

Package ‘calculus’

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Type Package

Title High Dimensional Numerical and Symbolic Calculus

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Description Efficient C++ optimized functions for numerical and symbolic calculus as described in Guidotti (2020) <arXiv:2101.00086>. It includes basic arithmetic, tensor calculus, Einstein summing convention, fast computation of the Levi-Civita symbol and generalized Kronecker delta, Taylor series expansion, multivariate Hermite polynomials, high-order derivatives, ordinary differential equations, differential operators (Gradient, Jacobian, Hessian, Divergence, Curl, Laplacian) and numerical integration in arbitrary orthogonal coordinate systems: cartesian, polar, spherical, cylindrical, parabolic or user defined by custom scale factors.

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URL <https://calculus.guidotti.dev>

BugReports <https://github.com/eguidotti/calculus/issues>

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Author Emanuele Guidotti [aut, cre] (<<https://orcid.org/0000-0002-8961-6623>>)

Maintainer Emanuele Guidotti <emanuele.guidotti@unine.ch>

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c2e

Characters to Expressions

Description

Converts characters to expressions.

Usage

c2e(x)

Arguments

x character.

Value

expression.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other utilities: [e2c\(\)](#), [evaluate\(\)](#), [wrap\(\)](#)

Examples

```
### convert characters
c2e("a")

### convert array of characters
c2e(array("a", dim = c(2,2)))
```

contraction

Numerical and Symbolic Tensor Contraction

Description

Sums over repeated indices in an array.

Usage

```
contraction(x, i = NULL, drop = TRUE)
```

Arguments

x	indexed array. See index .
i	subset of repeated indices to sum up. If NULL, the summation takes place on all the repeated indices.
drop	logical. Drop summation indices? If FALSE, keep dummy dimensions.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [delta\(\)](#), [diagonal\(\)](#), [einstein\(\)](#), [epsilon\(\)](#), [index\(\)](#)

Examples

```
### matrix trace
x <- matrix(letters[1:4], nrow = 2)
contraction(x)

### tensor trace
x <- array(1:27, dim = c(3,3,3))
contraction(x)

#### tensor contraction over repeated indices
x <- array(1:27, dim = c(3,3,3))
index(x) <- c("i", "i", "j")
contraction(x)

#### tensor contraction over specific repeated indices only
x <- array(1:16, dim = c(2,2,2,2))
index(x) <- c("i", "i", "k", "k")
contraction(x, i = "k")

#### tensor contraction keeping dummy dimensions
x <- array(letters[1:16], dim = c(2,2,2,2))
index(x) <- c("i", "i", "k", "k")
contraction(x, drop = FALSE)
```

cross

Numerical and Symbolic Cross Product

Description

Computes the cross product of $n - 1$ vectors of length n .

Usage

```
cross(...)

x %cross% y
```

Arguments

...	$n - 1$ vectors of length n .
x	numeric or character vector of length 3.
y	numeric or character vector of length 3.

Value

n -dimensional vector orthogonal to the $n - 1$ vectors.

Functions

- `%cross%`: binary operator for 3-dimensional cross products.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

Examples

```
### canonical basis 4-d
cross(c(1,0,0,0), c(0,1,0,0), c(0,0,0,1))
```

```
### canonical basis 3-d
cross(c(1,0,0), c(0,1,0))
```

```
### symbolic cross product 3-d
c(1,0,0) %cross% c(0,1,0)
```

```
### symbolic cross product 3-d
c("a","b","c") %cross% c(0,0,1)
```

 curl

Numerical and Symbolic Curl

Description

Computes the numerical curl of functions or the symbolic curl of characters in arbitrary **orthogonal coordinate systems**.

Usage

```
curl(
  f,
  var,
  params = list(),
  coordinates = "cartesian",
  accuracy = 4,
  stepsize = NULL,
  drop = TRUE
)

f %curl% var
```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a vector of scale factors for each variable.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the curl as a vector and not as an array for vector-valued functions.

Details

The curl of a vector-valued function F_i at a point is represented by a vector whose length and direction denote the magnitude and axis of the maximum circulation. In 2 dimensions, the `curl` is computed in arbitrary orthogonal coordinate systems using the scale factors h_i and the Levi-Civita symbol [epsilon](#):

$$\nabla \times F = \frac{1}{h_1 h_2} \sum_{ij} \epsilon_{ij} \partial_i (h_j F_j) = \frac{1}{h_1 h_2} \left(\partial_1 (h_2 F_2) - \partial_2 (h_1 F_1) \right)$$

In 3 dimensions:

$$(\nabla \times F)_k = \frac{h_k}{J} \sum_{ij} \epsilon_{ijk} \partial_i (h_j F_j)$$

where $J = \prod_i h_i$. In $m + 2$ dimensions, the `curl` is implemented in such a way that the formula reduces correctly to the previous cases for $m = 0$ and $m = 1$:

$$(\nabla \times F)_{k_1 \dots k_m} = \frac{h_{k_1} \dots h_{k_m}}{J} \sum_{ij} \epsilon_{ijk_1 \dots k_m} \partial_i (h_j F_j)$$

When F is an array of vector-valued functions $F_{d_1, \dots, d_n, j}$ the `curl` is computed for each vector:

$$(\nabla \times F)_{d_1 \dots d_n, k_1 \dots k_m} = \frac{h_{k_1} \dots h_{k_m}}{J} \sum_{ij} \epsilon_{ijk_1 \dots k_m} \partial_i (h_j F_{d_1 \dots d_n, j})$$

Value

Vector for vector-valued functions when `drop=TRUE`, array otherwise.

Functions

- `%curl%`: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [derivative\(\)](#), [divergence\(\)](#), [gradient\(\)](#), [hessian\(\)](#), [jacobian\(\)](#), [laplacian\(\)](#)

Examples

```
### symbolic curl of a 2-d vector field
f <- c("x^3*y^2", "x")
curl(f, var = c("x", "y"))

### numerical curl of a 2-d vector field in (x=1, y=1)
f <- function(x,y) c(x^3*y^2, x)
curl(f, var = c(x=1, y=1))

### numerical curl of a 3-d vector field in (x=1, y=1, z=1)
f <- function(x,y,z) c(x^3*y^2, x, z)
curl(f, var = c(x=1, y=1, z=1))

### vectorized interface
f <- function(x) c(x[1]^3*x[2]^2, x[1], x[3])
curl(f, var = c(1,1,1))

### symbolic array of vector-valued 3-d functions
f <- array(c("x*y", "x", "y*z", "y", "x*z", "z"), dim = c(2,3))
curl(f, var = c("x", "y", "z"))

### numeric array of vector-valued 3-d functions in (x=1, y=1, z=1)
f <- function(x,y,z) array(c(x*y, x, y*z, y, x*z, z), dim = c(2,3))
curl(f, var = c(x=1, y=1, z=1))

### binary operator
c("x*y", "y*z", "x*z") %curl% c("x", "y", "z")
```

delta

Generalized Kronecker Delta

Description

Computes the Generalized Kronecker Delta.

Usage

```
delta(n, p = 1)
```

Arguments

`n` number of elements for each dimension.
`p` order of the generalized Kronecker delta, $p=1$ for the standard Kronecker delta.

Value

array representing the generalized Kronecker delta tensor.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [contraction\(\)](#), [diagonal\(\)](#), [einstein\(\)](#), [epsilon\(\)](#), [index\(\)](#)

Examples

```
### Kronecker delta 3x3
delta(3)

### generalized Kronecker delta 3x3 of order 2
delta(3, p = 2)
```

derivative

Numerical and Symbolic Derivatives

Description

Computes symbolic derivatives based on the [D](#) function, or numerical derivatives based on finite differences.

Usage

```
derivative(
  f,
  var,
  params = list(),
  order = 1,
  accuracy = 4,
  stepsize = NULL,
  drop = TRUE,
```



```

    deparse = TRUE
)

```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See details.
params	list of additional parameters passed to f.
order	integer vector, giving the differentiation order for each variable. See details.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the array of derivatives without adding a dummy dimension when order is of length 1.
deparse	if TRUE, return character instead of expression.

Details

The function behaves differently depending on the arguments order, the order of differentiation, and var, the variable names with respect to which the derivatives are computed.

When multiple variables are provided and order is a single integer n , then the n -th order derivative is computed for each element of f with respect to each variable:

$$D = \partial^{(n)} \otimes F$$

that is:

$$D_{i,\dots,j,k} = \partial_k^{(n)} F_{i,\dots,j}$$

where F is the array of functions and $\partial_k^{(n)}$ denotes the n -th order partial derivative with respect to the k -th variable.

When order matches the length of var, it is assumed that the differentiation order is provided for each variable. In this case, each element is derived n_k times with respect to the k -th variable, for each of the m variables.

$$D_{i,\dots,j} = \partial_1^{(n_1)} \dots \partial_m^{(n_m)} F_{i,\dots,j}$$

The same applies when order is a named vector giving the differentiation order for each variable. For example, order = c(x=1,y=2) differentiates once with respect to x and twice with respect to y . A call with order = c(x=1,y=0) is equivalent to order = c(x=1).

To compute numerical derivatives or to evaluate symbolic derivatives at a point, the function accepts a named vector for the argument var; e.g. var = c(x=1,y=2) evaluates the derivatives in $x = 1$ and $y = 2$. For functions where the first argument is used as a parameter vector, var should be a numeric vector indicating the point at which the derivatives are to be calculated.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other derivatives: `taylor()`

Other differential operators: `curl()`, `divergence()`, `gradient()`, `hessian()`, `jacobian()`, `laplacian()`

Examples

```
### symbolic derivatives
derivative(f = "sin(x)", var = "x")

### numerical derivatives
f <- function(x) sin(x)
derivative(f = f, var = c(x=0))

### higher order derivatives
f <- function(x) sin(x)
derivative(f = f, var = c(x=0), order = 3)

### multivariate functions
## - derive once with respect to x
## - derive twice with respect to y
## - evaluate in x=0 and y=0
f <- function(x, y) y^2*sin(x)
derivative(f = f, var = c(x=0, y=0), order = c(1,2))

### vector-valued functions
## - derive each element twice with respect to each variable
## - evaluate in x=0 and y=0
f <- function(x, y) c(x^2, y^2)
derivative(f, var = c(x=0, y=0), order = 2)

### vectorized interface
f <- function(x) c(sum(x), prod(x))
derivative(f, var = c(0,0,0), order = 1)
```

diagonal

Tensor Diagonals

Description

Functions to extract or replace the diagonals of an array, or construct a diagonal array.

Usage

```
diagonal(x = 1, dim = rep(2, 2))
```

```
diagonal(x) <- value
```

Arguments

x	an array from which to extract the diagonals, or a vector giving the diagonal values to construct the array.
dim	the dimensions of the (square) array to construct when x is a vector.
value	vector giving the values of the diagonal entries.

Value

Vector of the diagonal entries of x if x is an array. If x is a vector, returns the diagonal array with the entries given by x.

Functions

- `diagonal<-`: set diagonals.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [contraction\(\)](#), [delta\(\)](#), [einstein\(\)](#), [epsilon\(\)](#), [index\(\)](#)

Examples

```
### 3x3 matrix
diagonal(x = 1, dim = c(3,3))

### 2x2x2 array
diagonal(x = 1:2, dim = c(2,2,2))

### extract diagonals
x <- diagonal(1:5, dim = c(5,5,5))
diagonal(x)

### set diagonals
x <- array(0, dim = c(2,2,2))
diagonal(x) <- 1:2
x
```

divergence

*Numerical and Symbolic Divergence***Description**

Computes the numerical divergence of functions or the symbolic divergence of characters in arbitrary **orthogonal coordinate systems**.

Usage

```
divergence(
  f,
  var,
  params = list(),
  coordinates = "cartesian",
  accuracy = 4,
  stepsize = NULL,
  drop = TRUE
)

f %divergence% var
```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a vector of scale factors for each variable.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the divergence as a scalar and not as an array for vector-valued functions.

Details

The divergence of a vector-valued function F_i produces a scalar value $\nabla \cdot F$ representing the volume density of the outward flux of the vector field from an infinitesimal volume around a given point. The divergence is computed in arbitrary orthogonal coordinate systems using the scale factors h_i :

$$\nabla \cdot F = \frac{1}{J} \sum_i \partial_i \left(\frac{J}{h_i} F_i \right)$$

where $J = \prod_i h_i$. When F is an array of vector-valued functions $F_{d_1 \dots d_n, i}$, the divergence is computed for each vector:

$$(\nabla \cdot F)_{d_1 \dots d_n} = \frac{1}{J} \sum_i \partial_i \left(\frac{J}{h_i} F_{d_1 \dots d_n, i} \right)$$

Value

Scalar for vector-valued functions when `drop=TRUE`, array otherwise.

Functions

- `%divergence%`: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [curl\(\)](#), [derivative\(\)](#), [gradient\(\)](#), [hessian\(\)](#), [jacobian\(\)](#), [laplacian\(\)](#)

Examples

```
### symbolic divergence of a vector field
f <- c("x^2", "y^3", "z^4")
divergence(f, var = c("x", "y", "z"))

### numerical divergence of a vector field in (x=1, y=1, z=1)
f <- function(x,y,z) c(x^2, y^3, z^4)
divergence(f, var = c(x=1, y=1, z=1))

### vectorized interface
f <- function(x) c(x[1]^2, x[2]^3, x[3]^4)
divergence(f, var = c(1,1,1))

### symbolic array of vector-valued 3-d functions
f <- array(c("x^2", "x", "y^2", "y", "z^2", "z"), dim = c(2,3))
divergence(f, var = c("x", "y", "z"))

### numeric array of vector-valued 3-d functions in (x=0, y=0, z=0)
f <- function(x,y,z) array(c(x^2,x,y^2,y,z^2,z), dim = c(2,3))
divergence(f, var = c(x=0, y=0, z=0))

### binary operator
c("x^2", "y^3", "z^4") %divergence% c("x", "y", "z")
```

e2c

Expressions to Characters

Description

Converts expressions to characters.

Usage

```
e2c(x)
```

Arguments

x expression.

Value

character.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other utilities: [c2e\(\)](#), [evaluate\(\)](#), [wrap\(\)](#)

Examples

```
### convert expressions
expr <- parse(text = "a")
e2c(expr)

### convert array of expressions
expr <- array(parse(text = "a"), dim = c(2,2))
e2c(expr)
```

`einstein`*Numerical and Symbolic Einstein Summation*

Description

Implements the Einstein notation for summation over repeated indices.

Usage

```
einstein(..., drop = TRUE)
```

Arguments

`...` arbitrary number of indexed arrays. See [index](#).
`drop` logical. Drop summation indices? If FALSE, keep dummy dimensions.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [contraction\(\)](#), [delta\(\)](#), [diagonal\(\)](#), [epsilon\(\)](#), [index\(\)](#)

Examples

```
### A{i,j} B{j,k}
a <- array(letters[1:6], dim = c(i=2, j=3))
b <- array(letters[1:3], dim = c(j=3, k=1))
einstein(a,b)
```

```
### A{i,j} B{j,k,k} C{k,l} D{j,k}
a <- array(1:10, dim = c(i=2, j=5))
b <- array(1:45, dim = c(j=5, k=3, k=3))
c <- array(1:12, dim = c(k=3, l=4))
d <- array(1:15, dim = c(j=5, k=3))
einstein(a,b,c,d)
```

epsilon

Levi-Civita Symbol

Description

Computes the Levi-Civita totally antisymmetric tensor.

Usage

```
epsilon(n)
```

Arguments

n number of dimensions.

Value

array representing the Levi-Civita symbol.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [contraction\(\)](#), [delta\(\)](#), [diagonal\(\)](#), [einstein\(\)](#), [index\(\)](#)

Examples

```
### Levi-Civita symbol in 2 dimensions  
epsilon(2)
```

```
### Levi-Civita symbol in 3 dimensions  
epsilon(3)
```

`evaluate`*Evaluate Characters and Expressions*

Description

Evaluates an array of characters or expressions.

Usage

```
evaluate(f, var, params = list(), vectorize = TRUE)
```

Arguments

<code>f</code>	array of characters or expressions to be evaluated.
<code>var</code>	named vector or data.frame in which <code>f</code> is to be evaluated.
<code>params</code>	list of additional parameters passed to <code>f</code> .
<code>vectorize</code>	logical. Use vectorization? If TRUE, it can significantly boost performance but <code>f</code> needs to handle the vector of inputs appropriately.

Value

Evaluated object. When `var` is a named vector, the return is an array with the same dimensions of `f`. When `var` is a data.frame, the return is a matrix with columns corresponding to the entries of `f` and rows corresponding to the rows of `var`.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other utilities: `c2e()`, `e2c()`, `wrap()`

Examples

```
### single evaluation
f <- array(letters[1:4], dim = c(2,2))
var <- c(a = 1, b = 2, c = 3, d = 4)
evaluate(f, var)

### multiple evaluation
f <- array(letters[1:4], dim = c(2,2))
var <- data.frame(a = 1:3, b = 2:4, c = 3:5, d = 4:6)
evaluate(f, var)

### multiple evaluation with additional parameters
f <- "a*sum(x)"
```

```

var <- data.frame(a = 1:3)
params <- list(x = 1:3)
evaluate(f, var, params)

### multiple evaluation of non-vectorized expressions
f <- "a*myf(x)"
myf <- function(x) if(x>0) 1 else -1
var <- data.frame(a = 1:3, x = -1:1)
evaluate(f, var, params = list(myf = myf), vectorize = FALSE)

```

gradient

Numerical and Symbolic Gradient

Description

Computes the numerical gradient of functions or the symbolic gradient of characters in arbitrary **orthogonal coordinate systems**.

Usage

```

gradient(
  f,
  var,
  params = list(),
  coordinates = "cartesian",
  accuracy = 4,
  stepsize = NULL,
  drop = TRUE
)

f %gradient% var

```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a vector of scale factors for each variable.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the gradient as a vector and not as an array for scalar-valued functions.

Details

The gradient of a scalar-valued function F is the vector $(\nabla F)_i$ whose components are the partial derivatives of F with respect to each variable i . The gradient is computed in arbitrary orthogonal coordinate systems using the scale factors h_i :

$$(\nabla F)_i = \frac{1}{h_i} \partial_i F$$

When the function F is a tensor-valued function F_{d_1, \dots, d_n} , the gradient is computed for each scalar component. In particular, it becomes the Jacobian matrix for vector-valued function.

$$(\nabla F_{d_1, \dots, d_n})_i = \frac{1}{h_i} \partial_i F_{d_1, \dots, d_n}$$

Value

Gradient vector for scalar-valued functions when drop=TRUE, array otherwise.

Functions

- %gradient%: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [curl\(\)](#), [derivative\(\)](#), [divergence\(\)](#), [hessian\(\)](#), [jacobian\(\)](#), [laplacian\(\)](#)

Examples

```
### symbolic gradient
gradient("x*y*z", var = c("x", "y", "z"))

### numerical gradient in (x=1, y=2, z=3)
f <- function(x, y, z) x*y*z
gradient(f = f, var = c(x=1, y=2, z=3))

### vectorized interface
f <- function(x) x[1]*x[2]*x[3]
gradient(f = f, var = c(1, 2, 3))

### symbolic vector-valued functions
f <- c("y*sin(x)", "x*cos(y)")
gradient(f = f, var = c("x", "y"))

### numerical vector-valued functions
f <- function(x) c(sum(x), prod(x))
```

```
gradient(f = f, var = c(0,0,0))

### binary operator
"x*y^2" %gradient% c(x=1, y=3)
```

hermite

*Hermite Polynomials***Description**

Computes univariate and multivariate Hermite polynomials.

Usage

```
hermite(order, sigma = 1, var = "x", transform = NULL)
```

Arguments

order	the order of the Hermite polynomial.
sigma	the covariance matrix of the Gaussian kernel.
var	character vector giving the variables of the polynomial.
transform	character vector representing a change of variables. See details.

Details

Hermite polynomials are obtained by differentiation of the Gaussian kernel:

$$H_\nu(x, \Sigma) = \exp\left(\frac{1}{2}x_i \Sigma_{ij} x_j\right) (-\partial_x)^\nu \exp\left(-\frac{1}{2}x_i \Sigma_{ij} x_j\right)$$

where Σ is a d -dimensional square matrix and $\nu = (\nu_1 \dots \nu_d)$ is the vector representing the order of differentiation for each variable $x = (x_1 \dots x_d)$. In the case where $\Sigma = 1$ and $x = x_1$ the formula reduces to the standard univariate Hermite polynomials:

$$H_\nu(x) = e^{\frac{x^2}{2}} (-1)^\nu \frac{d^\nu}{dx^\nu} e^{-\frac{x^2}{2}}$$

If transform is not NULL, the variables var x are replaced with transform $f(x)$ to compute the polynomials $H_\nu(f(x), \Sigma)$

Value

list of Hermite polynomials with components:

f the Hermite polynomial.

order the order of the Hermite polynomial.

terms data.frame containing the variables, coefficients and degrees of each term in the Hermite polynomial.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other polynomials: [taylor\(\)](#)

Examples

```
### univariate Hermite polynomials up to order 3
hermite(3)

### multivariate Hermite polynomials up to order 2
hermite(order = 2,
        sigma = matrix(c(1,0,0,1), nrow = 2),
        var = c('z1', 'z2'))

### multivariate Hermite polynomials with transformation of variables
hermite(order = 2,
        sigma = matrix(c(1,0,0,1), nrow = 2),
        var = c('z1', 'z2'),
        transform = c('z1+z2', 'z1-z2'))
```

hessian

Numerical and Symbolic Hessian

Description

Computes the numerical Hessian of functions or the symbolic Hessian of characters.

Usage

```
hessian(f, var, params = list(), accuracy = 4, stepsize = NULL, drop = TRUE)
```

```
f %hessian% var
```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
accuracy	degree of accuracy for numerical derivatives.

stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the Hessian as a matrix and not as an array for scalar-valued functions.

Details

In Cartesian coordinates, the Hessian of a scalar-valued function F is the square matrix of second-order partial derivatives:

$$(H(F))_{ij} = \partial_{ij} F$$

When the function F is a tensor-valued function F_{d_1, \dots, d_n} , the hessian is computed for each scalar component.

$$(H(F))_{d_1 \dots d_n, ij} = \partial_{ij} F_{d_1 \dots d_n}$$

It might be tempting to apply the definition of the Hessian as the Jacobian of the gradient to write it in arbitrary orthogonal coordinate systems. However, this results in a Hessian matrix that is not symmetric and ignores the distinction between vector and covectors in tensor analysis. The generalization to arbitrary coordinate system is not currently supported.

Value

Hessian matrix for scalar-valued functions when drop=TRUE, array otherwise.

Functions

- %hessian%: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [curl\(\)](#), [derivative\(\)](#), [divergence\(\)](#), [gradient\(\)](#), [jacobian\(\)](#), [laplacian\(\)](#)

Examples

```
### symbolic Hessian
hessian("x*y*z", var = c("x", "y", "z"))

### numerical Hessian in (x=1, y=2, z=3)
f <- function(x, y, z) x*y*z
hessian(f = f, var = c(x=1, y=2, z=3))
```

```

### vectorized interface
f <- function(x) x[1]*x[2]*x[3]
hessian(f = f, var = c(1, 2, 3))

### symbolic vector-valued functions
f <- c("y*sin(x)", "x*cos(y)")
hessian(f = f, var = c("x", "y"))

### numerical vector-valued functions
f <- function(x) c(sum(x), prod(x))
hessian(f = f, var = c(0,0,0))

### binary operator
"x*y^2" %hessian% c(x=1, y=3)

```

index

Tensor Indices

Description

Functions to get or set the names of the dimensions of an array.

Usage

```
index(x)
```

```
index(x) <- value
```

Arguments

x	array.
value	vector of indices.

Value

Vector of indices.

Functions

- `index<-`: set indices.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other tensor algebra: [contraction\(\)](#), [delta\(\)](#), [diagonal\(\)](#), [einstein\(\)](#), [epsilon\(\)](#)

Examples

```

### array with no indices
x <- array(1, dim = c(1, 3, 2))
index(x)

### indices on initialization
x <- array(1, dim = c(i=1, j=3, k=2))
index(x)

### set indices on the fly
x <- array(1, dim = c(1, 3, 2))
index(x) <- c("i", "j", "k")
index(x)

```

integral

Numerical Integration

Description

Computes the integrals of functions or characters in arbitrary **orthogonal coordinate systems**.

Usage

```

integral(
  f,
  bounds,
  params = list(),
  coordinates = "cartesian",
  relTol = 0.001,
  absTol = 1e-12,
  method = NULL,
  vectorize = NULL,
  drop = TRUE,
  verbose = FALSE,
  ...
)

```

Arguments

f	array of characters or a function returning a numeric array.
bounds	list containing the lower and upper bounds for each variable. If the two bounds coincide, or if a single number is specified, the corresponding variable is not integrated and its value is fixed.
params	list of additional parameters passed to f.

coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a character vector of scale factors for each variable.
relTol	the maximum relative tolerance.
absTol	the absolute tolerance.
method	the method to use. One of "mc", "hcubature", "pcubature", "cuhre", "divonne", "suave" or "vegas". Methods other than "mc" (naive Monte Carlo) require the cubature package to be installed (efficient integration in C). The default uses "hcubature" if cubature is installed or "mc" otherwise.
vectorize	logical. Use vectorization? If TRUE, it can significantly boost performance but f needs to handle the vector of inputs appropriately. The default uses FALSE if f is a function, TRUE otherwise.
drop	if TRUE, return the integral as a vector and not as an array for vector-valued functions.
verbose	logical. Print on progress?
...	additional arguments passed to <code>cubintegrate</code> , when method "hcubature", "pcubature", "cuhre", "divonne", "suave" or "vegas" is used.

Details

The function integrates seamlessly with **cubature** for efficient numerical integration in C. If the package **cubature** is not installed, the function implements a naive Monte Carlo integration by default. For arbitrary orthogonal coordinates $q_1 \dots q_n$ the integral is computed as:

$$\int J \cdot f(q_1 \dots q_n) dq_1 \dots dq_n$$

where $J = \prod_i h_i$ is the Jacobian determinant of the transformation and is equal to the product of the scale factors $h_1 \dots h_n$.

Value

list with components

value the final estimate of the integral.

error estimate of the modulus of the absolute error.

cuba cubature output when method "hcubature", "pcubature", "cuhre", "divonne", "suave" or "vegas" is used.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other integrals: `ode()`

Examples

```

### unidimensional integral
i <- integral("sin(x)", bounds = list(x = c(0,pi)))
i$value

### multidimensional integral
f <- function(x,y) x*y
i <- integral(f, bounds = list(x = c(0,1), y = c(0,1)))
i$value

### vector-valued integrals
f <- function(x,y) c(x, y, x*y)
i <- integral(f, bounds = list(x = c(0,1), y = c(0,1)))
i$value

### tensor-valued integrals
f <- function(x,y) array(c(x^2, x*y, x*y, y^2), dim = c(2,2))
i <- integral(f, bounds = list(x = c(0,1), y = c(0,1)))
i$value

### area of a circle
i <- integral(1,
             bounds = list(r = c(0,1), theta = c(0,2*pi)),
             coordinates = "polar")
i$value

### surface of a sphere
i <- integral(1,
             bounds = list(r = 1, theta = c(0,pi), phi = c(0,2*pi)),
             coordinates = "spherical")
i$value

### volume of a sphere
i <- integral(1,
             bounds = list(r = c(0,1), theta = c(0,pi), phi = c(0,2*pi)),
             coordinates = "spherical")
i$value

```

Description

Computes the numerical Jacobian of functions or the symbolic Jacobian of characters in arbitrary **orthogonal coordinate systems**.

Usage

```
jacobian(  
  f,  
  var,  
  params = list(),  
  coordinates = "cartesian",  
  accuracy = 4,  
  stepsize = NULL  
)  
  
f %jacobian% var
```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a vector of scale factors for each variable.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.

Details

The function is basically a wrapper for [gradient](#) with drop=FALSE.

Value

array.

Functions

- %jacobian%: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [curl\(\)](#), [derivative\(\)](#), [divergence\(\)](#), [gradient\(\)](#), [hessian\(\)](#), [laplacian\(\)](#)

Examples

```

### symbolic Jacobian
jacobian("x*y*z", var = c("x", "y", "z"))

### numerical Jacobian in (x=1, y=2, z=3)
f <- function(x, y, z) x*y*z
jacobian(f = f, var = c(x=1, y=2, z=3))

### vectorized interface
f <- function(x) x[1]*x[2]*x[3]
jacobian(f = f, var = c(1, 2, 3))

### symbolic vector-valued functions
f <- c("y*sin(x)", "x*cos(y)")
jacobian(f = f, var = c("x", "y"))

### numerical vector-valued functions
f <- function(x) c(sum(x), prod(x))
jacobian(f = f, var = c(0,0,0))

### binary operator
"x*y^2" %jacobian% c(x=1, y=3)

```

laplacian

Numerical and Symbolic Laplacian

Description

Computes the numerical Laplacian of functions or the symbolic Laplacian of characters in arbitrary **orthogonal coordinate systems**.

Usage

```

laplacian(
  f,
  var,
  params = list(),
  coordinates = "cartesian",
  accuracy = 4,
  stepsize = NULL,
  drop = TRUE
)

f %laplacian% var

```

Arguments

f	array of characters or a function returning a numeric array.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated. See derivative .
params	list of additional parameters passed to f.
coordinates	coordinate system to use. One of: cartesian, polar, spherical, cylindrical, parabolic, parabolic-cylindrical or a vector of scale factors for each variable.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
drop	if TRUE, return the Laplacian as a scalar and not as an array for scalar-valued functions.

Details

The Laplacian is a differential operator given by the divergence of the gradient of a scalar-valued function F , resulting in a scalar value giving the flux density of the gradient flow of a function. The laplacian is computed in arbitrary orthogonal coordinate systems using the scale factors h_i :

$$\nabla^2 F = \frac{1}{J} \sum_i \partial_i \left(\frac{J}{h_i^2} \partial_i F \right)$$

where $J = \prod_i h_i$. When the function F is a tensor-valued function $F_{d_1 \dots d_n}$, the laplacian is computed for each scalar component:

$$(\nabla^2 F)_{d_1 \dots d_n} = \frac{1}{J} \sum_i \partial_i \left(\frac{J}{h_i^2} \partial_i F_{d_1 \dots d_n} \right)$$

Value

Scalar for scalar-valued functions when drop=TRUE, array otherwise.

Functions

- `%laplacian%`: binary operator with default parameters.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other differential operators: [curl\(\)](#), [derivative\(\)](#), [divergence\(\)](#), [gradient\(\)](#), [hessian\(\)](#), [jacobian\(\)](#)

Examples

```

### symbolic Laplacian
laplacian("x^3+y^3+z^3", var = c("x","y","z"))

### numerical Laplacian in (x=1, y=1, z=1)
f <- function(x, y, z) x^3+y^3+z^3
laplacian(f = f, var = c(x=1, y=1, z=1))

### vectorized interface
f <- function(x) sum(x^3)
laplacian(f = f, var = c(1, 1, 1))

### symbolic vector-valued functions
f <- array(c("x^2","x*y","x*y","y^2"), dim = c(2,2))
laplacian(f = f, var = c("x","y"))

### numerical vector-valued functions
f <- function(x, y) array(c(x^2,x*y,x*y,y^2), dim = c(2,2))
laplacian(f = f, var = c(x=0,y=0))

### binary operator
"x^3+y^3+z^3" %laplacian% c("x","y","z")

```

 mx

Numerical and Symbolic Matrix Product

Description

Multiplies two numeric or character matrices, if they are conformable. If one argument is a vector, it will be promoted to either a row or column matrix to make the two arguments conformable. If both are vectors of the same length, it will return the inner product (as a matrix).

Usage

```
mx(x, y)
```

```
x %mx% y
```

Arguments

x numeric or character matrix.

y numeric or character matrix.

Value

matrix.

Functions

- %mx%: binary operator.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other matrix algebra: `mxdet()`, `mxinv()`, `mxtr()`

Examples

```
### numeric inner product
x <- 1:4
mx(x, x)

### symbolic inner product
x <- letters[1:4]
mx(x, x)

### numeric matrix product
x <- letters[1:4]
y <- diag(4)
mx(x, y)

### symbolic matrix product
x <- array(1:12, dim = c(3,4))
y <- letters[1:4]
mx(x, y)

### binary operator
x <- array(1:12, dim = c(3,4))
y <- letters[1:4]
x %mx% y
```

mxdet

Numerical and Symbolic Determinant

Description

Computes the determinant of a numeric or character matrix.

Usage

`mxdet(x)`

Arguments

x numeric or character matrix.

Value

numeric or character.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other matrix algebra: [mxinv\(\)](#), [mxtr\(\)](#), [mx\(\)](#)

Examples

```
### numeric matrix
x <- matrix(1:4, nrow = 2)
mxdet(x)

### symbolic matrix
x <- matrix(letters[1:4], nrow = 2)
mxdet(x)
```

mxinv

Numerical and Symbolic Matrix Inverse

Description

Computes the inverse of a numeric or character matrix.

Usage

```
mxinv(x)
```

Arguments

x numeric or character matrix.

Value

numeric or character matrix.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other matrix algebra: [mxdet\(\)](#), [mxtr\(\)](#), [mx\(\)](#)

Examples

```
### numeric matrix
x <- matrix(1:4, nrow = 2, byrow = TRUE)
mxinv(x)

### symbolic matrix
x <- matrix(letters[1:4], nrow = 2, byrow = TRUE)
mxinv(x)
```

mxtr

Numerical and Symbolic Matrix Trace

Description

Computes the trace of a numeric or character matrix.

Usage

```
mxtr(x)
```

Arguments

x numeric or character matrix.

Value

numeric or character.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other matrix algebra: [mxdet\(\)](#), [mxinv\(\)](#), [mx\(\)](#)

Examples

```
### numeric matrix
x <- matrix(1:4, nrow = 2)
mxtr(x)

### character matrix
x <- matrix(letters[1:4], nrow = 2)
mxtr(x)
```

ode

*Ordinary Differential Equations***Description**

Solves a numerical or symbolic system of ordinary differential equations.

Usage

```
ode(
  f,
  var,
  times,
  timevar = NULL,
  params = list(),
  method = "rk4",
  drop = FALSE
)
```

Arguments

f	vector of characters, or a function returning a numeric vector, giving the values of the derivatives in the ODE system at time timevar. See examples.
var	vector giving the initial conditions. See examples.
times	discretization sequence, the first value represents the initial time.
timevar	the time variable used by f, if any.
params	list of additional parameters passed to f.
method	the solver to use. One of "rk4" (Runge-Kutta) or "euler" (Euler).
drop	if TRUE, return only the final solution instead of the whole trajectory.

Value

Vector of final solutions if drop=TRUE, otherwise a matrix with as many rows as elements in times and as many columns as elements in var.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other integrals: [integral\(\)](#)

Examples

```
## =====
## Example: symbolic system
## System: dx = x dt
## Initial: x0 = 1
## =====
f <- "x"
var <- c(x=1)
times <- seq(0, 2*pi, by=0.001)
x <- ode(f, var, times)
plot(times, x, type = "l")

## =====
## Example: time dependent system
## System: dx = cos(t) dt
## Initial: x0 = 0
## =====
f <- "cos(t)"
var <- c(x=0)
times <- seq(0, 2*pi, by=0.001)
x <- ode(f, var, times, timevar = "t")
plot(times, x, type = "l")

## =====
## Example: multivariate time dependent system
## System: dx = x dt
##          dy = x*(1+cos(10*t)) dt
## Initial: x0 = 1
##          y0 = 1
## =====
f <- c("x", "x*(1+cos(10*t))")
var <- c(x=1, y=1)
times <- seq(0, 2*pi, by=0.001)
x <- ode(f, var, times, timevar = "t")
matplot(times, x, type = "l", lty = 1, col = 1:2)

## =====
## Example: numerical system
## System: dx = x dt
##          dy = y dt
## Initial: x0 = 1
##          y0 = 2
## =====
```

```

f <- function(x, y) c(x, y)
var <- c(x=1, y=2)
times <- seq(0, 2*pi, by=0.001)
x <- ode(f, var, times)
matplot(times, x, type = "l", lty = 1, col = 1:2)

## =====
## Example: vectorized interface
## System: dx = x dt
##          dy = y dt
##          dz = y*(1+cos(10*t)) dt
## Initial: x0 = 1
##          y0 = 2
##          z0 = 2
## =====
f <- function(x, t) c(x[1], x[2], x[2]*(1+cos(10*t)))
var <- c(1,2,2)
times <- seq(0, 2*pi, by=0.001)
x <- ode(f, var, times, timevar = "t")
matplot(times, x, type = "l", lty = 1, col = 1:3)

```

partitions

Integer Partitions

Description

Provides fast algorithms for generating integer partitions.

Usage

```
partitions(n, max = 0, length = 0, perm = FALSE, fill = FALSE, equal = T)
```

Arguments

n	positive integer.
max	maximum integer in the partitions.
length	maximum number of elements in the partitions.
perm	logical. Permute partitions?
fill	logical. Fill partitions with zeros to match length?
equal	logical. Return only partition of n? If FALSE, partitions of all integers less or equal to n are returned.

Value

list of partitions, or matrix if length>0 and fill=TRUE.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

Examples

```
### partitions of 4
partitions(4)

### partitions of 4 and permute
partitions(4, perm = TRUE)

### partitions of 4 with max element equal to 2
partitions(4, max = 2)

### partitions of 4 with 2 elements
partitions(4, length = 2)

### partitions of 4 with 3 elements, fill with zeros
partitions(4, length = 3, fill = TRUE)

### partitions of 4 with 2 elements, fill with zeros and permute
partitions(4, length = 2, fill = TRUE, perm = TRUE)

### partitions of all integers less or equal to 3
partitions(3, equal = FALSE)

### partitions of all integers less or equal to 3, fill to 2 elements and permute
partitions(3, equal = FALSE, length = 2, fill = TRUE, perm = TRUE)
```

taylor

Taylor Series Expansion

Description

Computes the Taylor series of functions or characters.

Usage

```
taylor(
  f,
  var,
  params = list(),
  order = 1,
  accuracy = 4,
  stepsize = NULL,
  zero = 1e-07
)
```

Arguments

f	character, or function returning a numeric scalar value.
var	vector giving the variable names with respect to which the derivatives are to be computed and/or the point where the derivatives are to be evaluated (the center of the Taylor series). See derivative .
params	list of additional parameters passed to f .
order	the order of the Taylor approximation.
accuracy	degree of accuracy for numerical derivatives.
stepsize	finite differences stepsize for numerical derivatives. It is based on the precision of the machine by default.
zero	tolerance used for deciding which derivatives are zero. Absolute values less than this number are set to zero.

Value

list with components:

f the Taylor series.

order the approximation order.

terms data.frame containing the variables, coefficients and degrees of each term in the Taylor series.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other polynomials: [hermite\(\)](#)

Other derivatives: [derivative\(\)](#)

Examples

```
### univariate taylor series (in x=0)
taylor("exp(x)", var = "x", order = 2)

### univariate taylor series of user-defined functions (in x=0)
f <- function(x) exp(x)
taylor(f = f, var = c(x=0), order = 2)

### multivariate taylor series (in x=0 and y=1)
taylor("x*(y-1)", var = c(x=0, y=1), order = 4)

### multivariate taylor series of user-defined functions (in x=0 and y=1)
f <- function(x,y) x*(y-1)
taylor(f, var = c(x=0, y=1), order = 4)
```

```
### vectorized interface
f <- function(x) prod(x)
taylor(f, var = c(0,0,0), order = 3)
```

wrap

Wrap Characters in Parentheses

Description

Wraps characters in round brackets.

Usage

```
wrap(x)
```

Arguments

x character.

Details

Characters are automatically wrapped when performing basic symbolic operations to prevent unwanted results. E.g.:

$$a + b * c + d$$

instead of

$$(a + b) * (c + d)$$

To disable this behaviour run `options(calculus.auto.wrap = FALSE)`.

Value

character.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other utilities: [c2e\(\)](#), [e2c\(\)](#), [evaluate\(\)](#)

Examples

```
### wrap characters
wrap("a+b")

### wrap array of characters
wrap(array(letters[1:9], dim = c(3,3)))
```

%diff%

Numerical and Symbolic Difference

Description

Elementwise difference of numeric or character arrays.

Usage

```
x %diff% y
```

Arguments

x	numeric or character array.
y	numeric or character array.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%div%\(\)](#), [%dot%\(\)](#), [%inner%\(\)](#), [%kronecker%\(\)](#), [%outer%\(\)](#), [%prod%\(\)](#), [%sum%\(\)](#)

Examples

```
### vector
x <- c("a+1", "b+2")
x %diff% x

### matrix
x <- matrix(letters[1:4], ncol = 2)
x %diff% x
```



```
### array
x <- array(letters[1:12], dim = c(2,2,3))
y <- array(1:12, dim = c(2,2,3))
x %diff% y
```

`%div%`*Numerical and Symbolic Division*

Description

Elementwise division of numeric or character arrays.

Usage

```
x %div% y
```

Arguments

x	numeric or character array.
y	numeric or character array.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%diff%](#)(), [%dot%](#)(), [%inner%](#)(), [%kronecker%](#)(), [%outer%](#)(), [%prod%](#)(), [%sum%](#)()

Examples

```
### vector
x <- c("a+1", "b+2")
x %div% x

### matrix
x <- matrix(letters[1:4], ncol = 2)
x %div% x

### array
x <- array(letters[1:12], dim = c(2,2,3))
y <- array(1:12, dim = c(2,2,3))
```

x %div% y

%dot%

Numerical and Symbolic Dot Product

Description

The dot product between arrays with different dimensions is computed by taking the inner product on the last dimensions of the two arrays.

Usage

x %dot% y

Arguments

x numeric or character array.
y numeric or character array.

Details

The dot product between two arrays A and B is computed as:

$$C_{i_1 \dots i_m} = \sum_{j_1 \dots j_n} A_{i_1 \dots i_m j_1 \dots j_n} B_{j_1 \dots j_n}$$

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%diff%\(\)](#), [%div%\(\)](#), [%inner%\(\)](#), [%kronecker%\(\)](#), [%outer%\(\)](#), [%prod%\(\)](#), [%sum%\(\)](#)

Examples

```
### inner product
x <- array(1:12, dim = c(3,4))
x %dot% x

### dot product
x <- array(1:24, dim = c(3,2,4))
y <- array(letters[1:8], dim = c(2,4))
x %dot% y
```

`%inner%`*Numerical and Symbolic Inner Product*

Description

Computes the inner product of two numeric or character arrays.

Usage

```
x %inner% y
```

Arguments

x	numeric or character array.
y	numeric or character array.

Details

The inner product between two arrays A and B is computed as:

$$C = \sum_{j_1 \dots j_n} A_{j_1 \dots j_n} B_{j_1 \dots j_n}$$

Value

numeric or character.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: `%diff%`(), `%div%`(), `%dot%`(), `%kronecker%`(), `%outer%`(), `%prod%`(), `%sum%`()

Examples

```
### numeric inner product
x <- array(1:4, dim = c(2,2))
x %inner% x

### symbolic inner product
x <- array(letters[1:4], dim = c(2,2))
x %inner% x
```

 %kronecker%

Numerical and Symbolic Kronecker Product

Description

Computes the generalised Kronecker product of two numeric or character arrays.

Usage

```
x %kronecker% y
```

Arguments

```
x          numeric or character array.
y          numeric or character array.
```

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%diff%](#)(), [%div%](#)(), [%dot%](#)(), [%inner%](#)(), [%outer%](#)(), [%prod%](#)(), [%sum%](#)()

Examples

```
### numeric Kronecker product
c(1,2) %kronecker% c(2,3)

### symbolic Kronecker product
array(1:4, dim = c(2,2)) %kronecker% c("a","b")
```

Description

Computes the outer product of two numeric or character arrays.

Usage

```
x %outer% y
```

Arguments

x	numeric or character array.
y	numeric or character array.

Details

The outer product between two arrays A and B is computed as:

$$C_{i_1 \dots i_m j_1 \dots j_n} = A_{i_1 \dots i_m} B_{j_1 \dots j_n}$$

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%diff%](#)(), [%div%](#)(), [%dot%](#)(), [%inner%](#)(), [%kronecker%](#)(), [%prod%](#)(), [%sum%](#)()

Examples

```
### numeric outer product
c(1,2) %outer% c(2,3)

### symbolic outer product
c("a","b") %outer% c("c","d")
```

%prod%

Numerical and Symbolic Product

Description

Elementwise product of numeric or character arrays.

Usage

```
x %prod% y
```

Arguments

x numeric or character array.
y numeric or character array.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: [%diff%](#)(), [%div%](#)(), [%dot%](#)(), [%inner%](#)(), [%kronecker%](#)(), [%outer%](#)(), [%sum%](#)()

Examples

```
### vector  
x <- c("a+1", "b+2")  
x %prod% x  
  
### matrix  
x <- matrix(letters[1:4], ncol = 2)  
x %prod% x  
  
### array  
x <- array(letters[1:12], dim = c(2,2,3))  
y <- array(1:12, dim = c(2,2,3))  
x %prod% y
```

Description

Elementwise sum of numeric or character arrays.

Usage

```
x %sum% y
```

Arguments

x numeric or character array.
y numeric or character array.

Value

array.

References

Guidotti, E. (2020). "calculus: High dimensional numerical and symbolic calculus in R". <https://arxiv.org/abs/2101.00086>

See Also

Other basic arithmetic: `%diff%`(), `%div%`(), `%dot%`(), `%inner%`(), `%kronecker%`(), `%outer%`(), `%prod%`()

Examples

```
### vector
x <- c("a+1", "b+2")
x %sum% x

### matrix
x <- matrix(letters[1:4], ncol = 2)
x %sum% x

### array
x <- array(letters[1:12], dim = c(2,2,3))
y <- array(1:12, dim = c(2,2,3))
x %sum% y
```

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