Package ‘SLEMI’

November 19, 2023

**Title**  Statistical Learning Based Estimation of Mutual Information

**Version**  1.0.2

**Description**  The implementation of the algorithm for estimation of mutual information and channel capacity from experimental data by classification procedures (logistic regression). Technically, it allows to estimate information-theoretic measures between finite-state input and multivariate, continuous output. Method described in Jetka et al. (2019) <doi:10.1371/journal.pcbi.1007132>.

**Depends**  R (>= 3.6.0)

**License**  GPL (>= 3)

**URL**  https://github.com/TJetka/SLEMI

**BugReports**  https://github.com/TJetka/SLEMI/issues

**Encoding**  UTF-8

**LazyData**  true

**Imports**  e1071, ggplot2, gridExtra, nnet, Hmisc, reshape2, stringr, doParallel, caret, corrplot, foreach, methods

**Suggests**  knitr, rmarkdown, testthat (>= 2.1.0), data.table, covr

**VignetteBuilder**  knitr

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**NeedsCompilation**  no

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capacity_logreg_algorithm

Main algorithm to calculate channel capacity by SLEMI approach

Description

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). It is recommended to conduct estimation by calling capacity_logreg_main.R.

Usage

```r
capacity_logreg_algorithm(
  data,
  signal = "signal",
  response = "response",
  side_variables = NULL,
  formula_string = NULL,
  model_out = TRUE,
  cc_maxit = 100,
  lr_maxit = 1000,
  MaxNWts = 5000
)
```

Arguments

data must be a data.frame object. Cannot contain NA values.
signal is a character object with names of columns of dataRaw to be treated as channel’s input.
response is a character vector with names of columns of dataRaw to be treated as channel’s output
side_variables (optional) is a character vector that indicates side variables’ columns of data, if NULL no side variables are included
formula_string (optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
model_out is the logical indicating if the calculated logistic regression model should be included in output list.

cc_maxit is the number of iteration of iterative optimisation of the algorithm to estimate channel capacity. Default is 100.

lr_maxit is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.

MaxNWts is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.

Value

a list with three elements:

- output$cc - channel capacity in bits
- output$p_opt - optimal probability distribution
- output$regression - confusion matrix of logistic regression predictions
- output$model - nnet object describing logistic regression model (if model_out=TRUE)

References


Examples

```r
tempdata=data_example1
outputCLR1=capacity_logreg_algorithm(data=tempdata, signal="signal", response="response", cc_maxit=3, model_out=FALSE, formula_string = "signal~response")
```

Description

The main wrapping function for basic usage of SLEMI package for estimation of channel capacity. Firstly, data is pre-processed (all arguments are checked, observation with NAs are removed, variables are scaled and centered (if scale=TRUE)). Then basic estimation is carried out and (if testing=TRUE) diagnostic tests are computed. If output directory path is given (output_path is not NULL), graphs visualising the data and the analysis are saved there, together with a compressed output object (as .rds file) with full estimation results.
capacity_logreg_main

Usage

capacity_logreg_main(
  dataRaw,
  signal = "input",
  response = NULL,
  output_path = NULL,
  side_variables = NULL,
  formula_string = NULL,
  cc_maxit = 100,
  lr_maxit = 1000,
  MaxNWts = 5000,
  testing = FALSE,
  model_out = TRUE,
  scale = TRUE,
  TestingSeed = 1234,
  testing_cores = 1,
  boot_num = 10,
  boot_prob = 0.8,
  sidevar_num = 10,
  traintest_num = 10,
  partition_trainfrac = 0.6,
  plot_width = 6,
  plot_height = 4,
  data_out = FALSE
)

Arguments

dataRaw: must be a data.frame object
signal: is a character object with names of columns of dataRaw to be treated as channel’s input.
response: is a character vector with names of columns of dataRaw to be treated as channel’s output
output_path: is the directory in which output will be saved
side_variables: (optional) is a character vector that indicates side variables’ columns of data, if NULL no side variables are included
formula_string: (optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
cc_maxit: is the number of iteration of iterative optimisation of the algorithm to estimate channel capacity. Default is 100.
lr_maxit: is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
MaxNWts: is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
model_out: is the logical indicating if the testing procedures should be executed
model_out is the logical indicating if the calculated logistic regression model should be included in output list
scale is a logical indicating if the response variables should be scaled and centered before fitting logistic regression
TestingSeed is the seed for random number generator used in testing procedures
testing_cores - number of cores to be used in parallel computing (via doParallel package)
boot_num is the number of bootstrap tests to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
boot_prob is the proportion of initial size of data to be used in bootstrap
sidevar_num is the number of re-shuffling tests of side variables to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
traintest_num is the number of overfitting tests to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
partition_trainfrac is the fraction of data to be used as a training dataset
plot_width - basic dimensions (width) of plots, in inches
plot_height - basic dimensions (height) of plots, in inches
data_out is the logical indicating if the data should be included in output list

Details

In a typical experiment aimed to quantify information flow a given signaling system, input values $x_1 \leq x_2 \ldots \leq x_m$, ranging from 0 to saturation are considered. Then, for each input level, $x_i$, $n_i$ observations are collected, which are represented as vectors

$$y_i^j \sim P(Y|X = x_i)$$

Within information theory the degree of information transmission is measured as the mutual information

$$MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X = x_i) \log_2 \frac{P(y|X = x_i)}{P(y)} dy,$$

where $P(y)$ is the marginal distribution of the output. MI is expressed in bits and $2^{\text{MI}}$ can be interpreted as the number of inputs that the system can resolve on average.

The maximization of mutual information with respect to the input distribution, $P(X)$, defines the information capacity, $C$. Formally,

$$C^\star = \max_{P(X)} MI(X,Y)$$

Information capacity is expressed in bits and $2^{C^\star}$ can be interpreted as the maximal number of inputs that the system can effectively resolve.

In contrast to existing approaches, instead of estimating, possibly highly dimensional, conditional output distributions $P(Y|X = x_i)$, we propose to estimate the discrete, conditional input distribution, $P(x_i|Y = y)$, which is known to be a simpler problem. Estimation of the MI using estimates of $P(x_i|Y = y)$, denoted here as $\hat{P}(x_i|Y = y)$, is possible as the MI, can be alternatively written as

$$MI(X,Y) = \sum_{i=1}^{m} \hat{P}(x_i) \int_{R^k} \hat{P}(y|X = x_i) \log_2 \frac{\hat{P}(y|X = x_i)}{\hat{P}(x_i)} dy$$
The expected value (as in above expression) with respect to distribution $P(Y|X = x_i)$ can be approximated by the average with respect to data

$$MI(X, Y) \approx \sum_{i=1}^{n} P(x_i) \frac{1}{n_i} \sum_{j=1}^{n_i} P(y|X = x_i) \log_2 \frac{\hat{P}(x_i|Y = y_j)}{P(x_i)} dy$$

Here, we propose to use logistic regression as $\hat{P}(x_i|Y = y_j)$. Specifically,

$$\log \frac{P(x_i|Y = y)}{P(x_i|Y = y_j)} \approx \alpha_i + \beta_i y$$

Following this approach, channel capacity can be calculated by optimising MI with respect to the input distribution, $P(X)$. However, this, potentially difficult problem, can be divided into two simpler maximization problems, for which explicit solutions exist. Therefore, channel capacity can be obtained from the two explicit solutions in an iterative procedure known as alternate maximization (similarly as in Blahut-Arimoto algorithm) [1].

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

**Value**

a list with several elements:

- `output$regression` - confusion matrix of logistic regression predictions
- `output$cc` - channel capacity in bits
- `output$p_opt` - optimal probability distribution
- `output$model` - nnet object describing logistic regression model (if model_out=TRUE)
- `output$params` - parameters used in algorithm
- `output$time` - computation time of calculations
- `output$testing` - a 2- or 4-element output list of testing procedures (if testing=TRUE)
- `output$testing_pv` - one-sided p-values of testing procedures (if testing=TRUE)
- `output$data` - raw data used in analysis

**References**


data_nfkb

Examples

```r
tempdata=data_example1
tempdata=data_example2
```

# For further details see vignette

---

Data from experiment with NFκB pathway

Description

In the paper describing methodological aspects of our algorithm we present the analysis of information transmission in NFκB pathway upon the stimulation of TNF-α. Experimental data from this experiment in the form of single-cell time series are attached to the package as a data.frame object and can be accessed using ‘data_nfkb’ variable. Each row of ‘data_nfkb’ represents a single observation of a cell. Column ‘signal’ indicates the level of TNF-α stimulation for a given cell, while columns ‘response’ gives the normalised ratio of nuclear and cytoplasmic transcription factor as described in Supplementary Methods of the corresponding publication. In the CRAN version of the package we included only a subset of the data (5 time measurements). For the full datasets, please access GitHub pages.

Usage

data_nfkb

Format

A data frame with 15632 rows and 6 variables:

- **signal** Level of TNFα stimulation
- **response_0** The concentration of normalised NFκB transcription factor, measured at time 0
- **response_3** The concentration of normalised NFκB transcription factor, measured at time 3
- **response_21** The concentration of normalised NFκB transcription factor, measured at time 21
- **response_90** The concentration of normalised NFκB transcription factor, measured at time 90
- **response_120** The concentration of normalised NFκB transcription factor, measured at time 120

Details

For each concentration, there are at least 1000 single-cell observation (with the exception of 0.5ng stimulation, where the number of identified cells is almost 900)
mi_logreg_algorithm

Main algorithm to calculate mutual information by SLEMI approach

**Source**

in-house experimental data

**Description**

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). It is recommended to conduct estimation by calling mi_logreg_main.R.

**Usage**

```r
mi_logreg_algorithm(
  data,
  signal = "signal",
  response = "response",
  side_variables = NULL,
  pinput = NULL,
  formula_string = NULL,
  lr_maxit = 1000,
  MaxNWts = 5000,
  model_out = TRUE
)
```

**Arguments**

- **data** must be a data.frame object. Cannot contain NA values.
- **signal** is a character object with names of columns of dataRaw to be treated as channel’s input.
- **response** is a character vector with names of columns of dataRaw to be treated as channel’s output
- **side_variables** (optional) is a character vector that indicates side variables’ columns of data, if NULL no side variables are included
- **pinput** is a numeric vector with prior probabilities of the input values. Uniform distribution is assumed as default (pinput=NULL).
- **formula_string** (optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
- **lr_maxit** is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
- **MaxNWts** is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
- **model_out** is the logical indicating if the calculated logistic regression model should be included in output list
Value

a list with three elements:

• output$mi - mutual information in bits
• output$pinput - prior probabilities used in estimation
• output$regression - confusion matrix of logistic regression model
• output$model - nnet object describing logistic regression model (if model_out=TRUE)

References


Examples

```r
## Estimate mutual information directly
temp_data=data_example1
output=mi_logreg_algorithm(data=data_example1,
                          signal = "signal",
                          response = "response")
```

### Description

The main wrapping function for basic usage of SLEMI package for estimation of mutual information. Firstly, data is pre-processed (all arguments are checked, observation with NAs are removed, variables are scaled and centered (if scale=TRUE)). Then basic estimation is carried out and (if testing=TRUE) diagnostic tests are computed. If output directory path is given (output_path is not NULL), graphs visualising the data and the analysis are saved there, together with a compressed output object (as .rds file) with full estimation results.

### Usage

```r
mi_logreg_main(
  dataRaw,
  signal = "input",
  response = NULL,
  output_path = NULL,
  side_variables = NULL,
  pinput = NULL,
  formula_string = NULL,
  lr_maxit = 1000,
)```
MaxNWts = 5000,

testing = FALSE,

model_out = TRUE,

scale = TRUE,

TestingSeed = 1234,

testing_cores = 1,

boot_num = 10,

boot_prob = 0.8,

sidevar_num = 10,

traintest_num = 10,

partition_trainfrac = 0.6,

plot_width = 6,

plot_height = 4,

data_out = FALSE

Arguments

dataRaw must be a data.frame object

signal is a character object with names of columns of dataRaw to be treated as channel’s input.

response is a character vector with names of columns of dataRaw to be treated as channel’s output

output_path is the directory in which output will be saved

side_variables (optional) is a character vector that indicates side variables’ columns of data, if NULL no side variables are included

pinput is a numeric vector with prior probabilities of the input values. Uniform distribution is assumed as default (pinput=NULL).

formula_string (optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.

lr_maxit is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.

MaxNWts is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.

testing is the logical indicating if the testing procedures should be executed

model_out is the logical indicating if the calculated logistic regression model should be included in output list

scale is a logical indicating if the response variables should be scaled and centered before fitting logistic regression

TestingSeed is the seed for random number generator used in testing procedures

testing_cores - number of cores to be used in parallel computing (via doParallel package)

boot_num is the number of bootstrap tests to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.
**mi_logreg_main**

**boot_prob** is the proportion of initial size of data to be used in bootstrap

**sidevar_num** is the number of re-shuffling tests of side variables to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.

**traintest_num** is the number of overfitting tests to be performed. Default is 10, but it is recommended to use at least 50 for reliable estimates.

**partition_trainfrac** is the fraction of data to be used as a training dataset

**plot_width** - basic dimensions (width) of plots, in inches

**plot_height** - basic dimensions (height) of plots, in inches

**data_out** is the logical indicating if the data should be included in output list

**Details**

In a typical experiment aimed to quantify information flow a given signaling system, input values \( x_1 \leq x_2 \ldots \leq x_m \), ranging from 0 to saturation are considered. Then, for each input level, \( x_i \), \( n_i \) observations are collected, which are represented as vectors

\[
y_i^j \sim P(Y | X = x_i)
\]

Within information theory the degree of information transmission is measured as the mutual information

\[
MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X = x_i) \log_2 \frac{P(y|X = x_i)}{P(y)} dy,
\]

where \( P(y) \) is the marginal distribution of the output. MI is expressed in bits and \( 2^{MI} \) can be interpreted as the number of inputs that the system can resolve on average.

In contrast to existing approaches, instead of estimating, possibly highly dimensional, conditional output distributions \( P(Y|X = x_i) \), we propose to estimate the discrete, conditional input distribution, \( P(x_i|Y = y) \), which is known to be a simpler problem. Estimation of the MI using estimates of \( P(x_i|Y = y) \), denoted here as \( \hat{P}(x_i|Y = y) \), is possible as the MI, can be alternatively written as

\[
MI(X,Y) = \sum_{i=1}^{m} P(x_i) \int_{R^k} P(y|X = x_i) \log_2 \frac{\hat{P}(x_i|Y = y)}{P(x_i)} dy
\]

The expected value (as in above expression) with respect to distribution \( P(Y|X = x_i) \) can be approximated by the average with respect to data

\[
MI(X,Y) \approx \sum_{i=1}^{m} P(x_i) \frac{1}{n_i} \sum_{j=1}^{n_i} P(y|X = x_i) \log_2 \frac{\hat{P}(x_i|Y = y_j)}{P(x_i)} dy
\]

Here, we propose to use logistic regression as \( \hat{P}(x_i|Y = y_j) \). Specifically,

\[
\log \frac{P(x_i|Y = y)}{P(x_m|Y = y)} \approx \alpha_i + \beta_i y
\]

Additional parameters: lr_maxit and maxNWts are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for
prob_discr_pairwise

Estimates probabilities of correct discrimination (PCDs) between each pair of input/signal values using a logistic regression model.

advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

Value

a list with several elements:

- output$regression - confusion matrix of logistic regression predictions
- output$mi - mutual information in bits
- output$model - nnet object describing logistic regression model (if model_out=TRUE)
- output$params - parameters used in algorithm
- output$time - computation time of calculations
- output$testing - a 2- or 4-element output list of testing procedures (if testing=TRUE)
- output$testing_pv - one-sided p-values of testing procedures (if testing=TRUE)
- output$data - raw data used in analysis

References


Examples

tempdata=data_example1
outputCLR1=mi_logreg_main(dataRaw=tempdata, signal="signal", response="response")

tempdata=data_example2
outputCLR2=mi_logreg_main(dataRaw=tempdata, signal="signal", response=c("X1","X2","X3"))

#For further details see vignette
**prob_discr_pairwise**

**Usage**

```r
prob_discr_pairwise(
  dataRaw,  # must be a data.frame object
  signal = "input",  
  response = NULL,  
  side_variables = NULL,  
  formula_string = NULL,  
  output_path = NULL,  
  scale = TRUE,  
  lr_maxit = 1000,  
  MaxNWts = 5000,  
  diagnostics = TRUE
)
```

**Arguments**

- **dataRaw**: must be a data.frame object
- **signal**: is a character object with names of columns of dataRaw to be treated as channel’s input.
- **response**: is a character vector with names of columns of dataRaw to be treated as channel’s output
- **side_variables**: (optional) is a character vector that indicates side variables’ columns of data, if NULL no side variables are included
- **formula_string**: (optional) is a character object that includes a formula syntax to use in logistic regression model. If NULL, a standard additive model of response variables is assumed. Only for advanced users.
- **output_path**: is a directory where a pie chart with calculated probabilities will be saved. If NULL, the graph will not be created.
- **scale**: is a logical indicating if the response variables should be scaled and centered before fitting logistic regression
- **lr_maxit**: is a maximum number of iteration of fitting algorithm of logistic regression. Default is 1000.
- **MaxNWts**: is a maximum acceptable number of weights in logistic regression algorithm. Default is 5000.
- **diagnostics**: is a logical indicating if details of logistic regression fitting should be included in output list

**Details**

In order to estimate PCDs, for a given pair of input values $x_i$ and $x_j$, we propose to fit a logistic regression model using response data corresponding to the two considered inputs, i.e. $y_u^l$, for $l \in \{i, j\}$ and $u$ ranging from 1 to $n_t$. To ensure that both inputs have equal contribution to the calculated discriminability, equal probabilities should be assigned, $P(X) = (P(x_i), P(x_j)) = (1/2, 1/2)$. Once the regression model is fitted, probability of assigning a given cellular response, $y$, to the correct input value is estimated as

$$\max\{\hat{P}_{tr}(x_i|Y = y; P(X)), \hat{P}_{tr}(x_j|Y = y; P(X))\}.$$
Note that \( P(x_j | Y = y) = 1 - P(x_i | Y = y) \) as well as \( \hat{P}_{lr}(x_j | Y = y; P(X)) = 1 - \hat{P}_{lr}(x_i | Y = y; P(X)) \). The average of the above probabilities over all observations \( y_l \) yields PCDs

\[
P_{CD_{x_i,x_j}} = \frac{1}{2} \frac{1}{n_i} \sum_{l=1}^{n_i} \max\{ \hat{P}_{lr}(x_i | Y = y_l; P(X)), \hat{P}_{lr}(x_j | Y = y_l; P(X)) \} + \frac{1}{2} \frac{1}{n_j} \sum_{l=1}^{n_j} \max\{ \hat{P}_{lr}(x_i | Y = y_l; P(X)), \hat{P}_{lr}(x_j | Y = y_l; P(X)) \}.
\]

Additional parameters: \( lr\_maxit \) and \( maxNWts \) are the same as in definition of multinom function from nnet package. An alternative model formula (using formula_string arguments) should be provided if data are not suitable for description by logistic regression (recommended only for advanced users). Preliminary scaling of data (argument scale) should be used similarly as in other data-driven approaches, e.g. if response variables are comparable, scaling (scale=FALSE) can be omitted, while if they represent different phenomenon (varying by units and/or magnitude) scaling is recommended.

Value

a list with two elements:

- output$prob_matr - a \( n \times n \) matrix, where \( n \) is the number of inputs, with probabilities of correct discrimination between pairs of input values.
- output$diagnostics - (if diagnostics=TRUE) a list corresponding to logistic regression models fitted for each pair of input values. Each element consists of three sub-elements: 1) nnet_model - nnet object summarising logistic regression model; 2) prob_lr - probabilities of assignment obtained from logistic regression model; 3) confusion_matrix - confusion matrix of classifier.

References


Examples

```r
## Calculate probabilities of discrimination for nfkb dataset
it=21 # choose from 0, 3, 6, ..., 120 for measurements at other time points
output=prob_discr_pairwise(dataRaw=data_nfkb[data_nfkb$signal%in%c("0ng","1ng","100ng"),],
                           signal = "signal",
                           response = paste0("response_",it))
```
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