Package ‘bivpois’

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Density computation of the bivariate Poisson distribution

bivpois-package

Bivariate Poisson Distribution

Description

The Bivariate Poisson Distribution.

Details

Package: bivpois
Type: Package
Version: 1.0
Date: 2023-10-18
License: GPL-2

Maintainers

Michail Tsagris <mtsagris@uoc.gr>.

Author(s)

Michail Tsagris <mtsagris@uoc.gr>.

References


Usage

dbp(x1, x2 = NULL, lambda, logged = TRUE)
Density computation of the bivariate Poisson distribution

Arguments

x1 Either a numerical vector with the values of the first variable or a matrix with 2 columns containing both variables. In the latter case, x2 must be NULL.

x2 A numerical vector with the values of the second. If x1 is a matrix with 2 columns containing both variables, x2 must be NULL.

lambda A vector with three numbers, the estimates of the $\lambda_i$.

logged Should the logarithm of the density values be computed? The default value is TRUE.

Details

The density of the bivariate Poisson distribution is computed.

Value

A vector with the logged density values.

Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

References


See Also

rbp

Examples

x <- rbp(300, c(3, 5, 2))
a <- bp.mle(x)
f <- dbp(x, lambda = a$lambda)
sum(f)
Goodness of fit test for the bivariate Poisson distribution

Description

Goodness of fit test for the bivariate Poisson distribution.

Usage

\begin{verbatim}
bp.gof(x1, x2 = NULL, R = 999)
bp.gof2(x1, x2 = NULL, R = 999)
\end{verbatim}

Arguments

- **x1**: Either a numerical vector with the values of the first variable or a matrix with 2 columns containing both variables. In the latter case, x2 must be NULL.
- **x2**: A numerical vector with the values of the second. If x1 is a matrix with 2 columns containing both variables, x2 must be NULL.
- **R**: The number of Monte Carlo replicates to use.

Details

Kocherlakota and Kocherlakota (1992) mention the following a goodness of fit test for the bivariate Poisson distribution, the index of dispersion test. They mention that Loukas and Kemp (1986) developed this test as an extension of the univariate dispersion test. They test for departures from the bivariate Poisson againsta alternatives which involve an increase in the generalised variance, the determinant of the covariance matrix of the two variables.

Rayner, Thas and Best (2009) mentions a revised version of this test whose test statistic is now given by

\[ I_{B^*} = \frac{n}{1 - r^2} \left( \frac{S_1^2}{\bar{x}_1} - 2r^2 \sqrt{\frac{S_1^2 S_2^2}{\bar{x}_1 \bar{x}_2}} + \frac{S_2^2}{\bar{x}_2} \right), \]

where \( n \) is the sample size, \( r \) is the sample Pearson correlation coefficient, \( S_1^2 \) and \( S_2^2 \) are the two sample variances and \( \bar{x}_1 \) and \( \bar{x}_2 \) are the two sample means. Under the null hypothesis the \( I_{B^*} \) follows asymptotically a \( \chi^2 \) with \( 2n - 3 \) degrees of freedom. However, I did some simulations and I saw that it does not perform very well in terms of the type I error. If you see the simulations in their book (page 132) you will see this. For this reason, the function calculates the p-value of the \( I_{B^*} \) using Monte Carlo (or parametric bootstrap).

The second function, \( \text{bp.gof2()} \), is a vectorised version of the first, and much faster. I put both of them here to show how one can vectorize a function and make it faster.
Maximum likelihood estimation of the bivariate Poisson distribution

Value

A list including:

- runtime: The duration of the algorithm.
- tab: The contingency table of the two variables.
- pvalue: The Monte-Carlo based estimated p-value.

Author(s)

Michail Tsagris.
R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

References


See Also

bp.mle

Examples

```r
x <- rbp(300, c(3, 5, 2))
bp.gof(x)
bp.gof2(x)
```

Maximum likelihood estimation of the bivariate Poisson distribution

Description

Maximum likelihood estimation of the bivariate Poisson distribution.

Usage

```
bp.mle(x1, x2 = NULL)
bp.mle2(x1, x2 = NULL)
```
Maximum likelihood estimation of the bivariate Poisson distribution

Arguments

x1 Either a numerical vector with the values of the first variable or a matrix with 2 columns containing both variables. In the latter case, x2 must be NULL.

x2 A numerical vector with the values of the second. If x1 is a matrix with 2 columns containing both variables, x2 must be NULL.

Details

Using the addition method (see function `rbp`) to simulate random values from the bivariate Poisson, its representation is given by

\[
P(X = x, Y = y) = e^{-(\lambda_1 + \lambda_2 + \lambda_3)} \frac{\lambda_1^x \lambda_2^y}{x! y!} \sum_{k=0}^{\min(x,y)} \frac{x!}{k!} \frac{y!}{k!} \left( \frac{\lambda_3}{\lambda_1 \lambda_2} \right)^k.
\]

The above form is found in Karlis and Ntzoufras (2003). This bivariate distribution allows for dependence between the two random variables. Marginally each random variable follows a Poisson distribution with \(E(X) = \lambda_1 + \lambda_3\) and \(E(Y) = \lambda_2 + \lambda_3\). In addition, \(Cov(X, Y) = \lambda_3\). If \(\lambda_3 = 0\), the above expression becomes a product of two Poisson distributions. Hence, \(\lambda_3\) is a measure of dependence between the two random variables.

The function bp.mle() returns a lot of information and is slower than bp.mle2(), which returns fewer information, but is faster.

Value

For the function bp.mle() a list including:

lambda A vector with the estimated values of \((\lambda_1, \lambda_2)\) and \(\lambda_3\). Note that \(\hat{\lambda}_1 = \bar{x}_1 - \lambda_3\) and \(\hat{\lambda}_1 = \bar{x}_1 - \lambda_3\), where \(\bar{x}_1\) and \(\bar{x}_2\) are the two sample means.

rho The estimated correlation coefficient, that is: \(\frac{\hat{\lambda}_3}{\sqrt{(\hat{\lambda}_1 + \hat{\lambda}_3)(\hat{\lambda}_2 + \hat{\lambda}_3)}}\).

ci The 95% Confidence intervals using the observed and the asymptotic information matrix.

loglik The log-likelihood values assuming independence \((\lambda_3 = 0)\) and assuming the bivariate Poisson distribution.

pvalue Three p-values for testing \(\lambda_3 = 0\). These are based on the log-likelihood ratio and two Wald tests using the observed and the asymptotic information matrix.

For the function bp.mle2() a list including:

lambda A vector with the estimated values of \((\lambda_1, \lambda_2)\) and \(\lambda_3\). Note that \(\hat{\lambda}_1 = \bar{x}_1 - \lambda_3\) and \(\hat{\lambda}_1 = \bar{x}_1 - \lambda_3\), where \(\bar{x}_1\) and \(\bar{x}_2\) are the two sample means.

loglik The log-likelihood values assuming independence \((\lambda_3 = 0)\) and assuming the bivariate Poisson distribution.
Profile likelihood of the covariance parameter of the bivariate Poisson distribution

Author(s)

Michail Tsagris.
R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

References


See Also

rbp

Examples

```r
x <- rbp( 300, c(3, 5, 2) )
bp.mle(x)
```

Description

Profile likelihood of the covariance parameter ($\lambda_3$) of the bivariate Poisson distribution.

Usage

```r
lambda3.profile(x1, x2 = NULL)
```

Arguments

- **x1**: Either a numerical vector with the values of the first variable or a matrix with 2 columns containing both variables. In the latter case, x2 must be NULL.
- **x2**: A numerical vector with the values of the second. If x1 is a matrix with 2 columns containing both variables, x2 must be NULL.

Details

The function plots the profile log-likelihood of $\lambda_3$ and computes the relevant 95% confidence interval for the parameter.
Random values generation from the bivariate Poisson distribution

Value

A plot with the profile log-likelihood of $\lambda_3$ and a vector with the 95% confidence interval for the parameter.

Author(s)

Michail Tsagris.
R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

References


See Also

bp.mle

Examples

```r
x <- rbp( 300, c(3, 5, 2) )
lambda3.profile(x)
```

---

Random values generation from the bivariate Poisson distribution

Description

Random values generation from the bivariate Poisson distribution.

Usage

```r
rbp(n, lambda)
```

Arguments

- `n` The sample size.
- `lambda` A vector with the three parameters, $(\lambda_1, \lambda_2, \lambda_3)$ of the Poisson distribution.

Details

In order to generate values from this distribution one needs three independent Poisson variables, $X_1 \sim \text{Po}(\lambda_1)$, $X_2 \sim \text{Po}(\lambda_2)$ and $X_3 \sim \text{Po}(\lambda_3)$. Then, $(X, Y) = (X_1 + X_3, X_2 + X_3) \sim \text{BP}(\lambda_1, \lambda_2, \lambda_3)$. 
Random values generation from the bivariate Poisson distribution

Value
A matrix with n rows and 2 columns.

Author(s)
Michail Tsagris.
R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

References

See Also
bp.mle

Examples
x <- rbp(300, c(3, 5, 2))
bp.mle(x)
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