Weighted Support Vector Machine Formulation

tx2155@columbia.edu

by Tianchen Xu

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The original formulation of unweighted SVM with linear kernel is as follows [Valdimir and Vapnik (1995)]:

$$\min_{\omega, \xi} \frac{1}{2} ||\omega||^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$$

s.t. $$y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i,$$
$$\langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*,$$
$$\xi_i, \xi_i^* \geq 0.$$  

The constant $C > 0$ determines the trade-off between the flatness of $f$ and the amount up to which deviations larger than $\varepsilon$ are tolerated. This corresponds to dealing with a so called $\varepsilon$-insensitive loss function $|\xi|_\varepsilon$ described by

$$|\xi|_\varepsilon = \begin{cases} 0, & \text{if } |\xi| \leq \varepsilon \\ |\xi| - \varepsilon, & \text{o/w.} \end{cases}$$

The corresponding weighted SVM with $W_i$ as individual weights:

$$\min_{\omega, \xi} \frac{1}{2} ||\omega||^2 + C \sum_{i=1}^{n} W_i (\xi_i + \xi_i^*)$$

s.t. $$y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i,$$
$$\langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*,$$
$$\xi_i, \xi_i^* \geq 0.$$  

Other kinds of weighted SVMs (with different kernels) have the similar formulation.

Available kernels:

<table>
<thead>
<tr>
<th>kernel</th>
<th>formula</th>
<th>parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>$u^T v$</td>
<td>(none)</td>
</tr>
<tr>
<td>polynomial</td>
<td>$(\gamma u^T v + c_0)^d$</td>
<td>$\gamma, d, c_0$</td>
</tr>
<tr>
<td>radial basis fct.</td>
<td>$\exp{-\gamma</td>
<td>u - v</td>
</tr>
<tr>
<td>sigmoid</td>
<td>$\tanh{\gamma u^T v + c_0}$</td>
<td>$\gamma, c_0$</td>
</tr>
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References