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cmna-package

Computational Methods for Numerical Analysis

Description
Provides the source and examples for *Computational Methods for Numerical Analysis with R*.

Details
This package provides a suite of simple implementations of standard methods from numerical analysis. The collection is designed to accompany *Computational Methods for Numerical Analysis with R* by James P. Howard, II. Together, these functions provide methods to support linear algebra, interpolation, integration, root finding, optimization, and differential equations.

Author(s)
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See Also
Useful links:
- [https://jameshoward.us/cmna/](https://jameshoward.us/cmna/)
- Report bugs at [https://github.com/k3jph/cmna-pkg/issues](https://github.com/k3jph/cmna-pkg/issues)

adaptint

Adaptive Integration

Description
Adaptive integration

Usage
adaptint(f, a, b, n = 10, tol = 1e-06)

Arguments
- f: function to integrate
- a: the a-bound of integration
- b: the b-bound of integration
- n: the maximum recursive depth
- tol: the maximum error tolerance
Details

The adaptint function uses Romberg’s rule to calculate the integral of the function $f$ over the interval from $a$ to $b$. The parameter $n$ sets the number of intervals to use when evaluating. Additional options are passed to the function $f$ when evaluating.

Value

the value of the integral

See Also

Other integration: `gaussint()`, `giniquintile()`, `mcint()`, `midpt()`, `revolution-solid`, `romberg()`, `simp38()`, `simp()`, `trap()`

Other newton-cotes: `giniquintile()`, `midpt()`, `romberg()`, `simp38()`, `simp()`, `trap()`

Examples

```r
f <- function(x) { sin(x)^2 + log(x) }
adaptint(f, 1, 10, n = 4)
adaptint(f, 1, 10, n = 5)
adaptint(f, 1, 10, n = 10)
```

beziers

Beziers curves

Description

Find the quadratic and cubic Beziers curve for the given points

Usage

```r
qbezier(x, y, t)
cbezier(x, y, t)
```

Arguments

- `x` a vector of x values
- `y` a vector of y values
- `t` a vector of t values for which the curve will be computed

Details

`qbezier` finds the quadratic Bezier curve for the given three points and `cbezier` finds the cubic Bezier curve for the given four points. The curve will be computed at all values in the vector `t` and a list of x and y values returned.
Value

a list composed of an x-vector and a y-vector

See Also

Other interp: `bilinear()`, `cubicspline()`, `linterp()`, `nn()`, `polyinterp()`, `pwiselinterp()`

Examples

```r
x <- c(1, 2, 3)
y <- c(2, 3, 5)
f <- qbezier(x, y, seq(0, 1, 1/100))

x <- c(-1, 1, 0, -2)
y <- c(-2, 2, -1, -1)
f <- cbezier(x, y, seq(0, 1, 1/100))
```

Description

Finds a bilinear interpolation bounded by four points

Usage

`bilinear(x, y, z, newx, newy)`

Arguments

- `x`: vector of two x values representing \( x_1 \) and \( x_2 \)
- `y`: vector of two y values representing \( y_1 \) and \( y_2 \)
- `z`: 2x2 matrix if z values
- `newx`: vector of new x values to interpolate
- `newy`: vector of new y values to interpolate

Details

`bilinear` finds a bilinear interpolation bounded by four corners

Value

a vector of interpolated z values at \( (x, y) \)
See Also

Other interp: `bezier`, `cubicspline()`, `linterp()`, `nn()`, `polyinterp()`, `pwiselinterp()`

Other algebra: `cubicspline()`, `division`, `fibonacci()`, `horner()`, `isPrime()`, `linterp()`, `nthroot()`, `polyinterp()`, `pwiselinterp()`, `quadratic()`

Examples

```r
x <- c(2, 4)
y <- c(4, 7)
z <- matrix(c(81, 84, 85, 89), nrow = 2)
newx <- c(2.5, 3, 3.5)
newy <- c(5, 5.5, 6)
bilinear(x, y, z, newx, newy)
```

---

bisection

The Bisection Method

Description

Use the bisection method to find real roots

Usage

```r
bisection(f, a, b, tol = 0.001, m = 100)
```

Arguments

- `f`: function to locate a root for
- `a`: the a bound of the search region
- `b`: the b bound of the search region
- `tol`: the error tolerance
- `m`: the maximum number of iterations

Details

The bisection method functions by repeatedly halving the interval between `a` and `b` and will return when the interval between them is less than `tol`, the error tolerance. However, this implementation also stops if after `m` iterations.

Value

the real root found

See Also

Other optimz: `goldsect`, `gradient`, `hillclimbing()`, `newton()`, `sa()`, `secant()`
Examples

```r
f <- function(x) { x^3 - 2 * x^2 - 159 * x - 540}
bisection(f, 0, 10)
```

Description

solve boundary value problems for ordinary differential equations

Usage

```r
bvpexample(x)
bvpexample10(x)
```

Arguments

- `x`: proposed initial x-value

Details

The `euler` method implements the Euler method for solving differential equations. The `codemidptvp` method solves initial value problems using the second-order Runge-Kutta method. The `rungekutta4` method is the fourth-order Runge-Kutta method.

Value

a data frame of x and y values

Examples

```r
bvpexample(-2)
bvpexample(-1)
bvpexample(0)
bvpexample(1)
bvpexample(2)
## (bvp.b <- bisection(bvpexample, 0, 1))
## (bvp.s <- secant(bvpexample, 0))
```
choleskymatrix | Cholesky Decomposition

**Description**
Decompose a matrix into the Cholesky

**Usage**
choleskymatrix(m)

**Arguments**
m | a matrix

**Details**
choleskymatrix decomposes the matrix m into the LU decomposition, such that m == L

**Value**
the matrix L

**See Also**
Other linear: detmatrix(), gds(), invmatrix(), iterativematrix, lumatrix(), refmatrix(), rowops, tridiagmatrix(), vecnorm()

**Examples**
(A <- matrix(c(5, 1, 2, 1, 9, 3, 2, 3, 7), 3))
(L <- choleskymatrix(A))
t(t(L) %% L)

---

cubicspline | Natural cubic spline interpolation

**Description**
Finds a piecewise linear function that interpolates the data points

**Usage**
cubicspline(x, y)
**Arguments**

- **x** a vector of x values
- **y** a vector of y values

**Details**

cubicspline finds a piecewise cubic spline function that interpolates the data points. For each x-y ordered pair. The function will return a list of four vectors representing the coefficients.

**Value**

a list of coefficient vectors

**See Also**

- Other interp: `bezier`, `bilinear()`, `linterp()`, `nn()`, `polyinterp()`, `pwiselinterp()`
- Other algebra: `bilinear()`, `division`, `fibonacci()`, `horner()`, `isPrime()`, `linterp()`, `nthroot()`, `polyinterp()`, `pwiselinterp()`, `quadratic()`

**Examples**

```r
x <- c(1, 2, 3)
y <- c(2, 3, 5)
f <- cubicspline(x, y)

x <- c(-1, 1, 0, -2)
y <- c(-2, 2, -1, -1)
f <- cubicspline(x, y)
```

---

**detmatrix**

*Calculate the determinant of the matrix*

**Description**

Calculate the determinant of the matrix

**Usage**

detmatrix(m)

**Arguments**

- **m** a matrix

**Details**

detmatrix calculates the determinant of the matrix given.
division

Value
the determinant

See Also
Other linear: `choleskymatrix()`, `gdls()`, `invmatrix()`, `iterativematrix`, `lumatrix()`, `refmatrix()`, `rowops`, `tridiagmatrix()`, `vecnorm()`

Examples
```r
A <- matrix(c(1, 2, -7, -1, -1, 1, 2, 1, 5), 3)
detmatrix(A)
```

Description
Algorithms for division that provide a quotient and remainder.

Usage
```r
naivediv(m, n)
longdiv(m, n)
```

Arguments
```r
m the dividend
n the divisor
```

Details
The `naivediv` divides `m` by `n` by using repeated division. The `longdiv` function uses the long division algorithm in binary.

Value
the quotient and remainder as a list

See Also
Other algebra: `bilinear()`, `cubicspline()`, `fibonacci()`, `horner()`, `isPrime()`, `linterp()`, `nthroot()`, `polyinterp()`, `pwiselinterp()`, `quadratic()`
Examples

\begin{verbatim}
a <- floor(runif(1, 1, 1000))
b <- floor(runif(1, 1, 100))
naivediv(a, b)
longdiv(a, b)
\end{verbatim}

---

<table>
<thead>
<tr>
<th>fibonacci</th>
<th>Fibonacci numbers</th>
</tr>
</thead>
</table>

Description

Return the \( n \)-th Fibonacci number

Usage

\品德\( \text{fibonacci}(n) \)

Arguments

\begin{verbatim}
n
\end{verbatim}

Details

This function is recursively implements the famous Fibonacci sequence. The function returns the \( n \)-th member of the sequence.

Value

the sequence element

See Also

Other algebra: \( \text{bilinear()}, \text{cubicspline()}, \text{division}, \text{horner()}, \text{isPrime()}, \text{linterp()}, \text{nthroot()}, \text{polyinterp()}, \text{pwiselinterp()}, \text{quadratic()} \)

Examples

\品德\( \text{fibonacci}(10) \)
Description

Finite differences formulas

Usage

\begin{align*}
\text{findiff}(f, x, h = x \times \sqrt{\text{Machine}\$\text{double}\$.\text{eps}}) \\
\text{symdiff}(f, x, h = x \times \text{Machine}\$\text{double}\$.\text{eps}^{(1/3)}) \\
\text{findiff2}(f, x, h) \\
\text{rdiff}(f, x, n = 10, h = 1e-04)
\end{align*}

Arguments

- \textit{f} function to differentiate
- \textit{x} the \textit{x}-value to differentiate at
- \textit{h} the step-size for evaluation
- \textit{n} the maximum number of convergence steps in \text{rdiff}

Details

The \text{findiff} formula uses the finite differences formula to find the derivative of \textit{f} at \textit{x}. The value of \textit{h} is the step size of the evaluation. The function \text{findiff2} provides the second derivative.

Value

the value of the derivative

Examples

\begin{align*}
\text{findiff}(\sin, \pi, 1e-3) \\
\text{symdiff}(\sin, \pi, 1e-3)
\end{align*}
gaussint

Gaussian integration method driver

Description

Use the Gaussian method to evaluate integrals

Usage

gaussint(f, x, w)
gauss.legendre(f, m = 5)
gauss.laguerre(f, m = 5)
gauss.hermite(f, m = 5)

Arguments

f function to integrate
x list of evaluation points
w list of weights
m number of evaluation points

Details

The gaussint function uses the Gaussian integration to evaluate an integral. The function itself is a driver and expects the integration points and associated weights as options.

Value

the value of the integral

See Also

Other integration: adaptint(), giniquintile(), mcint(), midpt(), revolution-solid, romberg(), simp38(), simp(), trap()

Examples

w = c(1, 1)
x = c(-1 / sqrt(3), 1 / sqrt(3))
f <- function(x) { x^3 + x + 1 }
gaussint(f, x, w)
gdls

Least squares with gradient descent

Description
Solve least squares with gradient descent

Usage

```
gdls(A, b, alpha = 0.05, tol = 1e-06, m = 1e+05)
```

Arguments

- `A`: a square matrix representing the coefficients of a linear system
- `b`: a vector representing the right-hand side of the linear system
- `alpha`: the learning rate
- `tol`: the expected error tolerance
- `m`: the maximum number of iterations

Details

gdls solves a linear system using gradient descent.

Value

the modified matrix

See Also

Other linear: `choleskymatrix()`, `detmatrix()`, `invmatrix()`, `iterativematrix`, `lumatrix()`, `refmatrix()`, `rowops`, `tridiagmatrix()`, `vecnorm()`

Examples

```
head(b <- iris$Sepal.Length)
head(A <- matrix(cbind(1, iris$Sepal.Width, iris$Petal.Length, iris$Petal.Width), ncol = 4))
gdls(A, b, alpha = 0.05, m = 10000)
```
giniquintile

Description

Calculate the Gini coefficient from quintile data

Usage

giniquintile(L)

Arguments

L vector of percentages at 20th, 40th, 60th, and 80th percentiles

Details

Calculate the Gini coefficient given the quintile data.

Value

the estimated Gini coefficient

References


See Also

Other integration: adaptint(), gaussint(), mcint(), midpt(), revolution-solid, romberg(), simp38(), simp(), trap()

Other newton-cotes: adaptint(), midpt(), romberg(), simp38(), simp(), trap()

Examples

L <- c(4.3, 9.8, 15.4, 22.7)
giniquintile(L)
Golden Section Search

Description

Use golden section search to find local extrema

Usage

```r
goldsectmin(f, a, b, tol = 0.001, m = 100)
goldsectmax(f, a, b, tol = 0.001, m = 100)
```

Arguments

- `f`: function to integrate
- `a`: the a bound of the search region
- `b`: the b bound of the search region
- `tol`: the error tolerance
- `m`: the maximum number of iterations

Details

The golden section search method functions by repeatedly dividing the interval between `a` and `b` and will return when the interval between them is less than `tol`, the error tolerance. However, this implementation also stop if after `m` iterations.

Value

the x value of the minimum found

See Also

Other optimz: `bisection()`, `gradient.hillclimbing()`, `newton()`, `sa()`, `secant()`

Examples

```r
f <- function(x) { x^2 - 3 * x + 3 }
goldsectmin(f, 0, 5)
```
**Description**

Use gradient descent to find local minima

**Usage**

```r
graddsc(fp, x, h = 0.001, tol = 1e-04, m = 1000)
gradasc(fp, x, h = 0.001, tol = 1e-04, m = 1000)
gd(fp, x, h = 100, tol = 1e-04, m = 1000)
```

**Arguments**

- `fp`: function representing the derivative of `f`
- `x`: an initial estimate of the minima
- `h`: the step size
- `tol`: the error tolerance
- `m`: the maximum number of iterations

**Details**

Gradient descent can be used to find local minima of functions. It will return an approximation based on the step size `h` and `fp`. The `tol` is the error tolerance, `x` is the initial guess at the minimum. This implementation also stops after `m` iterations.

**Value**

the `x` value of the minimum found

**See Also**

Other optimz: `bisection()`, `goldsect`, `hillclimbing()`, `newton()`, `sa()`, `secant()`

**Examples**

```r
fp <- function(x) { x^3 + 3 * x^2 - 1 }
graddsc(fp, 0)

f <- function(x) { (x[1] - 1)^2 + (x[2] - 1)^2 }
fp <- function(x) {
  x1 <- 2 * x[1] - 2
  x2 <- 8 * x[2] - 8
  return(c(x1, x2))
}
```
heat

**Heat Equation via Forward-Time Central-Space**

### Description

solve heat equation via forward-time central-space method

### Usage

heat(u, alpha, xdelta, tdelta, n)

### Arguments

- **u**
  - the initial values of u
- **alpha**
  - the thermal diffusivity coefficient
- **xdelta**
  - the change in x at each step in u
- **tdelta**
  - the time step
- **n**
  - the number of steps to take

### Details

The `heat` solves the heat equation using the forward-time central-space method in one-dimension.

### Value

a matrix of u values at each time step

### Examples

```r
c <- c(1, 2)
gd(fp, c(0, 0), 0.05)

heat

alpha <- 1
x0 <- 0
xdelta <- .05
x <- seq(x0, 1, xdelta)
u <- sin(x^4 * pi)
tdelta <- .001
n <- 25
z <- heat(u, alpha, xdelta, tdelta, n)
```
Description

Use hill climbing to find the global minimum

Usage

hillclimbing(f, x, h = 1, m = 1000)

Arguments

- **f**: function representing the derivative of \( f \)
- **x**: an initial estimate of the minimum
- **h**: the step size
- **m**: the maximum number of iterations

Details

Hill climbing

Value

the \( x \) value of the minimum found

See Also

Other optimz: \texttt{bisection()}, \texttt{goldsect}, \texttt{gradient}, \texttt{newton()}, \texttt{sa()}, \texttt{secant()}

Examples

```r
f <- function(x) {
}
hillclimbing(f, c(0,0))
hillclimbing(f, c(-1,-1))
hillclimbing(f, c(10,10))
```
**himmelblau**  
*Himmelblau Function*

**Description**  
Generate the Himmelblau function

**Usage**  
`himmelblau(x)`

**Arguments**  
`x`  
a vector of x-values

**Details**  
Generate the Himmelblau function

**Value**  
the value of the function at x.

---

**horner**  
*Horner’s rule*

**Description**  
Use Horner’s rule to evaluate a polynomial

**Usage**  
`horner(x, coefs)`  
`rhorner(x, coefs)`  
`naivepoly(x, coefs)`  
`betterpoly(x, coefs)`

**Arguments**  
`x`  
a vector of x values to evaluate the polynomial  
`coefs`  
vector of coefficients of x
invmatrix

Details
This function implements Horner's rule for fast polynomial evaluation. The implementation expects \( x \) to be a vector of \( x \) values at which to evaluate the polynomial. The parameter `coefs` is a vector of coefficients of \( x \). The vector order is such that the first element is the constant term, the second element is the coefficient of \( x \), the so forth to the highest degreed term. Terms with a 0 coefficient should have a 0 element in the vector.

The function `rhorner` implements the the Horner algorithm recursively.

The function `naivepoly` implements a polynomial evaluator using the straightforward algebraic approach.

The function `betterpoly` implements a polynomial evaluator using the straightforward algebraic approach with cached \( x \) terms.

Value
the value of the function at \( x \)

See Also
Other algebra: `bilinear()`, `cubicspline()`, `division`, `fibonacci()`, `isPrime()`, `linterp()`, `nthroot()`, `polyinterp()`, `pwiselinterp()`, `quadratic()`

Examples
```r
b <- c(2, 10, 11)
x <- 5
horner(x, b)
b <- c(-1, 0, 1)
x <- c(1, 2, 3, 4)
horner(x, b)
rhorner(x, b)
```

---

invmatrix

**Invert a matrix**

Description
Invert the matrix using Gaussian elimination

Usage

```
invmatrix(m)
```

Arguments

- `m` a matrix
isPrime

Details

invmatrix invertse the given matrix using Gaussian elimination and returns the result.

Value

the inverted matrix

See Also

Other linear: choleskymatrix(), detmatrix(), gdlsl(), iterativematrix, lumatrix(), refmatrix(), rowops, tridiagmatrix(), vecnorm()

Examples

A <- matrix(c(1, 2, -7, -1, -1, 1, 2, 1, 5), 3)
refmatrix(A)

isPrime Test for Primality

Description

Test the number given for primality.

Usage

isPrime(n)

Arguments

n

Details

This function tests n if it is prime through repeated division attempts. If a match is found, by finding a remainder of 0, FALSE is returned.

Value

boolean TRUE if n is prime, FALSE if not

See Also

Other algebra: bilinear(), cubicspline(), division, fibonacci(), horner(), interp(), nthroot(), polyinterp(), pwiselinterp(), quadratic()
Examples

isPrime(37)
isPrime(89)
isPrime(100)

Description

Solve a matrix using iterative methods.

Usage

jacobi(A, b, tol = 1e-06, maxiter = 100)

gaussseidel(A, b, tol = 1e-06, maxiter = 100)

cgmmatrix(A, b, tol = 1e-06, maxiter = 100)

Arguments

A       a square matrix representing the coefficients of a linear system
b       a vector representing the right-hand side of the linear system
tol     is a number representing the error tolerance
maxiter is the maximum number of iterations

Details

jacobi finds the solution using Jacobi iteration. Jacobi iteration depends on the matrix being diagonally-dominant. The tolerance is specified the norm of the solution vector.

gaussseidel finds the solution using Gauss-Seidel iteration. Gauss-Seidel iteration depends on the matrix being either diagonally-dominant or symmetric and positive definite.

cgmmatrix finds the solution using the conjugate gradient method. The conjugate gradient method depends on the matrix being symmetric and positive definite.

Value

the solution vector

See Also

Other linear: choleskymatrix(), detmatrix(), gdls(), invmatrix(), lumatrix(), refmatrix(), rowops, tridiagmatrix(), vecnorm()
### Examples

```r
A <- matrix(c(5, 2, 1, 2, 7, 3, 3, 4, 8), 3)
b <- c(40, 39, 55)
jacobi(A, b)
```

```
ivp  Initial value problems
```

### Description

solve initial value problems for ordinary differential equations

### Usage

- `euler(f, x0, y0, h, n)`
- `midptivp(f, x0, y0, h, n)`
- `rungekutta4(f, x0, y0, h, n)`
- `adamsbashforth(f, x0, y0, h, n)`

### Arguments

- `f`: function to integrate
- `x0`: the initial value of x
- `y0`: the initial value of y
- `h`: selected step size
- `n`: the number of steps

### Details

The `euler` method implements the Euler method for solving differential equations. The `midptivp` method solves initial value problems using the second-order Runge-Kutta method. The `rungekutta4` method is the fourth-order Runge-Kutta method.

### Value

a data frame of x and y values

### Examples

```r
f <- function(x, y) { y / (2 * x + 1) }
ivp.euler <- euler(f, 0, 1, 1/100, 100)
ivp.midpt <- midptivp(f, 0, 1, 1/100, 100)
ivp.rk4 <- rungekutta4(f, 0, 1, 1/100, 100)
```
Initial value problems for systems of ordinary differential equations

Description
solves initial value problems for systems ordinary differential equations

Usage
eulersys(f, x0, y0, h, n)

Arguments
f        function to integrate
x0       the initial value of x
y0       the vector initial values of y
h        selected step size
n        the number of steps

Details
The euler method implements the Euler method for solving differential equations. The \texttt{codemidptivp} method solves initial value problems using the second-order Runge-Kutta method. The \texttt{rungekutta4} method is the fourth-order Runge-Kutta method.

Value
 a data frame of \textit{x} and \textit{y} values

Examples
f <- function(x, y) { y / (2 * x + 1) }
ivp.euler <- euler(f, 0, 1, 1/100, 100)

Linear interpolation

Description
Finds a linear function between two points

Usage
\texttt{linterp(x1, y1, x2, y2)
Arguments

- \( x_1 \) x value of the first point
- \( y_1 \) y value of the first point
- \( x_2 \) x value of the second point
- \( y_2 \) y value of the second point

Details

linterp finds a linear function between two points.

Value

a linear equation’s coefficients

See Also

Other interp: bezier, bilinear(), cubicspline(), nn(), polyinterp(), pwiselinterp()
Other algebra: bilinear(), cubicspline(), division, fibonacci(), horner(), isPrime(), nthroot(), polyinterp(), pwiselinterp(), quadratic()

Examples

\[ f <- \text{linterp}(3, 2, 7, -2) \]

---

lumatrix \( L U \) Decomposition

Description

Decompose a matrix into lower- and upper-triangular matrices

Usage

lumatrix(m)

Arguments

- \( m \) a matrix

Details

lumatrix decomposes the matrix \( m \) into the LU decomposition, such that \( m == L \)

Value

list with matrices L and U representing the LU decomposition
mcint

Monte Carlo Integration

Description
Simple Monte Carlo Integraton

Usage
mcint(f, a, b, m = 1000)
mcint2(f, xdom, ydom, m = 1000)

Arguments
f       function to integrate
a       the lower-bound of integration
b       the upper-bound of integration
m       the number of subintervals to calculate
xdom    the domain on x of integration in two dimensions
ydom    the domain on y of integration in two dimensions

Details
The mcint function uses a simple Monte Carlo algorithm to estimate the value of an integral. The parameter m sets the total number of evaluation points. The parameter max.y is the maximum expected value of the range of function f. The mcint2 provides Monte Carlo integration in two dimensions.

Value
the value of the integral

See Also
Other integration: adaptint(), gaussint(), giniquintile(), midpt(), revolution-solid, romberg(), simp38(), simp(), trap()
Examples

```r
f <- function(x) { sin(x)^2 + log(x)}
m McInt(f, 0, 1)
m McInt(f, 0, 1, m = 10e6)
```

midpt  

rectangle method

Description

Use the rectangle method to integrate a function

Usage

```r
midpt(f, a, b, m = 100)
```

Arguments

- `f`: function to integrate
- `a`: the a-bound of integration
- `b`: the b-bound of integration
- `m`: the number of subintervals to calculate

Details

The `midpt` function uses the rectangle method to calculate the integral of the function `f` over the interval from `a` to `b`. The parameter `m` sets the number of intervals to use when evaluating the rectangles. Additional options are passed to the function `f` when evaluating.

Value

The value of the integral

See Also

Other integration: `adaptint()`, `gaussint()`, `giniquintile()`, `mcInt()`, `revolution-solid`, `romberg()`, `simp38()`, `simp()`, `trap()`

Other newton-cotes: `adaptint()`, `giniquintile()`, `romberg()`, `simp38()`, `simp()`, `trap()`

Examples

```r
f <- function(x) { sin(x)^2 + cos(x)^2 }
m McInt(f, -pi, pi, m = 10)
m McInt(f, -pi, pi, m = 100)
m McInt(f, -pi, pi, m = 1000)
```
Description

Use Newton's method to find real roots

Usage

newton(f, fp, x, tol = 0.001, m = 100)

Arguments

f 	function to integrate
fp 	function representing the derivative of f
x 	an initial estimate of the root
tol 	he error tolerance
m 	the maximum number of iterations

Details

Newton's method finds real roots of a function, but requires knowing the function derivative. It will return when the interval between them is less than tol, the error tolerance. However, this implementation also stops after m iterations.

Value

the real root found

See Also

Other optimz: bisection(), goldsect, gradient, hillclimbing(), sa(), secant()

Examples

f <- function(x) { x^3 - 2 * x^2 - 159 * x - 540 }
fp <- function(x) {3 * x^2 - 4 * x - 159}
newton(f, fp, 1)
Description

Find the nearest neighbor for a set of data points

Usage

```
nn(p, y, q)
```

Arguments

- **p**: matrix of variable values, each row is a data point
- **y**: vector of values, each entry corresponds to one row in `p`
- **q**: vector of variable values, each entry corresponds to one column of `p`

Details

`nn` finds the n-dimensional nearest neighbor for given datapoint

Value

an interpolated value for `q`

See Also

Other interp: `bezier`, `bilinear()`, `cubicspline()`, `linterp()`, `polyinterp()`, `pwiselinterp()`

Examples

```
p <- matrix(floor(runif(100, 0, 9)), 20)
y <- floor(runif(20, 0, 9))
q <- matrix(floor(runif(5, 0, 9)), 1)
nn(p, y, q)
```
nthroot

The n-th root formula

Description

Find the n-th root of real numbers

Usage

nthroot(a, n, tol = 1/1000)

Arguments

a a positive real number
n n
tol the permitted error tolerance

Details

The nthroot function finds the nth root of a via an iterative process.

Value

the root

See Also

Other algebra: bilinear(), cubicspline(), division, fibonacci(), horner(), isPrime(), linterp(), polyinterp(), pwiselinterp(), quadratic()

Examples

nthroot(100, 2)
nthroot(65536, 4)
nthroot(1000, 3)
Description

Finds a polynomial function interpolating the given points

Usage

polyinterp(x, y)

Arguments

x a vector of x values
y a vector of y values

Details

polyinterp finds a polynomial that interpolates the given points.

Value

a polynomial equation’s coefficients

See Also

Other interp: bezier, bilinear(), cubicspline(), linterp(), nn(), pwiselinterp()
Other algebra: bilinear(), cubicspline(), division, fibonacci(), horner(), isPrime(), linterp(), nthroot(), pwiselinterp(), quadratic()

Examples

x <- c(1, 2, 3)
y <- x^2 + 5 * x - 3
f <- polyinterp(x, y)
**pwiselinterp**  

*Piecewise linear interpolation*

**Description**

Finds a piecewise linear function that interpolates the data points.

**Usage**

```r
pwiselinterp(x, y)
```

**Arguments**

- `x`: a vector of x values
- `y`: a vector of y values

**Details**

`pwiselinterp` finds a piecewise linear function that interpolates the data points. For each x-y ordered pair, there function finds the unique line interpolating them. The function will return a data.frame with three columns.

- The column `x` is the upper bound of the domain for the given piece.
- The columns `m` and `b` represent the coefficients from the y-intercept form of the linear equation, $y = mx + b$.

The matrix will contain length(x) rows with the first row having m and b of NA.

**Value**

A matrix with the linear function components.

**See Also**

Other interp: `bezier`, `bilinear()`, `cubicspline()`, `linterp()`, `nn()`, `polyinterp()`  
Other algebra: `bilinear()`, `cubicspline()`, `division`, `fibonacci()`, `horner()`, `isPrime()`, `linterp()`, `nthroot()`, `polyinterp()`, `quadratic()`

**Examples**

```r
x <- c(5, 0, 3)  
y <- c(4, 0, 3)  
f <- pwiselinterp(x, y)
```
quadratic

The quadratic equation.

Description

Find the zeros of a quadratic equation.

Usage

quadratic(b2, b1, b0)

quadratic2(b2, b1, b0)

Arguments

b2 the coefficient of the x^2 term
b1 the coefficient of the x term
b0 the constant term

Details

quadratic and quadratic2 implement the quadratic equation from standard algebra in two different ways. The quadratic function is susceptible to cascading numerical error and the quadratic2 has reduced potential error.

Value

numeric vector of solutions to the equation

See Also

Other algebra: bilinear(), cubicspline(), division, fibonacci(), horner(), isPrime(), linterp(), nthroot(), polyinterp(), pwiselinterp()

Examples

quadratic(1, 0, -1)
quadratic(4, -4, 1)
quadratic2(1, 0, -1)
quadratic2(4, -4, 1)
Matrix to Row Echelon Form

Description
Transform a matrix to row echelon form.

Usage
refmatrix(m)
rrefmatrix(m)
solvematrix(A, b)

Arguments
m a matrix
A a square matrix representing the coefficients of a linear system in solvematrix
b a vector representing the right-hand side of the linear system in solvematrix

Details
refmatrix reduces a matrix to row echelon form. This is not a reduced row echelon form, though that can be easily calculated from the diagonal. This function works on non-square matrices.
rrefmatrix returns the reduced row echelon matrix.
solvematrix solves a linear system using rrefmatrix.

Value
the modified matrix

See Also
Other linear: choleskymatrix(), detmatrix(), gds(), invmatrix(), iterativematrix, lumatrix(), rowops, tridiagmatrix(), vecnorm()

Examples
A <- matrix(c(1, 2, -7, -1, -1, 1, 2, 1, 5), 3)
refmatrix(A)
**resizeImage**  
*Image resizing*

**Description**

Resize images using nearest neighbor and

**Usage**

`resizeImageNN(imx, width, height)`

`resizeImageBL(imx, width, height)`

**Arguments**

- `imx`: a 3-dimensional array containing image data
- `width`: the new width
- `height`: the new height

**Details**

The `resizeImageNN` function uses the nearest neighbor method to resize the image. Also, `resizeImageBL` uses bilinear interpolation to resize the image.

**Value**

a three-dimensional array containing the resized image.

---

**revolution-solid**  
*Volumes of solids of revolution*

**Description**

Find the volume of a solid of revolution

**Usage**

`shellmethod(f, a, b)`

`discmethod(f, a, b)`

**Arguments**

- `f`: function of revolution
- `a`: lower-bound of the solid
- `b`: upper-bound of the solid
Details

The functions `discmethod` and `shellmethod` implement the algorithms for finding the volume of solids of revolution. The `discmethod` function is suitable for volumes revolved around the x-axis and the `shellmethod` function is suitable for volumes revolved around the y-axis.

Value

the volume of the solid

See Also

Other integration: `adaptint()`, `gaussint()`, `giniquintile()`, `mcint()`, `midpt()`, `romberg()`, `simp38()`, `simp()`, `trap()`

Examples

```r
f <- function(x) { x^2 }
shellmethod(f, 1, 2)
discmethod(f, 1, 2)
```

---

**romberg**

**Romberg Integration**

Description

Romberg’s adaptive integration

Usage

```r
romberg(f, a, b, m, tab = FALSE)
```

Arguments

- `f`: function to integrate
- `a`: the lowerbound of integration
- `b`: the upperbound of integration
- `m`: the maximum number of iterations
- `tab`: if TRUE, return the table of values

Details

The `romberg` function uses Romberg’s rule to calculate the integral of the function `f` over the interval from `a` to `b`. The parameter `m` sets the number of intervals to use when evaluating. Additional options are passed to the function `f` when evaluating.
Value

the value of the integral

See Also

Other integration: adaptint(), gaussint(), giniquintile(), mcint(), midpt(), revolution-solid, simp38(), simp(), trap()

Other newton-cotes: adaptint(), giniquintile(), midpt(), simp38(), simp(), trap()

Examples

```r
f <- function(x) { sin(x)^2 + log(x)}
romberg(f, 1, 10, m = 3)
romberg(f, 1, 10, m = 5)
romberg(f, 1, 10, m = 10)
```

---

### rowops

#### Elementary row operations

**Description**

These are elementary operations for a matrix. They do not presume a square matrix and will work on any matrix. They use R’s internal row addressing to function.

**Usage**

```r
swaprows(m, row1, row2)
replacerow(m, row1, row2, k)
scalerow(m, row, k)
```

**Arguments**

- `m` a matrix
- `row1` a source row
- `row2` a destination row
- `k` a scaling factor
- `row` a row to modify

**Details**

`replacerow` replaces one row with the sum of itself and the multiple of another row. `swaprows` swap two rows in the matrix. `scalerow` scales all entries in a row by a constant.
Value
the modified matrix

See Also
Other linear: choleskymatrix(), detmatrix(), gdlss(), invmatrix(), iterativematrix, lumatrix(), refmatrix(), tridiagmatrix(), vecnorm()

Examples
n <- 5
A <- matrix(sample.int(10, n^2, TRUE) - 1, n)
A <- swaprows(A, 2, 4)
A <- replacerow(A, 1, 3, 2)
A <- scalerow(A, 5, 10)

Description
Use simulated annealing to find the global minimum

Usage
sa(f, x, temp = 10000, rate = 1e-04)

tpsa(x, temp = 100, rate = 1e-04)

Arguments
f function representing f
x an initial estimate of the minimum
temp the initial temperature
rate the cooling rate

Details
Simulated annealing finds a global minimum by mimicking the metallurgical process of annealing.

Value
the x value of the minimum found

See Also
Other optimz: bisection(), goldsect, gradient, hillclimbing(), newton(), secant()
Examples

```r
f <- function(x) { x^6 - 4 * x^5 - 7 * x^4 + 22 * x^3 + 24 * x^2 + 2 }
sa(f, 0)

f <- function(x) { (x[1] - 1)^2 + (x[2] - 1)^2 }
sa(f, c(0, 0), 0.05)
```

---

## secant

### Secant Method

**Description**

The secant method for root finding

**Usage**

```r
secant(f, x, tol = 0.001, m = 100)
```

**Arguments**

- `f`: function to integrate
- `x`: an initial estimate of the root
- `tol`: the error tolerance
- `m`: the maximum number of iterations

**Details**

The secant method for root finding extends Newton’s method to estimate the derivative. It will return when the interval between them is less than `tol`, the error tolerance. However, this implementation also stop if after `m` iterations.

**Value**

the real root found

**See Also**

Other optimz: `bisection()`, `goldsect`, `gradient`, `hillclimbing()`, `newton()`, `sa()`

**Examples**

```r
f <- function(x) { x^3 - 2 * x^2 - 159 * x - 540 }
secant(f, 1)
```
simp

Simpson’s rule

Description

Use Simpson’s rule to integrate a function

Usage

simp(f, a, b, m = 100)

Arguments

f function to integrate
a the a-bound of integration
b the b-bound of integration
m the number of subintervals to calculate

Details

The simp function uses Simpson’s rule to calculate the integral of the function f over the interval from a to b. The parameter m sets the number of intervals to use when evaluating. Additional options are passed to the function f when evaluating.

Value

the value of the integral

See Also

Other integration: adaptint(), gaussint(), giniquintile(), mcint(), midpt(), revolution-solid, romberg(), simp38(), trap()

Other newton-cotes: adaptint(), giniquintile(), midpt(), romberg(), simp38(), trap()

Examples

f <- function(x) { sin(x)^2 + cos(x)^2 }
simp(f, -pi, pi, m = 10)
simp(f, -pi, pi, m = 100)
simp(f, -pi, pi, m = 1000)
simp38

Simpson’s 3/8 rule

Description

Use Simpson’s 3/8 rule to integrate a function

Usage

\texttt{simp38(f, a, b, m = 100)}

Arguments

\begin{itemize}
\item \texttt{f} \quad \text{function to integrate}
\item \texttt{a} \quad \text{the a-bound of integration}
\item \texttt{b} \quad \text{the b-bound of integration}
\item \texttt{m} \quad \text{the number of subintervals to calculate}
\end{itemize}

Details

The \texttt{simp38} function uses Simpson’s 3/8 rule to calculate the integral of the function \( f \) over the interval from \( a \) to \( b \). The parameter \( m \) sets the number of intervals to use when evaluating. Additional options are passed to the function \( f \) when evaluating.

Value

the value of the integral

See Also

Other integration: \texttt{adaptint()}, \texttt{gaussint()}, \texttt{giniquintile()}, \texttt{mcint()}, \texttt{midpt()}, \texttt{revolution-solid}, \texttt{romberg()}, \texttt{simp()}, \texttt{trap()}

Other newton-cotes: \texttt{adaptint()}, \texttt{giniquintile()}, \texttt{midpt()}, \texttt{romberg()}, \texttt{simp()}, \texttt{trap()}

Examples

\begin{verbatim}
f <- function(x) { sin(x)^2 + log(x) }
simp38(f, 1, 10, m = 10)
simp38(f, 1, 10, m = 100)
simp38(f, 1, 10, m = 1000)
\end{verbatim}
Two summing algorithms

Description

Find the sum of a vector

Usage

naivesum(x)

kahansum(x)

pwisesum(x)

Arguments

x a vector of numbers to be summed

Details

naivesum calculates the sum of a vector by keeping a counter and repeatedly adding the next value to the interim sum. kahansum uses Kahan's algorithm to capture the low-order precision loss and ensure that the loss is reintegrated into the final sum. pwisesum is a recursive implementation of the piecewise summation algorithm that divides the vector in two and adds the individual vector sums for a result.

Value

the sum

Examples

```R
k <- 1:10^6
n <- sample(k, 1)
bound <- sample(k, 2)
bound.upper <- max(bound) - 10^6 / 2
bound.lower <- min(bound) - 10^6 / 2
x <- runif(n, bound.lower, bound.upper)
naivesum(x)
kahansum(x)
pwisesum(x)
```
Description

Use the trapezoid method to integrate a function

Usage

\[
\text{trap}(f, a, b, m = 100)
\]

Arguments

- **f**: function to integrate
- **a**: the a-bound of integration
- **b**: the b-bound of integration
- **m**: the number of subintervals to calculate

Details

The `trap` function uses the trapezoid method to calculate the integral of the function \( f \) over the interval from \( a \) to \( b \). The parameter \( m \) sets the number of intervals to use when evaluating the trapezoids. Additional options are passed to the function \( f \) when evaluating.

Value

the value of the integral

See Also

Other integration: `adaptint()`, `gaussint()`, `giniquintile()`, `mcint()`, `midpt()`, `revolution-solid`, `romberg()`, `simp38()`, `simp()`

Other newton-cotes: `adaptint()`, `giniquintile()`, `midpt()`, `romberg()`, `simp38()`, `simp()`

Examples

```r
f <- function(x) { sin(x)^2 + cos(x)^2 }
trap(f, -pi, pi, m = 10)
trap(f, -pi, pi, m = 100)
trap(f, -pi, pi, m = 1000)
```
tridiagmatrix

Solve a tridiagonal matrix

Description

use the tridiagonal matrix algorithm to solve a tridiagonal matrix

Usage

tridiagmatrix(L, D, U, b)

Arguments

L vector of entries below the main diagonal
D vector of entries on the main diagonal
U vector of entries above the main diagonal
b vector of the right-hand side of the linear system

Details

tridiagmatrix uses the tridiagonal matrix algorithm to solve a tridiagonal matrix.

Value

the solution vector

See Also

Other linear: choleskymatrix(), detmatrix(), gdls(), invmatrix(), iterativematrix, lumatrix(), refmatrix(), rowops, vecnorm()

vecnorm

Norm of a vector

Description

Find the norm of a vector

Usage

vecnorm(b)

Arguments

b a vector
Details
Find the norm of a vector

Value
the norm

See Also
Other linear: `choleskymatrix()`, `detmatrix()`, `gdls()`, `invmatrix()`, `iterativematrix`, `lumatrix()`, `refmatrix()`, `rowops`, `tridiagmatrix()`

Examples
```r
x <- c(1, 2, 3)
vecnorm(x)
```

---

**wave**  

*Wave Equation using*

---

**Description**
solve heat equation via forward-time central-space method

**Usage**
```r
wave(u, alpha, xdelta, tdelta, n)
```

**Arguments**
- `u`: the initial values of u
- `alpha`: the thermal diffusivity coefficient
- `xdelta`: the change in x at each step in u
- `tdelta`: the time step
- `n`: the number of steps to take

**Details**
The heat solves the heat equation using the forward-time central-space method in one-dimension.

**Value**
a matrix of u values at each time step
Examples

```r
speed <- 2
x0 <- 0
xdelta <- .05
x <- seq(x0, 1, xdelta)
m <- length(x)
u <- sin(x * pi * 2)
u[1:21] <- 0
tdelta <- .02
n <- 40
z <- wave(u, speed, xdelta, tdelta, n)
```

Description

Wilkinson’s polynomial

Usage

```r
wilkinson(x, w = 20)
```

Arguments

- `x` the x-value
- `w` the number of terms in the polynomial

Details

Wilkinson’s polynomial is a terrible joke played on numerical analysis. By tradition, the function is \( f(x) = (x - 1)(x - 2)\ldots(x - 20) \), giving a function with real roots at each integer from 1 to 20. This function is generalized and allows for \( n \) and the function value is \( f(x) = (x - 1)(x - 2)\ldots(x - n) \). The default of \( n \) is 20.

Value

the value of the function at \( x \)

Examples

```r
wilkinson(0)
```
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