Package ‘Rmpfr’

December 4, 2023

Title R MPFR - Multiple Precision Floating-Point Reliable

Version 0.9-4

Date 2023-12-04

DateNote Previous CRAN version 0.9-3 on 2023-07-24

Type Package

Description Arithmetic (via S4 classes and methods) for
arbitrary precision floating point numbers, including transcendental
(``special'') functions. To this end, the package interfaces to
the 'LGPL' licensed 'MPFR' (Multiple Precision Floating-Point Reliable) Library
which itself is based on the 'GMP' (GNU Multiple Precision) Library.

SystemRequirements gmp (>= 4.2.3), mpfr (>= 3.0.0), pdfcrop (part of
TexLive) is required to rebuild the vignettes.

SystemRequirementsNote 'MPFR' (MP Floating-Point Reliable Library,
https://www.mpfr.org/) and 'GMP' (GNU Multiple Precision
library, https://gmplib.org/), see >> README.md

Depends gmp (>= 0.6-1), R (>= 3.6.0)

Imports stats, utils, methods

Suggests MASS, Bessel, polynom, sfsmisc (>= 1.1-14)

SuggestsNote MASS, polynom, sfsmisc: only for vignette;

Enhances dfoptim, pracma, DPQ

EnhancesNote mentioned in Rd xrefs I used in example

URL https://rmpfr.r-forge.r-project.org/

BugReports https://r-forge.r-project.org/tracker/?group_id=386

License GPL (>= 2)

Encoding UTF-8

NeedsCompilation yes

Author Martin Maechler [aut, cre] (https://orcid.org/0000-0002-8685-9910),
Richard M. Heiberger [ctb] (formatHex(), *Bin, *Dec),
John C. Nash [ctb] (hjkMpfr(), origin of unirootR()),
Hans W. Borchers [ctb] (optimizeR(*, ```GoldenRatio''); origin of
hjkMpfr())
Maintainer  Martin Maechler <maechler@stat.math.ethz.ch>
Repository  CRAN
Date/Publication  2023-12-04 15:30:02 UTC

R topics documented:

Rmpfr-package .................................................. 3
array_or_vector-class ........................................ 6
asNumeric-methods ............................................. 7
atomicVector-class ............................................ 8
Bernoulli ....................................................... 9
Bessel_mpfr ................................................... 10
bind-methods .................................................. 11
chooseMpfr .................................................... 12
factorialMpfr ............................................... 14
formatHex ...................................................... 16
formatMpfr ..................................................... 19
frexpMpfr ..................................................... 21
gmp-conversions ............................................. 23
hjkMpfr ....................................................... 24
igamma .......................................................... 26
integrateR ..................................................... 28
is.whole ....................................................... 30
log1mexp ...................................................... 31
matmult ......................................................... 33
Mnumber-class ............................................... 35
mpfr ............................................................ 36
mpfr-class ..................................................... 38
mpfr-distr-etc ............................................... 44
mpfr-special-functions ..................................... 45
mpfr-utils .................................................... 47
mpfr.utils .................................................... 52
mpfrArray ..................................................... 53
mpfrMatrix ................................................... 54
mpfrMatrix-utils ............................................ 57
optimizeR ..................................................... 59
pbetaI ........................................................ 61
pmax .......................................................... 64
qnorml ......................................................... 65
Rmpfr-workarounds .......................................... 68
roundMpfr .................................................... 69
sapplyMpfr ................................................... 69
seqMpfr ......................................................... 71
str.mpfr ....................................................... 72
sumBinomMpfr ............................................... 73
unirootR ....................................................... 75
Description

Rmpfr provides S4 classes and methods for arithmetic including transcendental ("special") functions for arbitrary precision floating point numbers, here often called "mpfr - numbers". To this end, it interfaces to the LGPL'ed MPFR (Multiple Precision Floating-Point Reliable) Library which itself is based on the GMP (GNU Multiple Precision) Library.

Details

Package: Rmpfr
Title: R MPFR - Multiple Precision Floating-Point Reliable
Version: 0.9-4
Date: 2023-12-04
DateNote: Previous CRAN version 0.9-3 on 2023-07-24
Type: Package
Authors@R: c(person("Martin","Maechler", role = c("aut","cre"), email = "maechler@stat.math.ethz.ch", comment = "RMPFR - Multiple Precision Floating-Point Reliable"), person("Richard M.", "Heiberger", role = "ctb", email="hwborchers@googlemail.com", comment = "optimizeR(*, "GoldenRatio"); origin of hjkMpfr()") )
Description: Arithmetic (via S4 classes and methods) for arbitrary precision floating point numbers, including ... (Multiple Precision Floating-Point Reliable) Library which itself is based on the GMP (GNU Multiple Precision) Library.
SystemRequirements: gmp (>= 4.2.3), mpfr (>= 3.0.0), pdfcrop (part of TexLive) is required to rebuild the vignettes.
SystemRequirementsNote: 'MPFR' (MP Floating-Point Reliable Library, https://www.mpfr.org/) and 'GMP' (GNU Multiple Precision Library, https://gmplib.org/), see » README.md
Depends: gmp (>= 0.6-1), R (>= 3.6.0)
Imports: stats, utils, methods
Suggests: MASS, Bessel, polynom, sfsmisc (>= 1.1-14)
SuggestsNote: MASS, polynom, sfsmisc: only for vignette;
Enhances: dfoptim, pracma, DPQ
EnhancesNote: mentioned in Rd xrefs | used in example
URL: https://rmpfr.r-forge.r-project.org/
BugReports: https://r-forge.r-project.org/tracker/?group_id=386
License: GPL (>= 2)
Encoding: UTF-8
Author: Martin Maechler [aut, cre] (<https://orcid.org/0000-0002-8685-9910>), Richard M. Heiberger [ctb]
Maintainer: Martin Maechler <maechler@stat.math.ethz.ch>

Index of help topics:

.bigq2mpfr Conversion Utilities gmp <-> Rmpfr
Bernoulli Bernoulli Numbers in Arbitrary Precision
Bessel_mpfr Bessel functions of Integer Order in multiple precisions
Mnumber-class Class "Mnumber" and "mNumber" of "mpfr" and regular numbers and arrays from them
Rmpfr-package R MPFR - Multiple Precision Floating-Point Reliable
array_or_vector-class auxiliary class "array_or_vector"
asNumeric-methods Methods for 'asNumeric(<mpfr>)'
atomicVector-class Virtual Class "atomicVector" of Atomic Vectors
c.mfpr MPFR Number Utilities
cbind "mpfr'...' - Methods for Functions cbind(),
rbind()
chooseMfpr Binomial Coefficients and Pochhammer Symbol aka
Rising Factorial
determinant.mfprMatrix Functions for mfprMatrix Objects
factorialMfpr Factorial 'n!' in Arbitrary Precision
formatHex Flexibly Format Numbers in Binary, Hex and
Decimal Format
formatMfpr Formatting MPFR (multiprecision) Numbers
frexpMfpr Base-2 Representation and Multiplication of
Mfpr Numbers
getPrec Rmpfr - Utilities for Precision Setting,
Printing, etc
hjkMfpr Hooke-Jeeves Derivative-Free Minimization R
(working for MPFR)
igamma Incomplete Gamma Function
integrateR One-Dimensional Numerical Integration - in pure
R
is.whole.mfpr Whole ("Integer") Numbers
log1mexp Compute f(a) = log(1 +/- exp(-a)) Numerically
Optimally
matmult (MPFR) Matrix (Vector) Multiplication
mfpr Create "mfpr" Numbers (Objects)
mpfr-class Class "mpfr" of Multiple Precision Floating
Point Numbers
mpfrArray Construct "mpfrArray" almost as by 'array()
mpfrMatrix-class Classes "mpfrMatrix" and "mpfrArray"
optimizeR High Precision One-Dimensional Optimization
outer Base Functions etc, as an Rmpfr version
pbetaI Accurate Incomplete Beta / Beta Probabilities
For Integer Shapes
pmax Parallel Maxima and Minima
pnorm Distribution Functions with MPFR Arithmetic
qnormI Gaussian / Normal Quantiles 'qnorm()' via
Inversion
roundMfpr Rounding to Binary bits, "mpfr-internally"
sapplyMfpr Apply a Function over a "mpfr" Vector
seqMfpr "mpfr" Sequence Generation
str.mfpr Compactly Show STRucture of Rmpfr Number Object
sumBinomMfpr (Alternating) Binomial Sums via Mfpr
unirootR One Dimensional Root (Zero) Finding - in pure R
zeta Special Mathematical Functions (MPFR)
Further information is available in the following vignettes:

Maechler_useR_2011-abstr  useR-2011-abstract (source)
Rmpfr-pkg                Arbitrarily Accurate Computation with R Package Rmpfr (source)
log1mexp-note           Accurately Computing log(1 - exp(.)) – Assessed by Rmpfr (source)

The following (help pages) index does not really mention that we provide many methods for mathematical functions, including `gamma`, `digamma`, etc, namely, all of R’s S4 Math group (with the only exception of `trigamma`), see the list in the examples. Additionally also `pnorm`, the “error function”, and more, see the list in `zeta`, and further note the first vignette (below).

**Partial index:**

- `mpfr`  Create "mpfr" Numbers (Objects)
- `mpfrArray`  Construct "mpfrArray" almost as by `array()`
- `mpfr-class`  Class "mpfr" of Multiple Precision Floating Point Numbers
- `mpfrMatrix-class`  Classes "mpfrMatrix" and "mpfrArray"
- `Bernoulli`  Bernoulli Numbers in Arbitrary Precision
- `Bessel_mpfr`  Bessel functions of Integer Order in multiple precisions
- `c.mpfr`  MPFR Number Utilities
- `cbind`  "mpfr" . . . - Methods for Functions `cbind()`, `rbind()`
- `chooseMpfr`  Binomial Coefficients and Pochhammer Symbol aka Rising Factorial
- `factorialMpfr`  Factorial ‘n!’ in Arbitrary Precision
- `formatMpfr`  Formatting MPFR (multiprecision) Numbers
- `getPrec`  Rmpfr - Utilities for Precision Setting, Printing, etc
- `roundMpfr`  Rounding to Binary bits, "mpfr-internally"
- `seqMpfr`  "mpfr" Sequence Generation
- `sumBinomMpfr`  (Alternating) Binomial Sums via Rmpfr
- `zeta`  Special Mathematical Functions (MPFR)

- `integrateR`  One-Dimensional Numerical Integration - in pure R
- `unirootR`  One Dimensional Root (Zero) Finding - in pure R
- `optimizeR`  High Precisione One-Dimensional Optimization
- `hjkMpfr`  Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)

Further information is available in the following vignettes:

- `Rmpfr-pkg`  Arbitrarily Accurate Computation with R: The ’Rmpfr’ package (source, pdf)
- `log1mexp-note`  Accurately Computing log(1 - exp(.)) – Assessed by Rmpfr (source, pdf)
Author(s)

Martin Maechler

References

MPFR (MP Floating-Point Reliable Library), https://www.mpfr.org/
GMP (GNU Multiple Precision library), https://gmplib.org/
and see the vignettes mentioned above.

See Also

The R package gmp for big integer gmp and rational numbers (bigrational) on which Rmpfr depends.

Examples

```r
## Using "mpfr" numbers instead of regular numbers...
n1.25 <- mpfr(5, precBits = 256)/4
n1.25

## and then "everything" just works with the desired chosen precision:hig
n1.25 ^ c(1:7, 20, 30) ## fully precise; compare with
print(1.25 ^ 30, digits=19)

exp(n1.25)

## Show all math functions which work with "MPFR" numbers (1 exception: trigamma)
getGroupMembers("Math")

## We provide *many* arithmetic, special function, and other methods:
showMethods(classes = "mpfr")
showMethods(classes = "mpfrArray")
```

array_or_vector-class

Auxiliary Class "array_or_vector"

Description

"array_or_vector" is the class union of c("array", "matrix", "vector") and exists for its use in signatures of method definitions.

Details

Using "array_or_vector" instead of just "vector" in a signature makes an important difference: E.g., if we had setMethod(crossprod, c(x="mpfr", y="vector"), function(x,y) CPR(x,y)), a call crossprod(x, matrix(1:6, 2,3)) would extend into a call of CPR(x, as(y, "vector")) such that CPR(\(y\))'s second argument would simply be a vector instead of the desired 2 x 3 matrix.
Objects from the Class

A virtual Class: No objects may be created from it.

Examples

```
showClass("array_or_vector")
```

asNumeric-methods

Methods for

Methods for function \texttt{asNumeric} (in package \texttt{gmp}).

Usage

```r
## S4 method for signature 'mpfrArray'
asNumeric(x)
```

Arguments

\texttt{x} 

a “number-like” object, here, a \texttt{mpfr} or typically \texttt{mpfrArray} one.

Value

an R object of type (\texttt{typeof}) "numeric", a \texttt{matrix} or \texttt{array} if \texttt{x} had non-NULL dimension \texttt{dim()}.

Methods

\texttt{signature(x = "mpfrArray")} this method also dispatches for \texttt{mpfrMatrix} and returns a numeric array.

\texttt{signature(x = "mpfr")} for non-array/matrix, \texttt{asNumeric(x)} is basically the same as \texttt{as.numeric(x)}.

Author(s)

Martin Maechler

See Also

our lower level (non-generic) \texttt{toNum()}. Further, \texttt{asNumeric} (package \texttt{gmp}), standard R’s \texttt{as.numeric()}.  

Examples

```r
x <- (0:7)/8  # (exact)
X <- mpfr(x, 99)
stopifnot(identical(asNumeric(x), x),
          identical(asNumeric(X), x))

m <- matrix(1:6, 3,2)
(M <- mpfr(m, 99) / 5) ##-> "mpfrMatrix"
asNumeric(M) # numeric matrix
stopifnot(all.equal(asNumeric(M), m/5),
          identical(asNumeric(m), m))# remains matrix
```

---

atomicVector-class

Virtual Class "atomicVector" of Atomic Vectors

Description

The class "atomicVector" is a virtual class containing all atomic vector classes of base R, as also implicitly defined via `is.atomic`.

Objects from the Class

A virtual Class: No objects may be created from it.

Methods

In the `Matrix` package, the "atomicVector" is used in signatures where typically "old-style" "matrix" objects can be used and can be substituted by simple vectors.

Extends

The atomic classes "logical", "integer", "double", "numeric", "complex", "raw" and "character" are extended directly. Note that "numeric" already contains "integer" and "double", but we want all of them to be direct subclasses of "atomicVector".

Author(s)

Martin Maechler

See Also

`is.atomic, integer, numeric, complex`, etc.

Examples

`showClass("atomicVector")`
Bernoulli Numbers in Arbitrary Precision

Description
Computes the Bernoulli numbers in the desired (binary) precision. The computation happens via
the \texttt{zeta} function and the formula
\[ B_k = -k \zeta(1 - k), \]
and hence the only non-zero odd Bernoulli number is \( B_1 = +1/2 \). (Another tradition defines it,
equally sensibly, as \(-1/2\).)

Usage
\begin{verbatim}
Bernoulli(k, precBits = 128)
\end{verbatim}

Arguments
\begin{itemize}
\item \texttt{k} non-negative integer vector
\item \texttt{precBits} the precision in \textit{bits} desired.
\end{itemize}

Value
an \texttt{mpfr} class vector of the same length as \texttt{k}, with i-th component the \( k[i] \)-th Bernoulli number.

Author(s)
Martin Maechler

References
https://en.wikipedia.org/wiki/Bernoulli_number

See Also
\texttt{zeta} is used to compute them.
The next version of package \texttt{gmp} is to contain \texttt{BernoulliQ()}, providing exact Bernoulli numbers
as big rationals (class "bigq").

Examples
\begin{verbatim}
Bernoulli(0:10)
plot(as.numeric(Bernoulli(0:15)), type = "h")

curve(-x*zeta(1-x), -.2, 15.03, n=300,
    main = expression(-x %.% zeta(1-x)))
legend("top", paste(c("even","odd "), "Bernoulli numbers"),
    pch=c(1,3), col=2, pt.cex=2, inset=1/64)
\end{verbatim}
abline(h=0, v=0, lty=3, col="gray")
k <- 0:15; k[1] <- 1e-4
points(k, -k*zeta(1-k), col=2, cex=2, pch=1+2*(k%%2))

## They pretty much explode for larger k :
k2 <- 2*(1:120)
plot(k2, abs(as.numeric(Bernoulli(k2))), log = "y")
title("Bernoulli numbers exponential growth")

Bernoulli(10000)# - 9.0494239636 * 10^27677

---

**Bessel_mpfr**  
*Bessel functions of Integer Order in multiple precisions*

**Description**

Bessel functions of integer orders, provided via arbitrary precision algorithms from the MPFR library.

Note that the computation can be very slow when \( n \) and \( x \) are large (and of similar magnitude).

**Usage**

\[
\begin{align*}
\text{Ai}(x) \\
\text{j0}(x) \\
\text{j1}(x) \\
\text{jn}(n, x, \text{rnd.mode} = \text{c}("N","D","U","Z","A")) \\
\text{y0}(x) \\
\text{y1}(x) \\
\text{yn}(n, x, \text{rnd.mode} = \text{c}("N","D","U","Z","A"))
\end{align*}
\]

**Arguments**

- **x** a numeric or mpfr vector.
- **n** non-negative integer (vector).
- **rnd.mode** a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

**Value**

Computes multiple precision versions of the Bessel functions of integer order, \( J_n(x) \) and \( Y_n(x) \), and—when using MPFR library 3.0.0 or newer—also of the Airy function \( Ai(x) \). Note that currently \( Ai(x) \) is very slow to compute for large \( x \).

**See Also**

besselJ, and besselY compute the same bessel functions but for arbitrary real order and only precision of a bit more than ten digits.
Examples

```r
x <- (0:100)/8 # (have exact binary representation)
stopifnot(exprs = {
  all.equal(besselY(x, 0), b0Y <- y0(x))
  all.equal(besselJ(x, 1), b1J <- j1(x))
  all.equal(yn(0,x), bY0)
  all.equal(jn(1,x), bJ1)
})
```

```r
mpfrVersion() # now typically 4.1.0
if(mpfrVersion() >= "3.0.0") { ## Ai() not available previously
  print( aix <- Ai(x) )
  plot(x, aix, log="y", type="l", col=2)
  stopifnot(
    all.equal(Ai(0), 1/(3^(2/3) * gamma(2/3)))
    , # see https://dlmf.nist.gov/9.2.ii
    all.equal(Ai(100), mpfr("2.63448215208818448955055256064981561e-291"), tol=1e-37)
  )
  two3rd <- 2/mpfr(3, 144)
  print( all.equal(Ai(0), 1/(3^two3rd * gamma(two3rd)), tol=0) ) # 1.7....e-40
  if(Rmpfr:::doExtras()) withAutoprint({ # slowish:
    system.time(ai1k <- Ai(1000)) # 1.4 sec (on 2017 lynne)
    stopifnot(all.equal(print(log10(ai1k)),
                -9157.031193409585185582, tol=2e-16)) # seen 8.8..e-17 | 1.1..e-16
  })
} # ver >= 3.0
```

bind-methods

"mpfr" '...' - Methods for Functions cbind(), rbind()

Description

cbind and rbind methods for signature ... (see dotsMethods are provided for class Mnumber, i.e.,
for binding numeric vectors and class "mpfr" vectors and matrices ("mpfrMatrix") together.

Usage

```r
cbind(..., deparse.level = 1)
rbind(..., deparse.level = 1)
```

Arguments

... matrix-/vector-like R objects to be bound together, see the base documentation,
cbind.

deparse.level integer determining under which circumstances column and row names are built
from the actual arguments’ ‘expression’, see cbind.
chooseMpfr

Value

typically a ‘matrix-like’ object, here typically of class "mpfrMatrix".

Methods

... = "Mnumber" is used to (c|r)bind multiprecision "numbers" (inheriting from class "mpfr")
together, maybe combined with simple numeric vectors.
... = "ANY" reverts to cbind and rbind from package base.

Author(s)

Martin Maechler

See Also
cbind2, cbind, Documentation in base R’s methods package

Examples

```r
chooseMpfr(1, mpfr(6:3, 70)/7, 3:0)
```

chooseMpfr

Binomial Coefficients and Pochhammer Symbol aka Rising Factorial

Description

Compute binomial coefficients, chooseMpfr(a, n) being mathematically the same as choose(a, n),
but using high precision (MPFR) arithmetic.

chooseMpfr.all(n) means the vector choose(n, 1:n), using enough bits for exact computation
via MPFR. However, chooseMpfr.all() is now deprecated in favor of chooseZ from package
gmp, as that is now vectorized.

pochMpfr() computes the Pochhammer symbol or “rising factorial”, also called the “Pochhammer
function”, “Pochhammer polynomial”, “ascending factorial”, “rising sequential product” or “upper
factorial”,

\[ x^{(n)} = x(x + 1)(x + 2) \cdots (x + n - 1) = \frac{(x + n - 1)!}{(x - 1)!} = \frac{\Gamma(x + n)}{\Gamma(x - 1)}. \]

Usage

```r
chooseMpfr(a, n, rnd.mode = c("N","D","U","Z","A"))
chooseMpfr.all(n, precBits=NULL, k0=1, alternating=FALSE)
pochMpfr(a, n, rnd.mode = c("N","D","U","Z","A"))
```
chooseMpfr

Arguments

a a numeric or mpfr vector.

n an integer vector; if not of length one, n and a are recycled to the same length.

rnd.mode a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

precBits integer or NULL for increasing the default precision of the result.

k0 integer scalar

alternating logical, for chooseMpfr.all(), indicating if alternating sign coefficients should be returned, see below.

Value

For

chooseMpfr(), pochMpfr(): an mpfr vector of length \( \max(\text{length}(a), \text{length}(n)) \); chooseMpfr.all(n, k0): a mpfr vector of length \( n-k0+1 \), of binomial coefficients \( C_{n,m} \) or, if alternating is true, \( (-1)^m \cdot C_{n,m} \) for \( m \in k0:n \).

Note

Currently this works via a (C level) for(\( i \) in 1:\( n \))-loop which really slow for large \( n \), say \( 10^6 \), with computational cost \( O(n^2) \). In such cases, if you need high precision choose(a, n) (or Pochhammer(a, n)) for large \( n \), preferably work with the corresponding factorial(mpfr(...)), or gamma(mpfr(...))

See Also

choose(n, m) (base R) computes the binomial coefficient \( C_{n,m} \) which can also be expressed via Pochhammer symbol as \( C_{n,m} = (n-m+1)^{(m)}/m! \).

chooseZ from package gmp; for now, factorialMpfr.

For (alternating) binomial sums, directly use sumBinomMpfr, as that is potentially more efficient.

Examples

pochMpfr(100, 4) == 100*101*102*103 # TRUE
a <- 100:110
pochMpfr(a, 10) # exact (but too high precision)
x <- mpfr(a, 70)# should be enough
(px <- pochMpfr(x, 10)) # the same as above (needing only 70 bits)
stopifnot(pochMpfr(a, 10) == px,
px[1] == prod(mpfr(100:109, 100)))# used to fail

(c1 <- chooseMpfr(1000:997, 60)) # -> automatic "correct" precision
stopifnot(all.equal(c1, choose(1000:997, 60), tolerance=1e-12))

## --- Experimenting & Checking
n.set <- c(1:10, 20, 50:55, 100:105, 200:203, 300:303, 500:503,
699:702, 999:1001)
if(!Rmpfr:::doExtras()) {  # speed up: smaller set
n. <- n.set[-(1:10)]
n.set <- c(1:10, n.[c(TRUE, diff(n.) > 1)])
}
C1 <- C2 <- numeric(length(n.set))
for(i.n in seq_along(n.set)) {
cat(n <- n.set[i.n],":"")
C1[i.n] <- system.time(c.c <- chooseMpfr.all(n))[1]
C2[i.n] <- system.time(c.2 <- chooseMpfr(n, 1:n))[1]
stopifnot(is.whole(c.c), c.c == c.2,
  if(n > 60) TRUE else all.equal(c.c, choose(n, 1:n), tolerance = 1e-15))
cat(" [OK]\n")
}
matplot(n.set, cbind(C1,C2), type="b", log="xy",
xlab = "n", ylab = "system.time(.) [s]"
legend("topleft", c("chooseMpfr.all(n)", "chooseMpfr(n, 1:n)"),
pch=as.character(1:2), col=1:2, lty=1:2, bty="n")
## Currently, chooseMpfr.all() is faster only for large n (>= 300)
## That would change if we used C-code for the *.all() version

## If you want to measure more:
measureMore <- TRUE
measureMore <- FALSE
if(measureMore) {  # takes ~ 2 minutes (on "lynne", Intel i7-7700T, ~2019)
n.s <- 2^((5:20)
r <- lapply(n.s, function(n) {
  N <- ceiling(10000/n)
cat(sprintf("n=%9g => N=%d: ",n,N))
c <- system.time(C <- replicate(N, chooseMpfr(n, n/2)))
cat("[OK]\n")
cat(C=C, ct=ct/N)
})
print(ct.n <- t(sapply(r, "]var/quote/Var","ct")))
hasSfs <- requireNamespace("sfsmisc")
plot(ct.n[,"user.self"] ~ n.s, xlab=quote(n), ylab="system.time(.) [s]",
      main = "CPU Time for chooseMpfr(n, n/2)",
        log="xy", type = "b", axes = !hasSfs)
if(hasSfs) for(side in 1:2) sfsmisc::eaxis(side)
summary(fm <- lm(log(ct.n[,"user.self"]) ~ log(n.s), subset = n.s >= 10^4))
## --> slope = 2 ==> It's O(n^2)
nn <- 2^seq(11,21, by=1/16) ; Lcol <- adjustcolor(2, 1/2)
bet <- coef(fm)
lines(nn, exp(predict(fm, list(n.s = nn))), col=Lcol, lwd=3)
text(500000,1, substitute(AA %*% n^EE,
      list(AA = signif(exp(bet[1]),3),
        EE = signif(bet[2], 3))))
} # measure more
factorialMpfr

Description

Efficiently compute $n!$ in arbitrary precision, using the MPFR-internal implementation. This is mathematically (but not numerically) the same as $\Gamma(n + 1)$.

`factorialZ` (package `gmp`) should typically be used instead of `factorialMpfr()` nowadays. Hence, `factorialMpfr` now is somewhat deprecated.

Usage

```r
factorialMpfr(n, precBits = max(2, ceiling(lgamma(n+1)/log(2))),
               rnd.mode = c("N","D","U","Z","A"))
```

Arguments

- `n` non-negative integer (vector).
- `precBits` desired precision in bits (“binary digits”); the default sets the precision high enough for the result to be exact.
- `rnd.mode` a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see `mpfr`.

Value

a number of (S4) class `mpfr`.

See Also

- `factorial` and `gamma` in base R.
- `factorialZ` (package `gmp`), to replace `factorialMpfr`, see above.
- `chooseMpfr()` and `pochMpfr()` (on the same page).

Examples

```r
factorialMpfr(200)

n <- 1000:1010
f1000 <- factorialMpfr(n)
stopifnot(1e-15 > abs(as.numeric(1 - lfactorial(n)/log(f1000))))

## Note that---astonishingly--- measurements show only
## *small* efficiency gain of ~ 10% : over using the previous "technique"
system.time(replicate(8, f1e4 <- factorialMpfr(10000)))
system.time(replicate(8, f.1e4 <- factorial(mpfr(10000,
               prec=1+lfactorial(10000)/log(2))))))
```
Flexibly Format Numbers in Binary, Hex and Decimal Format

Description

Show numbers in binary, hex and decimal format. The resulting character-like objects can be back-transformed to "mpfr" numbers via mpfr().

Usage

formatHex(x, precBits = min(getPrec(x)), style = "+", expAlign = TRUE)

formatBin(x, precBits = min(getPrec(x)), scientific = TRUE,
          left.pad = "_", right.pad = left.pad, style = "+", expAlign = TRUE)

formatDec(x, precBits = min(getPrec(x)), digits = decdigits,
          nsmall = NULL, scientific = FALSE, style = "+",
          decimalPointAlign = TRUE, ...

Arguments

x                 a numeric or mpfr R object.
precBits          integer, the number of bits of precision, typically derived from x, see getPrec. Numeric, i.e., double precision numbers have 53 bits. For more detail, see mpfr.
style             a single character, to be used in sprintf’s format (fmt), immediately after the " sets a sign in the output, i.e., "+" or "+", where as style = " " may seem more standard.
expAlign          logical indicating if for scientific (“exponential”) representations the exponents should be aligned to the same width, i.e., zero-padded to the same number of digits.
scientific        logical indicating that formatBin should display the binary representation in scientific notation (mpfr(3, 5) is displayed as +0b1.1000p+1). When FALSE, formatBin will display the binary representation in regular format shifted to align binary points (mpfr(3, 5) is displayed +0b11.000).
...               additional optional arguments.

formatHex, formatBin: precBits is the only ... argument acted on. Other ... arguments are ignored.
formatDec: precBits is acted on. Any argument accepted by format (except nsmall) is acted on. Other ... arguments are ignored.
left.pad, right.pad
characters (one-character strings) that will be used for left- and right-padding of
the formatted string when scientific=FALSE. Do not change these unless for
display-only purpose !!

nsmall
only used when scientific is false, then passed to \texttt{format()} . If NULL, the
default is computed from the range of the non-zero values of \( x \).
digits
integer; the number of decimal digits displayed is the larger of this argument
and the internally generated value that is a function of \texttt{precBits} . This is related
to but different than digits in \texttt{format} .
decimalPointAlign
logical indicating if padding should be used to ensure that the resulting strings
align on the decimal point (". ").

Details
For the hexadecimal representation, when the precision is not larger than double precision, \texttt{sprintf()} is used directly, otherwise \texttt{formatMpfr()} is used and post processed. For the binary representation, the hexadecimal value is calculated and then edited by substitution of the binary representation of the hex characters coded in the \texttt{HextoBin} vector. For binary with \texttt{scientific=FALSE}, the result of the \texttt{scientific=TRUE} version is edited to align binary points. For the decimal representation, the hexadecimal value is calculated with the specified precision and then sent to the \texttt{format} function for \texttt{scientific=FALSE} or to the \texttt{sprintf} function for \texttt{scientific=TRUE}.

Value

a character vector (or matrix) like \( x \), say \( r \), containing the formatted representation of \( x \), with a \texttt{class}
(unless left.pad or right.pad were not "_" ). In that case, \texttt{formatHex()} and \texttt{formatBin()} return
class "Ncharacter"; for that, \texttt{mpfr(.)} has a method and will basically return \( x \), i.e., work as \texttt{inverse}
function.

Since \texttt{Rmpfr} version 0.6-2, the S3 class "Ncharacter" extends "character". \texttt{(class(.)} is
now of length two and \texttt{class(.[2]}} is "character"). There are simple \texttt{[} and \texttt{print} methods;
modifying or setting \texttt{dim} works as well.

Author(s)

Richard M. Heiberger <rmh@temple.edu>, with minor tweaking by Martin M.

References

R FAQ 7.31: Why doesn’t R think these numbers are equal? system.file("./../doc/FAQ")

See Also

\texttt{mpfr}, \texttt{sprintf}
Examples

```r
FourBits <- mpfr(matrix(0:31, 8, 4, dimnames = list(0:7, c(0,8,16,24))[,]), precBits=4) ## 4 significant bits

formatHex(FourBits)
formatBin(FourBits, style = " ")
formatBin(FourBits, scientific=FALSE)
formatDec(FourBits)

## as "Ncharacter" 'inherits from' "character", this now works too :
data.frame(Dec = c( formatDec(FourBits) ), formatHex(FourBits),
Bin = formatBin(FourBits, style = " "))

FBB <- formatBin(FourBits) ; clB <- class(FBB)
(nFBB <- mpfr(FBB))
stopifnot(class(FBB)[1] == "Ncharacter",
all.equal(nFBB, FourBits, tol=0))

FBH <- formatHex(FourBits) ; clH <- class(FBH)
(nFBH <- mpfr(FBH))
stopifnot(class(FBH)[1] == "Ncharacter",
all.equal(nFBH, FourBits, tol=0))

## Compare the different "formattings" (% details will change, i.e. improve!)
M <- mpfr(c(-Inf, -1.25, 1/(-Inf), NA, .5, 1:2, Inf), 3)
data.frame(fH = formatHex(M), f16 = format(M, base=16),
fB = formatBin(M), f2 = format(M, base= 2),
fD = formatDec(M), f10 = format(M), # base = 10 is default
fSci= format(M, scientific=TRUE),
fFix= format(M, scientific=FALSE))

## Other methods ("[", t()) also work :
stopifnot(dim(F1 <- FBB[,1, drop=FALSE]) == c(8,1), identical(class( F1), clB),
dim(t(F1)) == c(1,8), identical(class(t(F1)), clB),
is.null(dim(F.2 <- FBB[,2]),) == FALSE, identical(class( F.2), clB),
dim(F22 <- FBH[1:2, 3:4]) == c(2,2), identical(class(F22), clH),
identical(class(FBH[2,3]), clH), is.null(dim(FBH[2,3])),
identical(FBH[2,3:4], F22[2,:]),
identical(FBH[2,3], unname(FBH[,3][2])),
TRUE)

TenFrac <- matrix((1:10)/10, dimnames=list(1:10, expression(1/x)))
TenFrac9 <- mpfr(TenFrac, precBits=9) ## 9 significant bits
TenFrac9
formatHex(TenFrac9)
formatBin(TenFrac9)
formatBin(TenFrac9, scientific=FALSE)
formatDec(TenFrac9)
formatDec(TenFrac9, precBits=10)
```
formatMpfr

Formatting MPFR (multiprecision) Numbers

Description

Flexible formatting of “multiprecision numbers”, i.e., objects of class mpfr. formatMpfr() is also the mpfr method of the generic format function.

The formatN() methods for mpfr numbers renders them differently than their double precision equivalents, by appending ".M".

Function mpfr2str() is the low level work horse for formatMpfr() and hence all print()ing of “mpfr” objects.

Usage

formatMpfr(x, digits = NULL, trim = FALSE, scientific = NA, maybe.full = (!is.null(digits) && is.na(scientific)) || isFALSE(scientific),
   base = 10, showNeg0 = TRUE, max.digits = Inf,
   big.mark = "", big.interval = 3L,
   small.mark = ",", small.interval = 5L,
   decimal.mark = ".",
   exponent.char = if(base <= 14) "e" else if(base <= 36) "E" else "|e",
   exponent.plus = TRUE,
   zero.print = NULL, drop0trailing = FALSE,
   ...)

## S3 method for class 'mpfr'
formatN(x, drop0trailing = TRUE, ...)

.mpfr2str(x, digits = NULL, maybe.full = !is.null(digits), base = 10L)

Arguments

x
an MPFR number (vector or array).
digits
how many significant digits (in the base chosen!) are to be used in the result. The default, NULL, uses enough digits to represent the full precision, often one or two digits more than “you” would expect. For bases 2, 4, 8, 16, or 32, MPFR requires digits at least 2. For such bases, digits = 1 is changed into 2, with a message.
trim
logical: if FALSE, numbers are right-justified to a common width: if TRUE the leading blanks for justification are suppressed.
scientific
either a logical specifying whether MPFR numbers should be encoded in scientific format (“exponential representation”), or an integer penalty (see options("scipen"). Missing values correspond to the current default penalty.
maybe.full
logical, passed to .mpfr2str().
base an integer in 2, 3, ..., 62; the base ("basis") in which the numbers should be represented. Apart from the default base 10, binary (base = 2) or hexadecimal (base = 16) are particularly interesting.

showNeg0 logical indicating if "negative" zeros should be shown with a "-". The default, TRUE is intentionally different from format(<numeric>).

exponent.char the "exponent" character to be used in scientific notation. The default takes into account that for base $B \geq 15$, "e" is part of the (mantissa) digits and the same is true for "E" when $B \geq 37$.

exponent.plus logical indicating if "+" should be for positive exponents in exponential (aka "scientific") representation. This used to be hardcoded to FALSE; the new default is compatible to R's format()ing of numbers and helps to note visually when exponents are in use.

max.digits a (large) positive number to limit the number of (mantissa) digits, notably when digits is NULL (as by default). Otherwise, a numeric digits is preferred to setting max.digits (which should not be smaller than digits).

big.mark, big.interval, small.mark, small.interval, decimal.mark, zero.print, drop0trailing used for prettying decimal sequences, these are passed to prettyNum and that help page explains the details.

Value a character vector or array, say cx, of the same length as x. Since Rmpfr version 0.5-3 (2013-09), if x is an mpfrArray, then cx is a character array with the same dim and dimnames as x.

Note that in scientific notation, the integer exponent is always in decimal, i.e., base 10 (even when base is not 10), but of course meaning base powers, e.g., in base 32, "u.giE3" is the same as "ugi0" which is $32^3$ times "u.gi". This is in contrast, e.g., with sprintf("%a", x) where the powers after "p" are powers of 2.

Note Currently, formatMpfr(x, scientific = FALSE) does not work correctly, e.g., for x <- Const("pi", 128) * 2^c(-200, 200), i.e., it uses the scientific/exponential-style format. This is considered bogus and hopefully will change.

Author(s) Martin Maechler

References
The MPFR manual’s description of ‘mpfr_get_str()’ which is the C-internal workhorse for mpfr2str() (on which formatMpfr() builds).
See Also

`mpfr` for creation and the `mpfr` class description with its many methods. The `format` generic, and the `prettyNum` utility on which `formatMpfr` is based as well. The S3 generic function `formatN` from package `gmp`.

`mpfr_formatinfo(x)` provides the (cheap) non-string parts of `mpfr2str(x)`; the (base 2) exp exponents are also available via `mpfr2exp(x)`.

Examples

```r
## Printing of MPFR numbers uses formatMpfr() internally.
## Note how each components uses the "necessary" number of digits:
( x3 <- c(Const("pi", 168), mpfr(pi, 140), 3.14) )
format(x3[3], 15)
format(x3[3], 15, drop0 = TRUE)# "3.14" .. dropping the trailing zeros
x3[4] <- 2^30
x3[4] # automatically drops trailing zeros
format(x3[1], dig = 41, small.mark = "'") # (41 - 1 = ) 40 digits after "."

rbind(formatN( x3, digits = 15),
      formatN(as.numeric(x3), digits = 15))

(Zero <- mpfr(c(0,1/-Inf), 20)) # 0 and "-0"
xz <- c(Zero, 1:2, Const("pi", 120), -100*pi, -.00987)
format(xz, digits = 2)
format(xz, digits = 1, showNeg0 = FALSE)# "-0" no longer shown

## Output in other bases :
formatMpfr(mpfr(10^6, 40), base=32, drop0trailing=TRUE)
## "ugio"
mpfr("ugio", base=32) #-> `0000'000

## This now works: The large number shows "as" large integer:
x <- Const("pi", 128) * 2^c(-200,200)
formatMpfr(x, scientific = FALSE) # was 1.955...e-60 5.048...e+60

i32 <- mpfr(1:32, precBits = 64)
format(i32, base= 2, drop0trailing=TRUE)
format(i32, base= 16, drop0trailing=TRUE)
format(i1/32, base= 2, drop0trailing=TRUE)# using scientific notation for [17..32]
format(i1/32, base= 32)
format(i1/32, base= 62, drop0trailing=TRUE)
format(mpfr(2, 64)^-(1:16), base=16, drop0trailing=TRUE)
```

---

`frexpMpfr`  
`Base-2 Representation and Multiplication of Mpfr Numbers`
Description

MPFR - versions of the C99 (and POSIX) standard C (and C++) mathlib functions \texttt{frexp()} and \texttt{ldexp()}. \texttt{frexpMpfr(x)} computes base-2 exponent \(e\) and “mantissa”, or fraction \(r\), such that \(x = r \times 2^e\), where \(r \in [0.5, 1)\) (unless when \(x\) is in \(c(0, -\text{Inf}, \text{Inf}, \text{NaN})\) where \(r == x\) and \(e\) is 0), and \(e\) is integer valued. \texttt{ldexpMpfr(f, E)} is the inverse of \texttt{frexpMpfr()}: Given fraction or mantissa \(f\) and integer exponent \(E\), it returns \(x = f \times 2^E\). Viewed differently, it’s the fastest way to multiply or divide MPFR numbers with \(2^E\).

Usage

\begin{verbatim}
frexpMpfr(x, rnd.mode = c("N", "D", "U", "Z", "A"))
ldexpMpfr(f, E, rnd.mode = c("N", "D", "U", "Z", "A"))
\end{verbatim}

Arguments

- \(x\) numeric (coerced to \texttt{double}) vector.
- \(f\) numeric fraction (vector), in \([0.5, 1)\).
- \(E\) integer valued, exponent of 2, i.e., typically in \((-1024-50):1024\), otherwise the result will underflow to 0 or overflow to +/- Inf.
- \texttt{rnd.mode} a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see \texttt{mpfr}.

Value

\texttt{frexpMpfr} returns a list with named components \(r\) (of class mpfr) and \(e\) (integer valued, of type integer is small enough, “double” otherwise).

Author(s)

Martin Maechler

References

On unix-alikes, typically man \texttt{frexp} and man \texttt{ldexp}

See Also

Somewhat related, \texttt{.mpfr2exp()}, \texttt{frexp()} and \texttt{ldexp()} in package \texttt{DPQ}.

Examples

\begin{verbatim}
set.seed(47)
x <- c(0, 2^(-3:3), (-1:1)/0,
     sort(rlnorm(2^12, 10, 20) * sample(c(-1,1), 512, replace=TRUE)))
head(xM <- mpfr(x, 128), 11)
str(rFM <- frexpMpfr(xM))
\end{verbatim}
d.fr <- with(rFM, data.frame(x=x, r=asNumeric(r), e=e))
head(d.fr, 16)
tail(d.fr)
ar <- abs(rFM$r)
stopifnot(0.5 <= ar[is.finite(x) & x != 0],
ar[is.finite(x)] < 1,
is.integer(rFM$e))
ldx <- with(rFM, ldexpMpfr(r, e))
(iN <- which(is.na(x))) # 10
stopifnot(exprs = {
    all.equal(xM, ldx, tol = 2^-124) # allow 4 bits loss, but apart from the NA, even:
    identical(xM[-iN], ldx[-iN])
    is.na(xM [iN])
    is.na(ldx[iN])
})

---

**gmp-conversions**

Conversion Utilities gmp <-> Rmpfr

**Description**

Coerce from and to big integers (`bigz`) and `mpfr` numbers.
Further, coerce from big rationals (`bigq`) to `mpfr` numbers.

**Usage**

```r
.bigz2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.bigq2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.mpfr2bigz(x, mod = NA)
```

**Arguments**

- `x` an R object of class `bigz`, `bigq` or `mpfr` respectively.
- `precB` precision in bits for the result. The default, `NULL`, means to use the `minimal` precision necessary for correct representation.
- `rnd.mode` a 1-letter string specifying how `rounding` should happen at C-level conversion to MPFR, see details of `mpfr`.
- `mod` a possible modulus, see `as.bigz` in package `gmp`.

**Details**

Note that we also provide the natural (S4) coercions, `as(x, "mpfr")` for `x` inheriting from class "bigz" or "bigq".

**Value**

a numeric vector of the same length as `x`, of the desired class.
See Also

\texttt{mpfr()}, \texttt{as.bigz} and \texttt{as.bigq} in package \texttt{gmp}.

Examples

```r
S <- gmp::Stirling2(50,10)
show(S)
SS <- S * as.bigz(1:3)^128
stopifnot(all.equal(log2(SS[2]) - log2(S), 128, tolerance=1e-15),
          identical(SS, .mpfr2bigz(.bigz2mpfr(SS))))

.bigz2mpfr(S) # 148 bit precision
.bigz2mpfr(S, precB=256) # 256 bit

## rational --> mpfr:
sq <- SS / as.bigz(2)^100
MP <- as(sq, "mpfr")
stopifnot(identical(MP, .bigq2mpfr(sq)),
          SS == MP * as(2, "mpfr")^100)
```

hjkMpfr

\textit{Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)}

Description

An implementation of the Hooke-Jeeves algorithm for derivative-free optimization.

This is a slight adaption \texttt{hjk()} from package \texttt{dfoptim}.

Usage

```r
hjkMpfr(par, fn, control = list(), ...)
```

Arguments

- \texttt{par}  
  Starting vector of parameter values. The initial vector may lie on the boundary.  
  If \texttt{lower[i]=upper[i]} for some \texttt{i}, the \texttt{i}-th component of the solution vector will simply be kept fixed.

- \texttt{fn}  
  Nonlinear objective function that is to be optimized. A scalar function that takes a real vector as argument and returns a scalar that is the value of the function at that point.

- \texttt{control}  
  \texttt{list} of control parameters. See \texttt{Details} for more information.

- \texttt{...}  
  Additional arguments passed to \texttt{fn}. 

Details

Argument control is a list specifying changes to default values of algorithm control parameters. Note that parameter names may be abbreviated as long as they are unique.

The list items are as follows:

- **tol**: Convergence tolerance. Iteration is terminated when the step length of the main loop becomes smaller than tol. This does not imply that the optimum is found with the same accuracy. Default is 1.e-06.

- **maxfeval**: Maximum number of objective function evaluations allowed. Default is Inf, that is no restriction at all.

- **maximize**: A logical indicating whether the objective function is to be maximized (TRUE) or minimized (FALSE). Default is FALSE.

- **target**: A real number restricting the absolute function value. The procedure stops if this value is exceeded. Default is Inf, that is no restriction.

- **info**: A logical variable indicating whether the step number, number of function calls, best function value, and the first component of the solution vector will be printed to the console. Default is FALSE.

If the minimization process threatens to go into an infinite loop, set either maxfeval or target.

Value

A list with the following components:

- **par**: Best estimate of the parameter vector found by the algorithm.
- **value**: Value of the objective function at termination.
- **convergence**: Indicates convergence (TRUE) or not (FALSE).
- **feval**: Number of times the objective fn was evaluated.
- **niter**: Number of iterations (“steps”) in the main loop.

Note

This algorithm is based on the Matlab code of Prof. C. T. Kelley, given in his book “Iterative methods for optimization”. It has been implemented for package dfoptim with the permission of Prof. Kelley.

This version does not (yet) implement a cache for storing function values that have already been computed as searching the cache makes it slower.

Author(s)

Hans W Borchers <hwborchers@googlemail.com>; for Rmpfr: John Nash, June 2012. Modifications by Martin Maechler.

References

C.T. Kelley (1999), Iterative Methods for Optimization, SIAM.
See Also

Standard R’s `optim`, `optimizeR` provides one-dimensional minimization methods that work with `mpfr`-class numbers.

Examples

```r
## simple smooth example:
ff <- function(x) sum((x - c(2:4))^2)
str(rr <- hjkMpfr(rep(mpfr(0,128), 3), ff, control=list(info=TRUE)))

doX <- Rmpfr:::doExtras(); cat("doExtras: ", doX, "\n") # slow parts only if(doX)

## Hooke-Jeeves solves high-dim. Rosenbrock function (but slowly!)
rosenbrock <- function(x) {
  n <- length(x)
  sum (100*((x1 <- x[1:(n-1)])^2 - x[2:n])^2 + (x1 - 1)^2)
}
par0 <- rep(0, 10)
str(rb.db <- hjkMpfr(rep(0, 10), rosenbrock, control=list(info=TRUE)))
if(doX) {
  ## rosenbrook() is quite slow with mpfr-numbers:
  str(rb.M. <- hjkMpfr(mpfr(numeric(10), prec=128), rosenbrock,
                        control = list(tol = 1e-8, info=TRUE)))
}

## Hooke-Jeeves does not work well on non-smooth functions
nsf <- function(x) {
  max(f1, f2, f3)
}
par0 <- c(1, 1) # true min 7.2 at (1.2, 2.4)
h.d <- hjkMpfr(par0, nsf) # fmin=8 at xmin=(2,2)
if(doX) {
  ## and this is not at all better (but slower!)
  h.M <- hjkMpfr(mpfr(c(1,1), 128), nsf, control = list(tol = 1e-15))
}

## --> demo(hjkMpfr) # -> Fletcher's chebyquad function m = n -- residuals
```

---

**igamma**

**Incomplete Gamma Function**
**Description**
For MPFR version >= 3.2.0, the following MPFR library function is provided: mpfr_gamma_inc(a,x), the R interface of which is igamma(a,x), where igamma(a,x) is the “upper” incomplete gamma function

\[ \Gamma(a, x) := \Gamma(a) - \gamma(a, x), \]

where

\[ \gamma(a, x) := \int_{0}^{x} t^{a-1} e^{-t} dt, \]

and hence

\[ \Gamma(a, x) := \int_{x}^{\infty} t^{a-1} e^{-t} dt, \]

and

\[ \Gamma(a) := \gamma(a, \infty). \]

As R’s `pgamma(x,a)` is

\[ pgamma(x, a) := \gamma(a, x)/\Gamma(a), \]

we get

\[ igamma(a,x) == \gamma(a) * pgamma(x, a, lower.tail=FALSE) \]

**Usage**

\[ igamma(a, x, rnd.mode = c("N", "D", "U", "Z", "A")) \]

**Arguments**

- `a, x` an object of class mpfr or numeric.
- `rnd.mode` a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

**Value**

a numeric vector of “common length”, recycling along `a` and `x`.

**Author(s)**

R interface: Martin Maechler

**References**


**See Also**

R’s `gamma` (function) and `pgamma` (probability distribution).
Examples

```r
## show how close pgamma() is:
x <- c(seq(0, 20, by=1/4), 21:50, seq(55, 100, by=5))
if(mpfrVersion() >= "3.2.0") {
  print(all.equal(igamma(Const("pi", 80), x),
                pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
                tol=0, formatFUN = function(. , ...) format(. , digits = 7)) #-> 2.75e-16 (was 3.13e-16)
}
## and ensure *some* closeness:
stopifnot(exprs = {
  all.equal(igamma(Const("pi", 80), x),
            pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
            tol = 1e-15)
})
}
# only if MPFR version >= 3.2.0
```

integrateR

---

**One-Dimensional Numerical Integration - in pure R**

Description

Numerical integration of one-dimensional functions in pure R, with care so it also works for "mpfr"-numbers.

Currently, only classical Romberg integration of order \( ord \) is available.

Usage

```r
integrateR(f, lower, upper, ..., ord = NULL, rel.tol = .Machine$double.eps^0.25, abs.tol = rel.tol, max.ord = 19, verbose = FALSE)
```

Arguments

- **\( f \)**: an R function taking a numeric or "mpfr" first argument and returning a numeric (or "mpfr") vector of the same length. Returning a non-finite element will generate an error.
- **lower, upper**: the limits of integration. Currently must be finite. Do use "mpfr"-numbers to get higher than double precision, see the examples.
- **\( ... \)**: additional arguments to be passed to \( f \).
- **\( ord \)**: integer, the order of Romberg integration to be used. If this is NULL, as per default, and either rel.tol or abs.tol are specified, the order is increased until convergence.
- **rel.tol**: relative accuracy requested. The default is 1.2e-4, about 4 digits only, see the Note.
- **abs.tol**: absolute accuracy requested.
max.ord

only used, when neither ord or one of rel.tol, abs.tol are specified: Stop Romberg iterations after the order reaches max.ord; may prevent infinite loops or memory explosion.

verbose

logical or integer, indicating if and how much information should be printed during computation.

Details

Note that arguments after ... must be matched exactly.

For convergence, both relative and absolute changes must be smaller than rel.tol and abs.tol, respectively.

rel.tol cannot be less than max(50*.Machine$double.eps, 0.5e-28) if abs.tol <= 0.

Value

A list of class "integrateR" (as from standard R’s integrate()) with a print method and components

value

the final estimate of the integral.

abs.error

estimate of the modulus of the absolute error.

subdivisions

for Romberg, the number of function evaluations.

message

"OK" or a character string giving the error message.

call

the matched call.

Note

f must accept a vector of inputs and produce a vector of function evaluations at those points. The Vectorize function may be helpful to convert f to this form.

If you want to use higher accuracy, you must set lower or upper to "mpfr" numbers (and typically lower the relative tolerance, rel.tol), see also the examples.

Note that the default tolerances (rel.tol, abs.tol) are not very accurate, but the same as for integrate, which however often returns considerably more accurate results than requested. This is typically not the case for integrateR().

Note

We use practically the same print S3 method as print.integrate, provided by R, with a difference when the message component is not "Ok".

Author(s)

Martin Maechler

References

See Also

R’s standard, `integrate`, is much more adaptive, also allowing infinite integration boundaries, and typically considerably faster for a given accuracy.

Examples

```r
## See more from ?integrate
## this is in the region where integrate() can get problems:
integrateR(dnorm,0,2000)
integrateR(dnorm,0,2000, rel.tol=1e-15)
(Id <- integrateR(dnorm,0,2000, rel.tol=1e-15, verbose=TRUE))
Id$value == 0.5 # exactly

## Demonstrating that 'subdivisions' is correct:
Exp <- function(x) { .N <<- .N+ length(x); exp(x) }
.N <- 0; str(integrateR(Exp, 0,1, rel.tol=1e-10), digits=15); .N

### Using high-precision functions -----

## Polynomials are very nice:
integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, 5, verbose=TRUE)
# n= 1, 2^n= 2 | I = 46.04, abs.err = 98.9583
# n= 2, 2^n= 4 | I = 20, abs.err = 26.0417
# n= 3, 2^n= 8 | I = 20, abs.err = 7.10543e-15
## 20 with absolute error < 7.1e-15
## Now, using higher accuracy:
I <- integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, mpfr(5,128),
               rel.tol = 1e-20, verbose=TRUE)
I ; I$value ## all fine

## with floats:
integrateR(exp, 0 , 1, rel.tol=1e-15, verbose=TRUE)
## with "mpfr":
(I <- integrateR(exp, mpfr(0,200), 1, rel.tol=1e-25, verbose=TRUE))
(I.true <- exp(mpfr(1, 200)) - 1)
## true absolute error:
stopifnot(print(as.numeric(I.true - I$value)) < 4e-25)

## Want absolute tolerance check only (=> set 'rel.tol' very high, e.g. 1):
(Ia <- integrateR(exp, mpfr(0,200), 1, abs.tol = 1e-6, rel.tol=1, verbose=TRUE))

## Set 'ord' (but no '.tol') --> Using 'ord'; no convergence checking
(I <- integrateR(exp, mpfr(0,200), 1, ord = 13, verbose=TRUE))
```

---

**is.whole**  
*Whole ("Integer") Numbers*
log1mexp

Description
Check which elements of x[] are integer valued aka “whole” numbers, including MPFR numbers (class mpfr).

Usage
```r
## S3 method for class 'mpfr'
is.whole(x)
```

Arguments
- `x`: any R vector, here of class mpfr.

Value
logical vector of the same length as x, indicating where x[] is integer valued.

Author(s)
Martin Maechler

See Also
- `is.integer(x)` (base package) checks for the internal mode or class, not if x[i] are integer valued. The is.whole() methods in package gmp.

Examples
```r
is.integer(3) # FALSE, it's internally a double
is.whole(3)  # TRUE
x <- c(as(2,"mpfr") ^ 100, 3, 3.2, 1000000, 2^40)
is.whole(x) # one FALSE, only
```

log1mexp
Compute f(a) = \log(1 +/− \exp(-a)) Numerically Optimally

Description
Compute f(a) = \log(1 - \exp(-a)), respectively g(x) = \log(1 + \exp(x)) quickly numerically accurately.

Usage
```r
log1mexp(a, cutoff = log(2))
log1pexp(x, c0 = -37, c1 = 18, c2 = 33.3)
```
log1mexp

Arguments

- **a**: numeric (or mpfr) vector of positive values.
- **x**: numeric vector, may also be an “mpfr” object.
- **cutoff**: positive number; $\log(2)$ is “optimal”, but the exact value is unimportant, and anything in $[0.5, 1]$ is fine.
- **c0, c1, c2**: cutoffs for log1pexp; see below.

Value

$$
\text{log1mexp}(a) := f(a) = \log(1 - \exp(-a)) = \log(1 + \exp(-a)) = \log(-\expm1(-a))
$$
or, respectively,

$$
\text{log1pexp}(x) := g(x) = \log(1 + \exp(x)) = \log(1 + \exp(-|x|)) = \log(1 + \exp(-x))
$$

computed accurately and quickly.

Author(s)

Martin Maechler, May 2002; log1pexp() in 2012

References


Examples

```r
fExpr <- expression(
  DEF  = log(1 - exp(-a)),
  expm1 = log(-expm1(-a)),
  log1p = log1p(-exp(-a)),
  F    = log1mexp(a))
a. <- 2^seq(-58, 10, length = 256)
a <- a. ; str(fa <- do.call(cbind, as.list(fExpr)))
head(fa)# expm1() works here
tail(fa)# log1p() works here

## graphically:
lwd <- 1.5*(5:2); col <- adjustcolor(1:4, 0.4)
op <- par(mfcol=c(1,2), mgp = c(1.25, .6, 0), mar = .1+c(3,2,1,1))
matplot(a, fa, type = "l", log = "x", col=col, lwd=lwd)
legend("topleft", fExpr, col=col, lwd=lwd, lty=1:4, bty="n")
  # expm1() & log1mexp() work here

matplot(a, -fa, type = "l", log = "xy", col=col, lwd=lwd)
legend("left", paste("-", fExpr), col=col, lwd=lwd, lty=1:4, bty="n")
# log1p() & log1mexp() work here
```
par(op)

aM <- 2^seqMpfr(-58, 10, length=length(a.)) # => default prec = 128
a <- aM; dim(faM <- do.call(cbind, as.list(fExpr))) # 256 x 4, "same" as 'fa'
# Here, for small 'a' log1p() and even 'DEF' is still good enough
l_f <- asNumeric(log(-faM))
all.equal(l_f["F"], l_f["log1p"], tol=0) # see TRUE (Lnx 64-bit)
io <- a. < 80 # for these, the differences are small
all.equal(l_f[io,"F"], l_f[io,"expm1"], tol=0) # see 6.662e-9
all.equal(l_f[io,"F"], l_f[io,"DEF"], tol=0)
stopifnot(exprs = {
  all.equal(l_f["F"], l_f["log1p"], tol= 1e-15)
  all.equal(l_f[io,"F"], l_f[io,"expm1"], tol= 1e-7)
  all.equal(l_f[io,"F"], l_f[io,"DEF"], tol= 1e-7)
})
# For 128-bit prec, if we go down to 2^-130, "log1p" is no longer ok:
aM2 <- 2^seqMpfr(-130, 10, by = 1/2)
a <- aM2; fa2 <- do.call(cbind, as.list(fExpr))
head(asNumeric(fa2), 12)
tail(asNumeric(fa2), 12)
matplot(a, log(-fa2[,1:3]) -log(-fa2,"F")), type="l", log="x",
lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))
legend("top", colnames(fa2)[1:3], lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))
cols <- adjustcolor(2:4, 1/3); lwd <- 2*(3:1)-1
matplot(a, 1e-40+abs(log(-fa2[,1:3]) -log(-fa2,"F")), type="o", log="xy",
main = "log1mexp(a) -- approximation rel.errors, mpfr(*, prec=128)",
pch=21:23, cex=.6, bg=5:7, lty=1:2, lwd=lwd, col=cols)
legend("top", colnames(fa2)[1:3], bty="n", lty=1:2, lwd=lwd, col=cols,
pch=21:23, pt.cex=.6, pt.bg=5:7)

## -------------------------- log1pexp() [simpler] --------------------
curve(log1pexp, -10, 10, asp=1)
abline(0,1, h=0, v=0, lty=3, col="gray")

## Cutoff c1 for log1pexp() -- not often "needed":
curve(log1p(exp(x)) - log1pexp(x), 16, 20, n=2049)
## need for *some* cutoff:
x <- seq(700, 720, by=2)
cbind(x, log1p(exp(x)), log1pexp(x))

## Cutoff c2 for log1pexp():
curve((x+exp(-x)) - x, 20, 40, n=1025)
curve((x+exp(-x)) - x, 33.1, 33.5, n=1025)
Description

Matrix / vector multiplication of mpfr (and “simple” numeric) matrices and vectors.

matmult (x, y, fPrec = 2) or crossprod(x, y, fPrec = 2) use higher precision in underlying computations.

Usage

matmult(x, y, ...)

Arguments

- x, y: numeric or mpfrMatrix-classed R objects, i.e. semantically numeric matrices or vectors.
- ...: arguments passed to the hidden underlying .matmult.R() work horse which is also underlying the %*%, crossprod(), and tcrossprod() methods, see the mpfrMatrix class documentation:
  - fPrec: a multiplication factor, a positive number determining the number of bits precBits used for the underlying multiplication and summation arithmetic. The default is fPrec = 1. Setting fPrec = 2 doubles the precision which has been recommended, e.g., by John Nash.
  - precBits: the number of bits used for the underlying multiplication and summation arithmetic; by default precBits = fPrec * max(getPrec(x), getPrec(y)) which typically uses the same accuracy as regular mpfr-arithmetic would use.

Value

a (base R) matrix or mpfrMatrix, depending on the classes of x and y.

Note

Using matmult(x, y) instead of x %*% y, makes sense mainly if you use non-default fPrec or precBits arguments.

The crossprod(), and tcrossprod() function have the identical optional arguments fPrec or precBits.

Author(s)

Martin Maechler

See Also

%*%, crossprod, tcrossprod.
Class "Mnumber" and "mNumber" of "mpfr" and regular numbers and arrays from them

Methods

\%
signature(x = "mpfrMatrix", y = "Mnumber"):

\texttt{crossprod} signature(x = "mpfr", y = "Mnumber"):

\texttt{tcrossprod} signature(x = "Mnumber", y = "mpfr"): ...

e tcrossprod signature(x = "Mnumber", y = "mpfr"): ...

e etc. These are documented with the classes \texttt{mpfr} and or \texttt{mpfrMatrix}.

See Also

the \texttt{array_or_vector} sub class; \texttt{cbind-methods}.

Examples

## "Mnumber" encompasses (i.e., "extends") quite a few
## "vector / array - like" classes:
showClass("Mnumber")
stopifnot(extends("mpfrMatrix", "Mnumber"),
  extends("array", "Mnumber"))

Mnsub <- names(getClass("Mnumber")@subclasses)
(mNsub <- names(getClass("mNumber")@subclasses))

## mNumber has *one* subclass which is not in Mnumber:
setdiff(mNsub, Mnsub)# namely "numericVector"
## The following are only subclasses of "Mnumber", but not of "mNumber":
setdiff(Mnsub, mNsub)

---

**mpfr**

Create "mpfr" Numbers (Objects)

**Description**

Create multiple (i.e. typically high) precision numbers, to be used in arithmetic and mathematical computations with R.

**Usage**

```r
mpfr(x, precBits, ...)  
```  
## Default S3 method:  
```r
mpfr(x, precBits, base = 10,  
      rnd.mode = c("N","D","U","Z","A"), scientific = NA, ...)  
```  
```r
Const(name = c("pi", "gamma", "catalan", "log2"), prec = 120L,  
      rnd.mode = c("N","D","U","Z","A"))  
```  
```r
is.mpfr(x)  
```  

**Arguments**

- `x` : a numeric, mpfr, bigz, bigq, or character vector or array.
- `precBits, prec` : a number, the maximal precision to be used, in bits; i.e. 53 corresponds to double precision. Must be at least 2. If missing, `getPrec(x)` determines a default precision.
- `base` : (only when `x` is character) the base with respect to which `x[i]` represent numbers; base b must fulfill 2 ≤ b ≤ 62.
- `rnd.mode` : a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see details.
- `scientific` : (used only when `x` is the result of `formatBin()`, i.e., of class "Bcharacter") logical indicating that the binary representation of x is in scientific notation. When TRUE, `mpfr()` will substitute 0 for _; when NA, `mpfr()` will guess, and use TRUE when finding a "p" in x; see also `formatBin`.
- `name` : a string specifying the mpfrlib - internal constant computation. "gamma" is Euler’s gamma (γ), and "catalan" Catalan’s constant.
- `...` : potentially further arguments passed to and from methods.
Details

The "mpfr" method of mpfr() is a simple wrapper around roundMpfr().

MPFR supports the following rounding modes,

- **GMP_RNDN**: round to nearest (roundTiesToEven in IEEE 754-2008).
- **GMP_RNDZ**: round toward zero (roundTowardZero in IEEE 754-2008).
- **GMP_RNDU**: round toward plus infinity ("Up", roundTowardPositive in IEEE 754-2008).
- **GMP_RNDD**: round toward minus infinity ("Down", roundTowardNegative in IEEE 754-2008).
- **GMP_RNDA**: round away from zero (new since MPFR 3.0.0).

The 'round to nearest' ("N") mode, the default here, works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the "drift" phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

When x is character, mpfr() will detect the precision of the input object.

Value

an object of (S4) class mpfr, or for mpfr(x) when x is an array, mpfrMatrix, or mpfrArray which the user should just as a normal numeric vector or array.

is.mpfr() returns TRUE or FALSE.

Author(s)

Martin Maechler

References

The MPFR team. (202x). GNU MPFR – The Multiple Precision Floating-Point Reliable Library; see https://www.mpfr.org/mpfr-current/#doc or directly https://www.mpfr.org/mpfr-current/mpfr.pdf.

See Also

The class documentation mpfr contains more details. Use asNumeric to transform back to double precision ("numeric").

Examples

mpfr(pi, 120) ## the double-precision pi "translated" to 120-bit precision
pi. <- Const("pi", prec = 260) # pi "computed" to correct 260-bit precision
pi. # nicely prints 80 digits [260 * log10(2) ≈ 78.3 ~ 80]
Const("gamma", 128L) # 0.5772...
Const("catalan", 128L) # 0.9159...
x <- mpfr(0:7, 100)/7 # a more precise version of k/7, k=0,...,7
x
1 / x

## character input :
mpfr("2.718281828459045235360287471352662497757") - exp(mpfr(1, 150))
## ~- 4 * 10^-40
## Also works for NA, NaN, ...

cx <- c("1234567890123456", 345, "NA", "NaN", "Inf", "-Inf")
mpfr(cx)

## with some 'base' choices :
print(mpfr("111.1111", base=2)) * 2^4
mpfr("af21.01020300a0b0c", base=16)
## 68 bit prec. 44833.00393694653820642

mpfr("ugi0", base = 32) == 10^6  ## TRUE

## --- Large integers from package 'gmp' :
Z <- as.bigz(7)^c(1:200)
head(Z, 40)

## mpfr(Z) by default chooses the correct *maximal* default precision:
mZ. <- mpfr(Z)
## more efficiently chooses precision individually
m.Z <- mpfr(Z, precBits = frexpZ(Z)$exp)
## the precBits chosen are large enough to keep full precision:
stopifnot(identical(cZ <- as.character(Z),
    as(mZ.,"character")),
    identical(cZ, as(m.Z,"character")))

## compare mpfr-arithmetic with exact rational one:
stopifnot(all.equal(mpfr(as.bigq(355,113), 99),
    mpfr(355, 99) / 113, tol = 2^-98))

## look at different "rounding modes":
sapply(c("N", "D", "U", "Z", "A"), function(RND)
    mpfr(c(-1,1)/5, 20, rnd.mode = RND), simplify=FALSE)

symnum(sapply(c("N", "D", "U", "Z", "A"),
    function(RND) mpfr(0.2, prec = 5:15, rnd.mode = RND) < 0.2 ))

---

mpfr-class

**Class “mpfr” of Multiple Precision Floating Point Numbers**

**Description**

"mpfr" is the class of Multiple Precision Floatingpoint numbers with Reliable arithmetic.
For the high-level user, "mpfr" objects should behave as standard R's numeric vectors. They would just print differently and use the prespecified (typically high) precision instead of the double precision of 'traditional' R numbers (with class(.) == "numeric" and typeof(.) == "double").

```r
hypot(x,y)
```
computes the hypothenuse length \( z \) in a rectangular triangle with "leg" side lengths \( x \) and \( y \), i.e.,

\[
z = hypot(x, y) = \sqrt{x^2 + y^2},
\]
in a numerically stable way.

**Usage**

```r
hypot(x,y, rnd.mode = c("N","D","U","Z","A"))
```

**Arguments**

- `x, y`: an object of class mpfr.
- `rnd.mode`: a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

**Objects from the Class**

Objects are typically created by `mpfr(<number>, precBits)`.

`summary(<mpfr>)` returns an object of class "summaryMpfr" which contains "mpfr" but has its own `print` method.

**Slots**

Internally, "mpfr" objects just contain standard R lists where each list element is of class "mpfr1", representing one MPFR number, in a structure with four slots, very much parallelizing the C struct in the mpfr C library to which the Rmpfr package interfaces.

An object of class "mpfr1" has slots

- `prec`: "integer" specifying the maximal precision in bits.
- `exp`: "integer" specifying the base-2 exponent of the number.
- `sign`: "integer", typically -1 or 1, specifying the sign (i.e. sign(.)) of the number.
- `d`: an "integer" vector (of 32-bit “limbs”) which corresponds to the full mantissa of the number.

**Methods**

- `abs` signature(x = "mpfr"): ...
- `atan2` signature(y = "mpfr", x = "ANY"); and
- `atan2` signature(x = "ANY", y = "mpfr"): compute the arc-tangent of two arguments: \( \arctan(y, x) \) returns the angle between the x-axis and the vector from the origin to \((x, y)\), i.e., for positive arguments \( \arctan(y, x) = \arctan(y/x) \).
- `lbeta` signature(a = "ANY", b = "mpfrArray"), is \( \log(|B(a, b)|) \) where \( B(a, b) \) is the Beta function, `beta(a,b)`.
- `beta` signature(a = "mpfr", b = "ANY"),
beta signature(a = "mpfr", b = "mpfr"). .... etc: Compute the beta function \( B(a, b) \), using high precision, building on internal \( \text{gamma} \) or \( \text{lgamma} \). See the help for R's base function \( \text{beta} \) for more. Currently, there, \( a, b \geq 0 \) is required. Here, we provide (non-\( \text{NaN} \)) for all numeric \( a, b \).

When either \( a, b, \) or \( a + b \) is a negative integer, \( \Gamma(\cdot) \) has a pole there and is undefined (\( \text{NaN} \)). However the Beta function can be defined there as "limit", in some cases. Following other software such as SAGE, Maple or Mathematica, we provide finite values in these cases. However, note that these are not proper limits (two-dimensional in \((a, b)\)), but useful for some applications. E.g., \( B(a, b) \) is defined as zero when \( a + b \) is a negative integer, but neither \( a \) nor \( b \) is. Further, if \( a > b > 0 \) are integers, \( B(-a, b) = B(b, -a) \) can be seen as \((-1)^a B(a-b+1, b)\).

dim<- signature(x = "mpfr"): Setting a dimension \( \text{dim} \) on an \( \text{mpfr} \) object makes it into an object of class \"mpfrArray\" or (more specifically) \"mpfrMatrix\" for a length-2 dimension, see their help page; note that \( t(x) \) (below) is a special case of this.

**Ops** signature(e1 = "mpfr", e2 = "ANY"): ...

**Ops** signature(e1 = "ANY", e2 = "mpfr"): ...

**Arith** signature(e1 = "mpfr", e2 = "missing"): ...

**Arith** signature(e1 = "mpfr", e2 = "mpfr"): ...

**Arith** signature(e1 = "mpfr", e2 = "integer"): ...

**Arith** signature(e1 = "mpfr", e2 = "numeric"): ...

**Arith** signature(e1 = "integer", e2 = "mpfr"): ...

**Arith** signature(e1 = "numeric", e2 = "mpfr"): ...

**Compare** signature(e1 = "mpfr", e2 = "mpfr"): ...

**Compare** signature(e1 = "mpfr", e2 = "integer"): ...

**Compare** signature(e1 = "mpfr", e2 = "numeric"): ...

**Compare** signature(e1 = "integer", e2 = "mpfr"): ...

**Compare** signature(e1 = "numeric", e2 = "mpfr"): ...

**Logic** signature(e1 = "mpfr", e2 = "mpfr"): ...

**Summary** signature(x = "mpfr"): The S4 **Summary** group functions, max, min, range, prod, sum, any, and all are all defined for MPFR numbers. \text{mean}(x, \text{trim}) \text{for non-0 \text{trim} works analogously to mean.default.}

**median** signature(x = "mpfr"): works via

**quantile** signature(x = "mpfr"): a simple wrapper of the quantile.default method from \text{stats}.

**summary** signature(object = "mpfr"): modeled after \text{summary.default}, ensuring to provide the full "mpfr" range of numbers.

**Math** signature(x = "mpfr"): All the S4 **Math** group functions are defined, using multiple precision (MPFR) arithmetic, from \text{getGroupMembers("Math")}, these are (in alphabetical order): abs, sign, sqrt, ceiling, floor, trunc, cummax, cummin, cumprod, cumsum, exp, expm1, log, log10, log2, log1p, cos, cosh, sin, sinh, tan, tanh, acos, acosh, asin, asinh, atan, atanh, cospi, sinpi, tanpi, gamma, lgamma, digamma, and trigamma.

Currently, trigamma is not provided by the MPFR library and hence not yet implemented. Further, the cum*() methods are not yet implemented.
factorial signature(x = "mpfr"): this will round the result when x is integer valued. Note however that factorialMpfr(n) for integer n is slightly more efficient, using the MPFR function ‘mpfr_fac_ui’.

Math2 signature(x = "mpfr"): round(x,digits) and signif(x, digits) methods. Note that these do not change the formal precision (‘prec’ slot), and you may often want to apply roundMpfr() in addition or preference.

as.numeric signature(x = "mpfr"): ...

as.vector signature(x = "mpfrArray"): as for standard arrays, this “drops” the dim (and dimnames), i.e., transforms x into an ‘MPFR’ number vector, i.e., class mpfr.

[[ signature(x = "mpfr", i = "ANY"), and
[ signature(x = "mpfr", i = "ANY", j = "missing", drop = "missing"): subsetting aka “indexing” happens as for numeric vectors.

format signature(x = "mpfr"), further arguments digits = NULL, scientific = NA, etc: returns character vector of same length as x; when digits is NULL, with enough digits to recreate x accurately. For details, see formatMpfr.

is.finite signature(x = "mpfr"): ...

is.infinite signature(x = "mpfr"): ...

is.na signature(x = "mpfr"): ...

log signature(x = "mpfr"): ...

show signature(object = "mpfr"): ...

sign signature(x = "mpfr"): ...

Re, Im signature(z = "mpfr"): simply return z or 0 (as "mpfr" numbers of correct precision), as mpfr numbers are ‘real’ numbers.

Arg, Mod, Conj signature(z = "mpfr"): these are trivial for our ‘real’ mpfr numbers, but defined to work correctly when used in R code that also allows complex number input.

all.equal signature(target = "mpfr", current = "mpfr"),
all.equal signature(target = "mpfr", current = "ANY"), and
all.equal signature(target = "ANY", current = "mpfr"): methods for numerical (approximate) equality. all.equal of multiple precision numbers. Note that the default tolerance (argument) is taken to correspond to the (smaller of the two) precisions when both main arguments are of class "mpfr", and hence can be considerably less than double precision machine epsilon .Machine$double.eps.

coerce signature(from = "numeric", to = "mpfr"): as(. , "mpfr") coercion methods are available for character strings, numeric, integer, logical, and even raw. Note however, that mpfr(. , precBits, base) is more flexible.

coerce signature(from = "mpfr", to = "bigz"): coerces to biginteger, see bigz in package gmp.

coerce signature(from = "mpfr", to = "numeric"): ...

coerce signature(from = "mpfr", to = "character"): ...

unique signature(x = "mpfr"), and corresponding S3 method (such that unique(<mpfr>) works inside base functions), see unique. Note that duplicated() works for "mpfr" objects without the need for a specific method.
t signature(x = "mpfr"): makes x into an \( n \times 1 \) mpfrMatrix.

which.min signature(x = "mpfr"): gives the index of the first minimum, see which.min.

which.max signature(x = "mpfr"): gives the index of the first maximum, see which.max.

Note

Many more methods ("functions") automagically work for "mpfr" number vectors (and matrices, see the mpfrMatrix class doc), notably sort, order, quantile, rank.

Author(s)

Martin Maechler

See Also

The "mpfrMatrix" class, which extends the "mpfr" one.

roundMpfr to change precision of an "mpfr" object which is typically desirable instead of or in addition to signif() or round(); is.whole() etc.

Special mathematical functions such as some Bessel ones, e.g., \( j_n \); further, \( \text{zeta}(\cdot) = \zeta(\cdot), \text{Ei}(\cdot) \) etc. Bernoulli numbers and the Pochhammer function pochMpfr.

Examples

```r
## 30 digit precision
(x <- mpfr(c(2:3, pi), prec = 30 * log2(10)))
str(x) # str() displays *compact*ly => not full precision
x^2
x[1] / x[2] # 0.66666... ~ 30 digits

## indexing - as with numeric vectors
stopifnot(exprs = {
  identical(x[2], x[[2]]), # indexing "outside" gives NA (well: "mpfr-NaNaN" for now):
  is.na(x[5]), # whereas "[[" cannot index outside:
  inherits(tryCatch(x[5], error=identity), "error")
  ## and only select *one* element:
  inherits(tryCatch(x[[2:3]], error=identity), "error")
})

## factorial() & lfactorial would work automagically via \[l\]gamma(),
## but factorial() additionally has an "mpfr" method which rounds
f200 <- factorial(mpfr(200, prec = 1500)) # need high prec.!
f200
as.numeric(log2(f200)) # 1245.38 -- need precBits \(\geq 1246\) for full precision

##-- see factorialMpfr() for more such computations.

#--- "Underflow" **much** later -- exponents have \(30(\pm1)\) bits themselves:
```
mpfr.min.exp2 <- -(2^30 + 1)
two <- mpfr(2, 55)
stopifnot(two ^ mpfr.min.exp2 == 0)
## whereas
two ^ (mpfr.min.exp2 * (1 - 1e-15))
## 2.38256490488795107e-323228497  ["typically"]
##--- "Assert" that (sort), (order), (quantile), (rank), all work :

p <- mpfr(rpois(32, lambda=500), precBits=128)^10
defn <- as.numeric(log(p))
(sp <- summary(p))# using the print.summaryMpfr() method
stopifnot(all(diff(sort(p)) >= 0), 
idcntical(order(p), order(defn)),
idcntical(rank (p), rank (defn)),
all.equal(sapply(1:9, function(Typ) quantile(defn, type=Typ, names=FALSE)),
sapply(lapply(1:9, function(Typ) quantile( p, type=Typ, names=FALSE)),
function(x) as.numeric(log(x))),
tol = 1e-3),# quantiles: interpolated in orig. <--> log scale
TRUE)

m0 <- mpfr(numeric(), 99)
xy <- expand.grid(x = -2:2, y = -2:2) ; x <- xy[,"x"] ; y <- xy[,"y"]
a2. <- atan2(y,x)
stopifnot(identical(which.min(m0), integer(0)),
idcntical(which.max(m0), integer(0)),
all.equal(a2., atan2(as(y,"mpfr"), x)),
max(m0) == mpfr(-Inf, 53), # (53 is not a feature, but ok)
min(m0) == mpfr(+Inf, 53),
sum(m0) == 0, prod(m0) == 1)
## unique(), now even base::factor() "works" on <mpfr>:
set.seed(17)
p <- rlnorm(20) * mpfr(10, 100)^-999
pp <- sample(p, 50, replace=TRUE)
str(unique(pp)) # length 18 .. (from originally 20)
## Class 'mpfr' [package "Rmpfr"] of length 18 and precision 100
## 5.56520587824e-999 4.41636588227e-1000 ..
facp <- factor(pp)
str(facp) # the factor *levels* are a bit verbose :
# Factor w/ 18 levels "new(1\"mpfr\")", ...........
# At least *some* factor methods work :
stopifnot(exprs = {
  is.factor(facp)
  identical(unname(table(facp)),
    unname(table(asNumeric(pp * mpfr(10,100)^1000))))
})
## ((unfortunately, the expressions are wrong; should integer "L"))
#
## More useful: levels with which to *invert* factor() :
## -- this is not quite ok:
## simplified from 'utils':

defparse1 <- function(x, ...) paste(deparse(x, 500L, ...), collapse = " ")
if(FALSE) {
  str(pp.levs <- vapply(unclass(sort(unique(pp))), deparse1, ""))
  facp2 <- factor(pp, levels = pp.levs)
}

### mpfr-distr-etc: Distribution Functions with MPFR Arithmetic

**Description**

For some R standard (probability) density, distribution or quantile functions, we provide MPFR versions.

**Usage**

- `dpois(x, lambda, log = FALSE, useLog = )`
- `dbinom(x, size, prob, log = FALSE, useLog = )`
- `dnbinom(x, size, prob, mu, log = FALSE, useLog = any(x > 1e6))`
- `dnorm(x, mean = 0, sd = 1, log = FALSE)`
- `dgamma(x, shape, rate = 1, scale = 1/rate, log = FALSE)`
- `dt(x, df, ncp, log = FALSE)`
- `pnorm(q, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)`

**Arguments**

- `x,q, lambda, size, prob, mu, mean, sd, shape, rate, scale, df, ncp` numeric or mpfr vectors. All of these are “recycled” to the length of the longest one. For their meaning/definition, see the corresponding standard R (stats package) function.
- `log, log.p, lower.tail` logical, see `pnorm, dpois, etc.`.
- `useLog` logical with default depending on x etc, indicating if log-scale computation should be used even when `log = FALSE`, for performance or against overflow / underflow.

**Details**

`pnorm()` is based on `erf()` and `erfc()` which have direct MPFR counter parts and are both reparametrizations of `pnorm`, `erf(x) = 2*pnorm(sqrt(2)*x)` and `erfc(x) = 2*pnorm(sqrt(2)*x, lower=FALSE)`.

**Value**

A vector of the same length as the longest of `x, q, ...`, of class mpfr with the high accuracy results of the corresponding standard R function.
Note

E.g., for `pnorm(*, log.p = TRUE)` to be useful, i.e., not to underflow or overflow, you may want to extend the exponential range of MPFR numbers, using `.mpfr_erange_set()`, see the examples.

See Also

`pnorm`, `dt`, `dbinom`, `dnbinom`, `dgamma`, `dpois` in standard package `stats`.

`pbetaI(x, a, b)` is a `mpfr` version of `pbeta` only for `integer` `a` and `b`.

Examples

```r
x <- 1400+ 0:10
print(dpois(x, 1000), digits=18) ## standard R's double precision
(px <- dpois(mpfr(x, 120), 1000))## more accuracy for the same
px. <- dpois(mpfr(x, 120), 1000, useLog=TRUE)# (failed in 0.8-8)
stopifnot(all.equal(px, px., tol = 1e-31))
dpois(0:5, mpfr(10000, 80)) ## very small exponents (underflowing in dbl.prec.)

print(dbinom(0:8, 8, pr = 4 / 5), digits=18)
dbinom(0:8, 8, pr = 4/mpfr(5, 99)) -> dB; dB

print(dnorm( -5:5), digits=18)
dnorm(mpfr(-5:5, prec=99))
```

## For `pnorm()` in the extreme tails, need an exponent range
## larger than the (MPFR and Rmpfr) default:
(olderanges <- .mpfr_erange()) # typically -/+ 2^30:
log2(abs(olderanges)) # 30 30
.mpfr_erange_set(value = (1-2^-52)*.mpfr_erange(c("min.emin","max.emax")))
log2(abs(.mpfr_erange()))## 62 62 **iff** setup -- 2023-01: *not* on Winbuilder
tens <- mpfr(10^(4:7), 128)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # "works" (iff ...)
## "the" boundary:
pnorm(mpfr(~ 38581.371, 128), log.p=TRUE) # still does not underflow (but *.372 does)
## -744261105.59928382481986753129188937418 (iff ...)
.mpfr_erange()*.log(2) # the boundary
## Emin  Emax
## -3.196577e+18 3.196577e+18 (iff ...)

## reset to previous
.mpfr_erange_set( , old_eranges)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # all but first underflow to -Inf
```
Description

Special Mathematical Functions, supported by the MPFR Library.

Note that additionally, all the Math and Math2 group member functions are “mpfr-ified”, too; ditto, for many more standard R functions. See see the methods listed in mpfr (aka ?mpfr-class).

Usage

zeta(x)
Ei(x)
Li2(x)
erf(x)
erfc(x)

Arguments

x a numeric or mpfr vector.

Details

zeta(x) computes Riemann’s Zeta function \( \zeta(x) \) important in analytical number theory and related fields. The traditional definition is

\[ \zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}. \]

Ei(x) computes the exponential integral,

\[ \int_{-\infty}^{x} \frac{e^t}{t} dt. \]

Li2(x) computes the dilogarithm,

\[ \int_{0}^{x} \frac{-\log(1-t)}{t} dt. \]

erf(x) and erfc(x) are the error, respectively complementary error function which are both reparametrizations of pnorm, erf(x) = 2*pnorm(sqrt(2)*x) and erfc(x) = 2*pnorm(sqrt(2)*x, lower=FALSE), and hence Rmpfr provides its own version of pnorm.

Value

A vector of the same length as x, of class mpfr.

See Also

pnorm in standard package stats; the class description mpfr mentioning the generic arithmetic and mathematical functions (sin, log, etc) for which "mpfr" methods are available.

Note the (integer order, non modified) Bessel functions j0(), yn(), etc, named j0, yn etc, and Airy function Ai() in Bessel_mpfr.
Examples

curve(Ei, 0, 5, n=2001)

## As we now require (mpfrVersion() >= "2.4.0"):
curve(Li2, 0, 5, n=2001)
curve(Li2, -2, 13, n=2000); abline(h=0,v=0, lty=3)
curve(Li2, -200,400, n=2000); abline(h=0,v=0, lty=3)

curve(erf, -3,3, col = "red", ylim = c(-1,2))
curve(erfc, add = TRUE, col = "blue")
abline(h=0, v=0, lty=3)
legend(-3,1, c("erf(x)", "erfc(x)"), col = c("red","blue"), lty=1)

Description

This page documents utilities from package Rmpfr which are typically not called by the user, but may come handy in some situations.

Notably, the (base-2) maximal (and minimal) precision and the "erange", the range of possible (base-2) exponents of mpfr-numbers can be queried and partly extended.

Usage

gPrec(x, base = 10, doNumeric = TRUE, is.mpfr = NA, bigq. = 128L)
.getPrec(x)
gD(x)
mpfr_default_prec(prec)
## S3 method for class 'mpfrArray'
print(x, digits = NULL, drop0trailing = FALSE,
  right = TRUE,
  max.digits =getOption("Rmpfr.print.max.digits", 999L),
  exponent.plus =getOption("Rmpfr.print.exponent.plus", TRUE),
  ...
)
## S3 method for class 'mpfr'
print(x, digits = NULL, drop0trailing = TRUE,
  right = TRUE,
  max.digits =getOption("Rmpfr.print.max.digits", 999L),
  exponent.plus =getOption("Rmpfr.print.exponent.plus", TRUE),
  ...
)
toNum(from, rnd.mode = c('N','D','U','Z','A'))
mpfr2array(x, dim, dimnames = NULL, check = FALSE)

.mpfr2list(x, names = FALSE)

mpfrXport(x, names = FALSE)
mpfrImport(mxp)

.mpfr_formatinfo(x)
.mpfr2exp(x)

.mpfr_erange(kind = c("Emin", "Emax"), names = TRUE)
.mpfr_erange_set(kind = c("Emin", "Emax"), value)
.mpfr_erange_kinds
.mpfr_erange_is_int()
.mpfr_maxPrec()
.mpfr_minPrec()

.mpfr_gmp_numbits()
.mpfrVersion()

## Really Internal and low level, no error checking (for when you know ..)

.mpfr(x, precBits)
.mpfr.(x, precBits, rnd.mode)
.getSign(x)

.mpfr_negative(x)
.mpfr_sign(x)

..bigq2mpfr(x, precB = NULL, rnd.mode = c("N", "D", "U", "Z", "A"))
..bigz2mpfr(x, precB = NULL, rnd.mode = c("N", "D", "U", "Z", "A"))

**Arguments**

- **x**, from typically, an R object of class "mpfr", or "mpfrArray", respectively. For getPrec(), any number-like R object, or NULL.
- **base** (only when x is character) the base with respect to which x[i] represent numbers; base b must fulfill $2 \leq b \leq 62$.
- **doNumeric** logical indicating integer or double typed x should be accepted and a default precision be returned. Should typically be kept at default TRUE.
- **is.mpfr** logical indicating if class(x) is already known to be "mpfr"; typically should be kept at default, NA.
- **bigq.** for getPrec(), the precision to use for a big rational (class "bigq"); if not specified gives warning when used.
- **prec**, **precB**, **precBits** a positive integer, or missing.
- **drop0trailing** logical indicating if trailing "0"s should be omitted.
- **right** logical indicating print()ing should right justify the strings; see print.default() to which it is passed.
- **digits, ...** further arguments to print methods.
- **maxdigits** a number (possibly Inf) to limit the number of (mantissa) digits to be printed, simply passed to formatMpfr(). The default is finite to protect from printing very long strings which is often undesirable, notably in interactive use.
exponent.plus logical, simply passed to \texttt{formatMpfr()}. Was FALSE hardwired in Rmpfr versions before 0.8-0, and hence is allowed to be tweaked by an \texttt{options()} setting.

\texttt{rnd.mode} a 1-letter string specifying how \textit{rounding} should happen at C-level conversion to MPFR, see details of \texttt{mpfr}.

dim, dimnames for \texttt{"mpfrArray"} construction.

check logical indicating if the mpfrArray construction should happen with internal safety check. Previously, the implicit default used to be true.

names (for \texttt{.mpfr2list()}) \texttt{logical} or \texttt{character} vector, indicating if the list returned should have \texttt{names}. If character, it specifies the names; if true, the names are set to \texttt{format(x)}.

\texttt{mxp} an \texttt{"mpfrXport"} object, as resulting from \texttt{mpfrXport()}.  

\texttt{kind} a \texttt{character} string or vector, specifying the kind of \texttt{"erange"} value; must be an element of \texttt{.mpfr\_erange\_kinds}, i.e., one of \texttt{"Emin"}, \texttt{"Emax"}, \texttt{"min.emin"}, \texttt{"max.emin"}, \texttt{"min.emax"}, \texttt{"max.emax"}.

\texttt{value} \texttt{numeric}, for \texttt{.mpfr\_erange\_set()} one number per kind. Must be in range specified by the \texttt{\ast."emin"} and \texttt{\ast."emax"} \texttt{erange} values.

Details

The print method is currently built on the \texttt{format} method for class \texttt{mpfr}. This, currently does \texttt{not} format columns jointly which leads to suboptimally looking output. There are plans to change this.

Note that \texttt{formatMpfr()} which is called by \texttt{print()} (or \texttt{show()} or \texttt{R}'s implicit printing) uses \texttt{max.digits = Inf}, differing from our \texttt{print()}'s default on purpose. If you do want to see the full accuracy even in cases it is large, use \texttt{options(Rmpfr.print.max.digits = Inf)} or \texttt{(.. = 1e7)}, say.

The \texttt{.mpfr\_erange*} functions (and variable) allow to query and set the allowed range of values for the base-2 \texttt{exponents} of \texttt{"mpfr"} numbers. See the examples below and GNU MPFR library documentation on the C functions \texttt{mpfr\_get\_emin()}, \texttt{mpfr\_set\_emin()}, \texttt{mpfr\_get\_emin\_min()}, and \texttt{mpfr\_get\_emin\_max()}, (and those four with \texttt{\_emin} replaced by \texttt{\_emax} above).

Value

\texttt{getPrec(x)} returns a \texttt{integer} vector of length one or the same length as \texttt{x} when that is positive, whereas \texttt{getPrec(NULL)} returns \texttt{mpfr\_default\_prec()}, see below.

If you need to \texttt{change} the precision of \texttt{x}, i.e., need something like \texttt{"setPrec"}, use \texttt{roundMpfr()}.

\texttt{.getPrec(x)} is a simplified version of \texttt{getPrec()} which only works for \texttt{"mpfr"} objects \texttt{x}.

\texttt{getD(x)} is intended to be a fast version of \texttt{x@.Data}, and should not be used outside of lower level functions.

\texttt{mpfr\_default\_prec()} returns the current MPFR default precision, an \texttt{integer}. This is currently not made use of much in package \texttt{Rmpfr}, where functions have their own default precision where needed, and otherwise we'd rather not be dependent of such a \texttt{global} setting.

\texttt{mpfr\_default\_prec(prec)} \texttt{sets} the current MPFR default precision and returns the previous one; see above.

\texttt{.mpfr\_maxPrec()} and (less interestingly) \texttt{.mpfr\_minPrec()} give the maximal and minimal base-2 precision allowed in the current version of the MPFR library linked to by \texttt{R} package \texttt{Rmpfr}. The maximal precision is typically \(2^{63}\), i.e.,
all.equal(.mpfr_maxPrec(), 2^63)

is typically true.

toNum(m) returns a numeric array or matrix, when m is of class "mpfrArray" or "mpfrMatrix", respectively. It should be equivalent to as(m, "array") or ... "matrix". Note that the slightly more general asNumeric() is preferred now.

mpfr2array() a slightly more flexible alternative to dim(.) <- dd.

.mpfr2exp(x) returns the base-2 (integer valued) exponents of x, i.e., it is the \texttt{R} interface to MPFR C’s mpfr_get_exp(). The result is integer iff .mpfr_erange_is_int() is true, otherwise double. Note that the MPFR (4.0.1) manual says about mpfr_get_exp(): The behavior for NaN, infinity or zero is undefined.

.mpfr_erange_is_int() returns TRUE iff the .mpfr_erange(c("Emin","Emax")) range lies inside the range of \texttt{R}'s integer limits, i.e., has absolute values not larger than .Machine$integer.max (\(= 2^{31} - 1\)).

.mpfr_erange_set() invisibly (see \texttt{invisible()}) returns TRUE iff the change was successful.

.mpfr_gmp_numbits() returns the ‘GMP’ library “numb” size, which is either 32 or 64 bit (as integer, i.e., 64L or 32L). If it is not 64, you typically cannot enlarge the exponential range of mpfr numbers via .mpfr_erange(), see above.

.mpfrVersion() returns a string, the version of the ‘MPFR’ library we are linking to.

.mpfr_formatinfo(x) returns conceptually a subset of .mpfr2str()’s result, a list with three components

\textbf{exp} the base-2 exponents of x, identical to .mpfr2exp(x).

\textbf{finite} logical identical to is.finite(x).

\textbf{is.0} logical indicating if the corresponding x[i] is zero; identical to mpfrIs0(x)

(Note that .mpfr2str(x, ..., base)$exp is wrt base and is not undefined but ...)

.mpfr_sign(x) only works for mpfr objects, then identical to sign(x). Analogously, .mpfr_negative(x) is -x in that case. .getSign(x) is a low-level version of sign(x) returning -1 or +1, but not 0. Finally, .bigz2mpfr(x, ...) and .bigq2mpfr(x, ...) are fast ways to coerce bigz and bigq number objects (created by package \texttt{gmp}'s functionality) to our "mpfr" class.

\textbf{Note}

mpfrXport() and mpfrImport() are experimental and used to explore reported platform incompatibilities of \texttt{save()}d and \texttt{load()}d "mpfr" objects between Windows and non-Windows platforms.

In other words, the format of the result of mpfrXport() and hence the \texttt{mxp} argument to mpfrImport() are considered internal, not part of the API and subject to change.

\textbf{See Also}

Start using mpfr(.), and compute with these numbers.

\texttt{mpfrArray}(x) is for numeric ("non-mpfr") x, whereas mpfr2array(x) is for "mpfr" classed x, only.
Examples

getPrec(as(c(1, pi), "mpfr")) # 128 for both

(opr <- mpfr_default_prec()) ## typically 53, the MPFR system default
stopifnot(opr == (oprec <- mpfr_default_prec(70)),
70 == mpfr_default_prec())
## and reset it:
mpfr_default_prec(opr)

## Explore behavior of rounding modes 'rnd.mode':
x <- mpfr(10, 99)^512 # too large for regular (double prec. / numeric):
sapply(c("N", "D", "U", "Z", "A"), function(RM)
  sapply(list(-x, x), function(.) toNum(. , RM)))
## N  D  U  Z  A
## -Inf -Inf -1.797693e+308 -1.797693e+308 -Inf
## Inf 1.797693e+308 Inf 1.797693e+308 Inf

## Printing of "MPFR" matrices is less nice than R's usual matrix printing:
m <- outer(c(1, 3.14, -1024.5678), c(1, 1e-3, 10, 100))
m[3, 3] <- round(m[3, 3])
m
mpfr(m, 50)

B6 <- mpfr2array(Bernoulli(1:6, 60), c(2, 3),
  dimnames = list(LETTERS[1:2], letters[1:3]))
B6

## Ranges of (base 2) exponents of MPFR numbers:
.mpfr_erange() # the currently active range of possible base 2 exponents:

## A factory fresh setting fulfills
.mpfr_erange(c("Emin", "Emax")) == c(-1, 1) * (2^30 - 1)

## There are more 'kind's, the latter 4 showing how you could change the first two:
.mpfr_erange_kinds
.mpfr_erange(.mpfr_erange_kinds)
eLimits <- .mpfr_erange(c("min.emin", "max.emin", "min.emax", "max.emax"))
## Typically true in "current" MPFR versions:
eLimits == c(-1, 1, -1, 1) * (2^62 - 1)

## Looking at internal representation [for power users only!]:
i8 <- mpfr(-2:5, 32)
x4 <- mpfr(c(NA, NaN, -Inf, Inf), 32)
## The output of the following depends on the GMP "numb" size
## (32 bit vs. 64 bit), and may be even more platform specific:
str(.mpfr2list(i8))
str(.mpfr2list(x4, names = TRUE))
str(xp4 <- mpfrXport(x4, names = TRUE))
stopifnot(identical(x4, mpfrImport(mpfrXport(x4))),
Description

mpfrVersion() returns the version of the MPFR library which Rmpfr is currently linked to.

c(x,y,...) can be used to combine MPFR numbers in the same way as regular numbers IFF the
first argument x is of class mpfr.

mpfrIs0(.) uses the MPFR library in the documented way to check if (a vector of) MPFR numbers
are zero. It was called mpfr.is.0 which is strongly deprecated now.

.mpfr.is.whole(x) uses the MPFR library in the documented way to check if (a vector of) MPFR
numbers is integer valued. This is equivalent to x == round(x), but not at all to is.integer(as(x,
"numeric")).

You should typically rather use (the "mpfr" method of the generic function) is.whole(x) instead.
The former name mpfr.is.integer is deprecated now.

Usage

mpfrVersion()
mpfrIs0(x)

### S3 method for class 'mpfr'
c(...)

### S3 method for class 'mpfr'
diff(x, lag = 1L, differences = 1L, ...)

Arguments

x an object of class mpfr.

... for diff, further mpfr class objects or simple numbers (numeric vectors) which
are coerced to mpfr with default precision of 128 bits.

lag, differences

for diff(): exact same meaning as in diff()'s default method, diff.default.

Value

mpfrIs0 returns a logical vector of length length(x) with values TRUE iff the corresponding x[i]
is an MPFR representation of zero (0).

Similarly, .mpfr.is.whole and is.whole return a logical vector of length length(x).

mpfrVersion returns an object of S3 class "numeric_version", so it can be used in comparisons.
The other functions return MPFR number (vectors), i.e., extending class mpfr.
mpfrArray

See Also

str.mpfr for the str method. erf for special mathematical functions on MPFR.
The class description mpfr page mentions many generic arithmetic and mathematical functions for which "mpfr" methods are available.

Examples

mpfrVersion()

(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
mpfrIs0(x)  # one of them is
x[mpfrIs0(x)]  # but it may not have been obvious..
str(x)

x <- rep(-2:2, 5)
stopifnot(is.whole(mpfr(2, 500) ^ (1:200)),
  all.equal(diff(x), diff(as.numeric(x))))

mpfrArray

Construct "mpfrArray" almost as by 'array()'

Description

Utility to construct an R object of class mpfrArray, very analogously to the numeric array function.

Usage

mpfrArray(x, precBits, dim = length(x), dimnames = NULL,
  rnd.mode = c("N","D","U","Z","A"))

Arguments

x         numeric(like) vector, typically of length prod(dim) or shorter in which case it
          is recycled.
precBits   a number, the maximal precision to be used, in bits; i.e., 53 corresponds to
dim       double precision. Must be at least 2.
dimnames  the dimension of the array to be created, that is a vector of length one or more
giving the maximal indices in each dimension.
dimnames  giving the maximal indices in each dimension.
dimensiona either NULL or the names for the dimensions. This is a list with one component
             for each dimension, either NULL or a character vector of the length given by dim
             for that dimension.
rnd.mode   a 1-letter string specifying how rounding should happen at C-level conversion
to MPFR, see details of mpfr.

Value

an object of class "mpfrArray", specifically "mpfrMatrix" when length(dim) == 2.
See Also

`mpfr`, `array`, `asNumeric()` as “inverse” of `mpfrArray()`, to get back a numeric array.

`mpfr2array(x)` is for “mpfr” classed `x`, only, whereas `mpfrArray(x)` is for numeric (“non-mpfr”) `x`.

Examples

```r
## preallocating is possible here too
ma <- mpfrArray(NA, prec = 80, dim = 2:4)
validObject(A2 <- mpfrArray(1:24, prec = 64, dim = 2:4))

## recycles, gives an "mpfrMatrix" and dimnames :
mat <- mpfrArray(1:5, 64, dim = c(5,3), dimnames=list(NULL, letters[1:3]))
mat
asNumeric(mat)
stopifnot(identical(asNumeric(mat),
            matrix(1:5 +0, 5,3, dimnames=dimnames(mat))))

## Testing the apply() method :
apply(mat, 2, range)
apply(A2, 1:2, range)
apply(A2, 2:3, max)
(a2 <- apply(A2, 2, fivenum))
stopifnot(as(a2, "array")

(a2 <- as(A2, "array")
stopifnot(identical(as(a2, 2, range), "matrix") ==
    apply(a2, 2, range)
    , all.equal(fA2, apply(a2, 2, fivenum))
    , all.equal(apply(A2, 2, quantile),
        apply(a2, 2, quantile))
    , all.equal(A2, apply(A2, 2:3, identity) -> aA2, check.attributes=FALSE)
    , dim(A2) == dim(aA2)
)
```

mpfrMatrix

Classes "mpfrMatrix" and "mpfrArray"

Description

The classes "mpfrMatrix" and "mpfrArray" are, analogously to the base `matrix` and `array` functions and classes simply “numbers” of class `mpfr` with an additional `Dim` and `Dimnames` slot.

Objects from the Class

Objects should typically be created by `mpfrArray()`, but can also be created by `new("mpfrMatrix", ...)` or `new("mpfrArray", ...)`, or also by `t(x), dim(x) <- dd`, or `mpfr2array(x, dim=dd)` where `x` is an `mpfr` “number vector”.

A (slightly more flexible) alternative to `dim(x) <- dd` is `mpfr2array(x, dd, dimnames).`
**Slots**

- **Data**: as for the mpfr class, a "list" of mpfr numbers.
- **Dim**: of class "integer", specifying the array dimension.
- **Dimnames**: of class "list" and the same length as Dim, each list component either NULL or a character vector of length Dim[j].

**Extends**

Class "mpfrMatrix" extends "mpfrArray", directly.

Class "mpfrArray" extends class "mpfr", by class "mpfrArray", distance 2; class "list", by class "mpfrArray", distance 3; class "vector", by class "mpfrArray", distance 4.

**Methods**

- **Arith** signature(e1 = "mpfr", e2 = "mpfrArray"): ...
- **Arith** signature(e1 = "numeric", e2 = "mpfrArray"): ...
- **Arith** signature(e1 = "mpfrArray", e2 = "mpfrArray"): ...
- **Arith** signature(e1 = "mpfrArray", e2 = "mpfr"): ...
- **Arith** signature(e1 = "mpfrArray", e2 = "numeric"): ...
- **as.vector** signature(x = "mpfrArray", mode = "missing"): drops the dimension 'attribute', i.e., transforms x into a simple mpfr vector. This is an inverse of t(.) or dim(.) <- * on such a vector.
- **atan2** signature(y = "ANY", x = "mpfrArray"): ...
- **atan2** signature(y = "mpfrArray", x = "mpfrArray"): ...
- **<-* signature(x = "mpfrArray", x = "ANY"): ...
- **[** signature(x = "mpfrArray", i = "ANY", j = "ANY", value = "ANY"): ...
- **[** signature(x = "mpfrArray", i = "ANY", j = "ANY", drop = "ANY"): ...
- **[** signature(x = "mpfrArray", i = "ANY", j = "missing", drop = "missing"): "mpfrArray"s can be subset ("indexed") as regular R arrays.
- **%*%** signature(x = "mpfr", y = "mpfrMatrix"): Compute the matrix/vector product $xy$ when the dimensions (dim) of $x$ and $y$ match. If $x$ is not a matrix, it is treated as a 1-row or 1-column matrix (aka "row vector" or "column vector") depending on which one makes sense, see the documentation of the base function %*%.
- **%*%** signature(x = "mpfr", y = "Mnumber"): method definition for cases with one mpfr and any "number-like" argument are to use MPFR arithmetic as well.
- **%*%** signature(x = "mpfrMatrix", y = "mpfrMatrix").
- **%*%** signature(x = "mpfrMatrix", y = "mpfr"), etc. Further method definitions with identical semantic.
- **crossprod** signature(x = "mpfr", y = "missing"): Computes $x'x$, i.e., t(x) %*% x, typically more efficiently.
- **crossprod** signature(x = "mpfr", y = "mpfrMatrix"): Computes $x'y$, i.e., t(x) %*% y, typically more efficiently.
crossprod signature(x = "mpfrMatrix", y = "mpfrMatrix"): ...
crossprod signature(x = "mpfrMatrix", y = "mpfr"): ...
tcrossprod signature(x = "mpfr", y = "missing"): Computes \(xx', i.e., x \times^\ast t(x)\), typically more efficiently.
tcrossprod signature(x = "mpfrMatrix", y = "mpfrMatrix"): Computes \(xy', i.e., x \times^\ast t(y)\), typically more efficiently.
tcrossprod signature(x = "mpfrMatrix", y = "mpfr"): ...
tcrossprod signature(x = "mpfr", y = "mpfrMatrix"): ...
coerce signature(from = "mpfrArray", to = "array"): coerces from to a numeric array of the same dimension.

correct signature(from = "mpfrArray", to = "vector"): as for standard arrays, this “drops” the dim (and dimnames), i.e., returns an mpfr vector.

Compare signature(e1 = "mpfr", e2 = "mpfrArray"): ...

Compare signature(e1 = "numeric", e2 = "mpfrArray"): ...

Compare signature(e1 = "mpfrArray", e2 = "mpfr"): ...

Compare signature(e1 = "mpfrArray", e2 = "numeric"): ...

dim signature(x = "mpfrArray"): ...

dimnames<- signature(x = "mpfrArray"): ...

dimnames signature(x = "mpfrArray"): ...

show signature(object = "mpfrArray"): ...

sign signature(x = "mpfrArray"): ...

norm signature(x = "mpfrMatrix", type = "character"): computes the matrix norm of \(x\), see norm or the one in package Matrix.

t signature(x = "mpfrMatrix"): transpose the mpfrMatrix.

aperm signature(a = "mpfrArray"): aperm(a, perm) is a generalization of \(t(.)\) to permute the dimensions of an mpfrArray; it has the same semantics as the standard aperm() method for simple R arrays.

Author(s)
Martin Maechler

See Also

mpfrArray, also for more examples.

Examples

showClass("mpfrMatrix")
validObject(mm <- new("mpfrMatrix"))
validObject(aa <- new("mpfrArray"))
v6 <- mpfr(1:6, 128)
m6 <- new("mpfrMatrix", v6, Dim = c(2L, 3L))
validObject(m6)
m6

which(m6 == 3, arr.ind = TRUE) # --> (1, 2)

## Coercion back to "vector": Both of these work:
stopifnot(identical(as(m6, "mpfr"), v6),
  identical(as.vector(m6), v6)) # < but this is a "coincidence"

S2 <- m6[,-3] # 2 x 2
S3 <- cbind(m6[,c(1:2,10)]; s3 <- asNumeric(S3)
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tol=1e-15),
  all.equal(det(S3), det(s3), tol=1e-15))

## 2-column matrix indexing and replacement:
(sS <- S3[i2 <- cbind(1:2, 2:3)])
stopifnot(identical(asNumeric(sS), s3[i2]))
C3 <- S3; c3 <- s3
C3[i2] <- 10:11
c3[i2] <- 10:11
stopifnot(identical(asNumeric(C3), c3))

AA <- new("mpfrArray", as.vector(cbind(S3, -S3)), Dim=c(3L,3:2))
stopifnot(identical(AA[,1], S3), identical(AA[,2], -S3))
aa <- asNumeric(AA)

i3 <- cbind(3:1, 1:3, c(2L, 1:2))
i3 <- Rmpfr:::.mat2ind(i3, dim(AA), dimnames(AA))
stopifnot(aa[i3] == new("mpfr", getD(AA)[i33]))
stopifnot(identical(aa[i3], asNumeric(AA[i33])))
CA <- AA; ca <- aa
ca[i3] <- ca[i3] ^ 3
CA[i3] <- CA[i3] ^ 3

## scale():
S2. <- scale(S2)
stopifnot(all.equal(abs(as.vector(S2.)), rep(sqrt(1/mpfr(2, 128)), 4),
  tol = 1e-30))

## norm():
norm(S2)
stopifnot(identical(norm(S2), norm(S2, "1")),
  norm(S2, "1") == 6,
  norm(S2, "M") == 4,
  abs(norm(S2, "F") - 5.477225575051661) < 1e-15)
Description
determinant(x, ..) computes the determinant of the mpfr square matrix x. May work via coercion to "numeric", i.e., compute determinant(asNumeric(x), logarithm), if asNumeric is true, by default, if the dimension is larger than three. Otherwise, use precision precBits for the “accumulator” of the result, and use the recursive mathematical definition of the determinant (with computational complexity $n!$, where $n$ is the matrix dimension, i.e., very inefficient for all but small matrices!)

Usage

## S3 method for class 'mpfrMatrix'
determinant(x, logarithm = TRUE,
             asNumeric = (d[1] > 3), precBits = max(.getPrec(x)), ...)

Arguments

x an mpfrMatrix object of square dimension.
logarithm logical indicating if the log of the absolute determinant should be returned.
asNumeric logical ... if rather determinant(asNumeric(x), ...) should be computed.
precBits the number of binary digits for the result (and the intermediate accumulations).
... unused (potentially further arguments passed to methods).

Value

as determinant(), an object of S3 class "det", a list with components

modulus the (logarithm of) the absolute value (abs) of the determinant of x.
sign the sign of the determinant.

Author(s)

Martin Maechler

See Also
determinant in base R, which relies on a fast LU decomposition. mpfrMatrix

Examples

m6 <- mpfrArray(1:6, prec=128, dim = c(2L, 3L))
m6
S2 <- m6[,,-3] # 2 x 2
S3 <- rbind(m6, c(1:2,10))
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tolerance=1e-15),
          all.equal(det(S3), det(asNumeric(S3)), tolerance=1e-15))
**Description**

optimizeR searches the interval from lower to upper for a minimum of the function \( f \) with respect to its first argument.

**Usage**

```r
optimizeR(f, lower, upper, ..., tol = 1e-20,
         method = c("Brent", "GoldenRatio"),
         maximum = FALSE,
         precFactor = 2.0, precBits = -log2(tol) * precFactor,
         maxiter = 1000, trace = FALSE)
```

**Arguments**

- **f**: the function to be optimized. \( f(x) \) must work “in Rmpfr arithmetic” for optimizer() to make sense. The function is either minimized or maximized over its first argument depending on the value of maximum.
- **...**: additional named or unnamed arguments to be passed to \( f \).
- **lower**: the lower end point of the interval to be searched.
- **upper**: the upper end point of the interval to be searched.
- **tol**: the desired accuracy, typically higher than double precision, i.e., \( tol < 2 \times 10^{-16} \).
- **method**: character string specifying the optimization method.
- **maximum**: logical indicating if \( f() \) should be maximized or minimized (the default).
- **precFactor**: only for default precBits construction: a factor to multiply with the number of bits directly needed for tol.
- **precBits**: number of bits to be used for mpfr numbers used internally.
- **maxiter**: maximal number of iterations to be used.
- **trace**: integer or logical indicating if and how iterations should be monitored; if an integer \( k \), print every \( k \)-th iteration.

**Details**

"Brent": Brent(1973)’s simple and robust algorithm is a hybrid, using a combination of the golden ratio and local quadratic (“parabolic”) interpolation. This is the same algorithm as standard R’s optimize(), adapted to high precision numbers.

In smooth cases, the convergence is considerably faster than the golden section or Fibonacci ratio algorithms.

"GoldenRatio": The golden ratio method, aka ‘golden-section search’ works as follows: from a given interval containing the solution, it constructs the next point in the golden ratio between the interval boundaries.
Value

A list with components minimum (or maximum) and objective which give the location of the minimum (or maximum) and the value of the function at that point; iter specifying the number of iterations, the logical convergence indicating if the iterations converged and estim.prec which is an estimate or an upper bound of the final precision (in \( x \)). method the string of the method used.

Author(s)

"GoldenRatio" is based on Hans Werner Borchers’ golden_ratio (package pracma); modifications and "Brent" by Martin Maechler.

See Also

R’s standard optimize; for multivariate optimization, Rmpfr’s hjkMpfr(); for root finding, Rmpfr’s unirootR.

Examples

## The minimum of the Gamma (and lgamma) function (for x > 0):
Gmin <- optimizeR(gamma, .1, 3, tol = 1e-50)
str(Gmin, digits = 8)
## high precision chosen for "objective"; minimum has "estim.prec" = 1.79e-50
Gmin[c("minimum","objective")]
## it is however more accurate to 59 digits:
asNumeric(optimizeR(gamma, 1, 2, tol = 1e-100)$minimum - Gmin$minimum)

iG5 <- function(x) -exp(-(x-5)^2/2)
curve(iG5, 0, 10, 200)
o.dp <- optimize(iG5, c(0, 10)) #-> 5 of course
oM.gs <- optimizeR(iG5, 0, 10, method="Golden")
oM.Br <- optimizeR(iG5, 0, 10, method="Brent", trace=TRUE)
oM.gs$min ; oM.gs$iter
oM.Br$min ; oM.Br$iter
(doExtras <- Rmpfr:::doExtras())
if(doExtras) {## more accuracy {takes a few seconds}
oM.gs <- optimizeR(iG5, 0, 10, method="Golden", tol = 1e-70)
oM.Br <- optimizeR(iG5, 0, 10, tol = 1e-70)
}
rbind(Golden = c(err = as.numeric(oM.gs$min -5), iter = oM.gs$iter),
      Brent = c(err = as.numeric(oM.Br$min -5), iter = oM.Br$iter))
## ==> Brent is orders of magnitude more efficient !

## Testing on the sine curve with 40 correct digits:
sol <- optimizeR(sin, 2, 6, tol = 1e-40)
str(sol)
sol <- optimizeR(sin, 2, 6, tol = 1e-50,
                  precFactor = 3.0, trace = TRUE)
pi.. <- 2*sol$min/3
print(pi.., digits=51)
pbetaI

Accurate Incomplete Beta / Beta Probabilities For Integer Shapes

Description

For integers \( a, b \), \( I_{a,b}(x) \) aka \( \text{pbeta}(x, a, b) \) is a polynomial in \( x \) with rational coefficients, and hence arbitrarily accurately computable.

TODO (not yet): It’s sufficient for one of \( a, b \) to be integer such that the result is a finite sum (but the coefficients will no longer be rational, see Abramowitz and Stegun, 26.5.6 and *.7, p.944).

Usage
pbetaI(q, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE, precBits = NULL, useRational = !log.p && !is.mpfr(q) && is.null(precBits) && int2, rnd.mode = c("N","D","U","Z","A"))

Arguments

q called \( x \), above; vector of quantiles, in \([0, 1]\); can be numeric, or of class "mpfr" or also "\texttt{bigq}" ("big rational" from package \texttt{gmp}); in the latter case, if \texttt{log.p} = \texttt{FALSE} as by default, all computations are exact, using big rational arithmetic.

shape1, shape2 the positive Beta “shape” parameters, called \( a, b \), above. Must be integer valued for this function.

ncp unused, only for compatibility with \texttt{pbeta}, must be kept at its default, 0.

lower.tail logical; if TRUE (default), probabilities are \( P[X \leq x] \), otherwise, \( P[X > x] \).

log.p logical; if TRUE, probabilities \texttt{p} are given as \( \log(p) \).

precBits the precision (in number of bits) to be used in \texttt{sumBinomMpfr()}.

useRational optional logical, specifying if we should try to do everything in exact rational arithmetic, i.e, using package \texttt{gmp} functionality only, and return \texttt{bigq} numbers instead of \texttt{mpfr} numbers.

rnd.mode a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see \texttt{mpfr}.

Value

an "mpfr" vector of the same length as \texttt{q}.

Note

For upper tail probabilities, i.e., when \texttt{lower.tail=}FALSE, we may need large \texttt{precBits}, because the implicit or explicit \( 1 - P \) computation suffers from severe cancellation.

Author(s)

Martin Maechler

References


See Also

\texttt{pbeta, sumBinomMpfr chooseZ}.
pbetaI

Examples

x <- (0:12)/16 # not all the way up ..
a <- 7; b <- 788

p. <- pbetaI(x, a, b) ## a bit slower:
system.time(
  pp <- pbetaI(x, a, b, precBits = 2048)
) # 0.23 -- 0.50 sec
## Currently, the lower.tail=FALSE are computed "badly":
lp <- log(pp) ## = pbetaI(x, a, b, log.p=TRUE)
lIp <- log1p(-pp) ## = pbetaI(x, a, b, lower.tail=FALSE, log.p=TRUE)
Ip <- 1 - pp ## = pbetaI(x, a, b, lower.tail=FALSE)

if(Rmpfr:::doExtras()) { ## somewhat slow
  stopifnot(
    all.equal(lp, pbetaI(x, a, b, precBits = 2048, log.p=TRUE)),
    all.equal(lIp, pbetaI(x, a, b, precBits = 2048, lower.tail=FALSE, log.p=TRUE,
                   tol = 1e-230),
    all.equal( Ip, pbetaI(x, a, b, precBits = 2048, lower.tail=FALSE))
  )
}

rErr <- function(approx, true, eps = 1e-200) {
  true <- as.numeric(true) # for "mpfr"
  elseif(Mod(true) >= eps,
    ## relative error, catching '-Inf' etc :
    elseif(true == approx, 0, 1 - approx / true),
    ## else: absolute error (e.g. when true=0)
    true - approx)
}

rErr(pbeta(x, a, b), pp)
rErr(pbeta(x, a, b, lower=FALSE), Ip)
rErr(pbeta(x, a, b, log = TRUE), lp)
rErr(pbeta(x, a, b, lower=FALSE, log = TRUE), lIp)

a.EQ <- function(..., tol=1e-15) all.equal(..., tolerance=tol)
stopifnot(
  a.EQ(pp, pbeta(x, a, b)),
  a.EQ(lp, pbeta(x, a, b, log.p=TRUE)),
  a.EQ(lIp, pbeta(x, a, b, lower.tail=FALSE, log.p=TRUE)),
  a.EQ( Ip, pbeta(x, a, b, lower.tail=FALSE))
)

## When 'q' is a bigrational (i.e., class "bigq", package 'gmp'), everything
## is computed *exactly* with bigrational arithmetic:
(q4 <- as.bigq(1, 2^(0:4)))
pb4 <- pbetaI(q4, 10, 288, lower.tail=FALSE)
stopifnot( is.bigq(pb4) )
mpb4 <- as(pb4, "mpfr")
mpb4[1:2]
getPrec(mp4) # 128 349 1100 1746 2362
(pb. <- pbeta(asNumeric(q4), 10, 288, lower.tail=FALSE))
stopifnot(mpb4[1] == 0,
  all.equal(mpb4, pb., tol=4e-15))

qbetaI. <- function(p, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE,
  precBits = NULL, rnd.mode = c("N", "D", "U", "Z", "A"),
  tolerance = 1e-20, ...)
{
  if(is.na(a <- as.integer(shape1))) stop("a = shape1 is not coercable to finite integer")
  if(is.na(b <- as.integer(shape2))) stop("b = shape2 is not coercable to finite integer")
  unirootR(function(q) pbetaI(q, a, b, lower.tail=lower.tail, log.p=log.p,
    precBits=precBits, rnd.mode=rnd.mode) - p,
    interval = if(log.p) c(-double.xmax, 0) else 0:1,
    tol = tolerance, ...)}

(p <- 1 - mpfr(1,128)/20) # 'p' must be high precision
q95.1.3 <- qbetaI.(p, 1,3, tolerance = 1e-29) # -> ~29 digits accuracy
str(q95.1.3) ; roundMpfr(q95.1.3$root, precBits = 29 * log2(10))
## relative error is really small:
(relE <- asNumeric(1 - pbetaI(q95.1.3$root, 1,3) / p))
stopifnot(abs(relE) < 1e-28)

---

**pmax**

*Parallel Maxima and Minima*

**Description**

Returns the parallel maxima and minima of the input values.

The functions `pmin` and `pmax` have been made S4 generics, and this page documents the "... method for class "mNumber"", i.e., for arguments that are numeric or from class "mpfr".

**Usage**

```r
pmax(..., na.rm = FALSE)
pmin(..., na.rm = FALSE)
```

**Arguments**

- `...` numeric or arbitrary precision numbers (class `mpfr`).
- `na.rm` a logical indicating whether missing values should be removed.

**Details**

See `pmax`, the documentation of the base functions, i.e., default methods.

**Value**

vector-like, of length the longest of the input vectors; typically of class `mpfr`, for the methods here.
Methods

... = "ANY" the default method, really just base::pmin or base::pmax, respectively.

... = "mNumber" the method for mpfr arguments, mixed with numbers; designed to follow the same semantic as the default method.

See Also

The documentation of the base functions, pmin and pmax; also min and max; further, range (both min and max).

Examples

(pm <- pmin(1.35, mpfr(0:10, 77)))
stopifnot(pm == pmin(1.35, 0:10))

qnormI Gaussian / Normal Quantiles qnorm() via Inversion

Description

Compute Gaussian or Normal Quantiles qnorm(p, *) via inversion of our “mpfr-ified” arbitrary accurate pnorm(), using our unrootR() root finder.

Usage

qnormI(p, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE,
trace = 0, verbose = as.logical(trace),
tol,
useMpfr = any(prec > 53),
give.full = FALSE,
...)

Arguments

p vector of probabilities.
mean vector of means.
sd vector of standard deviations.
log.p logical; if TRUE, probabilities p are given as log(p).
lower.tail logical; if TRUE (default), probabilities are \( P[X \leq x] \) otherwise, \( P[X > x] \).
trace integer passed to unrootR(). If positive, information about a search interval extension will be printed to the console.
verbose logical indicating if progress details should be printed to the console.
tol optionally the desired accuracy (convergence tolerance); if missing or not finite, it is computed as \( 2^{−pr} + 2 \) where the precision \( pr \) is basically \( \max(getPrec(p+mean+sd)) \).
useMpfr logical indicating if mpfr arithmetic should be used.
give.full logical indicating if the full result of unirootR() should be returned (when applicable).

... optional further arguments passed to unirootR() such as maxiter, verbDigits, check.conv, warn.no.convergence, and epsC.

Value

If give.full is true, return a list, say r, of unirootR(.) results, with length(r) == length(p).
Otherwise, return a “numeric vector” like p, e.g., of class ”mpfr” when p is.

Author(s)

Martin Maechler

See Also

Standard R’s qnorm.

Examples

doX <- Rmpfr::.doExtras() # slow parts only if(doX)
cat("doExtras: ", doX, "\n")
p <- (0:32)/32
lp <- -c(1000, 500, 200, 100, 50, 20:1, 2^-(1:8))
if(doX) {
tol1 <- 2.3e-16
tolM <- 1e-20
tolRIlog <- 4e-14
} else { # use one more than a third of the points:
ip <- c(TRUE,FALSE, rep_len(c(TRUE,FALSE,FALSE), length(p)-2L))
p <- p[ip]
lp <- lp[ip]
tolI <- 1e-9
tolM <- 1e-12
tolRIlog <- 25*tolM
}

f.all.eq <- function(a,b)
  sub("\"Mean relative difference:\" ', '', format(all.equal(a, b, tol=0)))
for(logp in c(FALSE,TRUE)) {
  pp <- if(logp) lp else p
  mp <- mpfr(pp, precBits = if(doX) 80 else 64) # precBits = 128 gave "the same" as 80
  for(l.tail in c(FALSE,TRUE)) {
    qn <- qnorm (pp, lower.tail = l.tail, log.p = logp)
    qnI <- qnormI(pp, lower.tail = l.tail, log.p = logp, tol = tolI)
    qnM <- qnormI(mp, lower.tail = l.tail, log.p = logp, tol = tolM)
    cat(sprintf("Accuracy of qnorm(*, lower.t=%-5s, log.p=%-5s): %s || qnI: %s\n", l.tail, logp, qnM, qnM),
         f.all.eq(qnM, qnI))
  }
  stopifnot(exprs = {
    f.all.eq(qnM, qnM),
    f.all.eq(qnI, qnI))
}
all.equal(qn, qnI, tol = if(logp) tolRIlog else 4*tol1)
all.equal(qnM, qnI, tol = tol1)

## useMpfr, using mpfr() :
if(doX) {
p2 <- 2^-c(1:27, 5*(6:20), 20*(6:15))
e2 <- 88
} else {
p2 <- 2^-c(1:2, 7, 77, 177, 307)
e2 <- 60
}
system.time( pn2 <- pnorm(qnormI(mpfr(p2, e2))) ) # 4.1 or 0.68
all.equal(p2, pn2, tol = 0) # 5.48e-29 // 5.2e-18
2^-e2
stopifnot(all.equal(p2, pn2, tol = 6 * 2^-e2)) # '4 *' needed

## Boundary -- from limits in mpfr maximal exponent range!
## 1) Use maximal ranges:
(old_eranges <- .mpfr_erange()) # typically -/+ 2^30
(myERng <- (1-2^-52) * .mpfr_erange(c("min.emin","max.emax")))
(doIncr <- !isTRUE(all.equal(unname(myERng), unname(old_eranges))))
if(doIncr) .mpfr_erange_set(value = myERng)
log2(abs(.mpfr_erange()))# 62 62 if the above increase worked
(lrgOK <- all(log2(abs(.mpfr_erange()))) >= 62))

## The largest quantile for which our mpfr-ized qnorm() does *NOT* underflow :
cM <- if(doX) { "2528468770.343293436810768159197281514373932815851863149087539694969064" }
else "2528468770.34329343681"

(qM <- mpfr(cM))
(pM <- pnorm(-qM)) # precision 233 bits if(doX)
## 7.64890682545699845135633468495894619457903458325606933043966616334460003e-1388255822130839040
log(pM) # 233 bits: -3196577161300663205.857591962115614148120323933633827052786873078552904
if(lrgOK) withAutoprint({
try( qnormI(pM) ) ## Error: lower < upper not fulfilled (evt. TODO)
## but this works
print(qnI <- qnormI(log(pM), log.p=TRUE)) # -2528468770.343293436
all.equal(-qM, qnI, tol = 0) # << show how close; seen 1.084202e-19
stopifnot( all.equal(-qM, qnI, tol = 1e-18) )
})

if(FALSE) # this (**SLOW**) gives 21 x the *same* (wrong) result --- FIXME!
qnormI(log(pM) * (2:22), log.p=TRUE)
if(doX) ## Show how bad it is (currently ca. 220 iterations, and then *wrong*)
str(qnormI(round(log(pM)), log.p=TRUE, trace=1, give.full = TRUE))
if(requireNamespace("DPQ"))
  new("mpfr", as(DPQ::qnormR(pM, trace=1), "mpfr")) # as(*, "mpfr") also works for +/- Inf
  # qnormR1(p= 0, m=0, s=1, l.t.= 1, log= 0): q = -0.5
  # somewhat close to 0 or 1: r := sqrt(-lp) = 1.7879e+09
  # r > 5, using rational form R_3(t), for t=1.787897e+09 -- that is *not* accurate
  # [1] -94658744.369295865460...............  

  ## reset to previous status if needed
if(doIncr) .mpfr_erange_set( , old_eranges)

Rmpfr-workarounds Base Functions etc, as an Rmpfr version

Description

Functions from base etc which need a copy in the Rmpfr namespace so they correctly dispatch.

Usage

outer(X, Y, FUN = "+", ...)  

Arguments

X, Y, FUN,... See base package help: outer.

See Also

outer.

Examples

outer(1/mpfr(1:10, 70), 0:2)
roundMpfr

Rounding to Binary bits, "mpfr-internally"

Description

Rounding to binary bits, not decimal digits. Closer to the number representation, this also allows to increase or decrease a number’s precBits. In other words, it acts as setPrec(), see getPrec().

Usage

roundMpfr(x, precBits, rnd.mode = c("N","D","U","Z","A"))

Arguments

x an mpfr number (vector)
precBits integer specifying the desired precision in bits.
rnd.mode a 1-letter string specifying how rounding should happen at C-level conversion to MPFR, see mpfr.

Value

an mpfr number as x but with the new ‘precBits’ precision

See Also

The mpfr class group method Math2 implements a method for round(x, digits) which rounds to decimal digits.

Examples

(p1 <- Const("pi", 100)) # 100 bit prec
roundMpfr(p1, 120) # 20 bits more, but "random noise"
Const("pi", 120) # same "precision", but really precise

sapplyMpfr

Apply a Function over a "mpfr" Vector

Description

Users may be disappointed to note that sapply() or vapply() typically do not work with "mpfr" numbers.

This is a simple (but strong) approach to work around the problem, based on lapply().
Usage

```
sapplyMpfr(X, FUN, ..., drop_1_ = TRUE)
```

Arguments

- **X**: a vector, possibly of class "mpfr".
- **FUN**: a function returning an "mpfr" vector or even an "mpfrArray". May also be a function returning a numeric vector or array for numeric X, and which returns "mpfr(Array)" for an X argument inheriting from "mpfr".
- **...**: further arguments passed to `lapply`, typically further arguments to FUN.
- **drop_1_**: logical (with unusual name on purpose!) indicating if 1-column matrices ("mpfrMatrix") should be “dropped” to vectors ("mpfr"), the same as in base R's own `sapply`. This has been implicitly FALSE in `Rmpfr` versions 0.8-5 to 0.8-9 (Oct 2021 to June 2022), accidentally. Since `Rmpfr` 0.9-0, this has been made an argument with default TRUE to be compatible by default with R’s `sapply`.

Details

In the case `FUN(<length-1>)` returns an array or "mpfrArray", i.e., with two or more dimensions, `sapplyMpfr()` returns an "mpfrArray": this is analogous to `sapply(X, FUN, simplify = "array")` (rather than the default `sapply()` behaviour which returns a matrix also when a higher array would be more “logical”).

Value

an "mpfr" vector, typically of the same length as X.

Note

This may still not always work as well as `sapply()` does for atomic vectors. The examples seem to indicate that it typically does work as desired, since `Rmpfr` version 0.9-0.

If you want to transform back to regular numbers anyway, it maybe simpler and more efficient to use

```
res <- lapply(....)
sapply(res, asNumeric, simplify = "array")
```

instead of `sapplyMpfr()`.

Author(s)

Martin Maechler

See Also

`sapply, lapply`, etc.
Examples

```r
sapplyMpfr0 <- ## Originally, the function was simply defined as
  function (X, FUN, ...) new("mpfr", unlist(lapply(X, FUN, ...), recursive = FALSE))

(m1 <- sapply ( 3, function(k) (1:3)^k)) # 3 x 1 matrix (numeric)
(p1 <- sapplyMpfr(mpfr(3, 64), function(k) (1:3)^k))
stopifnot(m1 == p1, is(p1, "mpfrMatrix"), dim(p1) == c(3,1), dim(p1) == dim(m1))

k.s <- c(2, 5, 10, 20)
(mk <- sapply ( k.s, function(k) (1:3)^k)) # 3 x 4 "
(pm <- sapplyMpfr(mpfr(k.s, 64), function(k) (1:3)^k))
stopifnot(mk == pm, is(pm, "mpfrMatrix"), dim(pm) == 3:4, 3:4 == dim(mk))
  ## was *wrongly* 4x3 in Rmpfr 0.8-x

f5k <- function(k) outer(1:5, k+0:2, `^`)
(mk5 <- sapply ( k.s, f5k)) # matrix-valued
(ak5 <- sapply ( k.s, f5k, simplify = "array")  # what we want
(pm5 <- sapplyMpfr(mpfr(k.s, 64), f5k))
stopifnot(c(mk5) == c(ak5), ak5 == pm5, is(pm5, "mpfrArray"), is.array(ak5),
  dim(pm5) == dim(ak5), dim(pm5) == c(5,3,4))
```

```r
if(require("Bessel")) { # here X, is simple
  bI1 <- function(k) besselI.nuAsym(mpfr(1.31e9, 128), 10, expon.scaled=TRUE, k.max=k)
  bImp1 <- sapplyMpfr (0:4, bI1, drop_1_ = FALSE) # 1x5 mpfrMatrix -- as in DPQ 0.8-8
  bimp <- sapplyMpfr (0:4, bI1, drop_1_ = TRUE ) # 5 "mpfr" vector (by default)
  bImp0 <- sapplyMpfr0(0:4, bI1) # 5-vector
  stopifnot(identical(bImp, bImp0), bImp == bImp1,
    is(bImp, "mpfr"), is(bImp1, "mpfrMatrix"), dim(bImp1) == c(1,5))
}
```

seqMpfr

"mpfr" Sequence Generation

Description

Generate ‘regular’, i.e., arithmetic sequences. This is in lieu of methods for `seq` (dispatching on all three of `from`, `to`, and `by`).

Usage

```r
seqMpfr(from = 1, to = 1, by = ((to - from)/(length.out - 1)),
  length.out = NULL, along.with = NULL, ...)
```

Arguments

- `from, to`: the starting and (maximal) end value (numeric or "mpfr") of the sequence.
- `by`: number (numeric or "mpfr"): increment of the sequence.
- `length.out`: desired length of the sequence. A non-negative number, which will be rounded up if fractional.
- `along.with`: take the length from the length of this argument.
- `...`: arguments passed to or from methods.
Details

see seq (default method in package base), whose semantic we want to replicate (almost).

Value

a ‘vector’ of class "mpfr", when one of the first three arguments was.

Author(s)

Martin Maechler

See Also

The documentation of the base function seq; mpfr

Examples

```r
seqMpfr(0, 1, by = mpfr(0.25, prec=88))
seqMpfr(7, 3) # -> default prec.
```

---

**str.mpfr**

*Compactly Show STRucture of Rmpfr Number Object*

Description

The str method for objects of class mpfr produces a bit more useful output than the default method str.default.

Usage

```r
## S3 method for class 'mpfr'
str(object, nest.lev, internal = FALSE,
     give.head = TRUE, digits.d = 12, vec.len = NULL, drop0trailing=TRUE,
     width = getOption("width"), ...)
```

Arguments

- `object` an object of class mpfr.
- `nest.lev` for str(), typically only used when called by a higher level str().
- `internal` logical indicating if the low-level internal structure should be shown; if true (not by default), uses str(object@.Data).
- `give.head` logical indicating if the “header” should be printed.
sumBinomMpfr

the number of digits to be used, will be passed formatMpfr() and hence NULL will use "as many as needed", i.e. often too many. If this is a number, as per default, less digits will be used in case the precision (getPrec(object)) is smaller.

vec.len

the number of elements that will be shown. The default depends on the precision of object and width (since Rmpfr 0.6-0, it was 3 previously).

drop0trailing

logical, passed to formatMpfr() (with a different default here).

width

the (approximately) desired width of output, see options(width = ).

... further arguments, passed to formatMpfr().

See Also

.mpfr2list() puts the internal structure into a list, and its help page documents many more (low level) utilities.

Examples

(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
str(x)
str(list(pi = pi, x.mpfr = x))
str(x ^ 1000)
str(x ^ -1e4, digits=NULL) # full precision
str(x, internal = TRUE) # internal low-level (for experts)

uu <- Const("pi", 16)# unaccurate
str(uu) # very similar to just 'uu'

sumBinomMpfr (Alternating) Binomial Sums via Rmpfr

Compute (alternating) binomial sums via high-precision arithmetic. If $sBn(f, n) := \text{sumBinomMpfr}(n, f)$, (default alternating is true, and $n0 = 0$),

$$sBn(f, n) = \sum_{k=n0}^{n} (-1)^{(n-k)} \binom{n}{k} \cdot f(k) = \Delta^n f,$$

see Details for the $n$-th forward difference operator $\Delta^n f$. If alternating is false, the $(-1)^{(n-k)}$ factor is dropped (or replaced by 1) above.

Such sums appear in different contexts and are typically challenging, i.e., currently impossible, to evaluate reliably as soon as $n$ is larger than around 50 − 70.

Usage

sumBinomMpfr(n, f, n0 = 0, alternating = TRUE, precBits = 256,
             f.k = f(mpfr(k, precBits=precBits)))
Arguments

- **n**: upper summation index (integer).
- **f**: function to be evaluated at \( k \) for \( k \in n0:n \) (and which must return one value per \( k \)).
- **n0**: lower summation index, typically 0 (= default) or 1.
- **alternating**: logical indicating if the sum is alternating, see below.
- **precBits**: the number of bits for MPFR precision, see \texttt{mpfr}.
- **f.k**: can be specified instead of \( f \) and \( \text{precBits} \), and must contain the equivalent of its default, \( f(\text{mpfr}(k, \text{precBits}=\text{precBits})) \).

Details

The alternating binomial sum \( sB(f, n) := \text{sumBinom}(n, f, n0 = 0) \) is equal to the \( n \)-th forward difference operator \( \Delta^n f \),

\[
sB(f, n) = \Delta^n f,
\]

where

\[
\Delta^n f = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \cdot f(k),
\]

is the \( n \)-fold iterated forward difference \( \Delta f(x) = f(x + 1) - f(x) \) (for \( x = 0 \)).

The current implementation might be improved in the future, notably for the case where \( sB(f, n) = \text{sumBinomMpfr}(n, f, \star) \) is to be computed for a whole sequence \( n = 1, \ldots, N \).

Value

an \texttt{mpfr} number of precision \text{precBits}. \( s \). If \text{alternating} is true (as per default),

\[
s = \sum_{k=n0}^{n} (-1)^{k} \binom{n}{k} \cdot f(k),
\]

if \text{alternating} is false, the \((-1)^{k}\) factor is dropped (or replaced by 1) above.

Author(s)

Martin Maechler, after conversations with Christophe Dutang.

References


See Also

\texttt{chooseMpfr, chooseZ} from package \texttt{gmp}. 
Examples

```r
## "naive" R implementation:
sumBinom <- function(n, f, n0=0, ...) {
  k <- n0:n
  sum( choose(n, k) * (-1)^(n-k) * f(k, ...))
}

## compute sumBinomMpfr(.) for a whole set of 'n' values:
sumBin.all <- function(n, f, n0=0, precBits = 256, ...) {
  N <- length(n)
  precBits <- rep(precBits, length = N)
  ll <- lapply(seq_len(N), function(i)
    sumBinomMpfr(n[i], f, n0=n0, precBits=precBits[i], ...))
  sapply(ll, as, "double")
}

sumBin.all.R <- function(n, f, n0=0, ...) {
  sapply(n, sumBinom, f=f, n0=n0, ...)
}

n.set <- 5:80
system.time(res.R <- sumBin.all.R(n.set, f = sqrt)) ## instantaneous..
system.time(resMpfr <- sumBin.all(n.set, f = sqrt)) ## ~ 0.6 seconds

matplot(n.set, cbind(res.R, resMpfr), type = "l", lty=1,
  ylim = extendrange(resMpfr, f = 0.25), xlab = "n",
  main = "sumBinomMpfr(n, f = sqrt) vs. R double precision")
legend("topleft", leg=c("double prec.", "mpfr"), lty=1, col=1:2, bty = "n")
```

Description

The function `unirootR` searches the interval from lower to upper for a root (i.e., zero) of the function `f` with respect to its first argument.

`unirootR()` is “clone” of `uniroot()`, written entirely in R, in a way that it works with mpfr-numbers as well.

Usage

```r
unirootR(f, interval, ..., 
  lower = min(interval), upper = max(interval), 
  f.lower = f(lower, ...), f.upper = f(upper, ...), 
  extendInt = c("no", "yes", "downX", "upX"), 
  trace = 0, verbose = as.logical(trace),
  verbDigits = max(3, min(20, -log10(tol)/2)),
  tol = .Machine$double.eps*0.25, maxiter = 1000L,
  check.conv = FALSE,
  warn.no.convergence = !check.conv,
  epsC = NULL)
```
Arguments

- **f**
  - the function for which the root is sought.
- **interval**
  - a vector containing the end-points of the interval to be searched for the root.
- **...**
  - additional named or unnamed arguments to be passed to \( f \)
- **lower, upper**
  - the lower and upper end points of the interval to be searched.
- **f.lower, f.upper**
  - the same as \( f(\text{upper}) \) and \( f(\text{lower}) \), respectively. Passing these values from the caller where they are often known is more economical as soon as \( f() \) contains non-trivial computations.
- **extendInt**
  - character string specifying if the interval \( c(\text{lower}, \text{upper}) \) should be extended or directly produce an error when \( f() \) does not have differing signs at the end-points. The default, "no", keeps the search interval and hence produces an error. Can be abbreviated.
- **trace**
  - integer number; if positive, tracing information is produced. Higher values giving more details.
- **verbose**
  - logical (or integer) indicating if (and how much) verbose output should be produced during the iterations.
- **verbDigits**
  - used only if \( \text{verbose} \) is true, indicates the number of digits numbers should be printed with, using \( \text{format}(., \text{digits} = \text{verbDigits}) \).
- **tol**
  - the desired accuracy (convergence tolerance).
- **maxiter**
  - the maximum number of iterations.
- **check.conv**
  - logical indicating whether non convergence should be caught as an error, notably non-convergence in \( \text{maxiter} \) iterations should be an error instead of a warning.
- **warn.no.convergence**
  - if set to FALSE there’s no warning about non-convergence. Useful to just run a few iterations.
- **epsC**
  - positive number or \( \text{NULL} \) in which case a smart default is sought. This should specify the “achievable machine precision” for the given numbers and their arithmetic.
  
  The default will set this to \( .\text{Machine}$\text{double}$.\text{eps} \) for double precision numbers, and will basically use \( 2^\text{-\min(getPrec(f.lower), getPrec(f.upper))} \) when that works (as, e.g., for \text{mpfr}-numbers) otherwise.
  
  This is factually a lower bound for the achievable lower bound, and hence, setting \( \text{tol} \) smaller than \( \text{epsC} \) is typically non-sensical and produces a warning.

Details

Note that arguments after \( \ldots \) must be matched exactly.

Either interval or both lower and upper must be specified: the upper endpoint must be strictly larger than the lower endpoint. The function values at the endpoints must be of opposite signs (or zero), for \( \text{extendInt}="\text{no}" \), the default. Otherwise, if \( \text{extendInt}="\text{yes}" \), the interval is extended on both sides, in search of a sign change, i.e., until the search interval \([l, u]\) satisfies \( f(l) \cdot f(u) \leq 0 \).

If it is known how \( f \) changes sign at the root \( x_0 \), that is, if the function is increasing or decreasing there, \( \text{extendInt} \) can (and typically should) be specified as "upX" (for "upward crossing") or...
“downX”, respectively. Equivalently, define $S := \pm 1$, to require $S = \text{sign}(f(x_0 + \epsilon))$ at the solution. In that case, the search interval $[l, u]$ possibly is extended to be such that $S \cdot f(l) \leq 0$ and $S \cdot f(u) \geq 0$.

The function only uses R code with basic arithmetic, such that it should also work with “generalized” numbers (such as mpfr-numbers) as long the necessary Ops methods are defined for those.

The underlying algorithm assumes a continuous function (which then is known to have at least one root in the interval).

Convergence is declared either if $f(x) = 0$ or the change in $x$ for one step of the algorithm is less than tol (plus an allowance for representation error in $x$).

If the algorithm does not converge in maxiter steps, a warning is printed and the current approximation is returned.

$f$ will be called as $f(x, \ldots)$ for a (generalized) numeric value of $x$.

Value

A list with four components: root and $f$.root give the location of the root and the value of the function evaluated at that point. iter and estm.prec give the number of iterations used and an approximate estimated precision for root. (If the root occurs at one of the endpoints, the estimated precision is NA.)

Source

Based on zeroin() (in package rootoned) by John Nash who manually translated the C code in R’s zeroin.c and on uniroot() in R’s sources.

References

Brent, R. (1973), see uniroot.

See Also

R’s own (stats package) uniroot, polyroot for all complex roots of a polynomial; optimize, nlm.

Examples

```r
require(utils) # for str

## some platforms hit zero exactly on the first step:
## if so the estimated precision is 2/3.
## $f$ <- function (x,a) x - a
str(xmin <- unirootR(f, c(0, 1), tol = 0.0001, a = 1/3))

## handheld calculator example: fixpoint of cos(.):
rc <- unirootR(function(x) cos(x) - x, lower=-pi, upper=pi, tol = 1e-9)
rc$root

## the same with much higher precision:
rcM <- unirootR(function(x) cos(x) - x,
               interval= mpfr(c(-3,3), 300), tol = 1e-40)
```
rcM
x0 <- rcM$root
stopifnot(all.equal(cos(x0), x0,
  tol = 1e-40))## 40 digits accurate!

str(unirootR(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,
  tol = 0.0001), digits.d = 10)
str(unirootR(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,
  tol = 1e-10), digits.d = 10)

## A sign change of f(.), but not a zero but rather a "pole":
tan. <- function(x) tan(x * (Const("pi",200)/180))# == tan(<angle>)
(rtan <- unirootR(tan., interval = mpfr(c(80,100), 200), tol = 1e-40))
## finds 90 ("ok"), and now gives a warning

## Find the smallest value x for which exp(x) > 0 (numerically):
r <- unirootR(function(x) 1e80*exp(x)-1e-300, c(-1000,0), tol = 1e-15)
str(r, digits.d = 15) ##> around -745, depending on the platform.

exp(r$root) # = 0, but not for r$root * 0.999...
minexp <- r$root * (1 - 10*.Machine$double.eps)
exp(minexp) # typically denormalized

## --- using mpfr-numbers:

## Find the smallest value x for which exp(x) > 0 ("numerically");
## Note that mpfr-numbers underflow *MUCH* later than doubles:
## one of the smallest mpfr-numbers (see also ?mpfr-class ) :
(ep.M <- mpfr(2, 55) ^ - ((2^30 + 1) * (1 - 1e-15)))
r <- unirootR(function(x) 1e99* exp(x) - ep.M, mpfr(c(-1e20, 0), 200))
r # 97 iterations; f.root is very similar to ep.M

## interval extension 'extendInt' 

f1 <- function(x) (121 - x^2)/(x^2+1)
f2 <- function(x) exp(-x)*(x - 12)
tools::assertError(unirootR(f1, c(0,10)), verbose=TRUE)
##--> error: f() .. end points not of opposite sign

## where as 'extendInt="yes"' simply first enlarges the search interval:
u1 <- unirootR(f1, c(0,10),extendInt="yes", trace=1)
u2 <- unirootR(f2, mpfr(c(0,2), 128), extendInt="yes", trace=2, verbose=FALSE, tol = 1e-25)
stopifnot(all.equal(u1$root, 11, tolerance = 1e-5),
  all.equal(u2$root, 12, tolerance = 1e-23))

## The *danger* of interval extension:
## No way to find a zero of a positive function, but
## numerically, f(-|M|) becomes zero:
u3 <- unirootR(exp, c(0,2), extendInt="yes", trace=TRUE)

## Nonsense example (must give an error):
tools::assertCondition(unirootR(function(x) 1, 0:1, extendInt="yes"),
unirootR

"error", verbose=TRUE)
Index

* **Forward Difference**
  sumBinomMpfr, 73
* **Rice integral**
  sumBinomMpfr, 73
* **arithmetic**
  frexpMpfr, 21
* **arith**
  Bernoulli, 9
  chooseMpfr, 12
  factorialMpfr, 14
  formatHex, 16
  gmp-conversions, 23
  matmult, 33
  mpfr.utils, 52
  pbetaI, 61
  pmax, 64
  roundMpfr, 69
  sumBinomMpfr, 73
* **array**
  mpfrArray, 53
  mpfrMatrix-utils, 57
* **character**
  formatMpfr, 19
* **classes**
  array_or_vector-class, 6
  atomicVector-class, 8
  Mnumber-class, 35
  mpfr, 36
  mpfr-class, 38
  mpfrMatrix, 54
* **distribution**
  mpfr-distr-etc, 44
  pbetaI, 61
  qnormI, 65
* **manip**
  sapplyMpfr, 69
  seqMpfr, 71
* **math**
  Bessel_mpfr, 10
* igamma, 26
  integrateR, 28
  is.whole, 30
  log1mexp, 31
  mpfr-special-functions, 45
  qnormI, 65
* **methods**
  asNumeric-methods, 7
  bind-methods, 11
  pmax, 64
* **misc**
  Rmpfr-workarounds, 68
* **optimize**
  hjkMpfr, 24
  optimizeR, 59
  unirootR, 75
* **package**
  Rmpfr-package, 3
* **print**
  formatMpfr, 19
* **univar**
  pmax, 64
* **utilities**
  frexpMpfr, 21
  integrateR, 28
  mpfr-utils, 47
  str.mpfr, 72
  mpfr2bigz (gmp-conversions), 23
  mpfr2bigz (gmp-conversions), 23
  .Machine, 41, 50, 76
  mpfr2mpfr (gmp-conversions), 23
  mpfr2mpfr (gmp-conversions), 23
  .getPrec (mpfr-utils), 47
  .getSign (mpfr-utils), 47
  .matmult.R (matmult), 33
  mpfr (mpfr-utils), 47
  mpfr.is.whole (mpfr.utils), 52
  mpfr2bigz (gmp-conversions), 23
  mpfr2exp, 21, 22
INDEX

.mpf

.mpfr2exp (mpfr-utils), 47
.mpfr2list, 73
.mpfr2list (mpfr-utils), 47
.mpfr2str, 19, 50
.mpfr2str (formatMpfr), 19
.mpfrVersion (mpfr-utils), 47
.mpfr_negative (mpfr-utils), 47
.mpfr_sign (mpfr-utils), 47
.

[,mpfr,ANY,missing,missing-method (mpfr-class), 38
[,mpfrArray,ANY,ANY,ANY-method (mpfrMatrix), 54
[,mpfrArray,ANY,missing,missing-method (mpfrMatrix), 54
[,mpfrArray,matrix,missing,ANY-method (mpfrMatrix), 54
[<-,mpfr,ANY,missing,ANY-method (mpfr-class), 38
[<-,mpfr_ARRAY,missing,missing,ANY-method (mpfrMatrix), 54
[<-,mpfrArray,missing,missing,mpfr-method (mpfrMatrix), 54
[[,mpfr-method (mpfr-class), 38
%*%,Mnumber,mpfr-method (mpfrMatrix), 54
%*%,array_or_vector,mpfr-method (mpfr-class), 38
%*%,mpfr,Mnumber-method (mpfrMatrix), 54
%*%,mpfr,mpfrMatrix-method (mpfrMatrix), 54
%*%,mpfrMatrix,mpfr-method (mpfrMatrix), 54
%*%,mpfrMatrix,mpfrMatrix-method (mpfrMatrix), 54
%*%, 34, 55

abs, 40, 58
abs,mpfr-method (mpfr-class), 38
acos, 40
acosh, 40
Ai (Bessel_mpfr), 10
all, 40
all.equal, 41
all.equal,ANY,mpfr-method (mpfr-class), 38
all.equal,mpfr,ANY-method (mpfr-class), 38
all.equal,mpfr,mpfr-method (mpfr-class), 38
any, 40
aperm, 56
aperm,mpfrArray-method (mpfrMatrix), 54
apply,mpfrArray-method (mpfrMatrix), 54
Arg,mpfr-method (mpfr-class), 38
Arith,mpfr-method (mpfr-class), 38
Arith,Integer,mpfr-method (mpfr-class), 38
Arith,mpfr,mpfr-method (mpfr-class), 38
Arith,mpfr,Integer-method (mpfr-class), 38
Arith,mpfr,missing-method (mpfr-class), 38
Arith,mpfr,mpfrArray-method (mpfrMatrix), 54
Arith, mpfr, numeric-method (mpfr-class), 38
Arith, mpfrArray, mpfr-method (mpfrMatrix), 54
Arith, mpfrArray, mpfrArray-method (mpfrMatrix), 54
Arith, mpfrArray, numeric-method (mpfrMatrix), 54
Arith, numeric, mpfr-method (mpfr-class), 38
Arith, numeric, mpfrArray-method (mpfrMatrix), 54
Arith, numeric, mpfrArray-method (mpfrMatrix), 54
array, 5, 7, 20, 36, 41, 50, 53–56, 70
array_or_vector, 35
array_or_vector-class, 6
as, 41
as.bigq, 24
as.bigz, 23, 24
as.integer, mpfr-method (mpfr-class), 38
as.numeric, 7
as.numeric, mpfr-method (mpfr-class), 38
as.vector, mpfrArray, missing-method (mpfrMatrix), 54
as.vector, mpfrArray-method (mpfr-class), 38
asin, 40
asinh, 40
asNumeric, 7, 37, 50, 54, 58
asNumeric, mpfr-method (asNumeric-methods), 7
asNumeric, mpfrArray-method (asNumeric-methods), 7
asNumeric-methods, 7
atan, 39, 40
atan2, ANY, mpfr-method (mpfr-class), 38
atan2, ANY, mpfrArray-method (mpfr-class), 38
atan2, mpfr, ANY-method (mpfr-class), 38
atan2, mpfr, mpfr-method (mpfr-class), 38
atan2, mpfr, numeric-method (mpfr-class), 38
atan2, mpfrArray, ANY-method (mpfr-class), 38
atan2, mpfrArray, mpfrArray-method (mpfr-class), 38
atan2, numeric, mpfr-method (mpfr-class), 38
atanh, 40
atomicVector-class, 8
base::pmin, 65
Bernoulli, 5, 9, 42
Bessel_mpfr, 5, 10, 46
besselI, 10
besselY, 10
beta, 40
beta, ANY, mpfr-method (mpfr-class), 38
beta, ANY, mpfrArray-method (mpfr-class), 38
beta, mpfr, ANY-method (mpfr-class), 38
beta, mpfr, mpfr-method (mpfr-class), 38
beta, mpfr, numeric-method (mpfr-class), 38
beta, mpfrArray, ANY-method (mpfr-class), 38
beta, mpfrArray, mpfrArray-method (mpfr-class), 38
beta, numeric, mpfr-method (mpfr-class), 38
beta, numeric, mpfrArray-method (mpfr-class), 38
bigq, 23, 36, 62
bigRational, 6
bigz, 23, 36, 41
bind-methods, 11
c, 52
c.mpfr, 5
c.mpfr (mpfr.utils), 52
cbind, 5, 11, 12
cbind (bind-methods), 11
cbind, ANY-method (bind-methods), 11
cbind, Mnumber-method (bind-methods), 11
cbind-methods (bind-methods), 11
cbind2, 12
ceiling, 40
character, 36, 37, 41, 48, 49, 55, 59
choose, 12, 13
chooseMpf, 5, 12, 74
chooseZ, 12, 13, 62, 74
class, 8, 12, 17, 31, 39, 48, 64
coerce, array, mpfr-method (mpfr-class), 38
coerce, array, mpfrArray-method (mpfrMatrix), 54
coerce, bigq, mpfr-method (gmp-conversions), 23
coerce, bigz, mpfr-method (gmp-conversions), 23
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>coerce.character.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.integer.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.logical.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.matrix.mpfrMatrix-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr.bigz-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr.character-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr.integer-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr.mpfr1-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr.numeric-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr1.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfr1.numeric-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfrArray.array-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfrArray.matrix-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfrArray.vector-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>coerce.mpfrMatrix.matrix-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>coerce.numeric.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.numeric.mpfr1-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce.raw.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>coerce&lt;-, mpfrArray.vector-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>colMeans.mpfrArray-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>colSums.mpfrArray-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>Compare.array.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>Compare.integer.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>Compare.mpfr.array-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>Compare.mpfr.integer-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>complex</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Conj.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>Const.mpfr</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>cos</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>cosh</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>cospi</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>crossprod</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>crossprod.array_or_vector.mpfr-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>crossprod.Mnumber.mpfr-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfr.array_or_vector-method</td>
<td>(mpfr-class), 38</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfr.missing-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfr.Mnumber-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfr.mpfr-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfr.mpfrMatrix-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfrMatrix.mpfr-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>crossprod.mpfrMatrix.mpfrMatrix-method</td>
<td>(mpfrMatrix), 54</td>
<td></td>
</tr>
<tr>
<td>cummax</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>cummin</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>cumprod</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>cumsum</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>dbinom</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>dbinom.mpfr-distr-etc</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>determinant</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>determinant.mpfrMatrix</td>
<td>(mpfrMatrix-utils), 57</td>
<td></td>
</tr>
</tbody>
</table>
INDEX

dgamma. 45
dgamma (mpfr-distr-etc), 44
diag, mpfrMatrix-method (mpfrMatrix), 54
diag<-, mpfrMatrix-method (mpfrMatrix), 54
diff, 52
diff.default, 52
diff.mpfr (mpfr.utils), 52
digamma, 5, 40
dim, 7, 20, 40, 55
dim, mpfrArray-method (mpfrMatrix), 54
dim<-, mpfr-method (mpfr-class), 38
dimnames, 20
dimnames, mpfrArray-method (mpfrMatrix), 54
dimnames<-, mpfrArray-method (mpfrMatrix), 54
dbinom, 45
dbnbinom (mpfr-distr-etc), 44
dnorm (mpfr-distr-etc), 44
dotsMethods, 11
double, 48, 50
dpois, 44, 45
dpois (mpfr-distr-etc), 44
dt, 45
dt (mpfr-distr-etc), 44
duplicated, 41

Ei (mpfr-special-functions), 45
erf, 44, 53
erf (mpfr-special-functions), 45
erfc (mpfr-special-functions), 45
exp, 40
expml, 40

factorial, 13, 15
factorial, mpfr-method (mpfr-class), 38
factorialMpfr, 5, 13, 14, 41
factorialZ, 15
floor, 40
format, 16, 17, 19–21, 49, 76
format, mpfr-method (mpfr-class), 38
formatBin, 36
formatBin (formatHex), 16
formatDec (formatHex), 16
formatHex, 16
formatMpfr, 5, 17, 19, 41, 48, 49, 73
formatN, 21
formatN.mpfr (formatMpfr), 19

frexp, 22
frexpMpfr, 21
function, 70, 74

gamma, 5, 13, 15, 27, 40
getD (mpfr-utils), 47
getGroupMembers, 40
getPrec. 5, 16, 36, 65, 69, 73
getPrec (mpfr-utils), 47
gmp, 6
gmp-conversions, 23
golden_ratio, 60

hjk, 24
hjkMpfr, 5, 24, 60
hypot (mpfr-class), 38

igamma, 26
Im, mpfr-method (mpfr-class), 38
integer, 8, 13, 41, 48–50
integrate, 29, 30
integrateR, 5, 28
interactive, 48
invisible, 50

is.atomic, 8
is.finite, 50
is.finite, mpfr-method (mpfr-class), 38
is.finite, mpfrArray-method (mpfr-class), 38
is.infinite, mpfr-method (mpfr-class), 38
is.infinite, mpfrArray-method (mpfr-class), 38
is.integer, 31
is.mpfr, 36
is.na, mpfr-method (mpfr-class), 38
is.na, mpfrArray-method (mpfr-class), 38
is.nan, mpfr-method (mpfr-class), 38
is.nan, mpfrArray-method (mpfr-class), 38
is.whole, 30, 31, 42, 52

j0, 46
j0 (Bessel_mpfr), 10
j1 (Bessel_mpfr), 10
jn, 42
jn (Bessel_mpfr), 10

laply, 69, 70
lbeta, ANY, mpfr-method (mpfr-class), 38
lbeta, ANY, mpfrArray-method (mpfr-class), 38
INDEX

lbeta, mpfr, ANY-method (mpfr-class), 38
lbeta, mpfr, mpfr-method (mpfr-class), 38
lbeta, mpfr, numeric-method (mpfr-class), 38
lbeta, mpfrArray, ANY-method (mpfr-class), 38
lbeta, mpfrArray, mpfrArray-method (mpfr-class), 38
lbeta, numeric, mpfr-method (mpfr-class), 38
ldexpMpfr (frexpMpfr), 21
lgamma, 40
Li2 (mpfr-special-functions), 45
list, 22, 24, 25, 39, 55, 58, 60, 66, 73
load, 50
log, 40, 58
log, mpfr-method (mpfr-class), 38
log10, 40
log1mexp, 31
log1p, 40
log1pexp (log1mexp), 31
log2, 40
Logic, mpfr, mpfr-method (mpfr-class), 38
Logic, mpfr, numeric-method (mpfr-class), 38
Logic, numeric, mpfr-method (mpfr-class), 38
logical, 16, 19, 20, 41, 44, 49, 50, 62
Math, 40, 46
Math, mpfr-method (mpfr-class), 38
Math2, 46
Math2, mpfr-method (mpfr-class), 38
matmult, 33
matrix, 7, 34, 50, 54
max, 40
mean, 40
mean, mpfr-method (mpfr-class), 38
mean.default, 40
median, mpfr-method (mpfr-class), 38
min, 40, 65
missing, 36
Mnumber, 11
Mnumber-class, 35
Mnumber-class (Mnumber-class), 35
Mod, mpfr-method (mpfr-class), 38
mpfr-class, 5, 38
mpfr-distr (mpfr-distr-etc), 44
mpfr-distr-etc, 44
mpfr-special-functions, 45
mpfr-utils, 47
mpfr.is.0 (mpfr.utils), 52
mpfr.is.integer (mpfr.utils), 52
mpfr.utils, 52
mpfr1, 55
mpfr1-class (mpfr-class), 38
mpfr2array, 54
mpfr2array (mpfr-utils), 47
mpfr_default_prec (mpfr-utils), 47
mpfrArray, 5, 7, 20, 37, 40, 48–50, 53, 53, 54, 56, 70
mpfrArray-class (mpfrMatrix), 54
mpfrImport (mpfr-utils), 47
mpfrIs0, 50
mpfrIs0 (mpfr.utils), 52
mpfrMatrix, 7, 11, 12, 34, 35, 37, 42, 50, 53, 54, 58
mpfrMatrix-class, 5
mpfrMatrix-class (mpfrMatrix), 54
mpfrMatrix-utils, 57
mpfrVersion (mpfr.utils), 52
mpfrXport (mpfr-utils), 47
names, 49
NaN, 40
n1m, 77
norm, 56
norm, ANY, missing-method (mpfrMatrix), 54
norm, mpfrMatrix, character-method (mpfrMatrix), 54
NULL, 48, 55
numeric, 8, 10, 27, 34, 36, 37, 39, 41, 44, 46, 49, 52, 62, 70
numeric_version, 52
numericVector-class (Mnumber-class), 35
Ops, 77
Ops, ANY, mpfr-method (mpfr-class), 38
Ops, array, mpfr-method (mpfr-class), 38
Ops, bigq, mpfr-method (mpfr-class), 38
Ops, bign, mpfr-method (mpfr-class), 38
Ops, mpfr, ANY-method (mpfr-class), 38
Ops, mpfr, array-method (mpfr-class), 38
Ops, mpfr, mpfr-method (mpfr-class), 38
Ops, mpfr, bigq-method (mpfr-class), 38
Ops, mpfr, bign-method (mpfr-class), 38
<table>
<thead>
<tr>
<th>Function</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ops, mpfr, vector-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>Ops, vector, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>optim</td>
<td>26</td>
</tr>
<tr>
<td>optimize</td>
<td>59, 60, 77</td>
</tr>
<tr>
<td>optimizeR</td>
<td>5, 26, 59</td>
</tr>
<tr>
<td>options</td>
<td>19, 49, 73</td>
</tr>
<tr>
<td>order</td>
<td>42</td>
</tr>
<tr>
<td>outer</td>
<td>68</td>
</tr>
<tr>
<td>outer (Rmpfr-workarounds)</td>
<td>68</td>
</tr>
<tr>
<td>pbeta</td>
<td>45, 62</td>
</tr>
<tr>
<td>pbetaI</td>
<td>45, 61</td>
</tr>
<tr>
<td>pgamma</td>
<td>27</td>
</tr>
<tr>
<td>pmax</td>
<td>64, 64</td>
</tr>
<tr>
<td>pmax, ANY-method (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmax, mNumber-method (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmax-methods (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmin</td>
<td>65</td>
</tr>
<tr>
<td>pmin (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmin, ANY-method (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmin, mNumber-method (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pmin-methods (pmax)</td>
<td>64</td>
</tr>
<tr>
<td>pnorm</td>
<td>5, 44–46, 65</td>
</tr>
<tr>
<td>pnorm (mpfr-distr-etc)</td>
<td>44</td>
</tr>
<tr>
<td>pochMpfr</td>
<td>15, 42</td>
</tr>
<tr>
<td>pochMpfr (chooseMpfrr)</td>
<td>12</td>
</tr>
<tr>
<td>polyroot</td>
<td>77</td>
</tr>
<tr>
<td>prettyNum</td>
<td>20, 21</td>
</tr>
<tr>
<td>print</td>
<td>17, 19, 29, 39</td>
</tr>
<tr>
<td>print.default</td>
<td>48</td>
</tr>
<tr>
<td>print.inegrate</td>
<td>29</td>
</tr>
<tr>
<td>print.inegrateR</td>
<td>28</td>
</tr>
<tr>
<td>print.mpfr (mpfr-utils)</td>
<td>47</td>
</tr>
<tr>
<td>print.mpfr1 (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>print.mpfrArray (mpfr-utils)</td>
<td>47</td>
</tr>
<tr>
<td>print.Ncharacter (formatHex)</td>
<td>16</td>
</tr>
<tr>
<td>print.summaryMpfr (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>prod</td>
<td>40</td>
</tr>
<tr>
<td>qnorm</td>
<td>65, 66</td>
</tr>
<tr>
<td>qnormI</td>
<td>65</td>
</tr>
<tr>
<td>quantile</td>
<td>42</td>
</tr>
<tr>
<td>quantile, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>range</td>
<td>40, 65</td>
</tr>
<tr>
<td>rank</td>
<td>42</td>
</tr>
<tr>
<td>raw</td>
<td>41</td>
</tr>
<tr>
<td>rbind (bind-methods)</td>
<td>11</td>
</tr>
<tr>
<td>rbind, ANY-method (bind-methods)</td>
<td>11</td>
</tr>
<tr>
<td>rbind, mNumber-method (bind-methods)</td>
<td>11</td>
</tr>
<tr>
<td>rbind-methods (bind-methods)</td>
<td>11</td>
</tr>
<tr>
<td>Re, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>Rmpfr (Rmpfr-package)</td>
<td>3</td>
</tr>
<tr>
<td>Rmpfr-package</td>
<td>3</td>
</tr>
<tr>
<td>Rmpfr-workarounds</td>
<td>68</td>
</tr>
<tr>
<td>round</td>
<td>41, 69</td>
</tr>
<tr>
<td>roundMpfr</td>
<td>5, 37, 41, 42, 49, 69</td>
</tr>
<tr>
<td>rowMeans, mpfrArray-method (mpfrMatrix)</td>
<td>54</td>
</tr>
<tr>
<td>rowSums, mpfrArray-method (mpfrMatrix)</td>
<td>54</td>
</tr>
<tr>
<td>sample</td>
<td>69, 70</td>
</tr>
<tr>
<td>sampleMpfr</td>
<td>69</td>
</tr>
<tr>
<td>save</td>
<td>50</td>
</tr>
<tr>
<td>seq</td>
<td>71, 72</td>
</tr>
<tr>
<td>seqMpfr</td>
<td>5, 71</td>
</tr>
<tr>
<td>setPrec</td>
<td>(roundMpfr), 69</td>
</tr>
<tr>
<td>show, integrateR-method</td>
<td>28</td>
</tr>
<tr>
<td>show, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>show, mpfr1-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>show, mpfrArray-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>show, summaryMpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>sign</td>
<td>39, 40, 50</td>
</tr>
<tr>
<td>sign, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>sign, mpfrArray-method (mpfrMatrix)</td>
<td>54</td>
</tr>
<tr>
<td>signif</td>
<td>41</td>
</tr>
<tr>
<td>sin</td>
<td>40</td>
</tr>
<tr>
<td>sinh</td>
<td>40</td>
</tr>
<tr>
<td>sinpi</td>
<td>40</td>
</tr>
<tr>
<td>sort</td>
<td>42</td>
</tr>
<tr>
<td>sprintf</td>
<td>16, 17, 20</td>
</tr>
<tr>
<td>sqrt</td>
<td>40</td>
</tr>
<tr>
<td>str</td>
<td>53, 72</td>
</tr>
<tr>
<td>str.default</td>
<td>72</td>
</tr>
<tr>
<td>str.mpfr</td>
<td>53, 72</td>
</tr>
<tr>
<td>sum</td>
<td>40</td>
</tr>
<tr>
<td>sumBinomMpfr</td>
<td>5, 13, 62, 73</td>
</tr>
<tr>
<td>Summary</td>
<td>40</td>
</tr>
<tr>
<td>Summary, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>summary, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>summary.default</td>
<td>40</td>
</tr>
<tr>
<td>summaryMpfr-class (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>t, mpfr-method (mpfr-class)</td>
<td>38</td>
</tr>
<tr>
<td>t, mpfrMatrix-method (mpfrMatrix)</td>
<td>54</td>
</tr>
<tr>
<td>tan</td>
<td>40</td>
</tr>
</tbody>
</table>
INDEX

tanh, 40
tanpi, 40
tcrossprod, 34
tcrossprod, array_or_vector, mpfr-method
(mpfr-class), 38
tcrossprod, Mnumber, mpfr-method
(mpfrMatrix), 54
tcrossprod, mpfr, array_or_vector-method
(mpfr-class), 38
tcrossprod, mpfr, missing-method
(mpfrMatrix), 54
tcrossprod, mpfr, Mnumber-method
(mpfrMatrix), 54
tcrossprod, mpfr, mpfr-method
(mpfrMatrix), 54
tcrossprod, mpfrMatrix, mpfr-method
(mpfrMatrix), 54
tcrossprod, mpfrMatrix, mpfrMatrix-method
(mpfrMatrix), 54
toNum, 7
toNum (mpfr-utils), 47
trigamma, 5, 40
trunc, 40
typeof, 7, 39
unique, 41
unique, mpfr, ANY-method (mpfr-class), 38
unique, mpfr-method (mpfr-class), 38
unique.mpfr (mpfr-class), 38
uniroot, 75, 77
unirootR, 5, 60, 65, 66, 75
vapply, 69
vector, 55
Vectorize, 29
which.max, 42
which.max, mpfr-method (mpfr-class), 38
which.min, 42
which.min, mpfr-method (mpfr-class), 38
y0 (Bessel_mpfr), 10
y1 (Bessel_mpfr), 10
yn, 46
yn (Bessel_mpfr), 10
zeta, 5, 9, 42
zeta (mpfr-special-functions), 45