

# Package ‘TestCor’

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**Title** FWER and FDR Controlling Procedures for Multiple Correlation Tests

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**Description** Different multiple testing procedures for correlation tests are implemented. These procedures were shown to theoretically control asymptotically the Family Wise Error Rate (Roux (2018) <<https://tel.archives-ouvertes.fr/tel-01971574v1>>) or the False Discovery Rate (Cai & Liu (2016) <[doi:10.1080/01621459.2014.999157](https://doi.org/10.1080/01621459.2014.999157)>). The package gather four test statistics used in correlation testing, four FWER procedures with either single step or stepdown versions, and four FDR procedures.

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TestCor-package	<i>FWER and FDR controlling procedures for multiple correlation tests</i>
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## Description

The package compiles some multiple testing procedures which theoretically control asymptotically the FWER in the framework of correlation testing. Four tests statistics can be considered: the empirical correlation, the Student statistics, the Fisher's z-transform and the usual Gaussian statistics considering random variables  $(X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$ . Four methods are implemented: Bonferroni (1935)'s, Šidák (1967)'s, Romano & Wolf (2005)'s bootstrap and (Drton & Perlman (2007)'s procedure based on the asymptotic distributions of the test statistics, called Max-Tinfy. The package also includes some multiple testing procedures which are related to the control of the FDR : Cai & Liu (2016)'s procedures called LCT-N and LCT-B -which have been proven to control the FDR for correlation tests- and Benjamini & Hochberg (1995)'s -which has no theoretical results in correlation testing.

## Details

Consider  $\{\mathbf{X}_\ell = (X_{1\ell}, \dots, X_{p\ell}), \ell = 1, \dots, n\}$  a set of  $n$  independent and identically distributed  $R^p$ -valued random variables. Denote data the array containing  $\{\mathbf{X}_\ell, \ell = 1, \dots, n\}$ , with observation indexes  $\ell$  in row. The aim is to test simultaneously

$$(H_{0ij}) \text{Cor}(X_i, X_j) = 0 \text{ against } (H_{1ij}) \text{Cor}(X_i, X_j) \neq 0, \quad i, j = 1, \dots, p, \quad i < j.$$

Four tests statistics are implemented: the empirical correlation, the Student statistics, the Fisher's z-transform and the usual test statistics on expectancy considering the product of random variables. They are available in function eval\_stat. Next, two main types of procedures are available:

**Asymptotically FWER controlling procedures:** Bonferroni (1935)’s method, Šidák (1967)’s procedure, Romano & Wolf (2005)’s bootstrap procedure and Drton & Perlman (2007)’s procedure. A description of these methods can be found in Chapter 5 of Roux (2018). To apply these procedures, function `ApplyFwerCor` can be used as follows:

`ApplyFwerCor(data, alpha, stat_test, method)`, with `alpha` the desired level of control for FDR and `stat_test, method` respectively the kind of test statistic and the FDR controlling method. The function returns the list of indexes  $\{(i, j), i < j\}$  for which null hypothesis ( $H_{0ij}$ ) is rejected.

**Asymptotically FDR controlling procedures:** Cai & Liu(2016)’s two procedures and Benjamini & Hochberg (1995)’s procedure (with no theoretical proof for the latest). To apply these procedures, use function `ApplyFdrCor` as follows: `ApplyFdrCor(data, alpha, stat_test, method)` with `alpha` the desired level of control for FWER and `stat_test, method` respectively the kind of test statistic and the FDR controlling method. The function returns the list of indexes  $\{(i, j), i < j\}$  for which null hypothesis ( $H_{0ij}$ ) is rejected.

Functions `SimuFwer` and `SimuFdr` provide simulations of Gaussian random variables for a given correlation matrix and return estimated FWER, FDR, Power and true discovery rate obtained applying one of the procedure above. Some example of results obtained can be found in Chapter 6 of Roux (2018).

## Author(s)

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## References

- Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the royal statistical society. Series B (Methodological)*, 289-300, <https://doi.org/10.1111/j.2517-6161.1995.tb02031.x>.
- Bonferroni, C. E. (1935). Il calcolo delle assicurazioni su gruppi di teste. *Studi in onore del professore salvatore ortu carboni*, 13-60.
- Cai, T. T., & Liu, W. (2016). Large-scale multiple testing of correlations. *Journal of the American Statistical Association*, 111(513), 229-240, <https://doi.org/10.1080/01621459.2014.999157>.
- Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449, <https://doi.org/10.1214/088342307000000113>.
- Romano, J. P., & Wolf, M. (2005). Exact and approximate stepdown methods for multiple hypothesis testing. *Journal of the American Statistical Association*, 100(469), 94-108, <https://doi.org/10.1198/016214504000000539>.
- Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.
- Šidák, Z. (1967). Rectangular confidence regions for the means of multivariate normal distributions. *Journal of the American Statistical Association*, 62(318), 626-633.

## Examples

```
# Parameters for simulations
Nsimu <- 100          # number of Monte-Carlo simulations
seqn  <- seq(100,400,100) # sample sizes
p     <- 10           # number of random variables considered
rho   <- 0.3          # value of non-zero correlations
seed  <- 156724

corr_theo <- diag(1,p)      # the correlation matrix
corr_theo[1,2:p] <- rho
corr_theo[2:p,1] <- rho

# Parameters for multiple testing procedure
stat_test <- 'empirical'    # test statistics for correlation tests
method <- 'BootRW'         # FWER controlling procedure
SD <- FALSE                # logical determining if stepdown is applied
alpha <- 0.05              # FWER threshold
Nboot <- 100               # number of bootstrap or simulated samples

# Simulations and application of the chosen procedure
res <- matrix(0,nrow=length(seqn),ncol=5)
for(i in 1:length(seqn)){
  temp <- SimuFwer(corr_theo,n=seqn[i],Nsimu=Nsimu,alpha=alpha,stat_test=stat_test,
    method='BootRW',Nboot=Nboot,stepdown=SD,seed=seed)
  res[i,] <- temp
}
rownames(res) <- seqn
colnames(res) <- names(temp)

# Display results
par(mfrow=c(1,2))
plot(seqn,res[, 'fwer'],type='b',ylim=c(0,max(alpha*1.1,max(res[, 'fwer']))),
  main='FWER',ylab='fwer',xlab='number of observations')
plot(seqn,res[, 'sensitivity'],type='b',ylim=c(0,1.1),
  main='Power',ylab='sensitivity',xlab='number of observations')
```

---

ApplyFdrCor

*Applies multiple testing procedures built to control (asymptotically) the FDR for correlation testing.*

---

## Description

Applies multiple testing procedures built to control (asymptotically) the FDR for correlation testing. Some have no theoretical proofs for tests on a correlation matrix.

**Usage**

```

ApplyFdrCor(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  method = "LCTnorm",
  Nboot = 1000,
  vect = FALSE,
  arr.ind = FALSE
)

```

**Arguments**

data	matrix of observations
alpha	level of multiple testing
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr}) / \sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y) / \text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
method	choice between 'LCTnorm' and 'LCTboot' developped by Cai & Liu (2016), 'BH', traditional Benjamini-Hochberg's procedure Benjamini & Hochberg (1995)'s and 'BHboot', Benjamini-Hochberg (1995)'s procedure with bootstrap evaluation of p-values
Nboot	number of iterations for bootstrap p-values evaluation
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing rows and columns of significant correlations
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value**

Returns either

- logicals indicating if the corresponding correlation is significant, as a vector or a matrix depending on vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

- Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the royal statistical society. Series B (Methodological)*, 289-300.
- Cai, T. T., & Liu, W. (2016). Large-scale multiple testing of correlations. *Journal of the American Statistical Association*, 111(513), 229-240.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

### See Also

ApplyFwerCor

LCTnorm, LCTboot, BHCOR, BHBootCor

### Examples

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
res <- ApplyFdrCor(data,stat_test='empirical',method='LCTnorm')
# significant correlations, level alpha:
alpha <- 0.05
whichCor(res<alpha)
```

---

ApplyFwerCor

*Applies multiple testing procedures controlling (asymptotically) the FWER for tests on a correlation matrix.*

---

### Description

Applies multiple testing procedures controlling (asymptotically) the FWER for tests on a correlation matrix. Methods are described in Chapter 5 of Roux (2018).

### Usage

```
ApplyFwerCor(
  data,
  alpha = NULL,
  stat_test = "empirical",
  method = "Sidak",
  Nboot = 1000,
  stepdown = TRUE,
  vect = FALSE,
  logical = stepdown,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing (used if <code>logical=TRUE</code> )
<code>stat_test</code>	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr}) / \sqrt{1 - \text{corr}^2}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y) / \text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>method</code>	choice between 'Bonferroni', 'Sidak', 'BootRW', 'MaxTinfy'
<code>Nboot</code>	number of iterations for Monte-Carlo of bootstrap quantile evaluation
<code>stepdown</code>	logical, if TRUE a stepdown procedure is applied
<code>vect</code>	if TRUE returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing the adjusted p-values for each entry of the correlation matrix
<code>logical</code>	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if <code>stepdown=TRUE</code> and <code>logical=FALSE</code> , returns a list of successive p-values.
<code>arr.ind</code>	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value**

Returns either

- the adjusted p-values, as a vector or a matrix, depending on `vect`
- logicals indicating if the corresponding correlation is significant if `logical=TRUE`, as a vector or a matrix depending on `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

- Bonferroni, C. E. (1935). Il calcolo delle assicurazioni su gruppi di teste. Studi in onore del professore salvatore ortu carboni, 13-60.
- Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.
- Romano, J. P., & Wolf, M. (2005). Exact and approximate stepdown methods for multiple hypothesis testing. *Journal of the American Statistical Association*, 100(469), 94-108.
- Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.
- Šidák, Z. (1967). Rectangular confidence regions for the means of multivariate normal distributions. *Journal of the American Statistical Association*, 62(318), 626-633.

**See Also**

ApplyFwerCor\_SD, ApplyFdrCor

BonferroniCor, SidakCor, BootRWCOR, maxTinfyCor

BonferroniCor\_SD, SidakCor\_SD, BootRWCOR\_SD, maxTinfyCor\_SD

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values
(res <- ApplyFwerCor(data,stat_test='empirical',method='Bonferroni',stepdown=FALSE))
# significant correlations, level alpha:
alpha <- 0.05
whichCor(res<alpha)
```

---

ApplyFwerCor_oracle	<i>Applies an oracle version of MaxTinfy procedure described in Drton &amp; Perlman (2007) for correlation testing.</i>
---------------------	---

---

**Description**

Applies oracle MaxTinfy procedure described in Drton & Perlman (2007) which controls asymptotically the FWER for tests on a correlation matrix. It needs the true correlation matrix.

**Usage**

```
ApplyFwerCor_oracle(
  data,
  corr_theo,
  alpha = c(),
  stat_test = "empirical",
  method = "MaxTinfy",
  Nboot = 1000,
  stepdown = TRUE,
  vect = FALSE,
  logical = stepdown,
  arr.ind = FALSE
)
```



**Arguments**

data	matrix of observations
corr_theo	true matrix of correlations
alpha	level of multiple testing (used if logical=TRUE)
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
method	only 'MaxTinfy' implemented
Nboot	number of iterations for Monte-Carlo of bootstrap quantile evaluation
stepdown	logical, if TRUE a stepdown procedure is applied
vect	if TRUE returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing the adjusted p-values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if stepdown=TRUE and logical=FALSE, returns a list of successive p-values.
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value**

Returns either

- the adjusted p-values, as a vector or a matrix, depending on vect (unavailable with stepdown)
- logicals indicating if the corresponding correlation is significant if logical=TRUE, as a vector or a matrix depending on vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if arr.ind=TRUE.

Oracle estimation of the quantile is used, based on the true correlation matrix

**References**

- Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.
- Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

ApplyFwerCor  
maxTinfyCor, maxTinfyCor\_SD

**Examples**

```

n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values:
(res <- ApplyFwerCor_oracle(data,corr_theo,stat_test='empirical',Nboot=1000,stepdown=FALSE))
# significant correlations, level alpha:
alpha <- 0.05
whichCor(res<alpha)

```

BHBootCor

*Benjamini & Hochberg (1995)'s procedure for correlation testing with bootstrap evaluation of p-values.*

**Description**

Benjamini & Hochberg (1995)'s procedure on the correlation matrix entries with bootstrap evaluation of p-values (no theoretical proof of control).

**Usage**

```

BHBootCor(
  data,
  alpha = 0.05,
  stat_test = "2nd.order",
  Nboot = 100,
  vect = FALSE,
  arr.ind = FALSE
)

```

**Arguments**

data	matrix of observations
alpha	level of multiple testing
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
Nboot	number of iterations for bootstrap quantile evaluation
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value**

## Returns

- a vector or a matrix of logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, if `arr.ind=FALSE`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the royal statistical society. Series B (Methodological)*, 289-300.

**See Also**

`ApplyFdrCor`, `BHCor`

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
BHBootCor(data,alpha,stat_test='empirical',arr.ind=TRUE)
```

---

BHCor

*Benjamini & Hochberg (1995)'s procedure for correlation testing.*

---

**Description**

Benjamini & Hochberg (1995)'s procedure on the correlation matrix entries (no theoretical proof of control).

**Usage**

```
BHCor(
  data,
  alpha = 0.05,
  stat_test = "2nd.order",
  vect = FALSE,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing
<code>stat_test</code>	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{1 - \text{corr}^2}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>vect</code>	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
<code>arr.ind</code>	if TRUE, returns the indexes of the significant correlations, with respect to level <code>alpha</code>

**Value**

Returns

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the royal statistical society. Series B (Methodological)*, 289-300.

**See Also**

`ApplyFdrCor`, `BHBootCor`

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
BHCor(data,alpha,stat_test='empirical',arr.ind=TRUE)
```

BonferroniCor

*Bonferroni multiple testing procedure for correlations.***Description**

Bonferroni multiple testing procedure for correlations.

**Usage**

```
BonferroniCor(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  vect = FALSE,
  logical = FALSE,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing (used if <code>logical=TRUE</code> )
<code>stat_test</code>	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>vect</code>	if <code>TRUE</code> returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if <code>FALSE</code> , returns an array containing the adjusted p-values for each entry of the correlation matrix
<code>logical</code>	if <code>TRUE</code> , returns either a vector or a matrix where each element is equal to <code>TRUE</code> if the corresponding null hypothesis is rejected, and to <code>FALSE</code> if it is not rejected
<code>arr.ind</code>	if <code>TRUE</code> , returns the indexes of the significant correlations, with respect to level <code>alpha</code>

**Value**

Returns

- the adjusted p-values, as a vector or a matrix depending of the value of `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

## References

Bonferroni, C. E. (1935). Il calcolo delle assicurazioni su gruppi di teste. Studi in onore del professore salvatore ortu carboni, 13-60.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

## See Also

ApplyFwerCor, BonferroniCor\_SD

## Examples

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
corr_theo <- diag(1,p)
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values
res <- BonferroniCor(data,stat_test='empirical')
round(res,2)
# significant correlations with level alpha:
alpha <- 0.05
whichCor(res<alpha)
# directly
BonferroniCor(data,alpha,stat_test='empirical',arr.ind=TRUE)
```

---

BonferroniCor_SD	<i>Bonferroni multiple testing method for correlations with stepdown procedure.</i>
------------------	---

---

## Description

Bonferroni multiple testing method for correlations with stepdown procedure.

## Usage

```
BonferroniCor_SD(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  vect = FALSE,
  logical = TRUE,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing
<code>stat_test</code>	4 test statistics are available: <b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>vect</code>	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
<code>logical</code>	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if FALSE, returns a list of successive p-values : element <code>[[i+1]]</code> of the list giving the p-values evaluated on the non-rejected hypothesis at step <code>[[i]]</code> ; p-values are either as a vector or a list depending on <code>vect</code>
<code>arr.ind</code>	if TRUE, returns the indexes of the significant correlations, with respect to level <code>alpha</code>

**Value****Returns**

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Bonferroni, C. E. (1935). Il calcolo delle assicurazioni su gruppi di teste. Studi in onore del professore salvatore ortu carboni, 13-60.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

`ApplyFwerCor`, `BonferroniCor`

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
```

```

data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
BonferroniCor_SD(data,alpha,stat_test='empirical', arr.ind=TRUE)
# successive p-values
res <- BonferroniCor_SD(data,stat_test='empirical', logical=FALSE)
lapply(res,FUN=function(x){round(x,2)})
# successive rejections
lapply(res,FUN=function(x){whichCor(x<alpha)})

```

---

BootRWCOR

*Bootstrap multiple testing method of Romano & Wolf (2005) for correlations.*

---

### Description

Multiple testing method based on the evaluation of quantile by bootstrap in the initial dataset (Romano & Wolf (2005)).

### Usage

```

BootRWCOR(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  Nboot = 1000,
  vect = FALSE,
  logical = FALSE,
  arr.ind = FALSE
)

```

### Arguments

data	matrix of observations
alpha	level of multiple testing (used if logical=TRUE)
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr}) / \sqrt{1 - \text{corr}^2}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y) / \text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
Nboot	number of iterations for Monte-Carlo quantile evaluation
vect	if TRUE returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing the adjusted p-values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha



**Value**

Returns

- the adjusted p-values, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Romano, J. P., & Wolf, M. (2005). Exact and approximate stepdown methods for multiple hypothesis testing. *Journal of the American Statistical Association*, 100(469), 94-108.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

ApplyFwerCor, BootRWCOR\_SD

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values
res <- BootRWCOR(data,stat_test='empirical',Nboot=1000)
round(res,2)
# significant correlations with level alpha:
alpha <- 0.05
whichCor(res<alpha)
# directly
BootRWCOR(data,alpha,stat_test='empirical',Nboot=1000,arr.ind=TRUE)
```

---

BootRWCOR\_SD

*Bootstrap multiple testing method of Romano & Wolf (2005) for correlations, with stepdown procedure.*

---

**Description**

Multiple testing method based on the evaluation of quantile by bootstrap in the initial dataset (Romano & Wolf (2005)), with stepdown procedure.

**Usage**

```

BootRWCOR_SD(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  Nboot = 1000,
  vect = FALSE,
  logical = TRUE,
  arr.ind = FALSE
)

```

**Arguments**

data	matrix of observations
alpha	level of multiple testing
stat_test	4 test statistics are available: <b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
Nboot	number of iterations for Bootstrap quantile evaluation
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if FALSE, returns a list of successive p-values : element <code>[[i+1]]</code> of the list giving the p-values evaluated on the non-rejected hypothesis at step <code>[[i]]</code> ; p-values are either as a vector or a list depending on vect
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value****Returns**

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Romano, J. P., & Wolf, M. (2005). Exact and approximate stepdown methods for multiple hypothesis testing. *Journal of the American Statistical Association*, 100(469), 94-108.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

ApplyFwerCor, BootRWCOR

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
BootRWCOR_SD(data,alpha,stat_test='empirical', arr.ind=TRUE)
# successive p-values
res <- BootRWCOR_SD(data,stat_test='empirical', logical=FALSE)
lapply(res,FUN=function(x){round(x,2)})
# successive rejections
lapply(res,FUN=function(x){whichCor(x<alpha)})
```

---

covD2nd

---

*Returns the theoretical covariance of empirical correlations.*


---

**Description**

Returns the theoretical covariance of empirical correlations.

**Usage**

```
covD2nd(r)
```

**Arguments**

**r** a correlation matrix

**Value**

Returns the theoretical covariance of 2nd order statistics,  $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$  with  $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$ .

**See Also**

covDcor

**Examples**

```
p <- 10
corr_theo <- diag(1,p)
corr_theo[2:p,] <- 0.3
corr_theo[,2:p] <- 0.3
covD2nd(corr_theo)
```

---

 covDcor

*Returns the theoretical covariance of empirical correlations.*


---

**Description**

Returns the theoretical covariance of empirical correlations.

**Usage**

```
covDcor(r)
```

**Arguments**

`r` a correlation matrix

**Value**

Returns the theoretical covariance of empirical correlations.

**References**

Aitkin, M. A. (1969). Some tests for correlation matrices. *Biometrika*, 443-446.

**See Also**

covDcorNorm

**Examples**

```
p <- 10
corr_theo <- diag(1,p)
corr_theo[2:p,] <- 0.3
corr_theo[,2:p] <- 0.3
covDcor(corr_theo)
```

---

covDcorNorm	<i>Returns the theoretical covariance of test statistics for correlation testing.</i>
-------------	---

---

### Description

Returns the theoretical covariance of test statistics for correlation testing.

### Usage

```
covDcorNorm(cor_mat, stat_test = "empirical")
```

### Arguments

cor_mat	A correlation matrix
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(corr)$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1+corr)/(1-corr))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(corr) / \sqrt{(1-corr^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y) / \text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$

### Value

Returns the theoretical covariance of the test statistics.

### See Also

covDcor, covD2nd, eval\_stat

### Examples

```
p <- 10
corr_theo <- diag(1,p)
corr_theo[2:p,] <- 0.3
corr_theo[,2:p] <- 0.3
covDcorNorm(corr_theo, stat_test='student')
```

---

eval_stat	<i>Evaluates the test statistics for tests on correlation matrix entries.</i>
-----------	---

---

### Description

Evaluates the test statistics for tests on correlation matrix entries.

### Usage

```
eval_stat(data, type = "empirical")
```

**Arguments**

data                    matrix of observations  
 type                    **'empirical'**  $\sqrt{n} * \text{abs}(\text{corr})$   
                          **'fisher'**  $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$   
                          **'student'**  $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$   
                          **'2nd.order'**  $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$  with  $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$

**Value**

Returns the test statistics for correlation testing.

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
stat <- eval_stat(data,'fisher')
```

---

LCTboot

*Bootstrap procedure LCT-B proposed by Cai & Liu (2016) for correlation testing.*

---

**Description**

Bootstrap procedure LCT-B proposed by Cai & Liu (2016) for correlation testing.

**Usage**

```
LCTboot(
  data,
  alpha = 0.05,
  stat_test = "2nd.order",
  Nboot = 100,
  vect = FALSE,
  arr.ind = FALSE
)
```

**Arguments**

data                    matrix of observations  
 alpha                   level of multiple testing  
 stat\_test                **'empirical'**  $\sqrt{n} * \text{abs}(\text{corr})$   
                          **'fisher'**  $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$   
                          **'student'**  $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$   
                          **'2nd.order'**  $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$  with  $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$

Nboot	number of iterations for bootstrap quantile evaluation
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

## Value

### Returns

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

## References

Cai, T. T., & Liu, W. (2016). Large-scale multiple testing of correlations. *Journal of the American Statistical Association*, 111(513), 229-240.

## See Also

ApplyFdrCor, LCTNorm

## Examples

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
LCTboot(data,alpha,stat_test='empirical',Nboot=100,arr.ind=TRUE)
```

---

LCTnorm	<i>Procedure LCT-N proposed by Cai &amp; Liu (2016) for correlation testing.</i>
---------	--

---

## Description

Procedure LCT-N proposed by Cai & Liu (2016) for correlation testing.

**Usage**

```
LCTnorm(
  data,
  alpha = 0.05,
  stat_test = "2nd.order",
  vect = FALSE,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing
<code>stat_test</code>	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{1 - \text{corr}^2}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>vect</code>	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
<code>arr.ind</code>	if TRUE, returns the indexes of the significant correlations, with respect to level <code>alpha</code>

**Value**

Returns

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Cai, T. T., & Liu, W. (2016). Large-scale multiple testing of correlations. *Journal of the American Statistical Association*, 111(513), 229-240.

**See Also**

`ApplyFdrCor`, `LCTboot`

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
```



```

corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
LCTnorm(data,alpha,stat_test='empirical',arr.ind=TRUE)

```

---

maxTinfyCor

---

Multiple testing method of Drton & Perlman (2007) for correlations.

---

## Description

Multiple testing method based on the evaluation of quantile by simulation of observations from the asymptotic distribution (Drton & Perlman (2007)).

## Usage

```

maxTinfyCor(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  Nboot = 1000,
  OmegaChap = covDcorNorm(cor(data), stat_test),
  vect = FALSE,
  logical = FALSE,
  arr.ind = FALSE
)

```

## Arguments

data	matrix of observations
alpha	level of multiple testing (used if logical=TRUE)
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{1 - \text{corr}^2}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
Nboot	number of iterations for Monte-Carlo quantile evaluation
OmegaChap	matrix of covariance of empirical correlations used for quantile evaluation; optional, useful for oracle estimation and step-down
vect	if TRUE returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing the adjusted p-values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value****Returns**

- the adjusted p-values, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

ApplyFwerCor, maxTinfyCor\_SD

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values
res <- maxTinfyCor(data,stat_test='empirical',Nboot=1000)
round(res,2)
# significant correlations with level alpha:
alpha <- 0.05
whichCor(res<alpha)
# directly
res <- maxTinfyCor(data,alpha,stat_test='empirical',Nboot=1000,arr.ind=TRUE)
```

---

maxTinfyCor\_SD

*Multiple testing method of Drton & Perlman (2007) for correlations, with stepdown procedure.*

---

**Description**

Multiple testing method based on the evaluation of quantile by simulation of observations from the asymptotic distribution (Drton & Perlman (2007)), with stepdown procedure.

**Usage**

```
maxTinfyCor_SD(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  Nboot = 1000,
  OmegaChap = covDcorNorm(cor(data), stat_test),
  vect = FALSE,
  logical = TRUE,
  arr.ind = FALSE
)
```

**Arguments**

data	matrix of observations
alpha	level of multiple testing
stat_test	4 test statistics are available: <b>'empirical'</b> $\sqrt{n} * abs(corr)$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1+corr)/(1-corr))$ <b>'student'</b> $\sqrt{n-2} * abs(corr) / \sqrt{(1-corr^2)}$ <b>'2nd.order'</b> $\sqrt{n} * mean(Y) / sd(Y)$ with $Y = (X_i - mean(X_i))(X_j - mean(X_j))$
Nboot	number of iterations for Monte-Carlo quantile evaluation
OmegaChap	matrix of covariance of test statistics; optional, useful for oracle estimation and step-down
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if FALSE, returns a list of successive p-values : element <code>[[i+1]]</code> of the list giving the p-values evaluated on the non-rejected hypothesis at step <code>[[i]]</code> ; p-values are either as a vector or a list depending on vect
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value****Returns**

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

## References

Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

## See Also

ApplyFwerCor, maxTinfyCor

## Examples

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
maxTinfyCor_SD(data,alpha,stat_test='empirical', arr.ind=TRUE)
# successive p-values
res <- maxTinfyCor_SD(data,stat_test='empirical', logical=FALSE)
lapply(res,FUN=function(x){round(x,2)})
# successive rejections
lapply(res,FUN=function(x){whichCor(x<alpha)})
```

---

SidakCor

*Sidak multiple testing procedure for correlations.*

---

## Description

Sidak multiple testing procedure for correlations.

## Usage

```
SidakCor(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  vect = FALSE,
  logical = FALSE,
  arr.ind = FALSE
)
```

**Arguments**

<code>data</code>	matrix of observations
<code>alpha</code>	level of multiple testing (used if <code>logical=TRUE</code> )
<code>stat_test</code>	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
<code>vect</code>	if <code>TRUE</code> returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if <code>FALSE</code> , returns an array containing the adjusted p-values for each entry of the correlation matrix
<code>logical</code>	if <code>TRUE</code> , returns either a vector or a matrix where each element is equal to <code>TRUE</code> if the corresponding null hypothesis is rejected, and to <code>FALSE</code> if it is not rejected
<code>arr.ind</code>	if <code>TRUE</code> , returns the indexes of the significant correlations, with respect to level <code>alpha</code>

**Value****Returns**

- the adjusted p-values, as a vector or a matrix depending of the value of `vect`,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

Šidák, Z. (1967). Rectangular confidence regions for the means of multivariate normal distributions. Journal of the American Statistical Association, 62(318), 626-633.

**See Also**

`ApplyFwerCor`, `SidakCor_SD`

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# adjusted p-values
res <- SidakCor(data,stat_test='empirical')
round(res,2)
```

```
# significant correlations with level alpha:
alpha <- 0.05
whichCor(res<alpha)
# directly
SidakCor(data,alpha,stat_test='empirical',arr.ind=TRUE)
```

---

SidakCor_SD	<i>Sidak multiple testing method for correlations with stepdown procedure.</i>
-------------	--

---

### Description

Sidak multiple testing method for correlations with stepdown procedure.

### Usage

```
SidakCor_SD(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  vect = FALSE,
  logical = TRUE,
  arr.ind = FALSE
)
```

### Arguments

data	matrix of observations
alpha	level of multiple testing
stat_test	4 test statistics are available: <b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
vect	if TRUE returns a vector of TRUE/FALSE values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing TRUE/FALSE values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected if FALSE, returns a list of successive p-values : element <code>[[i+1]]</code> of the list giving the p-values evaluated on the non-rejected hypothesis at step <code>[[i]]</code> ; p-values are either as a vector or a list depending on vect
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

**Value****Returns**

- logicals, equal to TRUE if the corresponding element of the statistic vector is rejected, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**References**

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

Šidák, Z. (1967). Rectangular confidence regions for the means of multivariate normal distributions. Journal of the American Statistical Association, 62(318), 626-633.

**See Also**

ApplyFwerCor, SidakCor

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
alpha <- 0.05
# significant correlations:
SidakCor_SD(data,alpha,stat_test='empirical', arr.ind=TRUE)
# successive p-values
res <- SidakCor_SD(data,stat_test='empirical', logical=FALSE)
lapply(res,FUN=function(x){round(x,2)})
# successive rejections
lapply(res,FUN=function(x){whichCor(x<alpha)})
```

---

SimuFdr

*Simulates Gaussian data with a given correlation matrix and applies a FDR controlling procedure on the correlations.*

---

**Description**

Simulates Gaussian data with a given correlation matrix and applies a FDR controlling procedure on the correlations.

**Usage**

```

SimuFdr(
  corr_theo,
  n = 100,
  Nsimu = 1,
  alpha = 0.05,
  stat_test = "empirical",
  method = "LCTnorm",
  Nboot = 1000,
  seed = NULL
)

```

**Arguments**

corr_theo	the correlation matrix of Gaussian data simulated
n	sample size
Nsimu	number of simulations
alpha	level of multiple testing
stat_test	<b>'empirical'</b> $\sqrt{n} * abs(corr)$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1+corr)/(1-corr))$ <b>'student'</b> $\sqrt{n-2} * abs(corr) / \sqrt{1-corr^2}$ <b>'gaussian'</b> $\sqrt{n} * mean(Y) / sd(Y)$ with $Y = (X_i - mean(X_i))(X_j - mean(X_j))$
method	choice between 'LCTnorm' and 'LCTboot', developed by Cai & Liu (2016), 'BH', traditional Benjamini-Hochberg (1995)'s procedure, and 'BHboot', Benjamini-Hochberg (1995)'s procedure with bootstrap evaluation of pvalues
Nboot	number of iterations for Monte-Carlo of bootstrap quantile evaluation
seed	seed for the Gaussian simulations

**Value**

Returns a line vector containing estimated values for fwer, fdr, sensitivity, specificity and accuracy.

**References**

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the royal statistical society. Series B (Methodological)*, 289-300.

Cai, T. T., & Liu, W. (2016). Large-scale multiple testing of correlations. *Journal of the American Statistical Association*, 111(513), 229-240.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

**See Also**

ApplyFdrCor, SimuFwer



**Examples**

```

Nsimu <- 1000
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
alpha <- 0.05
SimuFdr(corr_theo,n,Nsimu,alpha,stat_test='empirical',method='LCTnorm')

```

---

SimuFwer	<i>Simulates Gaussian data with a given correlation matrix and applies a FWER controlling procedure on the correlations.</i>
----------	--

---

**Description**

Simulates Gaussian data with a given correlation matrix and applies a FWER controlling procedure on the correlations.

**Usage**

```

SimuFwer(
  corr_theo,
  n = 100,
  Nsimu = 1,
  alpha = 0.05,
  stat_test = "empirical",
  method = "Sidak",
  Nboot = 1000,
  stepdown = TRUE,
  seed = NULL
)

```

**Arguments**

corr_theo	the correlation matrix of Gaussian data simulated
n	sample size
Nsimu	number of simulations
alpha	level of multiple testing
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{1 - \text{corr}^2}$ <b>'gaussian'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
method	choice between 'Bonferroni', 'Sidak', 'BootRW', 'MaxTinfy'
Nboot	number of iterations for Monte-Carlo of bootstrap quantile evaluation
stepdown	logical, if TRUE a stepdown procedure is applied
seed	seed for the Gaussian simulations

**Value**

Returns a line vector containing estimated values for fwer, fdr, sensitivity, specificity and accuracy.

**References**

- Bonferroni, C. E. (1935). Il calcolo delle assicurazioni su gruppi di teste. Studi in onore del professore salvatore ortu carboni, 13-60.
- Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.
- Romano, J. P., & Wolf, M. (2005). Exact and approximate stepdown methods for multiple hypothesis testing. *Journal of the American Statistical Association*, 100(469), 94-108.
- Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.
- Westfall, P.H. & Young, S. (1993) Resampling-based multiple testing: Examples and methods for p-value adjustment, John Wiley & Sons, vol. 279.

**See Also**

ApplyFwerCor, SimuFwer\_oracle, SimuFdr

**Examples**

```
Nsimu <- 1000
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
alpha <- 0.05
SimuFwer(corr_theo,n,Nsimu,alpha,stat_test='empirical',method='Bonferroni',stepdown=FALSE)
```

---

SimuFwer_oracle	<i>Simulates Gaussian data with a given correlation matrix and applies oracle MaxTinfy on the correlations.</i>
-----------------	---

---

**Description**

Simulates Gaussian data with a given correlation matrix and applies oracle MaxTinfy (i.e. Drton & Perlman (2007)'s procedure with the true correlation matrix) on the correlations.

**Usage**

```
SimuFwer_oracle(
  corr_theo,
  n = 100,
  Nsimu = 1,
```

```

    alpha = 0.05,
    stat_test = "empirical",
    method = "MaxTinfy",
    Nboot = 1000,
    stepdown = TRUE,
    seed = NULL
)

```

### Arguments

corr_theo	the correlation matrix of Gaussian data simulated
n	sample size
Nsimu	number of simulations
alpha	level of multiple testing
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'gaussian'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
method	only 'MaxTinfy' available
Nboot	number of iterations for Monte-Carlo of bootstrap quantile evaluation
stepdown	logical, if TRUE a stepdown procedure is applied
seed	seed for the Gaussian simulations

### Value

Returns a line vector containing estimated values for fwer, fdr, sensitivity, specificity and accuracy.

### References

Drton, M., & Perlman, M. D. (2007). Multiple testing and error control in Gaussian graphical model selection. *Statistical Science*, 22(3), 430-449.

Roux, M. (2018). Graph inference by multiple testing with application to Neuroimaging, Ph.D., Université Grenoble Alpes, France, <https://tel.archives-ouvertes.fr/tel-01971574v1>.

### See Also

ApplyFwerCor\_Oracle, SimuFwer

### Examples

```

Nsimu <- 1000
n <- 50
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
alpha <- 0.05
SimuFwer_oracle(corr_theo,n,Nsimu,alpha,stat_test='empirical',stepdown=FALSE,Nboot=100)

```

---

UncorrectedCor	<i>Uncorrected testing procedure for correlations.</i>
----------------	--

---

## Description

Uncorrected testing procedure for correlations.

## Usage

```
UncorrectedCor(
  data,
  alpha = 0.05,
  stat_test = "empirical",
  vect = FALSE,
  logical = FALSE,
  arr.ind = FALSE
)
```

## Arguments

data	matrix of observations
alpha	level of multiple testing (used if logical=TRUE)
stat_test	<b>'empirical'</b> $\sqrt{n} * \text{abs}(\text{corr})$ <b>'fisher'</b> $\sqrt{n-3} * 1/2 * \log((1 + \text{corr})/(1 - \text{corr}))$ <b>'student'</b> $\sqrt{n-2} * \text{abs}(\text{corr})/\sqrt{(1 - \text{corr}^2)}$ <b>'2nd.order'</b> $\sqrt{n} * \text{mean}(Y)/\text{sd}(Y)$ with $Y = (X_i - \text{mean}(X_i))(X_j - \text{mean}(X_j))$
vect	if TRUE returns a vector of adjusted p-values, corresponding to <code>vectorize(cor(data))</code> ; if FALSE, returns an array containing the adjusted p-values for each entry of the correlation matrix
logical	if TRUE, returns either a vector or a matrix where each element is equal to TRUE if the corresponding null hypothesis is rejected, and to FALSE if it is not rejected
arr.ind	if TRUE, returns the indexes of the significant correlations, with respect to level alpha

## Value

Returns

- the non-adjusted p-values, as a vector or a matrix depending of the value of vect,
- an array containing indexes  $\{(i, j), i < j\}$  for which correlation between variables  $i$  and  $j$  is significant, if `arr.ind=TRUE`.

**Examples**

```

n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
# p-values
res <- UncorrectedCor(data,stat_test='empirical')
round(res,2)
# significant correlations with level alpha:
alpha <- 0.05
whichCor(res<alpha)
# directly
UncorrectedCor(data,alpha,stat_test='empirical',arr.ind=TRUE)

```

---

unvectorize	<i>Returns an upper-triangle matrix, without the diagonal, containing the elements of a given vector.</i>
-------------	---

---

**Description**

Returns an upper-triangle matrix, without the diagonal, containing the elements of a given vector.

**Usage**

```
unvectorize(vect)
```

**Arguments**

vect	A vector containing the upper triangle of a matrix, without the diagonal
------	--

**Value**

Returns an upper-triangle matrix where each entry is given by the vector containing the upper triangle of a matrix, without the diagonal.

**See Also**

vectorize

**Examples**

```
unvectorize(1:10)
```

---

vectorize	<i>Returns a vector containing the upper triangle of a matrix, without the diagonal.</i>
-----------	--

---

**Description**

Returns a vector containing the upper triangle of a matrix, without the diagonal.

**Usage**

```
vectorize(mat)
```

**Arguments**

mat	a square matrix
-----	-----------------

**Value**

Returns a vector containing the upper triangle of a matrix, without the diagonal.

**See Also**

unvectorize

**Examples**

```
vectorize(matrix(1:9,3,3))
```

---

whichCor	<i>Returns the indexes of an upper triangular matrix with logical entries.</i>
----------	--

---

**Description**

Returns the indexes of an upper triangular matrix with logical entries.

**Usage**

```
whichCor(mat)
```

**Arguments**

mat	A matrix with logical entries in the upper triangular part
-----	--

**Value**

Returns the indexes of the upper triangular part where the entries are TRUE

**Examples**

```
n <- 100
p <- 10
corr_theo <- diag(1,p)
corr_theo[1,3] <- 0.5
corr_theo[3,1] <- 0.5
data <- MASS::mvrnorm(n,rep(0,p),corr_theo)
res <- ApplyFwerCor(data,stat_test='empirical',method='Bonferroni',stepdown=FALSE)
# significant correlations, level alpha:
alpha <- 0.05
whichCor(res<alpha)
```

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