The `brr` package performs Bayesian inference on the rate ratio $\phi = \frac{\lambda}{\mu}$ in the *two Poisson samples model* given by two independent observations:

$$\begin{aligned}
    x &\sim \mathcal{P}(\lambda S) \\
y &\sim \mathcal{P}(\mu T)
\end{aligned}$$

where $\lambda$ and $\mu$ are the unknown incidence rates and $S$ and $T$ are the known observation-opportunity sizes, or, for short, the sample sizes. $S$ and $T$ are also called the times at risk when they represent some durations.

The `brr` package implements the *semi-conjugate family of prior distributions*. Precisely, for positive numbers $a$, $b$, $c$ and $d$, to be set by the user, the following independent prior distributions are assigned on $\mu$ and $\phi = \lambda/\mu$:

$$\mu \sim \mathcal{G}(a, b) \quad \text{and} \quad \phi \sim \frac{T + b}{S} \times \mathcal{B}'(c, d),$$

Then the joint posterior on $(\mu, \phi)$ is given by

$$(\mu \mid \phi, x, y) \sim \mathcal{G}(a + x + y, b + \phi S + T) \quad \text{and} \quad (\phi \mid x, y) \sim \frac{T + b}{S} \times \mathcal{B}'(c + x, a + d + y).$$

In particular:

- when $a = c = 0.5$ and $b = d = 0$, the prior is the reference prior, also called the *non-informative prior*;
- when $a, b > 0$, $c = 0.5$ and $d = 0$, the prior is the semi-reference prior, also called the *semi-informative prior*, that is to say the reference prior after the arbitrary Gamma prior distribution $\mathcal{G}(a, b)$ is assigned on $\mu$.

### Setting parameters with `brr`

Use the `Brr` function to set the prior parameters, the sample sizes, and the observed counts. One can proceed step by step, for example below we start by supplying the parameters $a$ and $b$ of the prior Gamma distribution on $\mu$:

```r
library(brr)
model <- Brr(a=2, b=3)
summary(model)
```
## Type of prior distribution: semi-informative prior

*Prior distribution on μ*: Gamma(a=2,b=3)

+--------+--------+--------+--------+--------+--------+
| mode   | mean   | sd     | Q1     | Q2     | Q3     |
+========+========+========+========+========+========+
| 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
+--------+--------+--------+--------+--------+--------+

*Prior distribution on ϕ*: Non-informative prior

*Sample sizes*

S (treated group): not supplied yet
T (control group): not supplied yet

*Observed counts*

x (treated group): not supplied yet
y (control group): not supplied yet

*Posterior distribution on ϕ*:
a, b, c, d, S, T, x and y must be supplied

Since c and d were not supplied, brr automatically considers the non-informative prior on ϕ. Equivalently the same brr object can be defined by Brr(a=2, b=3, c=NULL, d=NULL) or Brr(a=2, b=3, c=0.5, d=0).

The brr object is a function which can be used to update itself with new parameters, for example:

```r
model <- model(c=3, d=3, S=100, T=100)
summary(model)
```

## Type of prior distribution: informative prior

*Prior distribution on μ*: Gamma(a=2,b=3)

+--------+--------+--------+--------+--------+--------+
| mode   | mean   | sd     | Q1     | Q2     | Q3     |
+========+========+========+========+========+========+
| 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
+--------+--------+--------+--------+--------+--------+

*Prior distribution on ϕ*: Beta2(c=3,d=3,scale=1.03)

+--------+--------+-------+-------+------+-------+
|        |        |       |       |      |       |
+--------+--------+-------+-------+------+-------+
## | mode | mean  | sd   | Q1  | Q2  | Q3  |
## +========+========+=======+=======+======+=======+
## | 0.515  | 1.545  | 1.995 | 0.578 | 1.03 | 1.836 |
## +--------+--------+-------+-------+------|-------+
##
## *Sample sizes*
## S (treated group): 100
## T (control group): 100
##
## *Observed counts*
## x (treated group): not supplied yet
## y (control group): not supplied yet
##
## *Posterior distribution on \( \phi \)*:
## a, b, c, d, S, T, x and y must be supplied

Now that \( a, b, c, d, S \) and \( T \) have been supplied, the user can play with all functions related to the prior distributions.

For example, \( \text{dprior(model, "lambda", .)} \) evaluates the density of the prior distribution of \( \lambda \). This is convenient to generate a graphic with the \texttt{curve} function:

```r
par(mar=c(4, 3, 1, 1))
curve(dprior(model, "lambda", x), from=0, to=3, axes=FALSE, 
     xlab=expression(lambda), ylab=NA)
axis(1)
```

![Graph of the prior distribution of \( \lambda \)](image)

The \texttt{brr} package also provides a way to generate a plot with automatic aesthetics:

```r
par(mar=c(4, 3, 1, 1))
plot(model, dprior(lambda))
```
If you are not pleased with the automatic bounds of the interval over which the curve is plotted, set your bounds in the `bounds` argument of the `plot` function. You can also set usual graphical parameters such as `lwd`, `lty`, ...

**Posterior inference with `brr`**

Posterior inference is available after, obviously, setting the observed counts $x$ and $y$:

```r
model <- model(x=14, y=20)
summary(model)
```

```text
## Type of prior distribution: informative prior
##
## *Prior distribution on $\mu$*: Gamma($a=2,b=3$)
##
## | mode  | mean  | sd    | Q1    | Q2    | Q3    |
## |-------+-------+-------+-------+-------+-------|
## | 0.3333 | 0.6667 | 0.4714 | 0.3204 | 0.5594 | 0.8975 |
##
## *Prior distribution on $\phi$*: Beta2($c=3,d=3,scale=1.03$)
##
## | mode  | mean  | sd    | Q1    | Q2    | Q3    |
## |-------+-------+-------+-------+-------+-------|
## | 0.515 | 1.545 | 1.995 | 0.578 | 1.03  | 1.836 |
##
```
## *Sample sizes*
## S (treated group): 100
## T (control group): 100
##
## *Observed counts*
## x (treated group): 14
## y (control group): 20
##
## *Posterior distribution on ϕ*: Beta2(17,25,scale=1.03)
##
## +--------+--------+--------+--------+-------+--------+
## | mode | mean | sd | Q1 | Q2 | Q3 |
## +========+========+========+========+=======+========+
## | 0.6338 | 0.7296 | 0.2363 | 0.5613 | 0.696 | 0.8605 |
## +--------+--------+--------+--------+-------+--------+
##
## Pr('relative risk is greater than 1') = 0.876132340775555

Estimates are provided by the coef function:

```r
coeff(model)
```

## Estimates of ϕ
##
## mode : 0.6338462
## mean : 0.7295833
## median : 0.6959761
## intrinsic : 0.6966658
## intrinsic2 : 0.7033363

Posterior credibility intervals are provided by the confint function:

```r
confint(model)
```

## 95%-credibility intervals about ϕ
##
## +---------------------------------------------+
## | interval | lwr | upr |
## +=============================================+
## | equi-tailed | 0.3679 | 1.284 |
## +---------------------------------------------+
Predictions are provided by the `predict` function after adding the sample sizes of the future experiment:

```r
model <- model(Snew=500, Tnew=500)
predict(model)
```

```
# Predictions and 95%-credibility prediction intervals
#
## +-------+--------+----------+-------+-------+
## | obs | size | median | lwr | upr |
## +=======+========+==========+=======+=======+
## | xnew | 500 | 71 | 38 | 117 |
## +-------+--------+----------+-------+-------+
## | ynew | 500 | 102 | 62 | 156 |
## +-------+--------+----------+-------+-------+
```