(s)

Richard M. Heiberger <rmh@temple.edu>, with minor tweaking by Martin M.

References

R FAQ 7.31: Why doesn’t R think these numbers are equal? system.file("../..\/doc/FAQ")

See Also

mpfr, sprintf

Examples

FourBits <- mpfr(matrix(0:31, 8, 4, dimnames = list(0:7, c(0,8,16,24))),
precBits=4) ## 4 significant bits

FourBits

formatHex(FourBits)
formatBin(FourBits, style = " ")
formatBin(FourBits, scientific=FALSE)
formatDec(FourBits)

## as "Ncharacter", 'inherits from' "character", this now works too:
data.frame(Dec = c( formatDec(FourBits) ), formatHex(FourBits),
Bin = formatBin(FourBits, style = " "))

FBB <- formatBin(FourBits) ; clB <- class(FBB)
(nFBB <- mpfr(FBB))
stopifnot(class(FBB)[1] == "Ncharacter",
all.equal(nFBB, FourBits, tol=0))

FBH <- formatHex(FourBits) ; clH <- class(FBH)
(nFBH <- mpfr(FBH))
stopifnot(class(FBH)[1] == "Ncharacter",
all.equal(nFBH, FourBits, tol=0))
formatMpfr

Formatting MPFR (multiprecision) Numbers

Description

Flexible formatting of “multiprecision numbers”, i.e., objects of class mpfr. formatMpfr() is also the mpfr method of the generic format function.

The formatN() methods for mpfr numbers renders them differently than their double precision equivalents, by appending ".M".

Function mpfr2str() is the low level work horse for formatMpfr() and hence all print()ing of "mpfr" objects.

Usage

formatMpfr(x, digits = NULL, trim = FALSE, scientific = NA,
maybe.full = !is.null(digits) & is.na(scientific),
base = 10, showNeg0 = TRUE, max.digits = Inf,
big.mark = ",", big.interval = 3L,
small.mark = ",", small.interval = 5L,
decimal.mark = ",",
exponent.char = if(base <= 14) "e" else if(base <= 36) "E" else "|e",

Arguments

- **x**: an MPFR number (vector or array).
- **digits**: how many significant digits (in the base chosen!) are to be used in the result. The default, NULL, uses enough digits to represent the full precision, often one or two digits more than “you” would expect. For bases 2,4,8,16, or 32, MPFR requires digits at least 2. For such bases, digits = 1 is changed into 2, with a message.
- **trim**: logical; if FALSE, numbers are right-justified to a common width: if TRUE the leading blanks for justification are suppressed.
- **scientific**: either a logical specifying whether MPFR numbers should be encoded in scientific format (“exponential representation”), or an integer penalty (see options("scipen")). Missing values correspond to the current default penalty.
- **maybe.full**: logical, passed to .mpfr2str().
- **base**: an integer in \(2, 3, \ldots, 62\); the base (“basis”) in which the numbers should be represented. Apart from the default base 10, binary (base = 2) or hexadecimal (base = 16) are particularly interesting.
- **showNeg0**: logical indicating if “negative” zeros should be shown with a "-". The default, TRUE is intentionally different from format(<numeric>).
- **exponent.char**: the “exponent” character to be used in scientific notation. The default takes into account that for base \(B \geq 15\), "e" is part of the (mantissa) digits and the same is true for "E" when \(B \geq 37\).
- **exponent.plus**: logical indicating if "+" should be for positive exponents in exponential (aka “scientific”) representation. This used to be hardcoded to FALSE; the new default is compatible to R’s format()ing of numbers and helps to note visually when exponents are in use.
- **max.digits**: a (large) positive number to limit the number of (mantissa) digits, notably when digits is NULL (as by default). Otherwise, a numeric digits is preferred to setting max.digits (which should not be smaller than digits).
- **big.mark, big.interval, small.mark, small.interval, decimal.mark, zero.print, drop0trailing**: used for prettying decimal sequences, these are passed to prettyNum and that help page explains the details.
- **...**: further arguments passed to or from other methods.

Value

A character vector or array, say cx, of the same length as x. Since Rmpfr version 0.5-3 (2013-09), if x is an mpfrArray, then cx is a character array with the same dim and dimnames as x.
Note that in scientific notation, the integer exponent is always in decimal, i.e., base 10 (even when base is not 10), but of course meaning base powers, e.g., in base 32, "ugi0" is the same as \(32^{1}\) times "ugi". This is in contrast, e.g., with \texttt{sprintf("%a",x)} where the powers after "p" are powers of \(2\).

**Author(s)**

Martin Maechler

**References**

The MPFR manual’s description of ‘\texttt{mpfr\_get\_str()}’ which is the C-internal workhorse for \texttt{.mpfr\_2str()} (on which \texttt{formatMpfr()} builds).

**See Also**

\texttt{mpfr} for creation and the \texttt{mpfr} class description with its many methods. The \texttt{format} generic, and the \texttt{prettyNum} utility on which \texttt{formatMpfr()} is based as well. The S3 generic function \texttt{formatN} from package \texttt{gmp}.

\texttt{.mpfr\_formatinfo(x)} provides the (cheap) non-string parts of \texttt{.mpfr\_2str(x)}; the (base 2) exp exponents are also available via \texttt{.mpfr\_2exp(x)}.

**Examples**

```r
## Printing of MPFR numbers uses formatMpfr() internally.
## Note how each components uses the "necessary" number of digits:
(x3 <- c(Const("pi", 168), mpfr(pi, 140), 3.14))
format(x3[3], 15)
format(x3[3], 15, drop0 = TRUE)# "3.14" .. dropping the trailing zeros
x3[4] <- 2^30
x3[4] # automatically drops trailing zeros
format(x3[1], dig = 41, small.mark = "\'/quotesingle.Var") # (41 - 1 = ) 40 digits after "."

rbind(formatN(x3, digits = 15),
      formatN(as.numeric(x3), digits = 15))

(Zero <- mpfr(c(0, 1/-Inf), 20)) # 0 and "-0"
xx <- c(Zero, 1:2, Const("pi", 120), -100*pi, -.00987)
format(xx, digits = 2)
format(xx, digits = 1, showNeg0 = FALSE)# "-0" no longer shown

## Output in other bases :
formatMpfr(mpfr(10^6, 40), base=32, drop0trailing=TRUE)
## "ugi0"
mpfr("ugi0", base=32) #-> 1'000'000

i32 <- mpfr(1:32, precBits = 64)
format(i32, base=2, drop0trailing=TRUE)
format(i32, base=16, drop0trailing=TRUE)
format(1/i32, base=2, drop0trailing=TRUE)# using scientific notation for [17..32]
```
format(1/i32, base=32)
format(1/i32, base=62, drop0trailing=TRUE)
format(mpfr(2, 64)^-(1:16), base=16, drop0trailing=TRUE)

Description

Coerce from and to big integers (bigz) and mpfr numbers.
Further, coerce from big rationals (bigq) to mpfr numbers.

Usage

.bigz2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.bigq2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.mpfr2bigz(x, mod = NA)

Arguments

x an R object of class bigz, bigq or mpfr respectively.
precB precision in bits for the result. The default, NULL, means to use the minimal precision necessary for correct representation.
rnd.mode a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see details of mpfr.
mod a possible modulus, see as.bigz in package gmp.

Details

Note that we also provide the natural (S4) coercions, as(x,"mpfr") for x inheriting from class "bigz" or "bigq".

Value

a numeric vector of the same length as x, of the desired class.

See Also

mpfr(), as.bigz and as.bigq in package gmp.
Examples

```r
S <- gmp::Stirling2(50, 10)
show(S)
SS <- S * as.bigz(1:3)^128
stopifnot(all.equal(log2(SS[2]) - log2(S), 128, tolerance=1e-15),
          identical(SS, .mpfr2bigz(.bigz2mpfr(SS))))

.bigz2mpfr(S)   # 148 bit precision
.bigz2mpfr(S, precB=256) # 256 bit

## rational --> mpfr:
sq <- SS / as.bigz(2)^100
MP <- as(sq, "mpfr")
stopifnot(identical(MP, .bigq2mpfr(sq)),
          SS == MP * as(2, "mpfr")^100)
```

Description

An implementation of the Hooke-Jeeves algorithm for derivative-free optimization. This is a slight adaption `hjk()` from package `dfoptim`.

Usage

```r
hjkMpfr(par, fn, control = list(), ...)
```

Arguments

- `par` Starting vector of parameter values. The initial vector may lie on the boundary. If `lower[i]=upper[i]` for some `i`, the `i`-th component of the solution vector will simply be kept fixed.
- `fn` Nonlinear objective function that is to be optimized. A scalar function that takes a real vector as argument and returns a scalar that is the value of the function at that point.
- `control` `list` of control parameters. See `Details` for more information.
- `...` Additional arguments passed to `fn`.

Details

Argument `control` is a list specifying changes to default values of algorithm control parameters. Note that parameter names may be abbreviated as long as they are unique.

The list items are as follows:

- `tol` Convergence tolerance. Iteration is terminated when the step length of the main loop becomes smaller than `tol`. This does *not* imply that the optimum is found with the same accuracy. Default is 1.e-06.
hjkMpfr

maxfeval  Maximum number of objective function evaluations allowed. Default is Inf, that is no restriction at all.

maximize  A logical indicating whether the objective function is to be maximized (TRUE) or minimized (FALSE). Default is FALSE.

target  A real number restricting the absolute function value. The procedure stops if this value is exceeded. Default is Inf, that is no restriction.

info  A logical variable indicating whether the step number, number of function calls, best function value, and the first component of the solution vector will be printed to the console. Default is FALSE.

If the minimization process threatens to go into an infinite loop, set either maxfeval or target.

Value

A list with the following components:

- par  Best estimate of the parameter vector found by the algorithm.
- value  value of the objective function at termination.
- convergence  indicates convergence (TRUE) or not (FALSE).
- feval  number of times the objective fn was evaluated.
- niter  number of iterations (“steps”) in the main loop.

Note

This algorithm is based on the Matlab code of Prof. C. T. Kelley, given in his book “Iterative methods for optimization”. It has been implemented for package dfoptim with the permission of Prof. Kelley.

This version does not (yet) implement a cache for storing function values that have already been computed as searching the cache makes it slower.

Author(s)

Hans W Borchers <hwborchers@googlemail.com>; for Rmpfr: John Nash, June 2012. Modifications by Martin Maechler.

References

C.T. Kelley (1999), Iterative Methods for Optimization, SIAM.

See Also

Standard R’s optim; optimizeR provides one-dimensional minimization methods that work with mpfr-class numbers.
Examples

## simple smooth example:
```r
ff <- function(x) sum((x - c(2:4))^2)
str(rr <- hjkMpfr(rep(mpfr(0,128), 3), ff, control=list(info=TRUE)))
```

## Hooke-Jeeves solves high-dim. Rosenbrock function {but slowly!}
```r
rosenbrock <- function(x) {
  n <- length(x)
  sum (100*((x1 <- x[1:(n-1)])^2 - x[2:n])^2 + (x1 - 1)^2)
}
par0 <- rep(0, 10)
str(rb.db <- hjkMpfr(rep(0, 10), rosenbrock, control=list(info=TRUE)))
```

## rosenbrook() is quite slow with mpfr-numbers:
```r
str(rb.M. <- hjkMpfr(mpfr(numeric(10), prec=128), rosenbrock, control = list(tol = 1e-8, info=TRUE)))
```

## Hooke-Jeeves does not work well on non-smooth functions
```r
nsf <- function(x) {
  max(f1, f2, f3)
}
par0 <- c(1, 1) # true min 7.2 at (1.2, 2.4)
h.d <- hjkMpfr(par0, nsf) # fmin=8 at xmin=(2,2)
```

## and this is not at all better (but slower!)
```r
h.M <- hjkMpfr(mpfr(c(1,1), 128), nsf, control = list(tol = 1e-15))
```

## --> demo(hjkMpfr) # -> Fletcher's chebyquad function m = n -- residuals

---

### igamma

**Incomplete Gamma Function**

**Description**

For MPFR version >= 3.2.0, the following MPFR library function is provided: 
`mpfr_gamma_inc(a,x)`, the R interface of which is `igamma(a,x)`, where `igamma(a,x)` is the “upper” incomplete gamma function

\[ \Gamma(a, x) := \Gamma(a) - \gamma(a, x), \]
where
\[ \gamma(a, x) := \int_0^x t^{a-1}e^{-t}dt, \]
and hence
\[ \Gamma(a, x) := \int_x^{\infty} t^{a-1}e^{-t}dt, \]
and
\[ \Gamma(a) := \gamma(a, \infty). \]

As R’s \texttt{pgamma(x, a)} is
\[ \text{pgamma}(x, a) := \frac{\gamma(a, x)}{\Gamma(a)}, \]
we get
\[ \text{igamma}(a, x) \equiv \text{gamma}(a) \times \text{pgamma}(x, a, \text{lower.tail}=\text{FALSE}) \]

Usage
\[ \text{igamma}(a, x, \text{rnd.mode} = \text{c("N", "D", "U", "Z", "A"))} \]

Arguments
- \texttt{a, x} an object of class \texttt{mpfr} or \texttt{numeric}.
- \texttt{rnd.mode} a 1-letter string specifying how \texttt{rounding} should happen at C-level conversion to MPFR, see \texttt{mpfr}.

Value
a numeric vector of “common length”, recycling along \texttt{a} and \texttt{x}.

Author(s)
R interface: Martin Maechler

References
NIST Digital Library of Mathematical Functions, section 8.2. \url{https://dlmf.nist.gov/8.2.i}

See Also
R’s \texttt{gamma (function)} and \texttt{pgamma (probability distribution)}. 
**Examples**

```r
## show how close pgamma() is :
x <- c(seq(0,20, by=1/4), 21:50, seq(55, 100, by=5))
if(mpfrVersion() >= "3.2.0") {
  print(
    all.equal(igamma(Const("pi", 80), x),
      pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
      tol=0, formatFUN = function(., ...) format(., digits = 7)) #-> 3.13e-16
  )
  ## and ensure *some* closeness:
  stopifnot(exprs = {
    all.equal(igamma(Const("pi", 80), x),
      pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
      tol = 1e-15)
  })
} # only if MPFR version >= 3.2.0
```

---

**integrateR**

*One-Dimensional Numerical Integration - in pure R*

**Description**

Numerical integration of one-dimensional functions in pure R, with care so it also works for "mpfr"-numbers.

Currently, only classical Romberg integration of order ord is available.

**Usage**

```r
integrateR(f, lower, upper, ..., ord = NULL,
  rel.tol = .Machine$double.eps^0.25, abs.tol = rel.tol,
  max.ord = 19, verbose = FALSE)
```

**Arguments**

- `f`  
an R function taking a numeric or "mpfr" first argument and returning a numeric (or "mpfr") vector of the same length. Returning a non-finite element will generate an error.

- `lower`, `upper`  
the limits of integration. Currently must be finite. Do use "mpfr"-numbers to get higher than double precision, see the examples.

- `...`  
additional arguments to be passed to `f`.

- `ord`  
integer, the order of Romberg integration to be used. If this is NULL, as per default, and either `rel.tol` or `abs.tol` are specified, the order is increased until convergence.

- `rel.tol`  
relative accuracy requested. The default is 1.2e-4, about 4 digits only, see the Note.

- `abs.tol`  
absolute accuracy requested.
**max.ord**
only used, when neither ord nor one of rel.tol, abs.tol are specified: Stop Romberg iterations after the order reaches max.ord; may prevent infinite loops or memory explosion.

**verbose**
logical or integer, indicating if and how much information should be printed during computation.

### Details
Note that arguments after ... must be matched exactly.

For convergence, both relative and absolute changes must be smaller than rel.tol and abs.tol, respectively.

rel.tol cannot be less than max(50*.Machine$double.eps,0.5e-28) if abs.tol <= 0.

### Value
A list of class "integrateR" (as from standard R's integrate()) with a print method and components

- **value**
the final estimate of the integral.

- **abs.error**
estimate of the modulus of the absolute error.

- **subdivisions**
for Romberg, the number of function evaluations.

- **message**
"OK" or a character string giving the error message.

- **call**
the matched call.

### Note
f must accept a vector of inputs and produce a vector of function evaluations at those points. The Vectorize function may be helpful to convert f to this form.

If you want to use higher accuracy, you must set lower or upper to "mpfr" numbers (and typically lower the relative tolerance, rel.tol), see also the examples.

Note that the default tolerances (rel.tol, abs.tol) are not very accurate, but the same as for integrate, which however often returns considerably more accurate results than requested. This is typically not the case for integrateR().

### References
See Also

R's standard, `integrate`, is much more adaptive, also allowing infinite integration boundaries, and typically considerably faster for a given accuracy.

Examples

```r
## See more from ?integrate
## this is in the region where integrate() can get problems:
integrateR(dnorm,0,2000)
integrateR(dnorm,0,2000, rel.tol=1e-15)
(Id <- integrateR(dnorm,0,2000, rel.tol=1e-15, verbose=TRUE))
Id$value == 0.5 # exactly

## Demonstrating that 'subdivisions' is correct:
Exp <- function(x) { .N <<- .N+ length(x); exp(x) }
.N <- 0; str(integrateR(Exp, 0,1, rel.tol=1e-10), digits=15); .N

## Using high-precision functions ------
## Polynomials are very nice:
integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, 5, verbose=TRUE)
# n= 1, 2^n= 2 | I = 46.04, abs.err = 98.9583
# n= 2, 2^n= 4 | I = 20, abs.err = 26.0417
# n= 3, 2^n= 8 | I = 20, abs.err = 7.10543e-15
## 20 with absolute error < 7.1e-15
## Now, using higher accuracy:
I <- integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, mpfr(5,128),
  rel.tol = 1e-20, verbose=TRUE)
I ; I$value ## all fine

## with floats:
integrateR(exp, 0 , 1, rel.tol=1e-15, verbose=TRUE)
## with "mpfr":
(I <- integrateR(exp, mpfr(0,200), 1, rel.tol=1e-25, verbose=TRUE))
(I.true <- exp(mpfr(1, 200)) - 1)
## true absolute error:
stopifnot(print(as.numeric(I.true - I$value)) < 4e-25)

## Want absolute tolerance check only (=> set 'rel.tol' very high, e.g. 1):
(Ia <- integrateR(exp, mpfr(0,200), 1, abs.tol = 1e-6, rel.tol=1, verbose=TRUE))

## Set 'ord' (but no '.tol') --> Using 'ord'; no convergence checking
(I <- integrateR(exp, mpfr(0,200), 1, ord = 13, verbose=TRUE))
```

---

**is.whole**

*Whole ("Integer") Numbers*
Description

Check which elements of `x[]` are integer valued aka “whole” numbers, including MPFR numbers (class `mpfr`).

Usage

```r
## S3 method for class 'mpfr'
is.whole(x)
```

Arguments

- `x`: any R vector, here of class `mpfr`.

Value

logical vector of the same length as `x`, indicating where `x[]` is integer valued.

Author(s)

Martin Maechler

See Also

- `is.integer(x)` (base package) checks for the internal mode or class, not if `x[i]` are integer valued.
- The `is.whole()` methods in package `gmp`.

Examples

```r
is.integer(3) # FALSE, it's internally a double
is.whole(3)  # TRUE
x <- c(as(2,"mpfr") ^ 100, 3, 3.2, 1000000, 2^40)
is.whole(x) # one FALSE, only
```

---

**log1mexp**

*Compute f(a) = log(1 +/- exp(-a)) Numerically Optimally*

Description

Compute f(a) = log(1 - exp(-a)), respectively g(x) = log(1 + exp(x)) quickly numerically accurately.

Usage

```r
log1mexp(a, cutoff = log(2))
log1pexp(x, c0 = -37, c1 = 18, c2 = 33.3)
```
Arguments

- **a**: numeric (or mpfr) vector of positive values.
- **x**: numeric vector, may also be an "mpfr" object.
- **cutoff**: positive number; \( \log(2) \) is "optimal", but the exact value is unimportant, and anything in \([0.5, 1]\) is fine.
- **c0, c1, c2**: cutoffs for `log1pexp`; see below.

Value

\[
\log1mexp(a) := f(a) = \log(1 - \exp(-a)) = \log1p(-\exp(-a)) = \log(-\expm1(-a))
\]

or, respectively,

\[
\log1pexp(x) := g(x) = \log(1 + \exp(x)) = \log1p(\exp(x))
\]

computed accurately and quickly.

Author(s)

Martin Maechler, May 2002; `log1pexp()` in 2012

References


Examples

```r
fExpr <- expression(
  DEF = log(1 - exp(-a)),
  expm1 = log(-expm1(-a)),
  log1p = log1p(-exp(-a)),
  F = log1mexp(a))

a <- 2^seq(-58, 10, length = 256)
fa <- do.call(cbind, as.list(fExpr))
head(fa)# expm1() works here
tail(fa)# log1p() works here

## graphically:
lwd <- 1.5*(5:2); col <- adjustcolor(1:4, 0.4)
op <- par(mfcol=c(1,2), mgp = c(1.25, .6, 0), mar = .1+c(3,2,1,1))
matplot(a, fa, type = "l", log = "x", col=col, lwd=lwd)
legend("topleft", fExpr, col=col, lwd=lwd, lty=1:4, bty="n")
# expm1() & log1mexp() work here
matplot(a, -fa, type = "l", log = "xy", col=col, lwd=lwd)
legend("left", paste("-",fExpr), col=col, lwd=lwd, lty=1:4, bty="n")
# log1p() & log1mexp() work here
par(op)
```
aM <- 2^seqMpfr(-58, 10, length=length(a.)) # => default prec = 128
a <- aM; dim(faM <- do.call(cbind, as.list(fExpr))) # 256 x 4, "same" as 'fa'
## Here, for small 'a' log1p() and even 'DEF' is still good enough
l_f <- asNumeric(log(-faM))
all.equal(l_f[,"F"], l_f[,"log1p"], tol=0) # see TRUE (Lnx 64-bit)
io <- a. < 80 # for these, the differences are small
all.equal(l_f[io,"F"], l_f[io,"expm1"], tol=0) # see 6.662e-9
all.equal(l_f[io,"F"], l_f[io,"DEF"], tol=0)
stopifnot(exprs = {
  all.equal(l_f[,"F"], l_f[,"log1p"], tol=1e-15)
  all.equal(l_f[io,"F"], l_f[io,"expm1"], tol=1e-7)
  all.equal(l_f[io,"F"], l_f[io,"DEF"], tol=1e-7)
})
## For 128-bit prec, if we go down to 2^-130, "log1p" is no longer ok:
aM2 <- 2^seqMpfr(-130, 10, by = 1/2)
a <- aM2; fa2 <- do.call(cbind, as.list(fExpr))
head(asNumeric(fa2), 12)
tail(asNumeric(fa2), 12)
matplot(a, log(-fa2[,1:3]) - log(-fa2,"F")), type="l", log="x",
  lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))
legend("top", colnames(fa2)[1:3], lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))
cols <- adjustcolor(2:4, 1/3); lwd <- 2*(3:1)-1
matplot(a, 1e-40+abs(log(-fa2[,1:3]) - log(-fa2,"F")), type="o", log="xy",
  main = "log1mexp(a) -- approximation rel.errors, mpfr(*, prec=128)",
  pch=21:23, cex=.6, bg=5:7, lty=1:2, lwd=lwd, col=cols)
legend("top", colnames(fa2)[1:3], bty="n", lty=1:2, lwd=lwd, col=cols,
  pch=21:23, pt.cex=.6, pt.bg=5:7)

## -------------------------- log1pexp() [simpler] --------------------------
curve(log1pexp, -10, 10, asp=1)
abline(0,1, h=0,v=0, lty=3, col="gray")
## Cutoff c1 for log1pexp() -- not often "needed":
curve(log1p(exp(x)) - log1pexp(x), 16, 20, n=2049)
## need for *some* cutoff:
x <- seq(700, 720, by=2)
cbind(x, log1p(exp(x)), log1pexp(x))
## Cutoff c2 for log1pexp():
curve((x+exp(-x)) - x, 20, 40, n=1025)
curve((x+exp(-x)) - x, 33.1, 33.5, n=1025)
Description

Matrix / vector multiplication of mpfr (and “simple” numeric) matrices and vectors.

matmult (x, y, fPrec = 2) or crossprod(x, y, fPrec = 2) use higher precision in underlying computations.

Usage

matmult(x, y, ...)

Arguments

x, y numeric or mpfrMatrix-classed R objects, i.e. semantically numeric matrices or vectors.

... arguments passed to the hidden underlying .matmult.R() work horse which is also underlying the %*%, crossprod(), and tcrossprod() methods, see the mpfrMatrix class documentation:

fPrec a multiplication factor, a positive number determining the number of bits precBits used for the underlying multiplication and summation arithmetic. The default is fPrec = 1. Setting fPrec = 2 doubles the precision which has been recommended, e.g., by John Nash.

precBits the number of bits used for the underlying multiplication and summation arithmetic; by default precBits = fPrec * max(getPrec(x), getPrec(y)) which typically uses the same accuracy as regular mpfr-arithmetic would use.

Value

a (base R) matrix or mpfrMatrix, depending on the classes of x and y.

Note

Using matmult(x, y) instead of x %*% y, makes sense mainly if you use non-default fPrec or precBits arguments.

The crossprod(), and tcrossprod() function have the identical optional arguments fPrec or precBits.

Author(s)

Martin Maechler

See Also

%*%, crossprod, tcrossprod.
Examples

```r
## FIXME: add example
## 1) matmult() <-> %*%
## 2) crossprod(), tcrossprod() %<-> .mpfrMatrix-class.Rd examples (!)
```

### Description

Classes "Mnumber" "mNumber" are class unions of "mpfr" and regular numbers and arrays from them. Its purpose is for method dispatch, notably defining a `cbind(...)` method where ... contains objects of one of the member classes of "Mnumber".

Classes "mNumber" is considerably smaller is it does not contain "matrix" and "array" since these also extend "character" which is not really desirable for generalized numbers. It extends the simple "numericVector" class by mpfr* classes.

### Methods

```r
%*% signature(x = "mpfrMatrix", y = "Mnumber"): ...
crossprod signature(x = "mpfr", y = "Mnumber"): ...
tcrossprod signature(x = "Mnumber", y = "mpfr"): ...
```

etc. These are documented with the classes `mpfr` and or `mpfrMatrix`.

### See Also

the `array_or_vector` sub class; `cbind-methods`.

### Examples

```r
## "Mnumber" encompasses (i.e., "extends") quite a few
## "vector / array - like" classes:
showClass("Mnumber")
stopifnot(extends("mpfrMatrix", "Mnumber"),
  extends("array", "Mnumber"))

Mnsub <- names(getClass("Mnumber")@subclasses)
(mNsub <- names(getClass("mNumber")@subclasses))
## mNumber has *one* subclass which is not in Mnumber:
```
mpfr

Create "mpfr" Numbers (Objects)

Description

Create multiple (i.e. typically high) precision numbers, to be used in arithmetic and mathematical computations with R.

Usage

mpfr(x, precBits, ...)  
## Default S3 method:  
mpfr(x, precBits, base = 10,  
rnd.mode = c("N","D","U","Z","A"), scientific = NA, ...)  
Const(name = c("pi","gamma","catalan","log2"), prec = 120L,  
rnd.mode = c("N","D","U","Z","A"))  
is.mpfr(x)

Arguments

x a numeric, mpfr, bigz, bigq, or character vector or array.
precBits, prec a number, the maximal precision to be used, in bits; i.e. 53 corresponds to double precision. Must be at least 2. If missing, getPrec(x) determines a default precision.
base (only when x is character) the base with respect to which x[i] represent numbers; base b must fulfill $2 \leq b \leq 62$.
rnd.mode a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see details.
scientific (used only when x is the result of formatBin(), i.e., of class "Bcharacter"). logical indicating that the binary representation of x is in scientific notation. When TRUE, mpfr() will substitute 0 for _; when NA, mpfr() will guess, and use TRUE when finding a "p" in x; see also formatBin.
name a string specifying the mpfrlib - internal constant computation. "gamma" is Euler's gamma ($\gamma$), and "catalan" Catalan's constant.
... potentially further arguments passed to and from methods.
Details
The "mpfr" method of mpfr() is a simple wrapper around roundMpfr().
MPFR supports the following rounding modes,

GMP_RNDN: round to nearest (roundTiesToEven in IEEE 754-2008).
GMP_RNDZ: round toward zero (roundTowardZero in IEEE 754-2008).
GMP_RNDA: round away from zero (new since MPFR 3.0.0).

The 'round to nearest' ("N") mode, the default here, works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the "drift" phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

When x is character, mpfr() will detect the precision of the input object.

Value
an object of (S4) class mpfr, or for mpfr(x) when x is an array, mpfrMatrix, or mpfrArray which the user should just as a normal numeric vector or array.
is.mpfr() returns TRUE or FALSE.

Author(s)
Martin Maechler

References
The MPFR team. (201x). GNU MPFR – The Multiple Precision Floating-Point Reliable Library; see https://www.mpfr.org/mpfr-current/#doc or directly https://www.mpfr.org/mpfr-current/mpfr.pdf.

See Also
The class documentation mpfr contains more details. Use asNumeric to transform back to double precision ("numeric").

Examples
mpfr(pi, 120) ## the double-precision pi "translated" to 120-bit precision

pi. <- Const("pi", prec = 260) # pi "computed" to correct 260-bit precision
pi. # nicely prints 80 digits [260 * log10(2) ~= 78.3 ~ 80]

Const("gamma", 128L) # 0.5772...
Const("catalan", 128L) # 0.9159...
x <- mpfr(0:7, 100)/7 # a more precise version of k/7, k=0,...,7
x

## character input :
mpfr("2.718281828459045235360287471352662497757") - exp(mpfr(1, 150))
## ~ 4 * 10^-40

## Also works for NA, NaN, ... :
cx <- c("1234567890123456", 345, "NA", "NaN", "Inf", "-Inf")
mpfr(cx)

## with some 'base' choices :
print(mpfr("111.1111", base=2)) * 2^4
mpfr("af21.01020300a0b0c", base=16)
## 68 bit prec. 44833.00393694653820642

mpfr("ugi0", base = 32) == 10^6  ## TRUE

## Large integers from package 'gmp' :
Z <- as.bigz(7)^c(1:200)
head(Z, 40)

## mpfr(Z) by default chooses the correct *maximal* default precision:
mZ. <- mpfr(Z)
## more efficiently chooses precision individually
m.Z <- mpfr(Z, precBits = frexpZ(Z)$exp)
## the precBits chosen are large enough to keep full precision:
stopifnot(identical(cZ <- as.character(Z),
                      as(mZ.,"character")),
          identical(cZ, as(m.Z,"character")))

## compare mpfr-arithmetic with exact rational one:
stopifnot(all.equal(mpfr(as.bigq(355,113), 99),
                    mpfr(355, 99) / 113, tol = 2^-98))

## look at different "rounding modes":
sapply(c("N", "D","U","Z","A"), function(RND)
       mpfr(c(-1,1)/5, 20, rnd.mode = RND, simplify=FALSE))
symnum(sapply(c("N", "D","U","Z","A"),
              function(RND) mpfr(0.2, prec = 5:15, rnd.mode = RND < 0.2 ))

---

Class "mpfr" of Multiple Precision Floating Point Numbers

Description

"mpfr" is the class of Multiple Precision Floatingpoint numbers with Reliable arithmetic.
For the high-level user, "mpfr" objects should behave as standard R's numeric vectors. They would just print differently and use the prespecified (typically high) precision instead of the double precision of 'traditional' R numbers (with class(.) == "numeric" and typeof(.) == "double").

\[ z = \text{hypot}(x, y) = \sqrt{x^2 + y^2}, \]

in a numerically stable way.

**Usage**

```r
hypot(x, y, rnd.mode = c("N", "D", "U", "Z", "A"))
```

**Arguments**

- `x, y`: an object of class mpfr.
- `rnd.mode`: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

**Objects from the Class**

Objects are typically created by mpfr(<number>, precBits).
summary(<mpfr>) returns an object of class "summaryMpfr" which contains "mpfr" but has its own print method.

**Slots**

Internally, "mpfr" objects just contain standard R lists where each list element is of class "mpfr1", representing one MPFR number, in a structure with four slots, very much parallelizing the C struct in the mpfr C library to which the Rmpfr package interfaces.

An object of class "mpfr1" has slots

- `prec`: "integer" specifying the maximal precision in bits.
- `exp`: "integer" specifying the base-2 exponent of the number.
- `sign`: "integer", typically -1 or 1, specifying the sign (i.e. sign(.)) of the number.
- `d`: an "integer" vector (of 32-bit “limbs”) which corresponds to the full mantissa of the number.

**Methods**

- `abs` signature(x = "mpfr"): ...
- `atan2` signature(y = "mpfr", x = "ANY"), and
- `atan2` signature(x = "ANY", y = "mpfr"): compute the arc-tangent of two arguments: atan2(y,x) returns the angle between the x-axis and the vector from the origin to \((x, y), \text{i.e., for positive arguments atan2(y,x) == atan(y/x)).\`
- `lbeta` signature(a = "ANY", b = "mpfrArray"): is \(\log(|\text{B}(a, b)|)\) where \(\text{B}(a, b)\) is the Beta function, beta(a,b).
- `beta` signature(a = "mpfr", b = "ANY").
beta  signature(a = "mpfr", b = "mpfr"), etc: Compute the beta function $B(a, b)$, using high precision, building on internal `gamma` or `lgamma`. See the help for R's base function `beta` for more. Currently, there, $a, b \geq 0$ is required. Here, we provide (non-`NaN`) for all numeric $a, b$.

When either $a, b$, or $a + b$ is a negative integer, $\Gamma(.)$ has a pole there and is undefined (`NaN`). However the Beta function can be defined there as “limit”, in some cases. Following other software such as SAGE, Maple or Mathematica, we provide finite values in these cases. However, note that these are not proper limits (two-dimensional in $(a, b)$), but useful for some applications. E.g., $B(a, b)$ is defined as zero when $a + b$ is a negative integer, but neither $a$ nor $b$ is.

Further, if $a > b > 0$ are integers, $B(-a, b) = B(b, -a)$ can be seen as $(-1)^b \cdot B(a - b + 1, b)$.

dim< signature(x = "mpfr"): Setting a dimension `dim` on an "mpfr" object makes it into an object of class "mpfrArray" or (more specifically) "mpfrMatrix" for a length-2 dimension, see their help page; note that `t(x)` (below) is a special case of this.

Ops  signature(e1 = "mpfr", e2 = "ANY"): ...
Ops  signature(e1 = "ANY", e2 = "mpfr"): ...
Arith signature(e1 = "mpfr", e2 = "missing"): ...
Arith signature(e1 = "mpfr", e2 = "mpfr"): ...
Arith signature(e1 = "mpfr", e2 = "integer"): ...
Arith signature(e1 = "mpfr", e2 = "numeric"): ...
Arith signature(e1 = "integer", e2 = "mpfr"): ...
Arith signature(e1 = "numeric", e2 = "mpfr"): ...
Compare signature(e1 = "mpfr", e2 = "mpfr"): ...
Compare signature(e1 = "mpfr", e2 = "integer"): ...
Compare signature(e1 = "mpfr", e2 = "numeric"): ...
Compare signature(e1 = "integer", e2 = "mpfr"): ...
Compare signature(e1 = "numeric", e2 = "mpfr"): ...
Logic signature(e1 = "mpfr", e2 = "mpfr"): ...

Summary  signature(x = "mpfr"): The S4 `Summary` group functions, `max`, `min`, `range`, `prod`, `sum`, `any`, and `all` are all defined for MPFR numbers. `mean(x, trim)` for non-0 trim works analogously to `mean.default`.

median  signature(x = "mpfr"): works via

quantile  signature(x = "mpfr"): a simple wrapper of the `quantile.default` method from `stats`.

summary  signature(object = "mpfr"): modeled after `summary.default`, ensuring to provide the full "mpfr" range of numbers.

Math  signature(x = "mpfr"): All the S4 `Math` group functions are defined, using multiple precision (MPFR) arithmetic, from `getGroupMembers("Math")`, these are (in alphabetical order): `abs`, `sign`, `sqrt`, `ceiling`, `floor`, `trunc`, `cummax`, `cummin`, `cumprod`, `cumsum`, `exp`, `expm1`, `log`, `log10`, `log2`, `log1p`, `cos`, `cosh`, `sin`, `sinh`, `tan`, `tanh`, `acos`, `acosh`, `asin`, `asinh`, `atan`, `atanh`, `gamma`, `lgamma`, `digamma`, and `trigamma`.

Currently, trigamma is not provided by the MPFR library and hence not yet implemented. Further, the `cum*()` methods are not yet implemented.

factorial  signature(x = "mpfr"): this will `round` the result when `x` is integer valued. Note however that `factorialMpfr(n)` for integer `n` is slightly more efficient, using the MPFR function 'mpfr_fac_ui'.
**Math2** signature(x = "mpfr"): `round(x,digits)` and `signif(x,digits)` methods. Note that these do not change the formal precision (’prec’ slot), and you may often want to apply `roundMpfr()` in addition or preference.

`as.numeric` signature(x = "mpfr"): ...

`as.vector` signature(x = "mpfrArray"): as for standard arrays, this “drops” the dim (and dimnames), i.e., transforms x into an ‘MPFR’ number vector, i.e., class mpfr.

[[ signature(x = "mpfr", i = "ANY"), and
[ signature(x = "mpfr", i = "ANY", j = "missing", drop = "missing"): subsetting aka “indexing” happens as for numeric vectors.

`format` signature(x = "mpfr"), further arguments digits = NULL, scientific = NA, etc: returns character vector of same length as x; when digits is NULL, with enough digits to recreate x accurately. For details, see `formatMpfr`.

`is.finite` signature(x = "mpfr"): ...

`is.infinite` signature(x = "mpfr"): ...

`is.na` signature(x = "mpfr"): ...

`is.nan` signature(x = "mpfr"): ...

`log` signature(x = "mpfr"): ...

`show` signature(object = "mpfr"): ...

`sign` signature(x = "mpfr"): ...

`Re, Im` signature(z = "mpfr"): simply return z or 0 (as ”mpfr” numbers of correct precision), as mpfr numbers are ‘real’ numbers.

`Arg, Mod, Conj` signature(z = "mpfr"): these are trivial for our ‘real’ mpfr numbers, but defined to work correctly when used in R code that also allows complex number input.

`all.equal` signature(target = "mpfr", current = "mpfr"),
`all.equal` signature(target = "mpfr", current = "ANY"), and
`all.equal` signature(target = "ANY", current = "mpfr"): methods for numerical (approximate) equality, all.equal of multiple precision numbers. Note that the default tolerance (argument) is taken to correspond to the (smaller of the two) precisions when both main arguments are of class “mpfr”, and hence can be considerably less than double precision machine epsilon .Machine$double.eps.

`coerce` signature(from = "numeric", to = "mpfr"): as(.,"mpfr") coercion methods are available for character strings, numeric, integer, logical, and even raw. Note however, that mpfr(.,precBits,base) is more flexible.

`coerce` signature(from = "mpfr", to = "bigz"): coerces to biginteger, see bigz in package gmp.

`coerce` signature(from = "mpfr", to = "numeric"): ...

`coerce` signature(from = "mpfr", to = "character"): ...

`unique` signature(x = "mpfr"), and corresponding S3 method (such that unique(<mpfr>) works inside base functions), see unique.

Note that duplicated() works for "mpfr" objects without the need for a specific method.

`t` signature(x = "mpfr"): makes x into an n × 1 mpfrMatrix.

`which.min` signature(x = "mpfr"): gives the index of the first minimum, see which.min.

`which.max` signature(x = "mpfr"): gives the index of the first maximum, see which.max.
Note

Many more methods (“functions”) automagically work for "mpfr" number vectors (and matrices, see the mpfrMatrix class doc), notably sort, order, quantile, rank.

Author(s)

Martin Maechler

See Also

The "mpfrMatrix" class, which extends the "mpfr" one.

roundMpfr to change precision of an "mpfr" object which is typically desirable instead of or in addition to signif() or round(); is.whole() etc.

Special mathematical functions such as some Bessel ones, e.g., jn; further, zeta(.) (= ζ(.)), Ei() etc. Bernoulli numbers and the Pochhammer function pochMpfr.

Examples

```r
## 30 digit precision
str(x <- mpfr(c(2:3, pi), prec = 30 * log2(10)))
x^2
x[1] / x[2] # 0.66666... ~ 30 digits

## indexing - as with numeric vectors
stopifnot(identical(x[2], x[[2]]),
  ## indexing "outside" gives NA (well: "mpfr-NaN" for now):
  is.na(x[5]),
  ## whereas "[" cannot index outside:
  is(try(x[[5]]), "try-error"),
  ## and only select *one* element:
  is(try(x[[2:3]]), "try-error")

## factorial() & ifactorial would work automagically via [l]gamma(),
## but factorial() additionally has an "mpfr" method which rounds
f200 <- factorial(mpfr(200, prec = 1500)) # need high prec!.
f200
as.numeric(log2(f200))# 1245.38 -- need precBits >= 1246 for full precision

##--- "Underflow" **much** later -- exponents have 30(+1) bits themselves:

mpfr.min.exp2 <- - (2^30 + 1)
two <- mpfr(2, 55)
stopifnot(two ^ mpfr.min.exp2 == 0)
## whereas
two ^ (mpfr.min.exp2 * (1 - 1e-15))
## 2.382564904887951070e-3232284977 "typically"

##--- "Assert" that {sort}, {order}, {quantile}, {rank}, all work :
```

p <- mpfr(rpois(32, lambda=500), precBits=128)^10
np <- as.numeric(log(p))
(sp <- summary(p))# using the print.summaryMpfr() method
stopifnot(all(diff(sort(p)) >= 0),
  identical(order(p), order(np)),
  identical(rank(p), rank(np)),
  all.equal(sapply(1:9, function(Typ) quantile(np, type=Typ, names=FALSE)),
    sapply(lapply(1:9, function(Typ) quantile(p, type=Typ, names=FALSE)),
      function(x) as.numeric(log(x))),
    tol = 1e-3),# quantiles: interpolated in orig. <--> log scale
  TRUE)
m0 <- mpfr(numeric(), 99)
xy <- expand.grid(x = -2:2, y = -2:2) ; x <- xy[,"x"] ; y <- xy[,"y"]
a2. <- atan2(y,x)
stopifnot(identical(which.min(m0), integer(0)),
  identical(which.max(m0), integer(0)),
  all.equal(a2., atan2(as(y,"mpfr"), x)),
  max(m0) == mpfr(-Inf, 53), # (53 is not a feature, but ok)
  min(m0) == mpfr(+Inf, 53),
  sum(m0) == 0, prod(m0) == 1)
## unique(), now even base::factor() "works" on <mpfr> :
set.seed(17)
p <- rlnorm(20) * mpfr(10, 100)^-999
pp <- sample(p, 50, replace=TRUE)
str(unique(pp)) # length 18 .. (from originally 20)
## Class [package "Rmpfr"] of length 18 and precision 100
## 5.56520587824e-999 4.41636588227e-1000 ..
fACP <- factor(pp)
str(facp) # the factor *levels* are a bit verbose :
# Factor w/ 18 levels "new("mpfr1", ...........): 
## At least *some* factor methods work :
stopifnot(exprs = {
  is.factor(facp)
  identical(unname(table(facp)),
    unname(table(asNumeric(pp * mpfr(10,100)^1000))))
})
## ((unfortunately, the expressions are wrong; should integer "L"))
## More useful: levels with which to *invert* factor() :
## -- this is not quite ok:
## simplified from 'utils':
deparse1c <- function(x, ...) paste(deparse(x, 500L, ...), collapse = " ")
if(FALSE) {
  str(pp.levs <- vapply(unclass(sort(unique(pp))),
    deparse1c, "")
  facp2 <- factor(pp, levels = pp.levs)
Distribution Functions etc (MPFR)

Description

For some \(\text{R}\) standard (probability) density, distribution or quantile functions, we provide MPFR versions.

Usage

\begin{verbatim}
dpois (x, lambda, log = FALSE, useLog = )
dbinom (x, size, prob, log = FALSE, useLog = )
dnbinom(x, size, prob, mu, log = FALSE, useLog = any(x > 1e6))
dnorm (x, mean = 0, sd = 1, log = FALSE)
dgamma(x, shape, rate = 1, scale = 1/rate, log = FALSE)

pnorm(q, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
\end{verbatim}

Arguments

\begin{verbatim}
x,q, lambda, size, prob, mu, mean, sd, shape, rate, scale

log, log.p, lower.tail

useLog
\end{verbatim}

numeric or \texttt{mpfr} vectors. All of these are “recycled” to the length of the longest one. For their meaning/definition, see the corresponding standard \texttt{R} (\texttt{stats} package) function.

log, log.p, lower.tail

logical, see \texttt{pnorm, dpois}, etc.

useLog

logical with default depending on \(x\) etc, indicating if log-scale computation should be used even when \(\text{log} = \text{FALSE}\), for performance or against overflow / underflow.

Details

\texttt{pnorm()} is based on \texttt{erf()} and \texttt{erfc()} which have direct MPFR counter parts and are both reparametrizations of \texttt{pnorm}, \(\text{erf}(x) = 2*\text{pnorm}(\sqrt{2}*x)\) and \(\text{erfc}(x) = 2* \text{pnorm}(\sqrt{2}*x, \text{lower} = \text{FALSE})\).

Value

A vector of the same length as the longest of \(x, q, \ldots\), of class \texttt{mpfr} with the high accuracy results of the corresponding standard \texttt{R} function.

Note

E.g., for \texttt{pnorm(*, log.p = TRUE)} to be useful, i.e., not to underflow or overflow, you may want to extend the exponential range of MPFR numbers, using \texttt{.mpfr_erange_set()}, see the examples.
See Also

\texttt{pnorm, dbinom, dnbinom, dgamma, dpois} in standard package \texttt{stats}.

\texttt{pbetaI(x,a,b)} is a \texttt{mpfr} version of \texttt{pbeta} only for integer \texttt{a} and \texttt{b}.

Examples

\begin{verbatim}
x <- 1400+ 0:10
print(dpois(x, 1000), digits =18) ## standard R's double precision
dpois(mpfr(x, 120), 1000)# more accuracy for the same

dpois(0:5, mpfr(10000, 80)) # very small exponents (underflowing in dbl.prec.)

print(dbinom(0:8, 8, pr = 4 / 5), digits=18)
  dbinom(0:8, 8, pr = 4/mpfr(5, 99)) -> dB; dB

print(dnorm( -5:5), digits=18)
  dnorm(mpfr(-5:5, prec=99))

## For pnorm() in the extreme tails, need an exponent range
## larger than the (MPFR and Rmpfr) default:
(old_eranges <- .mpfr_erange()) # typically +/- 2^30
.mpfr_erange_set(value = (1-2^-52)*.mpfr_erange(c("min.emin","max.emax")))
tens <- mpfr(10^(4:7), 128)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # "works"
## reset to previous
.mpfr_erange_set( , old_eranges)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # all but first underflow to -Inf
\end{verbatim}
Details

zeta(x) computes Riemann’s Zeta function $\zeta(x)$ important in analytical number theory and related fields. The traditional definition is

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}.$$

Ei(x) computes the exponential integral,

$$\int_{-\infty}^{x} \frac{e^t}{t} dt.$$

Li2(x) computes the dilogarithm,

$$\int_{0}^{x} \frac{-\log(1-t)}{t} dt.$$

erf(x) and erfc(x) are the error, respectively complementary error function which are both reparametrizations of pnorm, erf(x) = 2*pnorm(sqrt(2)*x) and erfc(x) = 2*pnorm(sqrt(2)*x, lower=FALSE), and hence Rmpfr provides its own version of pnorm.

Value

A vector of the same length as x, of class mpfr.

See Also

pnorm in standard package stats; the class description mpfr mentioning the generic arithmetic and mathematical functions (sin, log, ..., etc) for which "mpfr" methods are available.

Note the (integer order, non modified) Bessel functions $j_0()$, $y_n()$, etc, named j0, yn etc, and Airy function Ai in Bessel_mpf.
Description

This page documents utilities from package `Rmpfr` which are typically not called by the user, but may come handy in some situations.

Notably, the (base-2) maximal (and minimal) precision and the “range”, the range of possible (base-2) exponents of `mpfr`-numbers can be queried and partly extended.

Usage

```r
getPrec(x, base = 10, doNumeric = TRUE, is.mpfr = NA, bigq. = 128L)
.mpPrec(x)
getD(x)
mpfr_default_prec(prec)
## S3 method for class 'mpfrArray'
print(x, digits = NULL, drop0trailing = FALSE, 
  right = TRUE, 
  max.digits =getOption("Rmpfr.print.max.digits", 999L), 
  exponent.plus =getOption("Rmpfr.print.exponent.plus", TRUE), 
  ...)
## S3 method for class 'mpfr'
print(x, digits = NULL, drop0trailing = TRUE, 
  right = TRUE, 
  max.digits =getOption("Rmpfr.print.max.digits", 999L), 
  exponent.plus =getOption("Rmpfr.print.exponent.plus", TRUE), 
  ...)
toNum(from, rnd.mode = c('N','D','U','Z','A'))
mpfr2array(x, dim, dimnames = NULL, check = FALSE)

.mpfr2list(x, names = FALSE)
mpfrXport(x, names = FALSE)
mpfrImport(mxp)

.mpfr_formatinfo(x)
.mpfr2exp(x)

.mpfr_erange(kind = c("Emin", "Emax"), names = TRUE)
.mpfr_erange_set(kind = c("Emin", "Emax"), value)
.mpfr_erange_kinds
.mpfr_erange_is_int()
.mpfr_maxPrec()
.mpfr_minPrec()
```
Arguments

- **x**, from typically, an R object of class "mpfr", or "mpfrArray", respectively. For getPrec(), any number-like R object, or **NULL**.
- **base** (only when x is **character**) the base with respect to which x[i] represent numbers; base b must fulfill $2 \leq b \leq 62$.
- **doNumeric** logical indicating **integer** or **double** typed x should be accepted and a default precision be returned. Should typically be kept at default **TRUE**.
- **is.mpfr** logical indicating if `class(x)` is already known to be "mpfr"; typically should be kept at default, **NA**.
- **bigq.** for getPrec(), the precision to use for a big rational (class "bigq"); if not specified gives warning when used.
- **prec** a positive integer, or missing.
- **drop0trailing** logical indicating if trailing "0"s should be omitted.
- **right** logical indicating `print()`ing should right justify the strings; see `print.default()` to which it is passed.
- **digits, ...** further arguments to print methods.
- **max.digits** a number (possibly Inf) to limit the number of (mantissa) digits to be printed, simply passed to `formatMpfr()`. The default is finite to protect from printing very long strings which is often undesirable, notably in interactive use.
- **exponent.plus** logical, simply passed to `formatMpfr()`. Was FALSE hardwired in Rmpfr versions before 0.8-0, and hence is allowed to be tweaked by an `options()` setting.
- **rnd.mode** a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see details of `mpfr`.
- **dim, dimnames** for "mpfrArray" construction.
- **check** logical indicating if the mpfrArray construction should happen with internal safety check. Previously, the implicit default used to be true.
- **names** (for `mpfr2list()`) **logical** or **character** vector, indicating if the list returned should have **names**. If character, it specifies the names; if true, the names are set to `format(x)`.
- **mxp** an "mpfrXport" object, as resulting from `mpfrXport()`.
- **kind** a **character** string or vector, specifying the kind of "erange" value; must be an element of `mpfr_erange_kinds`, i.e., one of "Emin", "Emax", "min.emin", "max.emin", "min.emax", "max.emax".
- **value** **numeric**, for `mpfr_erange_set()` one number per kind. Must be in range specified by the \*."emin" and \*."emax" erange values.

Details

The print method is currently built on the `format` method for class `mpfr`. This, currently does not format columns jointly which leads to suboptimally looking output. There are plans to change this.

Note that `formatMpfr()` which is called by `print()` (or `show()` or R’s implicit printing) uses `max.digits = Inf`, differing from our `print()`’s default on purpose. If you do want to see the full
accuracy even in cases it is large, use `options(Rmpfr.print.max.digits = Inf)` or `.. = 1e7`, say.

The `.mpfr_erange*` functions (and variable) allow to query and set the allowed range of values for the base-2 exponents of "mpfr" numbers. See the examples below and GNU MPFR library documentation on the C functions `mpfr_get_emin()`, `mpfr_set_emin()`, `mpfr_get_emin_min()` and `mpfr_get_emin_max()`, (and those four with '_emin' replaced by '_emax' above).

**Value**

`getPrec(x)` returns a integer vector of length one or the same length as `x` when that is positive, whereas `getPrec(NULL)` returns `mpfr_default_prec()`, see below.

If you need to change the precision of `x`, i.e., need something like “setPrec”, use `roundMpfr()`.

`getD(x)` is intended to be a fast version of `getPrec()` which only works for "mpfr" objects `x`.

`getD(x)` is intended to be a fast version of `x@.Data`, and should not be used outside of lower level functions.

`mpfr_default_prec()` returns the current MPFR default precision, an integer. This is currently not made use of much in package `Rmpfr`, where functions have their own default precision where needed, and otherwise we’d rather not be dependent of such a global setting.

`mpfr_default_prec(prec)` sets the current MPFR default precision and returns the previous one; see above.

`.mpfr_maxPrec()` and (less interestingly) `.mpfr_minPrec()` give the maximal and minimal base-2 precision allowed in the current version of the MPFR library linked to by R package `Rmpfr`. The maximal precision is typically $2^{63}$, i.e.,

```
all.equal(.mpfr_maxPrec(), 2^63)
```

is typically true.

toNum(m) returns a numeric array or matrix, when `m` is of class "mpfrArray" or "mpfrMatrix", respectively. It should be equivalent to as(m,"array") or ... "matrix". Note that the slightly more general `asNumeric()` is preferred now.

`mpfr2array()` a slightly more flexible alternative to `dim(.) <- dd`.

`mpfr2exp(x)` returns the base-2 (integer valued) exponents of `x`, i.e., it is the R interface to MPFR C’s `mpfr_get_exp()`. The result is integer iff `.mpfr_erange_is_int()` is true, otherwise double. Note that the MPFR (4.0.1) manual says about `mpfr_get_exp()`: The behavior for NaN, infinity or zero is undefined.

`.mpfr_erange_is_int()` returns TRUE iff the `.mpfr_erange(c("Emin","Emax"))` range lies inside the range of R’s integer limits, i.e., has absolute values not larger than `.Machine$integer.max (= 2^{31} - 1)`.

`.mpfr_erange_set()` invisibly (see `invisible()`) returns TRUE iff the change was successful.

`.mpfr_formatinfo(x)` returns conceptually a subset of `.mpfr2str()`’s result, a list with three components

```
exp the base-2 exponents of x, identical to .mpfr2exp(x).
finite logical identical to is.finite(x).
is.0 logical indicating if the corresponding x[i] is zero; identical to mpfrIs0(x).
```

(Note that `.mpfr2str(x,...,base)$exp` is wrt base and is not undefined but...
Note

`mpfrXport()` and `mpfrImport()` are experimental and used to explore reported platform incompatibilities of `save()`d and `load()`ed "mpfr" objects between Windows and non-Windows platforms.

In other words, the format of the result of `mpfrXport()` and hence the `mxp` argument to `mpfrImport()` are considered internal, not part of the API and subject to change.

See Also

Start using `mpfr(..)`, and compute with these numbers.

`mpfrArray(x)` is for numeric ("non-mpfr") x, whereas `mpfr2array(x)` is for "mpfr" classed x, only.

Examples

```r
getPrec(as(c(1,pi), "mpfr")) # 128 for both

(opr <- mpfr_DEFAULT_prec()) ## typically 53, the MPFR system default
stopifnot(opr == (oprec <- mpfr_DEFAULT_prec(70)),
70 == mpfr_DEFAULT_prec())

## and reset it:
mpfr_DEFAULT_prec(opr)

## Explore behavior of rounding modes 'rnd.mode':
exp <- mpfr(10,99)^512 # too large for regular (double prec. / numeric):
sapply(c("N", "D", "U", "Z", "A"), function(RM)
sapply(list(-exp,exp), function(.) toNum(. , RM)))

## N D U Z A
## -Inf -Inf -1.797693e+308 -1.797693e+308 -Inf
## Inf 1.797693e+308 Inf 1.797693e+308 Inf

## Printing of "MPFR" matrices is less nice than R's usual matrix printing:
m <- outer(c(1, 3.14, -1024.5678), c(1, 1e-3, 10,100))
m[3,3] <- round(m[3,3])
m
mpfr(m, 50)

B6 <- mpfr2array(Bernoulli(1:6, 60), c(2,3),
dimnames = c(LETTERS[1:2], letters[1:3]))

## Ranges of (base 2) exponents of MPFR numbers:
.mpfr_erange() # the currently active range of possible base 2 exponents:

## A factory fresh setting fulfills
.mpfr_erange(c("Emin","Emax")) == c(-1,1) * (2^30 - 1)

## There are more 'kind's, the latter 4 showing how you could change the first two :
.mpfr_erange_kinds
.mpfr_erange(.mpfr_erange_kinds)
eLimits <- .mpfr_erange(c("min.emin", "max.emin", "min.emax", "max.emax"))
```
## Typically true in "current" MPFR versions:
eLimits == c(-1,1, -1,1) * (2^62 - 1)

## Looking at internal representation [for power users only!]:

```r
i8 <- mpfr(-2:5, 32)
x4 <- mpfr(c(NA, NaN, -Inf, Inf), 32)
## The output of the following depends on the GMP "numb" size
## (32 bit vs. 64 bit), and may be even more platform specifics:
str(.mpfr2list(i8))
str(.mpfr2list(x4, names = TRUE))

str(xp4 <- mpfrXport(x4, names = TRUE))
stopifnot(identical(x4, mpfrImport(mpfrXport(x4))),
           identical(i8, mpfrImport(mpfrXport(i8)))))
```

---

### mpfr.utils

**MPFR Number Utilities**

#### Description

`mpfrVersion()` returns the version of the MPFR library which **Rmpfr** is currently linked to.

`c(x,y,...)` can be used to combine MPFR numbers in the same way as regular numbers **IFF** the first argument `x` is of class `mpfr`.

`mpfrIs0(.)` uses the MPFR library in the documented way to check if (a vector of) MPFR numbers are zero. It was called `mpfr.is.0` which is strongly deprecated now.

`.mpfr.is.whole(x)` uses the MPFR library in the documented way to check if (a vector of) MPFR numbers is integer valued. This is equivalent to `x == round(x)`, but **not** at all to `is.integer(as(x,"numeric"))`. You should typically rather use (the "mpfr" method of the generic function) `is.whole(x)` instead. The former name `mpfr.is.integer` is deprecated now.

#### Usage

```r
mpfrVersion()
mpfrIs0(x)

## S3 method for class 'mpfr'
c(...)
## S3 method for class 'mpfr'
diff(x, lag = 1L, differences = 1L, ...)
```
Arguments

- **x**: an object of class `mpfr`.

... further `mpfr` class objects or simple numbers (numeric vectors) which are coerced to `mpfr` with default precision of 128 bits.

- **lag, differences**: for `diff()`: exact same meaning as in `diff()`'s default method, `diff.default`.

Value

- `mpfrIs0` returns a logical vector of length `length(x)` with values `TRUE` iff the corresponding `x[i]` is an MPFR representation of zero (0).
- `mpfr.is.whole` and `is.whole` return a logical vector of length `length(x)`.
- `mpfrVersion` returns an object of S3 class "numeric_version", so it can be used in comparisons.
- The other functions return MPFR number (vectors), i.e., extending class `mpfr`.

See Also

- `str.mpfr` for the `str` method. `erf` for special mathematical functions on MPFR.

The class description `mpfr` page mentions many generic arithmetic and mathematical functions for which "mpfr" methods are available.

Examples

```r
mpfrVersion()

(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
mpfrIs0(x) # one of them is
x[mpfrIs0(x)] # but it may not have been obvious..
str(x)

x <- rep(-2:2, 5)
stopifnot(is.whole(mpfr(2, 500) ^ (1:200)),
  all.equal(diff(x), diff(as.numeric(x))))
```

---

### mpfrArray

Construct "mpfrArray" almost as by 'array()'

Description

Utility to construct an R object of class `mpfrArray`, very analogously to the numeric `array` function.

Usage

```r
mpfrArray(x, precBits, dim = length(x), dimnames = NULL,
  rnd.mode = c("N","D","U","Z","A"))
```
Arguments

- **x**: numeric(like) vector, typically of length `prod(dim)` or shorter in which case it is recycled.
- **precBits**: a number, the maximal precision to be used, in **bits**; i.e., 53 corresponds to double precision. Must be at least 2.
- **dim**: the dimension of the array to be created, that is a vector of length one or more giving the maximal indices in each dimension.
- **dimnames**: either NULL or the names for the dimensions. This is a list with one component for each dimension, either NULL or a character vector of the length given by `dim` for that dimension.
- **rnd.mode**: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see details of mpfr.

Value

an object of class "mpfrArray", specifically "mpfrMatrix" when `length(dim) == 2`.

See Also

mpfr, array; asNumeric() as “inverse” of mpfrArray(), to get back a numeric array.

Examples

```r
## preallocating is possible here too
ma <- mpfrArray(NA, prec = 80, dim = 2:4)
validObject(A2 <- mpfrArray(1:24, prec = 64, dim = 2:4))

## recycles, gives an "mpfrMatrix" and dimnames:
mat <- mpfrArray(1:5, 64, dim = c(5,3), dimnames=list(NULL, letters[1:3]))
mat
asNumeric(mat)
stopifnot(identical(asNumeric(mat),
                     matrix(1:5 +0, 5,3, dimnames=dimnames(mat))))

## Testing the apply() method:
apply(mat, 2, range)
apply(A2, 1:2, range)
apply(A2, 2:3, max)
(fA2 <- apply(A2, 2, fivenum))
a2 <- as(A2, "array")
stopifnot(as(apply(A2, 2, range), "matrix") ==
          apply(a2, 2, range),
          all.equal(fA2, apply(a2, 2, fivenum)),
          all.equal(apply(A2, 2, quantile),
                     apply(a2, 2, quantile)),
          all.equal(A2, apply(A2, 2:3, identity) -> aA2, check.attributes=FALSE),
          dim(A2) == dim(aA2))
```
Classes "mpfrMatrix" and "mpfrArray"

Description

The classes "mpfrMatrix" and "mpfrArray" are, analogously to the base matrix and array functions and classes simply "numbers" of class mpfr with an additional Dim and Dimnames slot.

Objects from the Class

Objects should typically be created by mpfrArray(), but can also be created by new("mpfrMatrix",...) or new("mpfrArray",...), or also by t(x), dim(x) <-dd, or mpfr2array(x, dim=dd) where x is a an mpfr "number vector".

A (slightly more flexible) alternative to dim(x) <-dd is mpfr2array(x, dd, dimnames).

Slots

.Data: as for the mpfr class, a "list" of mpfr1 numbers.
Dim: of class "integer", specifying the array dimension.
Dimnames: of class "list" and the same length as Dim, each list component either NULL or a character vector of length Dim[j].

Extends

Class "mpfrMatrix" extends "mpfrArray", directly.
Class "mpfrArray" extends class "mpfr", by class "mpfrArray", distance 2; class "list", by class "mpfrArray", distance 3; class "vector", by class "mpfrArray", distance 4.

Methods

Arith signature(e1 = "mpfr", e2 = "mpfrArray"): ...
Arith signature(e1 = "numeric", e2 = "mpfrArray"): ...
Arith signature(e1 = "mpfrArray", e2 = "mpfrArray"): ...
Arith signature(e1 = "mpfrArray", e2 = "mpfr"): ...
Arith signature(e1 = "mpfrArray", e2 = "numeric"): ...

as.vector signature(x = "mpfrArray", mode = "missing"): drops the dimension 'attribute', i.e., transforms x into a simple mpfr vector. This is an inverse of t(.) or dim(.) <-* on such a vector.

atan2 signature(y = "ANY", x = "mpfrArray"): ...
atan2 signature(y = "mpfrArray", x = "mpfrArray"): ...
atan2 signature(y = "mpfrArray", x = "ANY"): ...
mpfrMatrix

[< signature(x = "mpfrArray", i = "ANY", j = "ANY", value = "ANY"): ...
[ signature(x = "mpfrArray", i = "ANY", j = "ANY", drop = "ANY"): ...
[ signature(x = "mpfrArray", i = "ANY", j = "missing", drop = "missing"): "mpfrArray" can be subset ("indexed") as regular R arrays.

%%signature(x = "mpfr", y = "mpfrMatrix"): Compute the matrix/vector product \( xy \) when the dimensions (dim) of \( x \) and \( y \) match. If \( x \) is not a matrix, it is treated as a 1-row or 1-column matrix (aka "row vector" or "column vector") depending on which one makes sense, see the documentation of the base function **%

%%signature(x = "mpfr", y = "Mnumber"): method definition for cases with one mpfr and any "number-like" argument are to use MPFR arithmetic as well.

%%signature(x = "mpfrMatrix", y = "mpfrMatrix"),
%%signature(x = "mpfrMatrix", y = "mpfr"), etc. Further method definitions with identical semantic.

crossprod signature(x = "mpfr", y = "missing"): Computes \( x'x \), i.e., \( t(x) \times x \), typically more efficiently.
crossprod signature(x = "mpfr", y = "mpfrMatrix"): Computes \( x'y \), i.e., \( t(x) \times y \), typically more efficiently.
crossprod signature(x = "mpfrMatrix", y = "mpfrMatrix"): ...
crossprod signature(x = "mpfrMatrix", y = "mpfr"): ...
tcrossprod signature(x = "mpfr", y = "missing"): Computes \( xx' \), i.e., \( x \times x' \), typically more efficiently.
tcrossprod signature(x = "mpfrMatrix", y = "mpfrMatrix"): Computes \( xy' \), i.e., \( x \times y' \), typically more efficiently.
tcrossprod signature(x = "mpfrMatrix", y = "mpfr"): ...
tcrossprod signature(x = "mpfr", y = "mpfrMatrix"): ...
crossprod signature(e1 = "mpfr", e2 = "mpfrArray"): ...
crossprod signature(e1 = "numeric", e2 = "mpfrArray"): ...
crossprod signature(e1 = "mpfrArray", e2 = "mpfr"): ...
crossprod signature(e1 = "mpfrArray", e2 = "numeric"): ...
dim signature(x = "mpfrArray"): ...
dimnames<- signature(x = "mpfrArray"): ...
dimnames signature(x = "mpfrArray"): ...
show signature(object = "mpfrArray"): ...
sign signature(x = "mpfrArray"): ...
norm signature(x = "mpfrMatrix", type = "character"): computes the matrix norm of \( x \), see norm or the one in package Matrix.
t signature(x = "mpfrMatrix"): transpose the mpfrMatrix.
aperm signature(a = "mpfrArray"): aperm(a, perm) is a generalization of t(.) to permute the dimensions of an mpfrArray; it has the same semantics as the standard aperm() method for simple R arrays.
Author(s)

Martin Maechler

See Also

`mpfrArray`, also for more examples.

Examples

```r
showClass("mpfrMatrix")
validObject(mm <- new("mpfrMatrix"))
validObject(aa <- new("mpfrArray"))

v6 <- mpfr(1:6, 128)
m6 <- new("mpfrMatrix", v6, Dim = c(2L, 3L))
validObject(m6)
m6
which(m6 == 3, arr.ind = TRUE) # |--> (1, 2)
## Coercion back to "vector": Both of these work:
stopifnot(identical(as(m6, "mpfr"), v6),
          identical(as.vector(m6), v6)) # < but this is a "coincidence"

S2 <- m6[, , -3] # 2 x 2
S3 <- rbind(m6, c(1:2, 10)); s3 <- asNumeric(S3)
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tol=1e-15),
          all.equal(det(S3), det(s3), tol=1e-15))

## 2-column matrix indexing and replacement:
(sS <- S3[i2 <- cbind(1:2, 2:3)])
stopifnot(identical(asNumeric(sS), s3[i2]))
C3 <- S3; c3 <- s3
C3[i2] <- 10:11
c3[i2] <- 10:11
stopifnot(identical(asNumeric(C3), c3))

AA <- new("mpfrArray", as.vector(cbind(S3, -S3)), Dim=c(3L,3:2))
stopifnot(identical(AA[, , 1], S3), identical(AA[, , 2], -S3))
aa <- asNumeric(AA)

i3 <- cbind(3:1, 1:3, c(2L, 1:2))
i13 <- Rmpfr:::.mat2ind(i3, dim(AA), dimnames(AA))
stopifnot(aa[i3] == new("mpfr", getD(AA)[i13]))
stopifnot(identical(aa[i3], asNumeric(AA[i13])))
CA <- AA; ca <- aa
c azi <- ca[i3] * 3
CA[i3] <- CA[i3] * 3

## scale():
```
S2. <- scale(S2)
sstopifnot(all.equal(abs(as.vector(S2.)), rep(sqrt(1/mpfr(2, 128)), 4),
tol = 1e-30))

## norm() :
norm(S2)
sstopifnot(identical(norm(S2), norm(S2, "1")),
  norm(S2, "I") == 6,
  norm(S2, "H") == 4,
  abs(norm(S2, "F") - 5.477225575051661) < 1e-15)

### mpfrMatrix-utils

**mpfrMatrix-utils**

**Functions for mpfrMatrix Objects**

**Description**

determinant(x,..) computes the determinant of the mpfr square matrix x. May work via coercion to "numeric", i.e., compute determinant(asNumeric(x),logarithm), if asNumeric is true, by default, if the dimension is larger than three. Otherwise, use precision precBits for the "accumulator" of the result, and use the recursive mathematical definition of the determinant (with computational complexity n!, where n is the matrix dimension, i.e., very inefficient for all but small matrices!)

**Usage**

```r
## S3 method for class 'mpfrMatrix'
determinant(x, logarithm = TRUE,
  asNumeric = (d[1] > 3), precBits = max(.getPrec(x)), ...)
```

**Arguments**

- **x** an mpfrMatrix object of square dimension.
- **logarithm** logical indicating if the log of the absolute determinant should be returned.
- **asNumeric** logical .. .. if rather determinant(asNumeric(x),...) should be computed.
- **precBits** the number of binary digits for the result (and the intermediate accumulations).
- **...** unused (potentially further arguments passed to methods).

**Value**

as determinant(), an object of S3 class "det", a list with components

- **modulus** the (logarithm of) the absolute value (abs) of the determinant of x.
- **sign** the sign of the determinant.

**Author(s)**

Martin Maechler
See Also
determinant in base R, which relies on a fast LU decomposition. mpfrMatrix

Examples

```r
m6 <- mpfrArray(1:6, prec=128, dim = c(2L, 3L))
m6
S2 <- m6[,,-3] # 2 x 2
S3 <- rbind(m6, c(1:2,10))
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tolerance=1e-15),
  all.equal(det(S3), det(asNumeric(S3)), tolerance=1e-15))
```

optimizeR

**High Precision One-Dimensional Optimization**

Description

optimizeR searches the interval from lower to upper for a minimum of the function \( f \) with respect to its first argument.

Usage

```r
optimizeR(f, lower, upper, ..., tol = 1e-20,
  method = c("Brent", "GoldenRatio"),
  maximum = FALSE,
  precFactor = 2.0, precBits = -log2(tol) * precFactor,
  maxiter = 1000, trace = FALSE)
```

Arguments

- **f** the function to be optimized. \( f(x) \) must work “in Rmpfr arithmetic” for optimizer() to make sense. The function is either minimized or maximized over its first argument depending on the value of maximum.
- **...** additional named or unnamed arguments to be passed to \( f \).
- **lower** the lower end point of the interval to be searched.
- **upper** the upper end point of the interval to be searched.
- **tol** the desired accuracy, typically higher than double precision, i.e., \( \text{tol} < 2\text{e-16} \).
- **method** character string specifying the optimization method.
- **maximum** logical indicating if \( f() \) should be maximized or minimized (the default).
- **precFactor** only for default precBits construction: a factor to multiply with the number of bits directly needed for \( \text{tol} \).
- **precBits** number of bits to be used for mpfr numbers used internally.
maxiter  maximal number of iterations to be used.
trace    integer or logical indicating if and how iterations should be monitored; if an
         integer $k$, print every $k$-th iteration.

Details

"Brent": Brent(1973)'s simple and robust algorithm is a hybrid, using a combination of the golden
ratio and local quadratic ("parabolic") interpolation. This is the same algorithm as standard
R's optimize(), adapted to high precision numbers.
In smooth cases, the convergence is considerably faster than the golden section or Fibonacci
ratio algorithms.

"GoldenRatio": The golden ratio method, aka 'golden-section search' works as follows: from a
given interval containing the solution, it constructs the next point in the golden ratio between
the interval boundaries.

Value

A list with components minimum (or maximum) and objective which give the location of the
minimum (or maximum) and the value of the function at that point; iter specifying the number of
iterations, the logical convergence indicating if the iterations converged and estim.prec which is
an estimate or an upper bound of the final precision (in $x$). method the string of the method used.

Author(s)

"GoldenRatio" is based on Hans Werner Borchers' golden_ratio (package pracma); modifications and "Brent" by Martin Maechler.

See Also

R's standard optimize; for multivariate optimization, Rmpfr's hjkMpfr(); for root finding, Rmpfr's
unirootR.

Examples

## The minimum of the Gamma (and lgamma) function (for $x > 0$):
Gmin <- optimizeR(gamma, .1, 3, tol = 1e-50)
str(Gmin, digits = 8)
## high precision chosen for "objective"; minimum has "estim.prec" = 1.79e-50
Gmin[,c("minimum","objective")]
## it is however more accurate to 59 digits:
asNumeric(optimizeR(gamma, 1, 2, tol = 1e-100)$minimum - Gmin$minimum)

iG5 <- function(x) -exp(-(x-5)^2/2)
curve(iG5, 0, 10, 200)
o.dp <- optimize (iG5, c(0, 10)) #-> 5 of course
o.M.gs <- optimizeR(iG5, 0, 10, method="Golden")
O.M.Br <- optimizeR(iG5, 0, 10, method="Brent", trace=TRUE)
O.M.gs$min ; O.M.gs$iter
O.M.Br$min ; O.M.Br$iter
(doExtras <- Rmpfr:::doExtras())
if(doExtras) {## more accuracy (takes a few seconds)
oM.gs <- optimizeR(iG5, 0, 10, method="Golden", tol = 1e-70)
oM.Br <- optimizeR(iG5, 0, 10, tol = 1e-70)
}
rbind(Golden = c(err = as.numeric(oM.gs$min - 5), iter = oM.gs$iter),
       Brent = c(err = as.numeric(oM.Br$min - 5), iter = oM.Br$iter))
## ==> Brent is orders of magnitude more efficient!

## Testing on the sine curve with 40 correct digits:
sol <- optimizeR(sin, 2, 6, tol = 1e-40)
str(sol)
sol <- optimizeR(sin, 2, 6, tol = 1e-50,
                  precFactor = 3.0, trace = TRUE)
pi.. <- 2*sol$min/3
print(pi.., digits=51)
stopifnot(all.equal(pi.., Const("pi", 256), tolerance = 10*1e-50))

if(doExtras) { # considerably more expensive
  ## a harder one:
  f.sq <- function(x) sin(x-2)^4 + sqrt(pmax(0,(x-1)*(x-4)))*(x-2)^2
  curve(f.sq, 0, 4.5, n=1000)
  msq <- optimizeR(f.sq, 0, 5, tol = 1e-50, trace=5)
  str(msq) # ok
  stopifnot(abs(msq$minimum - 2) < 1e-49)

  ## find the other local minimum: -- non-smooth ==> Golden ratio -section is used
  msq2 <- optimizeR(f.sq, 3.5, 5, tol = 1e-50, trace=10)
  stopifnot(abs(msq2$minimum - 4) < 1e-49)

  ## and a local maximum:
  msq3 <- optimizeR(f.sq, 3, 4, maximum=TRUE, trace=2)
  stopifnot(abs(msq3$maximum - 3.57) < 1e-2)
}#end {doExtras}

##----- "impossible" one to get precisely -------------------------

ff <- function(x) exp(-1/(x-8)^2)
curve(exp(-1/(x-8)^2), -3, 13, n=1001)
(opt. <- optimizeR(function(x) exp(-1/(x-8)^2), -3, 13, trace = 5))
## -> close to 8 (but not very close!)
ff(opt.$minimum) # gives 0
if(doExtras) {
  ## try harder ... in vain..
  str(opt1 <- optimizeR(ff, -3,13, tol = 1e-60, precFactor = 4))
  print(opt1$minimum, digits=20)
  ## still just 7.99998038 or 8.000036655 (depending on method)
}
Description

For integers \(a, b\), \(I_x(a, b)\) aka \(pbeta(x, a, b)\) is a polynomial in \(x\) with rational coefficients, and hence arbitrarily accurate computable.

Usage

\[
\text{pbetaI}(q, \text{shape1}, \text{shape2}, \text{ncp} = 0, \text{lower.tail} = \text{TRUE}, \text{log.p} = \text{FALSE}, \\
\text{precBits} = \text{NULL}, \\
\text{useRational} = \text{!log.p} && \text{!is.mpfr}(q) && \text{is.null(precBits)}, \\
\text{rnd.mode} = \text{c("N","D","U","Z","A")})
\]

Arguments

- \(q\) called \(x\), above; vector of quantiles, in \([0, 1]\); can be numeric, or of class "mpfr" or also "bigq" ("big rational" from package \texttt{gmp}); in the latter case, if \(\text{log.p} = \text{FALSE}\) as by default, all computations are exact, using big rational arithmetic.
- \(\text{shape1}, \text{shape2}\) the positive Beta "shape" parameters, called \(a, b\), above. \textbf{Must} be integer valued for this function.
- \(\text{ncp}\) unused, only for compatibility with \texttt{pbeta}, must be kept at its default, 0.
- \(\text{lower.tail}\) logical; if \text{TRUE} (default), probabilities are \(P[X \leq x]\), otherwise, \(P[X > x]\).
- \(\text{log.p}\) logical; if \text{TRUE}, probabilities \(p\) are given as \(\log(p)\).
- \(\text{precBits}\) the precision (in number of bits) to be used in \texttt{sumBinomMpfr()}.
- \(\text{useRational}\) optional logical, specifying if we should try to do everything in exact rational arithmetic, i.e, using package \texttt{gmp} functionality only, and return \texttt{bigq} numbers instead of \texttt{mpfr} numbers.
- \(\text{rnd.mode}\) a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see \texttt{mpfr}.

Value

an "mpfr" vector of the same length as \(q\).

Note

For upper tail probabilities, i.e., when \(\text{lower.tail}=\text{FALSE}\), we may need large \texttt{precBits}, because the implicit or explicit \(1 - P\) computation suffers from severe cancellation.

Author(s)

Martin Maechler
See Also

`pbeta`, `sumBinomMpfr`, `chooseZ`.

Examples

```r
x <- (0:12)/16 # not all the way up ..
a <- 7; b <- 788

p. <- pbetaI(x, a, b) ## a bit slower:
system.time(
  pp <- pbetaI(x, a, b, precBits = 2048)
) # 0.23 -- 0.50 sec
## Currently, the lower.tail=FALSE are computed "badly":
lp <- log(pp)  ## = pbetaI(x, a, b, log.p=TRUE)
lIp <- log1p(-pp)  ## = pbetaI(x, a, b, lower.tail=FALSE, log.p=TRUE)
Ip <- 1 - pp  ## = pbetaI(x, a, b, lower.tail=FALSE)

if(Rmpfr:::doExtras()) { ## somewhat slow
  stopifnot(
    all.equal(lp, pbetaI(x, a, b, precBits = 2048, log.p=TRUE)),
    all.equal(lIp, pbetaI(x, a, b, precBits = 2048, lower.tail=FALSE, log.p=TRUE),
      tol = 1e-230),
    all.equal( Ip, pbetaI(x, a, b, lower.tail=FALSE))
  )
}

rErr <- function(approx, true, eps = 1e-200) {
  true <- as.numeric(true) # for "mpfr"
  ifelse(Mod(true) >= eps,
    ## relative error, catching '-Inf' etc :
    ifelse(true == approx, 0, 1 - approx / true),
    ## else: absolute error (e.g. when true=0)
    true - approx)
}

rErr(pbeta(x, a, b), pp)
rErr(pbeta(x, a, b, lower=FALSE), Ip)
rErr(pbeta(x, a, b, log = TRUE), lp)
rErr(pbeta(x, a, b, lower=FALSE, log = TRUE), lIp)

a.EQ <- function(..., tol=1e-15) all.equal(..., tolerance=tol)
stopifnot(
  a.EQ(pp, pbeta(x, a, b)),
  a.EQ(lp, pbeta(x, a, b, log.p=TRUE)),
  a.EQ(lIp, pbeta(x, a, b, lower.tail=FALSE, log.p=TRUE)),
  a.EQ( Ip, pbeta(x, a, b, lower.tail=FALSE))
)
```

## When 'q' is a bigrational (i.e., class "bigq", package 'gmp'), everything
## is computed *exactly* with bigrational arithmetic:
```
(q4 <- as.bigq(1, 2^16))
pb4 <- pbetaI(q4, 10, 288, lower.tail=FALSE)
```
stopifnot( is.bigq(pb4) )
mpb4 <- as(pb4, "mpfr")
mpb4[1:2]
getPrec(mp4) # 128 349 1100 1746 2362
(pb. <- pbeta(asNumeric(q4), 10, 288, lower.tail=FALSE))
stopifnot(mp4[1] == 0,
    all.equal(mp4, pb., tol=4e-15))

qbetaI. <- function(p, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE,
    precBits = NULL, rnd.mode = c("N", "D", "U", "Z", "A"),
    tolerance = 1e-20, ...)
{
    if(is.na(a <- as.integer(shape1))) stop("a = shape1 is not coercable to finite integer")
    if(is.na(b <- as.integer(shape2))) stop("b = shape2 is not coercable to finite integer")
    unirootR(function(q) pbetaI(q, a, b, lower.tail=lower.tail, log.p=log.p,
        precBits=precBits, rnd.mode=rnd.mode) - p,
        interval = if(log.p) c(-double.xmax, 0) else 0:1,
        tol = tolerance, ...)
} # end(qbetaI)

(p <- 1 - mpfr(1,128)/20) # 'p' must be high precision
q95.1.3 <- qbetaI.(p, 1,3, tolerance = 1e-29) # -> ~29 digits accuracy
str(q95.1.3) ; roundMpfr(q95.1.3$root, precBits = 29 * log2(10))
## relative error is really small:
(relE <- asNumeric(1 - pbetaI(q95.1.3$root, 1,3) / p))
stopifnot(abs(relE) < 1e-28)

---

**pmax**

*Parallel Maxima and Minima*

**Description**

Returns the parallel maxima and minima of the input values.

The functions *pmin* and *pmax* have been made S4 generics, and this page documents the “... method for class "mNumber"", i.e., for arguments that are numeric or from class "mpfr".

**Usage**

```r
pmax(..., na.rm = FALSE)
pmin(..., na.rm = FALSE)
```

**Arguments**

- `...`: numeric or arbitrary precision numbers (class *mpfr*).
- `na.rm`: a logical indicating whether missing values should be removed.

**Details**

See *pmax*, the documentation of the base functions, i.e., default methods.
Value

vector-like, of length the longest of the input vectors; typically of class `mpfr`, for the methods here.

Methods

... = "ANY"  the default method, really just `base::pmin` or `base::pmax`, respectively.
... = "mNumber"  the method for `mpfr` arguments, mixed with numbers; designed to follow the same semantic as the default method.

See Also

The documentation of the `base` functions, `pmin` and `pmax`; also `min` and `max`; further, `range` (both min and max).

Examples

```r
(pm <- pmin(1.35, mpfr(0:10, 77)))
stopifnot(pm == pmin(1.35, 0:10))
```

Description

Functions from `base` etc which need a copy in the `Rmpfr` namespace so they correctly dispatch.

Usage

outer(X, Y, FUN = "*", ...)

Arguments

X, Y, FUN, ...  See `base` package help: outer.

See Also

outer.

Examples

```r
outer(1/mpfr(1:10, 70), 0:2)
```
roundMpfr

Rounding to Binary bits, “mpfr-internally”

Description

Rounding to binary bits, not decimal digits. Closer to the number representation, this also allows to increase or decrease a number’s precBits. In other words, it acts as setPrec(), see getPrec().

Usage

roundMpfr(x, precBits, rnd.mode = c("N","D","U","Z","A"))

Arguments

- x: an mpfr number (vector)
- precBits: integer specifying the desired precision in bits.
- rnd.mode: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

Value

an mpfr number as x but with the new ‘precBits’ precision

See Also

The mpfr class group method Math2 implements a method for round(x,digits) which rounds to decimal digits.

Examples

(p1 <- Const("pi", 100)) # 100 bit prec
roundMpfr(p1, 120) # 20 bits more, but "random noise"
Const("pi", 120) # same "precision", but really precise

sapplyMpfr

Apply a Function over a "mpfr" Vector

Description

Users may be disappointed to note that sapply() or vapply() typically do not work with "mpfr" numbers.

This is a simple (but strong) approach to work around the problem, based on lapply().

Note that this is not yet as flexible as sapply() for atomic vectors.
seqMpfr

Usage

sapplyMpfr(X, FUN, ...)

Arguments

X       a vector, possibly of class "mpfr".
FUN     a function returning an "mpfr" number. (TODO: A function returning a vector of mpfr numbers or even "mpfrArray").
...     further arguments passed to lapply, typically further arguments to FUN.

Value

an "mpfr" vector, typically of the same length as X.

Note

Another workaround could be to use

```r
res <- lapply(....)
sapply(res, asNumeric)
```

Author(s)

Martin Maechler

Examples

```r
## The function is simply defined as
function (X, FUN, ...) 
  new("mpfr", unlist(lapply(X, FUN, ...), recursive = FALSE))

if(require("Bessel")) # here X, is simple
  bImp <- sapplyMpfr(0:4, function(k)
    besselI.nuAsym(mpfr(1.31e9, 128), 10, expon.scaled=TRUE, k.max=k))
```

Description

Generate ‘regular’, i.e., arithmetic sequences. This is in lieu of methods for seq (dispatching on all three of from, to, and by).

Usage

```r
seqMpfr(from = 1, to = 1, by = ((to - from)/(length.out - 1)),
         length.out = NULL, along.with = NULL, ...)
```
**str.mpfr**  

**Arguments**
- `from, to` the starting and (maximal) end value (numeric or "mpfr") of the sequence.
- `by` number (numeric or "mpfr"): increment of the sequence.
- `length.out` desired length of the sequence. A non-negative number, which will be rounded up if fractional.
- `along.with` take the length from the length of this argument.
- `...` arguments passed to or from methods.

**Details**
- see `seq` (default method in package base), whose semantic we want to replicate (almost).

**Value**
- a `vector` of class "mpfr", when one of the first three arguments was.

**Author(s)**
- Martin Maechler

**See Also**
- The documentation of the base function `seq; mpfr`

**Examples**
```r
seqMpfr(0, 1, by = mpfr(0.25, prec=88))
seqMpfr(7, 3) # -> default prec.
```

---

**str.mpfr**  

**Compactly Show STRucture of Rmpfr Number Object**

**Description**
- The `str` method for objects of class `mpfr` produces a bit more useful output than the default method `str.default`.

**Usage**
```
# S3 method for class 'mpfr'
str(object, nest.lev, internal = FALSE,
     give.head = TRUE, digits.d = 12, vec.len = NULL, drop0trailing=TRUE, 
     width = getOption("width"), ...)
Arguments

object an object of class mpfr.
nest.lev for str(), typically only used when called by a higher level str().
internal logical indicating if the low-level internal structure should be shown; if true (not by default), uses str(object@.Data).
give.head logical indicating if the “header” should be printed.
digits.d the number of digits to be used, will be passed formatMpfr() and hence NULL will use “as many as needed”, i.e. often too many. If this is a number, as per default, less digits will be used in case the precision (getPrec(object)) is smaller.
vec.len the number of elements that will be shown. The default depends on the precision of object and width (since Rmpfr 0.6-0, it was 3 previously).
drop0trailing logical, passed to formatMpfr() (with a different default here).
width the (approximately) desired width of output, see options(width = .).
... further arguments, passed to formatMpfr().

See Also

.mpfr2list() puts the internal structure into a list, and its help page documents many more (low level) utilities.

Examples

(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
str(x)
str(list(pi = pi, x.mpfr = x))
str(x ^ 1000)
str(x ^ -1e4, digits=NULL) # full precision
str(x, internal = TRUE) # internal low-level (for experts)

uu <- Const("pi", 16)# unaccurate
str(uu) # very similar to just ‘uu’

sumBinomMpfr

(Alternating) Binomial Sums via Rmpfr

Description

Compute (alternating) binomial sums via high-precision arithmetic. If \( sBn(f, n) := \sum_{k=n0}^{n} (-1)^{n-k} \binom{n}{k} \cdot f(k) = \Delta^n f, \)
see Details for the $n$-th forward difference operator $\Delta^n f$. If alternating is false, the $(-1)^{n-k}$ factor is dropped (or replaced by 1) above.

Such sums appear in different contexts and are typically challenging, i.e., currently impossible, to evaluate reliably as soon as $n$ is larger than around $50-70$.

Usage

```r
sumBinomMpfr(n, f, n0 = 0, alternating = TRUE, precBits = 256,
             f.k = f(mpfr(k, precBits=precBits)))
```

Arguments

- `n` upper summation index (integer).
- `f` function to be evaluated at $k$ for $k$ in $n0:n$ (and which must return one value per $k$).
- `n0` lower summation index, typically 0 (= default) or 1.
- `alternating` logical indicating if the sum is alternating, see below.
- `precBits` the number of bits for MPFR precision, see `mpfr`.
- `f.k` can be specified instead of `f` and `precBits`, and must contain the equivalent of its default, `f(mpfr(k,precBits=precBits))`.

Details

The alternating binomial sum $sB(f, n) := sumBinom(n, f, n0 = 0)$ is equal to the $n$-th forward difference operator $\Delta^n f$, $sB(f, n) = \Delta^n f,$ where

$$\Delta^n f = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \cdot f(k),$$

is the $n$-fold iterated forward difference $\Delta f(x) = f(x + 1) - f(x)$ (for $x = 0$).

The current implementation might be improved in the future, notably for the case where $sB(f, n) = sumBinomMpfr(n, f, *)$ is to be computed for a whole sequence $n = 1, \ldots, N$.

Value

an `mpfr` number of precision `precBits`. $s$. If alternating is true (as per default),

$$s = \sum_{k=n0}^{n} (-1)^{k} \binom{n}{k} \cdot f(k),$$

if alternating is false, the $(-1)^k$ factor is dropped (or replaced by 1) above.

Author(s)

Martin Maechler, after conversations with Christophe Dutang.
References


See Also

chooseMpfr, chooseZ from package gmp.

Examples

## "naive" R implementation:

```r
sumBinom <- function(n, f, n0=0, ...) {
  k <- n0:n
  sum( choose(n, k) * (-1)^(n-k) * f(k, ...))
}
```

## compute sumBinomMpfr(.) for a whole set of 'n' values:

```r
sumBin.all <- function(n, f, n0=0, precBits = 256, ...) {
  N <- length(n)
  precBits <- rep(precBits, length = N)
  ll <- lapply(seq_len(N), function(i)
    sumBinomMpfr(n[i], f, n0=n0, precBits=precBits[i], ...))
  sapply(ll, as, "double")
}
```

## compute sumBin.all.R for a whole set of 'n' values:

```r
sumBin.all.R <- function(n, f, n0=0, ...) sapply(n, sumBinom, f=f, n0=n0, ...)
```

n.set <- 5:80

```r
system.time(res.R <- sumBin.all.R(n.set, f = sqrt)) ## instantaneous..
system.time(resMpfr <- sumBin.all (n.set, f = sqrt)) ## ~ 0.6 seconds
```

```r
matplot(n.set, cbind(res.R, resMpfr), type = "l", lty=1, ylim = extendrange(resMpfr, f = 0.25), xlab = "n", main = "sumBinomMpfr(n, f = sqrt) vs. R double precision")
legend("topleft", leg=c("double prec.", "mpfr"), lty=1, col=1:2, bty = "n")
```

---

**unirootR**

*One Dimensional Root (Zero) Finding – in pure R*

Description

The function `unirootR` searches the interval from `lower` to `upper` for a root (i.e., zero) of the function `f` with respect to its first argument.

`unirootR()` is “clone” of `uniroot()`, written entirely in `R`, in a way that it works with `mpfr`-numbers as well.
Usage

unirootR(f, interval, ..., 
lower = min(interval), upper = max(interval),
f.lower = f(lower, ...), f.upper = f(upper, ...),
verbose = FALSE,
tol = .Machine$double.eps^0.25, maxiter = 1000,
warn.no.convergence = TRUE,
epsC = NULL)

Arguments

f the function for which the root is sought.
interval a vector containing the end-points of the interval to be searched for the root.
... additional named or unnamed arguments to be passed to f
lower, upper the lower and upper end points of the interval to be searched.
f.lower, f.upper the same as f(upper) and f(lower), respectively. Passing these values from the caller where they are often known is more economical as soon as f() contains non-trivial computations.
verbose logical (or integer) indicating if (and how much) verbose output should be produced during the iterations.
tol the desired accuracy (convergence tolerance).
maxiter the maximum number of iterations.
warn.no.convergence if set to FALSE there’s no warning about non-convergence. Useful to just run a few iterations.
epsC positive number or NULL in which case a smart default is sought. This should specify the “achievable machine precision” for the given numbers and their arithmetic.

The default will set this to .Machine$double.eps for double precision numbers, and will basically use $2^{-\min(getPrec(f.lower),getPrec(f.upper))}$ when that works (as, e.g., for mpfr-numbers) otherwise.

This is factually a lower bound for the achievable lower bound, and hence, setting tol smaller than epsC is typically non-sensical sense and produces a warning.

Details

Note that arguments after ... must be matched exactly.

Either interval or both lower and upper must be specified: the upper endpoint must be strictly larger than the lower endpoint. The function values at the endpoints must be of opposite signs (or zero).

The function only uses R code with basic arithmetic, such that it should also work with “generalized” numbers (such as mpfr-numbers) as long the necessary Ops methods are defined for those.
The underlying algorithm assumes a continuous function (which then is known to have at least one root in the interval).

Convergence is declared either if \( f(x) = 0 \) or the change in \( x \) for one step of the algorithm is less than \( \text{tol} \) (plus an allowance for representation error in \( x \)).

If the algorithm does not converge in \( \text{maxiter} \) steps, a warning is printed and the current approximation is returned.

\( f \) will be called as \( f(x, \ldots) \) for a (generalized) numeric value of \( x \).

**Value**

A list with four components: \( \text{root} \) and \( f.\text{root} \) give the location of the root and the value of the function evaluated at that point. \( \text{iter} \) and \( \text{estim.prec} \) give the number of iterations used and an approximate estimated precision for \( \text{root} \). (If the root occurs at one of the endpoints, the estimated precision is NA.)

**Source**

Based on \texttt{zeroin()} (in package \texttt{rootoned}) by John Nash who manually translated the C code in \texttt{R}'s \texttt{zeroin.c} and on \texttt{uniroot()} in \texttt{R}'s sources.

**References**

Brent, R. (1973), see \texttt{uniroot}.

**See Also**

\texttt{polyroot} for all complex roots of a polynomial; \texttt{optimize, nlm}.

**Examples**

```r
require(utils) # for str

## some platforms hit zero exactly on the first step:
## if so the estimated precision is 2/3.
\( f \) <- function (x,a) x - a
str(xmin <- unirootR(f, c(0, 1), tol = 0.0001, a = 1/3))

## handheld calculator example: fixpoint of cos(.):
\( rc \) <- unirootR(function(x) cos(x) - x, lower=-pi, upper=pi, tol = 1e-9)
\( rc \$ \text{root} \)

## the same with much higher precision:
\( rcM \) <- unirootR(function(x) cos(x) - x,
  interval = mpfr(c(-3,3), 300), tol = 1e-40)
\( rcM \$ \text{root} \)
x0 <- rcM$\text{root}
stopifnot(all.equal(cos(x0), x0,
  tol = 1e-40))## 40 digits accurate!
str(unirootR(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,
```
\texttt{str(uniroot(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,}
\texttt{tol = 1e-10 ), digits.d = 10)}

## A sign change of f(.), but not a zero but rather a "pole":
\texttt{tan. <- function(x) tan(x * (Const("pi",200)/180))# == tan( <angle> )}
\texttt{(rtan <- uniroot(tan., interval = mpfr(c(80,100), 200), tol = 1e-40))}
\texttt{## finds 90 ("ok"), and now gives a warning}

\texttt{## Find the smallest value x for which exp(x) > 0 (numerically):}
\texttt{r <- uniroot(function(x) 1e80*exp(x)-1e-300, c(-1000,0), tol = 1e-15)}
\texttt{str(r, digits.d = 15) ##> around -745, depending on the platform.}
\texttt{exp(r$root) # = 0, but not for r$root * 0.999...}
\texttt{minexp <- r$root * (1 - 10*.Machine$double.eps)}
\texttt{exp(minexp) # typically denormalized}

## --- using mpfr-numbers :

\texttt{## Find the smallest value x for which exp(x) > 0 ("numerically");}
\texttt{## Note that mpfr-numbers underflow *MUCH* later than doubles:}
\texttt{## one of the smallest mpfr-numbers (see also ?mpfr-class ) :}
\texttt{(ep.M <- mpfr(2, 55) ^ - ((2^30 + 1) * (1 - 1e-15)))}
\texttt{r <- uniroot(function(x) 1e99* exp(x) - ep.M, mpfr(c(-1e20, 0), 200))}
\texttt{r # 97 iterations; f.root is very similar to ep.M}
Index

* Forward Difference
  - sumBinomMpfr, 66
* Rice integral
  - sumBinomMpfr, 66
* arith
  - Bernoulli, 8
  - chooseMpfr, 12
  - factorialMpfr, 14
  - formatHex, 15
  - gmp-conversions, 21
  - matmult, 31
  - mpfr.utils, 49
  - pbetaI, 59
  - pmax, 61
  - roundMpfr, 63
  - sumBinomMpfr, 66
* array
  - mpfrArray, 50
  - mpfrMatrix-utils, 55
* character
  - formatMpfr, 18
* classes
  - array_or_vector-class, 6
  - atomicVector-class, 8
  - Mnumber-class, 33
  - mpfr, 34
  - mpfr-class, 36
  - mpfrMatrix, 52
* distribution
  - mpfr-distr-etc, 42
  - pbetaI, 59
* manip
  - sapplyMpfr, 63
  - seqMpfr, 64
* math
  - Bessel_mpfr, 10
  - igamma, 24
  - integrateR, 26
  - is.whole, 28
  - log1mexp, 29
  - mpfr-special-functions, 43
* methods
  - asNumeric-methods, 7
  - bind-methods, 11
  - pmax, 61
* misc
  - Rmpfr-workarounds, 62
* optimize
  - hjkMpfr, 22
  - optimizeR, 56
  - unirootR, 68
* package
  - Rmpfr-package, 3
* print
  - formatMpfr, 18
* univar
  - pmax, 61
* utilities
  - integrateR, 26
  - mpfr-utils, 45
  - str.mpfr, 65
  - .Machine, 39, 47, 69
  - .bigz2mpfr (gmp-conversions), 21
  - .bigz2mpfr (gmp-conversions), 21
  - .getPrec (mpfr-utils), 45
  - .matmult.R (matmult), 31
  - .mpfr.is.whole (mpfr.utils), 49
  - .mpfr2bigz (gmp-conversions), 21
  - .mpfr2exp, 20
  - .mpfr2exp (mpfr-utils), 45
  - .mpfr2list, 66
  - .mpfr2list (mpfr-utils), 45
  - .mpfr2str, 19, 47
  - .mpfr2str (formatMpfr), 18
  - .mpfr_erange (mpfr-utils), 45
  - .mpfr_erange_is_int (mpfr-utils), 45
  - .mpfr_erange_kinds (mpfr-utils), 45
  - .mpfr_erange_set, 42
INDEX

.mpfr_erange_set (mpfr-utils), 45
.mpfr_formatinfo, 20
.mpfr_formatinfo (mpfr-utils), 45
.mpfr_maxPrec (mpfr-utils), 45
.mpfr_minPrec (mpfr-utils), 45
[[], mpfr, ANY, missing, missing-method
(mpfr-class), 36
[[], mpfrArray, ANY, ANY, ANY-method
(mpfrMatrix), 52
[[], mpfrArray, ANY, missing, missing-method
(mpfrMatrix), 52
[[], mpfrArray, matrix, missing, missing-method
(mpfrMatrix), 52
[<-, mpfr, ANY, missing, ANY-method
(mpfr-class), 36
[<-, mpfr, ANY, missing, mpfr-method
(mpfr-class), 36
[<-, mpfr, missing, missing, ANY-method
(mpfr-class), 36
[<-, mpfrArray, ANY, ANY, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, ANY, ANY, mpfr-method
(mpfrMatrix), 52
[<-, mpfrArray, ANY, missing, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, ANY, missing, mpfr-method
(mpfrMatrix), 52
[<-, mpfrArray, matrix, missing, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, matrix, missing, mpfr-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, mpfr-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, missing, ANY-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, missing, mpfr-method
(mpfrMatrix), 52
[<-, mpfrArray, missing, missing, mpfrArray-method
(mpfrMatrix), 52
%*%, mpfr, mpfr-method
(mpfrMatrix), 52
%*%, mpfrArray, mpfr-method
(mpfrMatrix), 52
%*%, mpfrArray, mpfrMatrix-method
(mpfrMatrix), 52
%*%, 32, 53
abs, 38, 55
abs, mpfr-method (mpfr-class), 36
acos, 38
acosh, 38
Ai (Bessel_mpfr), 10
all, 38
all.equal, 39
all.equal, ANY, mpfr-method (mpfr-class), 36
all.equal, mpfr, ANY-method (mpfr-class), 36
all.equal, mpfr, mpfr-method
(mpfr-class), 36
any, 38
aperm, 53
aperm, mpfrArray-method (mpfrMatrix), 52
apply, mpfrArray-method (mpfrMatrix), 52
Arg, mpfr-method (mpfr-class), 36
Arith, array, mpfr-method (mpfr-class), 36
Arith, integer, mpfr-method (mpfr-class), 36
Arith, mpfr, array-method (mpfr-class), 36
Arith, mpfr, integer-method (mpfr-class), 36
Arith, mpfr, missing-method (mpfr-class), 36
Arith, mpfr, mpfr-method (mpfr-class), 36
Arith, mpfrArray-method
(mpfrMatrix), 52
Arith, mpfrArray, mpfr-method
(mpfrMatrix), 52
Arith, mpfrArray, mpfrArray-method
(mpfrMatrix), 52
Arith, mpfrArray, numeric-method
(mpfrMatrix), 52
Arith, numeric, mpfr-method (mpfr-class), 36
Arith, numeric, mpfrArray-method
(mpfrMatrix), 52
array, 5, 7, 19, 34, 39, 47, 50–53
INDEX

array_or_vector, 33
array_or_vector-class, 6
as, 39
as.bigq, 21
as.bigz, 21
as.integer, mpfr-method (mpfr-class), 36
as.vector, mpfrArray, missing-method (mpfrMatrix), 52
as.vector, mpfrArray-method (mpfr-class), 36
asin, 38
asinh, 38
asNumeric, 7, 35, 47, 51, 55
asNumeric, mpfr-method (asNumeric-methods), 7
asNumeric, mpfrArray-method (asNumeric-methods), 7
asNumeric-methods, 7
atan, 37, 38
atan2, ANY, mpfr-method (mpfr-class), 36
atan2, mpfr, ANY-method (mpfr-class), 36
atan2, mpfr, numeric-method (mpfr-class), 36
atan2, mpfrArray, ANY-method (mpfr-class), 36
atan2, mpfrArray, mpfrArray-method (mpfr-class), 36
atan2, numeric, mpfr-method (mpfr-class), 36
atanh, 38
atanh
atomicVector-class, 8
base::pmin, 62
Bernoulli, 5, 8, 40
Bessel_mpfr, 5, 10, 44
besselJ, 10
besselY, 10
beta, 38
beta, ANY, mpfr-method (mpfr-class), 36
beta, ANY, mpfrArray-method (mpfr-class), 36
beta, mpfr, ANY-method (mpfr-class), 36
beta, mpfr, mpfr-method (mpfr-class), 36
beta, mpfrArray, ANY-method (mpfr-class), 36
beta, mpfrArray, mpfrArray-method (mpfr-class), 36
bigq, 21, 34, 59
bigz, 21, 34, 39
bind-methods, 11
c, 49
c.mpfr, 5
c.mpfr (mpfr.utils), 49
cbind, 5, 11, 12
cbind (bind-methods), 11
cbind, ANY-method (bind-methods), 11
cbind, Mnumber-method (bind-methods), 11
cbind-methods (bind-methods), 11
cbind2, 12
ceiling, 38
class, 8, 11, 17, 29, 37, 46, 61
coerce, array, mpfr-method (mpfr-class), 36
coerce, array, mpfrArray-method (mpfrMatrix), 52
coerce, bigq, mpfr-method (gmp-conversions), 21
coerce, bigz, mpfr-method (gmp-conversions), 21
coerce, character, mpfr-method (mpfr-class), 36
coerce, character, mpfr-method (mpfr-class), 36
coerce, integer, mpfr-method (mpfr-class), 36
coerce, logical, mpfr-method (mpfr-class), 36
coerce, matrix, mpfrMatrix-method (mpfrMatrix), 52
coerce, mpfr(bigz-method (mpfr-class), 36
coerce, mpfr,(character-method (mpfr-class), 36
coerce, mpfr, integer-method (mpfr-class), 36
INDEX

coerce, mpfr, mpfr1-method (mpfr-class), 36
coerce, mpfr, numeric-method (mpfr-class), 36
coerce, mpfr1, mpfr-method (mpfr-class), 36
coerce, mpfr1, numeric-method (mpfr-class), 36
coerce, mpfrArray, array-method (mpfrMatrix), 52
coerce, mpfrArray, matrix-method (mpfrMatrix), 52
coerce, mpfrArray, vector-method (mpfrMatrix), 52
coerce, mpfrMatrix, matrix-method (mpfrMatrix), 52
coerce, numeric, mpfr-method (mpfr-class), 36
coerce, numeric, mpfr1-method (mpfr-class), 36
coerce, raw, mpfr-method (mpfr-class), 36
coerce<-, mpfrArray, vector-method (mpfrMatrix), 52
colMeans, mpfrArray-method (mpfrMatrix), 52
colSums, mpfrArray-method (mpfrMatrix), 52
Compare, array, mpfr-method (mpfr-class), 36
Compare, integer, mpfr-method (mpfr-class), 36
Compare, mpfr, array-method (mpfr-class), 36
Compare, mpfr, integer-method (mpfr-class), 36
Compare, mpfr, mpfr-method (mpfr-class), 36
Compare, mpfr, mpfrArray-method (mpfrMatrix), 52
Compare, mpfr, numeric-method (mpfr-class), 36
Compare, mpfrArray, mpfr-method (mpfrMatrix), 52
Compare, mpfrArray, numeric-method (mpfrMatrix), 52
Compare, numeric, mpfr-method (mpfr-class), 36
Compare, numeric, mpfrArray-method (mpfrMatrix), 52
colMeans, mpfrArray-method (mpfrMatrix), 52
complex, 8
Conj, mpfr-method (mpfr-class), 36
Const (mpfr), 34
cos, 38
cosh, 38
crossprod, 32
crossprod, array_or_vector, mpfr-method (mpfr-class), 36
crossprod, Mnumber, mpfr-method (mpfrMatrix), 52
crossprod, mpfr, array_or_vector-method (mpfr-class), 36
crossprod, mpfr, missing-method (mpfrMatrix), 52
crossprod, mpfr, Mnumber-method (mpfrMatrix), 52
crossprod, mpfr, mpfr-method (mpfrMatrix), 52
crossprod, mpfr, mpfrMatrix-method (mpfrMatrix), 52
crossprod, mpfrMatrix, mpfrMatrix-method (mpfrMatrix), 52
crossprod, mpfrMatrix, mpfrMatrix-method (mpfrMatrix), 52
cummax, 38
cummin, 38
cumprod, 38
cumsum, 38
dbinom, 43
dbinom (mpfr-distr-etc), 42
determinant, 55, 56
determinant.mpfrMatrix (mpfrMatrix-utils), 55
dgamma, 43
dgamma (mpfr-distr-etc), 42
diag, mpfrMatrix-method (mpfrMatrix), 52
diag<-, mpfrMatrix-method (mpfrMatrix), 52
diff, 50
diff.default, 50
diff.mpfr (mpfrMatrix), 52
digamma, 5, 38
dim, 7, 19, 38, 53
dim, mpfrArray-method (mpfrMatrix), 52
dim<-, mpfr-method (mpfr-class), 36
dimnames, 19
dimnames.mpfrArray-method(mpfrMatrix), 52
dimnames<-mpfrArray-method
  (mpfrMatrix), 52
dnbinom, 43
dnbinom(mpfr-distr-etc), 42
dnorm(mpfr-distr-etc), 42
dotsMethods, 11
double, 46, 47
dpois, 42, 43
lapply, 63, 64
lgamma, 38
list, 22, 23, 37, 52, 55, 57, 66
load, 48
log, 38, 55
log,mpfr-method(mpfr-class), 36
log10, 38
Im,mpfr-method(mpfr-class), 36
integer, 8, 12, 39, 46, 47
integrate, 27, 28
integrateR, 5, 26
interactive, 46
invisible, 47
is.atomic, 8
is.finite, 47
is.finite.mpfr-method(mpfr-class), 36
is.finite.mpfrArray-method
  (mpfr-class), 36
is.infinite.mpfr-method(mpfr-class), 36
is.infinite.mpfrArray-method
  (mpfr-class), 36
is.integer, 29
is.mpfr, 34
is.na.mpfr-method(mpfr-class), 36
is.na,mpfrArray-method(mpfr-class), 36
is.nan.mpfr-method(mpfr-class), 36
is.nan,mpfrArray-method(mpfr-class), 36
is.whole, 28, 29, 40, 49
j0, 44
j0(Bessel_mpfr), 10
ej1(Bessel_mpfr), 10
jn, 40
jn(Bessel_mpfr), 10
lbeta,ANY,mpfr-method(mpfr-class), 36
lbeta,ANY,mpfrArray-method
  (mpfr-class), 36
lbeta,mpfr,ANY-method(mpfr-class), 36
lbeta,mpfr,mpfr-method(mpfr-class), 36
lbeta,mpfr,numerich metod(mpfr-class), 36
lbeta,mpfrArray,ANY-method
  (mpfr-class), 36
lbeta,mpfrArray,mpfrArray-method
  (mpfr-class), 36
lbeta,numerich,mpfr-method(mpfr-class), 36
lgamma, 38
Li2(mpfr-special-functions), 43
load, 48
log, 38, 55
log,mpfr-method(mpfr-class), 36
log10, 38

INDEX

log1mexp, 29
log1p, 38
log1pexp (log1mexp), 29
log2, 38
Logic, mpfr, mpfr-method (mpfr-class), 36
Logic, mpfr, numeric-method (mpfr-class), 36
Logic, numeric, mpfr-method (mpfr-class), 36
logical, 16, 19, 39, 42, 46, 47, 59

Math, 38
Math, mpfr-method (mpfr-class), 36
Math2, mpfr-method (mpfr-class), 36
mapply, 31
matrix, 7, 32, 47, 52
max, 38
mean, 38
mean, mpfr-method (mpfr-class), 36
mean.default, 38
median, mpfr-method (mpfr-class), 36
min, 38, 62
missing, 34
Mnumber, 11
Mnumber-class, 33
mNumber-class (Mnumber-class), 33
Mod, mpfr-method (mpfr-class), 36
mpfr-class, 5, 36
mpfr-distr (mpfr-distr-etc), 42
mpfr-distr-etc, 42
mpfr-special-functions, 43
mpfr-utils, 45
mpfr.is.0 (mpfr.utils), 49
mpfr.is.integer (mpfr.utils), 49
mpfr.utils, 49
mpfr1, 52
mpfr1-class (mpfr-class), 36
mpfr2array, 51, 52
mpfr2array (mpfr-utils), 45
mpfr_default_prec (mpfr-utils), 45
mpfrArray, 5, 7, 19, 35, 38, 46–48, 50, 50, 51, 52, 54, 64
mpfrArray-class (mpfrMatrix), 52
mpfrImport (mpfr-utils), 45
mpfrIs0, 47
mpfrIs0 (mpfr.utils), 49

mpfrMatrix, 7, 11, 32, 33, 35, 39, 40, 47, 51, 52, 55, 56
mpfrMatrix-class, 5
mpfrMatrix-class (mpfrMatrix), 52
mpfrMatrix-utils, 55
mpfrVersion (mpfr.utils), 49
mpfrXport (mpfr-utils), 45

names, 46
NaN, 38
nlm, 70
norm, 53
norm, ANY, missing-method (mpfrMatrix), 52
norm, mpfrMatrix, character-method (mpfrMatrix), 52
NULL, 46, 52
numeric, 8, 10, 25, 32, 34, 35, 37, 39, 42, 43, 46, 50, 59
numeric_version, 50
numericVector-class (Mnumber-class), 33

Ops, 69
Ops, ANY, mpfr-method (mpfr-class), 36
Ops, array, mpfr-method (mpfr-class), 36
Ops, bigq, mpfr-method (mpfr-class), 36
Ops, bigz, mpfr-method (mpfr-class), 36
Ops, mpfr, ANY-method (mpfr-class), 36
Ops, mpfr, array-method (mpfr-class), 36
Ops, mpfr, bigq-method (mpfr-class), 36
Ops, mpfr, bigz-method (mpfr-class), 36
Ops, mpfr, vector-method (mpfr-class), 36
Ops, vector, mpfr-method (mpfr-class), 36
optim, 23
optimize, 57, 70
optimizeR, 5, 23, 56
options, 19, 46, 47, 66
order, 40
outer, 62
outer (Rmpfr-workarounds), 62

pgamma, 25
pmax, 61, 61
pmax, ANY-method (pmax), 61
pmax, mNumber-method (pmax), 61
pmax-methods (pmax), 61
pmin, 62
pmin (pmax), 61
INDEX

pmin, ANY-method (pmax), 61
pmin, mNumber-method (pmax), 61
pmin-methods (pmax), 61
pnorm, 5, 42–44
pnorm (mpfr-distr-etc), 42
pochMpfr, 15, 40
pochMpfr (chooseMpfr), 12
polyroot, 70
prettyNum, 19, 20
print, 17, 18, 27, 37
print.default, 46
print.integrate, 27
print.integrateR (integrateR), 26
print.mpfr (mpfr-utils), 45
print.mpfr1 (mpfr-class), 36
print.mpfrArray (mpfr-utils), 45
print.Ncharacter (formatHex), 15
print.summaryMpfr (mpfr-class), 36
prod, 38
quantile, 40
quantile, mpfr-method (mpfr-class), 36
range, 38, 62
rank, 40
raw, 39
rbind, 11
rbind, ANY-method (bind-methods), 11
rbind, Mnumber-method (bind-methods), 11
rbind-methods (bind-methods), 11
Re, mpfr-method (mpfr-class), 36
Rmpfr (Rmpfr-package), 3
Rmpfr-package, 3
Rmpfr-workarounds, 62
round, 38, 39, 63
roundMpfr, 5, 35, 39, 40, 47, 63
rowMeans, mpfrArray-method (mpfrMatrix), 52
rowSums, mpfrArray-method (mpfrMatrix), 52
sapply, 63
sapplyMpfr, 63
save, 48
seq, 64, 65
seqMpfr, 5, 64
setPrec (roundMpfr), 63
show, integrateR-method (integrateR), 26
show, mpfr-method (mpfr-class), 36
show, mpfr1-method (mpfr-class), 36
show, mpfrArray-method (mpfrMatrix), 52
show, summaryMpfr-method (mpfr-class), 36
sign, 37, 38
sign, mpfr-method (mpfr-class), 36
sign, mpfrArray-method (mpfrMatrix), 52
signif, 39
sin, 38
sindh, 38
sort, 40
sprintf, 16, 17, 20
sqrt, 38
str, 50, 65, 66
str.default, 65
str.mpfr, 50, 65
sum, 38
sumBinomMpfr, 5, 13, 59, 60, 66
Summary, 38
Summary, mpfr-method (mpfr-class), 36
summary, mpfr-method (mpfr-class), 36
summary.default, 38
summaryMpfr-class (mpfr-class), 36
t, mpfr-method (mpfr-class), 36
t, mpfrMatrix-method (mpfrMatrix), 52
tan, 38
tanh, 38
tcrossprod, 32
tcrossprod, array_or_vector, mpfr-method (mpfr-class), 36
tcrossprod, Mnumber, mpfr-method (mpfrMatrix), 52
tcrossprod, mpfr, array_or_vector-method (mpfr-class), 36
tcrossprod, mpfr, missing-method (mpfrMatrix), 52
tcrossprod, mpfr, Mnumber-method (mpfrMatrix), 52
tcrossprod, mpfr, mpfr-method (mpfrMatrix), 52
tcrossprod, mpfr, mpfrMatrix-method (mpfrMatrix), 52
tcrossprod, mpfrMatrix, mpfr-method (mpfrMatrix), 52
tcROSSprod, mpfrMatrix, mpfrMatrix-method (mpfrMatrix), 52
toNum, 7
toNum (mpfr-utils), 45
trigamma, 5, 38
trunc, 38
typeof, 7, 37

unique, 39
unique, mpfr, ANY-method (mpfr-class), 36
unique, mpfr-method (mpfr-class), 36
unique.mpfr (mpfr-class), 36
unroot, 68, 70
unrootR, 5, 57, 68

vapply, 63
vector, 52
Vectorize, 27

which.max, 39
which.max, mpfr-method (mpfr-class), 36
which.min, 39
which.min, mpfr-method (mpfr-class), 36

y0 (Bessel_mpfr), 10
y1 (Bessel_mpfr), 10
yn, 44
yn (Bessel_mpfr), 10

zeta, 5, 8, 9, 40
zeta (mpfr-special-functions), 43