Package ‘SuperGauss’

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Title Superfast Likelihood Inference for Stationary Gaussian Time Series

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Description Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as \( O(n^2) \) in the number of time series observations. This package provides a `superfast` \( O(n \log^2 n) \) algorithm written in C++, crossing over with Durbin-Levinson around \( n = 300 \). Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

License GPL-3

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Description

Likelihood evaluations for stationary Gaussian time series are typically obtained via the Durbin-Levinson algorithm, which scales as $O(n^2)$ in the number of time series observations. This package provides a "superfast" $O(n \log^2 n)$ algorithm written in C++, crossing over with Durbin-Levinson around $n = 300$. Efficient implementations of the score and Hessian functions are also provided, leading to superfast versions of inference algorithms such as Newton-Raphson and Hamiltonian Monte Carlo. The C++ code provides a Toeplitz matrix class packaged as a header-only library, to simplify low-level usage in other packages and outside of R.

Details

While likelihood calculations with stationary Gaussian time series generally scale as $O(N^2)$ in the number of observations, this package implements an algorithm which scales as $O(N \log^2 N)$. "Superfast" algorithms for loglikelihood gradients and Hessians are also provided. The underlying C++ code is distributed through a header-only library found in the installed package’s include directory.

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acf2incr

Convert position autocorrelations to increment autocorrelations.

Examples

# Superfast inference for the timescale parameter
# of the exponential autocorrelation function
exp_acf <- function(lambda) exp(-(1:N-1)/lambda)

# simulate data
lambda0 <- 1
N <- 1000
X <- rnormtz(n = 1, acf = exp_acf(lambda0))

# loglikelihood function
# allocate memory for a NormalToeplitz distribution object
NTz <- NormalToeplitz$new(N)
loglik <- function(lambda) {
  NTz$logdens(z = X, acf = exp_acf(lambda))
  ## dSnorm(X = X, acf = Toep, log = TRUE)
}

# maximum likelihood estimation
optimize(f = loglik, interval = c(.2, 5), maximum = TRUE)

acf2incr(acf = exp(-(0:10)))

Description

Convert the autocorrelation of a stationary sequence \( x = (x_1, ..., x_N) \) to that of its increments, \( dx = (x_2 - x_1, ..., x_N - x_{(N-1)}) \).

Usage

acf2incr(acf)

Arguments

acf Length-N vector of position autocorrelations.

Value

Length \( N-1 \) vector of increment autocorrelations.

Examples

acf2incr(acf = exp(-(0:10)))
acf2msd

Convert autocorrelation of stationary increments to mean squared displacement of positions.

Description

Converts the autocorrelation of a stationary increment sequence $dx = (x_1 - x_0, ..., x_N - x_{N-1})$ to the mean squared displacement (MSD) of the corresponding positions, i.e., $MSD_i = E[(x_i - x_0)^2]$.

Usage

acf2msd(acf)

Arguments

acf

Length-N autocorrelation vector of a stationary increment sequence.

Value

Length-N MSD vector of the corresponding positions.

Examples

acf2msd(acf = exp(-(0:10)))

Cholesky

Cholesky multiplication with Toeplitz variance matrices.

Description

Multiplies the Cholesky decomposition of the Toeplitz matrix with another matrix, or solves a system of equations with the Cholesky factor.

Usage

cholZX(Z, acf)

cholXZ(X, acf)

Arguments

Z

Length-N or N x p matrix of residuals.

acf

Length-N autocorrelation vector of the Toeplitz variance matrix.

X

Length-N or N x p matrix of observations.
Details

If $C = t(chol(toeplitz(acf)))$, then cholZX() computes $C \times Z$ and cholZX() computes solve($C$, $X$). Both functions use the Durbin-Levinson algorithm.

Value

Size $N \times p$ residual or observation matrix.

Examples

```r
N <- 10
p <- 2
acf <- exp(-(1:N - 1))

Z <- matrix(rnorm(N * p), N, p)
cholZX(Z = Z, acf = acf) - (t(chol(toeplitz(acf))) %*% Z)

X <- matrix(rnorm(N * p), N, p)
cholXZ(X = X, acf = acf) - solve(t(chol(toeplitz(acf))), X)
```

Circulant

Constructor and methods for Circulant matrix objects.

Description

Constructor and methods for Circulant matrix objects.

Methods

Public methods:

- Circulant$new()
- Circulant$size()
- Circulant$set_acf()
- Circulant$get_acf()
- Circulant$set_psd()
- Circulant$get_psd()
- Circulant$has_acf()
- Circulant$prod()
- Circulant$solve()
- Circulant$log_det()
- Circulant$clone()

Method new(): Class constructor.

Usage:

Circulant$new(N, uacf, upsdx)
Arguments:
N Size of Circulant matrix.
uacf Optional vector of Nu = floor(N/2)+1 unique elements of the autocorrelation.
upsd Optional vector of Nu = floor(N/2)+1 unique elements of the PSD.

Returns: A Circulant object.

Method `size()`: Get the size of the Circulant matrix.

Usage:
Circulant$size()

Returns: Size of the Circulant matrix.

Method `set_acf()`: Set the autocorrelation of the Circulant matrix.

Usage:
Circulant$set_acf(uacf)

Arguments:
uacf Vector of Nu = floor(N/2)+1 unique elements of the autocorrelation.

Method `get_acf()`: Get the autocorrelation of the Circulant matrix.

Usage:
Circulant$get_acf()

Returns: The complete autocorrelation vector of length N.

Method `set_psd()`: Set the PSD of the Circulant matrix.

Usage:
Circulant$set_psd(upsd)

Arguments:
upsd Vector of Nu = floor(N/2)+1 unique elements of the psd.

Method `get_psd()`: Get the PSD of the Circulant matrix.

Usage:
Circulant$get_psd()

Returns: The complete PSD vector of length N.

Method `has_acf()`: Check whether the autocorrelation of the Circulant matrix has been set.

Usage:
Circulant$has_acf()

Returns: Logical; TRUE if Circulant$set_acf() has been called.

Method `prod()`: Circulant matrix-matrix product.

Usage:
Circulant$prod(x)
**dnormtz**

*Density of a multivariate normal with Toeplitz variance matrix.*

**Description**

Density of a multivariate normal with Toeplitz variance matrix.

**Usage**

`dnormtz(X, mu, acf, log = FALSE, method = c("gschur", "ltz"))`

**Arguments**

- `X` Vector of length N or N x n matrix, of which each column is a multivariate observation.
- `mu` Vector or matrix of mean values of compatible dimensions with `X`. Defaults to all zeros.
- `acf` Vector of length N containing the first column of the Toeplitz variance matrix.
- `log` Logical; whether to return the multivariate normal density on the log scale.
- `method` Which calculation method to use. Choices are: `gschur` for a modified version of the Generalized Schur algorithm of Ammar & Gragg (1988), or `ltz` for the Levinson-Trench-Zohar method. The former scales as O(N log^2 N) whereas the latter scales as O(N^2) and should only be used for N < 300.
Value

Vector of \( n \) (log-)densities, one for each column of \( X \).

Examples

```r
# simulate data
N <- 10 # length of each time series
n <- 3 # number of time series
theta <- 0.1
lambda <- 2
mu <- theta^2 * rep(1, N)
acf <- exp(-lambda * (1:N - 1))
X <- rnormtz(n, acf = acf) + mu

# evaluate log-density
dnormtz(X, mu, acf, log = TRUE)
```

fbm_msd

Mean square displacement of fractional Brownian motion.

Description

Mean square displacement of fractional Brownian motion.

Usage

```
fbm_msd(tseq, H)
```

Arguments

tseq

Length-\( N \) vector of timepoints.

H

Hurst parameter (between 0 and 1).

Details

The mean squared displacement (MSD) of a stochastic process \( X_t \) is defined as

\[
MSD(t) = E[(X_t - X_0)^2].
\]

Fractional Brownian motion (fBM) is a continuous Gaussian process with stationary increments, such that its covariance function is entirely defined the MSD, which in this case is MSD(t) = t^p(2H).

Value

Length-\( N \) vector of mean square displacements.

Examples

```
fbm_msd(tseq = 1:10, H = 0.4)
```
Matern autocorrelation function.

Usage
matern_acf(tseq, lambda, nu)

Arguments
- `tseq`: Vector of \( N \) time points at which the autocorrelation is to be calculated.
- `lambda`: Timescale parameter.
- `nu`: Smoothness parameter.

Details
The Matern autocorrelation is given by
\[
ACF(t) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left( \frac{\sqrt{2\nu} t}{\lambda} \right)^\nu K_\nu \left( \frac{\sqrt{2\nu} t}{\lambda} \right),
\]
where \( K_\nu(x) \) is the modified Bessel function of second kind.

Value
An autocorrelation vector of length \( N \).

Examples
matern_acf(tseq = 1:10, lambda = 1, nu = 3/2)

Convert mean square displacement of positions to autocorrelation of increments.

Usage
msd2acf(msd)
Arguments

msd  Length-N MSD vector, i.e., excluding $x_0$ which is assumed to be zero.

Value

Length-N autocorrelation vector.

Examples

# autocorrelation of fBM increments
msd2acf(msd = fbm_msd(tseq = 0:10, H = .3))

NormalCirculant Multivariate normal with Circulant variance matrix.

Description

Provides methods for the Normal-Circulant (NCt) distribution, which for a random vector $z$ of length $N$ is defined as

$$z \sim \text{NCt}(\text{uacf}) \iff z \sim \text{Normal}(0, \text{toeplitz}(\text{acf})), $$

where $\text{uacf}$ are the $\text{Nu} = \lfloor N/2 \rfloor + 1$ unique elements of the autocorrelation vector $\text{acf}$, i.e.,

$$\text{acf} = (\text{uacf}, \text{rev}((\text{uacf}[2:(\text{Nu}-1)])), \quad N \text{ even},$$

$$= (\text{uacf}, \text{rev}((\text{uacf}[2: \text{Nu}]))), \quad N \text{ odd}. $$

Methods

Public methods:

• NormalCirculant$new()
• NormalCirculant$size()
• NormalCirculant$logdens()
• NormalCirculant$grad_full()
• NormalCirculant$clone()

Method new(): Class constructor.

Usage:
NormalCirculant$new(N)

Arguments:
N  Size of the NCt random vector.

Returns: A NormalCirculant object.

Method size(): Get the size of the NCt random vector.

Usage:
NormalCirculant

NormalCirculant$size()

*Returns:* Size of the NCt random vector.

**Method** logdens(): Log-density function.

*Usage:*
NormalCirculant$logdens(z, uacf)

*Arguments:*
- **z** Density argument. A vector of length N or an N x n_obs matrix where each column is an N-dimensional observation.
- **uacf** A vector of length \( N_u = \text{floor}(N/2) \) containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.

*Returns:* A scalar or vector of length n_obs containing the log-density of the NCt evaluated at its arguments.

**Method** grad_full(): Full gradient of log-density function.

*Usage:*
NormalCirculant$grad_full(z, uacf, calc_dldz = TRUE, calc_dldu = TRUE)

*Arguments:*
- **z** Density argument. A vector of length N.
- **uacf** A vector of length \( N_u = \text{floor}(N/2) \) containing the first half of the autocorrelation (i.e., first row/column) of the Circulant variance matrix.
- **calc_dldz** Whether or not to calculate the gradient with respect to z.
- **calc_dldu** Whether or not to calculate the gradient with respect to uacf.

*Returns:* A list with elements:
- **ldens** The log-density evaluated at z and uacf.
- **dldz** The length-N gradient vector with respect to z, if calc_dldz = TRUE.
- **dldu** The length-\( N_u = \text{floor}(N/2)+1 \) gradient vector with respect to uacf, if calc_dldu = TRUE.

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*
NormalCirculant$clone(deep = FALSE)

*Arguments:*
- **deep** Whether to make a deep clone.
NormalToeplitz

Multivariate normal with Toeplitz variance matrix.

Description

Provides methods for the Normal-Toeplitz (NTz) distribution defined as

\[ z \sim \text{NTz}(\text{acf}) \iff z \sim \text{Normal}(0, \text{toeplitz}(\text{acf})), \]

i.e., for a multivariate normal with mean zero and variance \( T_z = \text{toeplitz}(\text{acf}) \).

Methods

Public methods:

- `NormalToeplitz$new()`
- `NormalToeplitz$size()`
- `NormalToeplitz$logdens()`
- `NormalToeplitz$grad()`
- `NormalToeplitz$hess()`
- `NormalToeplitz$grad_full()`
- `NormalToeplitz$clone()`

Method `new()`: Class constructor.

Usage:

`NormalToeplitz$new(N)`

Arguments:

- `N` Size of the NTz random vector.

Returns: A `NormalToeplitz` object.

Method `size()`: Get the size of the NTz random vector.

Usage:

`NormalToeplitz$size()`

Returns: Size of the NTz random vector.

Method `logdens()`: Log-density function.

Usage:

`NormalToeplitz$logdens(z, acf)`

Arguments:

- `z` Density argument. A vector of length \( N \) or an \( N \times n_{\text{obs}} \) matrix where each column is an \( N \)-dimensional observation.
- `acf` A vector of length \( N \) containing the autocorrelation (i.e., first row/column) of the Toeplitz variance matrix.
Returns: A scalar or vector of length \( n_{\text{obs}} \) containing the log-density of the NTz evaluated at its arguments.

**Method** \texttt{grad()}: Gradient of the log-density with respect to parameters.

**Usage:**
\[
\text{NormalToeplitz}\$\texttt{grad}(z, \ dz, \ acf, \ dacf, \ full\_out = \text{FALSE})
\]

**Arguments:**
- \( z \): Density argument. A vector of length \( N \).
- \( dz \): An \( N \times n_{\text{theta}} \) matrix containing the gradient \( dz/d\theta \).
- \( acf \): A vector of length \( N \) containing the autocorrelation of the Toeplitz variance matrix.
- \( dacf \): An \( N \times n_{\text{theta}} \) matrix containing the gradient \( dacf/d\theta \).
- \( full\_out \): If \texttt{TRUE}, returns the log-density as well (see 'Value').

**Returns:** A vector of length \( n_{\text{theta}} \) containing the gradient of the NTz log-density with respect to \( \theta \), or a list with elements \texttt{ldens} and \texttt{grad} consisting of the log-density and the gradient vector.

**Method** \texttt{hess()}: Hessian of log-density with respect to parameters.

**Usage:**
\[
\text{NormalToeplitz}\$\texttt{hess}(z, \ dz, \ d2z, \ acf, \ dacf, \ d2acf, \ full\_out = \text{FALSE})
\]

**Arguments:**
- \( z \): Density argument. A vector of length \( N \).
- \( dz \): An \( N \times n_{\text{theta}} \) matrix containing the gradient \( dz/d\theta \).
- \( d2z \): An \( N \times n_{\text{theta}} \times n_{\text{theta}} \) array containing the Hessian \( d^2z/d\theta^2 \).
- \( acf \): A vector of length \( N \) containing the autocorrelation of the Toeplitz variance matrix.
- \( dacf \): An \( N \times n_{\text{theta}} \) matrix containing the gradient \( dacf/d\theta \).
- \( d2acf \): An \( N \times n_{\text{theta}} \times n_{\text{theta}} \) array containing the Hessian \( dacf^2/d\theta^2 \).
- \( full\_out \): If \texttt{TRUE}, returns the log-density and its gradient as well (see 'Value').

**Returns:** An \( n_{\text{theta}} \times n_{\text{theta}} \) matrix containing the Hessian of the NTz log-density with respect to \( \theta \), or a list with elements \texttt{l dens}, \texttt{grad}, and \texttt{hess} consisting of the log-density, its gradient (a vector of size \( n_{\text{theta}} \)), and the Hessian matrix, respectively.

**Method** \texttt{grad_full()}: Full gradient of log-density function.

**Usage:**
\[
\text{NormalToeplitz}\$\texttt{grad\_full}(z, \ acf, \ calc\_dldz = \text{TRUE}, \ calc\_dl da = \text{TRUE})
\]

**Arguments:**
- \( z \): Density argument. A vector of length \( N \).
- \( acf \): A vector of length \( N \) containing the autocorrelation of the Toeplitz variance matrix.
- \( calc\_dl dz \): Whether or not to calculate the gradient with respect to \( z \).
- \( calc\_dl da \): Whether or not to calculate the gradient with respect to \( acf \).

**Returns:** A list with elements:
- \texttt{l dens} The log-density evaluated at \( z \) and \( acf \).
- \texttt{dl dz} The length-\( N \) gradient vector with respect to \( z \), if \texttt{calc\_dl dz} = \texttt{TRUE}. 

dl da The length-N gradient vector with respect to \( \text{acf} \), if \( \text{calc\_dlda} = \text{TRUE} \).

**Method** clone(): The objects of this class are cloneable with this method.

*Usage:*

`NormalToeplitz$clone(\text{deep} = \text{FALSE})`

*Arguments:*

deep Whether to make a deep clone.

---

**pex\_acf**

*Power-exponential autocorrelation function.*

---

**Description**

Power-exponential autocorrelation function.

**Usage**

```
pex\_acf(tseq, lambda, rho)
```

**Arguments**

- **tseq** Vector of \( N \) time points at which the autocorrelation is to be calculated.
- **lambda** Timescale parameter.
- **rho** Power parameter.

**Details**

The power-exponential autocorrelation function is given by:

\[
\text{ACF}(t) = \exp \left\{ -(t/\lambda)^\rho \right\}.
\]

**Value**

An autocorrelation vector of length \( N \).

**Examples**

```
pex\_acf(tseq = 1:10, lambda = 1, rho = 2)
```
Simulate a stationary Gaussian time series.

Usage

rnormtz(n = 1, acf, Z, fft = TRUE, nkeep, tol = 1e-06)

Arguments

n
Number of time series to generate.

acf
Length-N vector giving the autocorrelation of the series.

Z
Optional size (2N-2) x n or N x n matrix of iid standard normals, to use in the FFT and Durbin-Levinson methods, respectively.

fft
Logical; whether or not to use the O(N log N) FFT-based algorithm of Wood and Chan (1994) or the more stable O(N^2) Durbin-Levinson algorithm. See Details.

nkeep
Length of time series. Defaults to N = length(acf). See Details.

tol
Relative tolerance on negative eigenvalues. See Details.

Details

The FFT method fails when the embedding circulant matrix is not positive definite. This is typically due to one of two things:

1. Roundoff error can make tiny eigenvalues appear negative. For this purpose, argument tol can be used to replace all negative eigenvalues by tol * ev_max, where ev_max is the largest eigenvalue.

2. The autocorrelation is decaying too slowly on the given timescale. To mitigate this, argument nkeep can be used to supply a longer acf than is required, and keep only the first nkeep time series observations. For consistency, nkeep also applies to Durbin-Levinson method.

Value

Length-nkeep vector or size nkeep x n matrix with time series as columns.

Examples

```r
N <- 10
acf <- exp(-(1:N - 1)/N)
rnormtz(n = 3, acf = acf)
```
Defunct functions in SuperGauss.

The following functions have been removed from the SuperGauss package

- `rSnorm()` Please use `rnormtz()` instead.
- `dSnorm()` Please use `dnormtz()` instead.
- `Snorm.grad()` Please use the `grad()` method in the `NormalToeplitz` class.
- `Snorm.hess()` Please use the `hess()` method in the `NormalToeplitz` class.

---

Toeplitz matrix multiplication.

Description

Efficient matrix multiplication with Toeplitz matrix and arbitrary matrix or vector.

Usage

```r
toepl.mult(acf, X)
```

Arguments

- `acf` Length-N vector giving the first column (or row) of the Toeplitz matrix.
- `X` Vector or matrix of compatible dimensions with `acf`.

Value

An N-row matrix corresponding to `toeplitz(acf) %*% X`.

Examples

```r
N <- 20
d <- 3
acf <- exp(-(1:N))
X <- matrix(rnorm(N*d), N, d)
toepl.mult(acf, X)
```
Description
The `Toeplitz` class contains efficient methods for linear algebra with symmetric positive definite (i.e., variance) Toeplitz matrices.

Usage

```r
isToeplitz(x)
asToeplitz(x)
```

```
# S3 method for class 'Toeplitz'
dim(x)
```

Arguments

- `x` An R object.

Details

An N x N Toeplitz matrix $T_z$ is defined by its length-N "autocorrelation" vector $acf$, i.e., first row/column $T_z$. Thus, for the function `stats::toeplitz()`, we have $T_z = \text{toeplitz}(acf)$.

It is assumed that $acf$ defines a valid (i.e., positive definite) variance matrix. The matrix multiplication methods still work when this is not the case but the other methods do not (return values typically contain NaNs).

`as.Toeplitz(x)` attempts to convert its argument to a `Toeplitz` object by calling `Toeplitz$new(acf = x)`. `is.Toeplitz(x)` checks whether its argument is a `Toeplitz` object.

Methods

Public methods:
- `Toeplitz$new()`
- `Toeplitz$print()`
- `Toeplitz$size()`
- `Toeplitz$set_acf()`
- `Toeplitz$get_acf()`
- `Toeplitz$has_acf()`
- `Toeplitz$prod()`
- `Toeplitz$solve()`
- `Toeplitz$log_det()`
- `Toeplitz$trace_grad()`
- `Toeplitz$trace_hess()`
- **Toeplitz$\text{clone}()**

**Method** new(): Class constructor.

**Usage:**
Toeplitz$new(N, acf)

**Arguments:**
N Size of Toeplitz matrix.
acf Autocorrelation vector of length N.

**Returns:** A Toeplitz object.

**Method** print(): Print method.

**Usage:**
Toeplitz$print()

**Method** size(): Get the size of the Toeplitz matrix.

**Usage:**
Toeplitz$size()

**Returns:** Size of the Toeplitz matrix. \text{ncol}, \text{nrow}, and \text{dim} methods for Toeplitz objects also work as expected.

**Method** set_acf(): Set the autocorrelation of the Toeplitz matrix.

**Usage:**
Toeplitz$set_acf(acf)

**Arguments:**
acf Autocorrelation vector of length N.

**Method** get_acf(): Get the autocorrelation of the Toeplitz matrix.

**Usage:**
Toeplitz$get_acf()

**Returns:** The autocorrelation vector of length N.

**Method** has_acf(): Check whether the autocorrelation of the Toeplitz matrix has been set.

**Usage:**
Toeplitz$has_acf()

**Returns:** Logical; TRUE if Toeplitz$set_acf() has been called.

**Method** prod(): Toeplitz matrix-matrix product.

**Usage:**
Toeplitz$prod(x)

**Arguments:**
x Vector or matrix with N rows.

**Returns:** The matrix product $Tz \times x$. $Tz \times x$ and $x \times Tz$ also work as expected.
Method solve(): Solve a Toeplitz system of equations.

Usage:
Toeplitz$solve(x, method = c("gschur", "pcg"), tol = 1e-10)

Arguments:
x  Optional vector or matrix with N rows.
tol  Tolerance level for the pcg method.

Returns:  The solution in z to the system of equations Tz %*% z = x. If x is missing, returns the inverse of Tz. solve(Tz, x) and solve(Tz, x, method, tol) also work as expected.

Method log_det(): Calculate the log-determinant of the Toeplitz matrix.

Usage:
Toeplitz$log_det()

Returns:  The log-determinant log(det(Tz)). determinant(Tz) also works as expected.

Method trace_grad(): Computes the trace-gradient with respect to Toeplitz matrices.

Usage:
Toeplitz$trace_grad(acf2)
Arguments:
acf2  Length-N autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns:  Computes the trace of solve(Tz, toeplitz(acf2)).
This is used in the computation of the gradient of log(det(Tz(theta))) with respect to theta.

Method trace_hess(): Computes the trace-Hessian with respect to Toeplitz matrices.

Usage:
Toeplitz$trace_hess(acf2, acf3)
Arguments:
acf2  Length-N autocorrelation vector of the second Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.
acf3  Length-N autocorrelation vector of the third Toeplitz matrix. This matrix must be symmetric but not necessarily positive definite.

Returns:  Computes the trace of solve(Tz, toeplitz(acf2)) %*% solve(Tz, toeplitz(acf3)).
This is used in the computation of the Hessian of log(det(Tz(theta))) with respect to theta.

Method clone(): The objects of this class are cloneable with this method.

Usage:
Toeplitz$clone(deep = FALSE)
Arguments:
deep  Whether to make a deep clone.
Examples

# construct a Toeplitz matrix
acf <- exp(-(1:5))
Tz <- Toeplitz$new(acf = acf)
# alternatively, can allocate space first
Tz <- Toeplitz$new(N = length(acf))
Tz$set_acf(acf = acf)

# basic methods
Tz$get_acf() # extract the acf
dim(Tz) # == c(nrow(Tz), ncol(Tz))
Tz # print method

# linear algebra methods
X <- matrix(rnorm(10), 5, 2)
Tz %*% X
t(X) %*% Tz
solve(Tz, X)
determinant(Tz) # log-determinant
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