Package ‘cmvnorm’

January 31, 2022

Type Package
Title The Complex Multivariate Gaussian Distribution
Version 1.0-7
Depends emulator (>= 1.2-21)
Suggests knitr
Imports elliptic
Maintainer Robin K. S. Hankin <hankin.robin@gmail.com>
Description Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.
VignetteBuilder knitr
License GPL-2
URL https://github.com/RobinHankin/cmvnorm
BugReports https://github.com/RobinHankin/cmvnorm/issues
NeedsCompilation no
Author Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>)
Repository CRAN
Date/Publication 2022-01-31 00:00:02 UTC

R topics documented:
cmvnorm-package ................................................. 2
corr_complex .................................................. 3
isHermitian ..................................................... 5
Mvnorm .......................................................... 6
setreal .......................................................... 7
var ............................................................. 8
wishart .......................................................... 9

Index 11
The Complex Multivariate Gaussian Distribution

Description

Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.

Details

The DESCRIPTION file:

- Package: cmvnorm
- Type: Package
- Title: The Complex Multivariate Gaussian Distribution
- Version: 1.0-7
- Authors@R: person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@gmail.com"), comment = c(ORCID = "0000-0001-5982-0415"))
- Depends: emulator (>= 1.2-21)
- Suggests: knitr
- Imports: elliptic
- Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>
- Description: Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.
- VignetteBuilder: knitr
- License: GPL-2
- URL: https://github.com/RobinHankin/cmvnorm
- BugReports: https://github.com/RobinHankin/cmvnorm/issues
- Author: Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>)

Index of help topics:

- `Im<-` Manipulate real or imaginary components of an object
- `Mvncnorm` Multivariate complex Gaussian density and random deviates
- `cmvnorm-package` The Complex Multivariate Gaussian Distribution
- `corr_complex` Complex Gaussian processes
- `ishHermitian` Is a Matrix Hermitian?
- `var` Variance and standard deviation of complex vectors
- `wishart` The complex Wishart distribution

Generalizing the real multivariate Gaussian distribution to the complex case is not straightforward but one common approach is to replace the real symmetric variance matrix with a Hermitian positive-definite matrix. The `cmvnorm` package provides some functionality for the resulting density function.
corr_complex

Author(s)
NA
Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

References

• N. R. Goodman 1963. “Statistical analysis based on a certain multivariate complex Gaussian

number 1.

Examples

S1 <- 4+diag(5)
S2 <- S1
S2[1,5] <- 4+1i
S2[5,1] <- 4-1i  # Hermitian

rcmvnorm(10,sigma=S1)
rcmvnorm(10,mean=rep(1i,5),sigma=S2)
dcmvnorm(rep(1,5),sigma=S2)

corr_complex Complex Gaussian processes

Description

Various utilities for investigating complex Gaussian processes

Usage

corr_complex(z1, z2 = NULL, distance.function = complex_CF, means =
NULL, scales = NULL, pos.def.matrix = NULL)
complex_CF(z1,z2, means, pos.def.matrix)
scales.likelihood.complex(pos.def.matrix, scales, means, zold, z,
give_log = TRUE, func = regressor.basis)
interpolant.quick.complex(x, d, zold, Ainv, scales = NULL, pos.def.matrix = NULL,
means=NULL, func = regressor.basis, give.Z = FALSE,
distance.function = corr_complex, ...)
Arguments

\[ z, z_1, z_2 \]  
Points in \( C^n \)

distance.function

Function giving the (complex) covariance between two points in \( C^n \)

means, pos.def.matrix, scales

In function complex_CF(), the mean and covariance matrix of the distribution whose characteristic function is used to give the covariance matrix; scales is used to specify the diagonal of pos.def.matrix if the off-diagonal elements are zero

\[ z_{old}, d, \text{give\_log, func, x, Ainv, give\_Z,...} \]

Direct analogues of the arguments in interpolant() and scales.likelihood() in the emulator package

Details

- Function complex_CF() returns a (slightly reparameterized) characteristic function of a complex Gaussian distribution. The covariance is given by

\[
c(t) = \exp(i \text{Re}(t^* \mu) - t^* B t)
\]

where \( t = x - x' \) is interpreted as the distance between two observations, \( \mu \) is the mean of the distribution (which is in general a complex vector), and \( B \) a positive-definite matrix.

- Function corr_complex() is the complex analogue of corr.matrix(). It returns a matrix with entry \((i,j)\) equal to the covariance of the process at observation \( i \) and observation \( j \), or \( \text{cov}(\eta(x_i), \eta(x_j)) \). The elements are calculated by complex_CF().

This function includes only a single method, that of nested calls to apply(). I could not figure out how to generalize method 1 of corr.matrix() to the complex case.

- Function scales.likelihood.complex() is a complex version of scales.likelihood() which takes a positive definite matrix and a mean. The formula used is

\[
(\sigma^2)^{-\frac{1}{2}} |A|^{-\frac{1}{2}} |H^* A^{-1} H|^{-\frac{1}{2}}
\]

. Here and elsewhere, \( A^* \) means the complex conjugate of the transpose.

- Function interpolant.quick.complex() is a complex version of interpolant.quick().

\[
h(x)^* \hat{\beta} + t(x)^* A^{-1} (y - H \hat{\beta})
\]

This is the complex version of Oakley’s equation 2.30 or Hankin’s equation 5.

More details are given in the package vignette.

Author(s)

Robin K. S. Hankin

References


**isHermitian**  

**Examples**

```r
complex_CF(c(1,1i),c(1,-1i),means=c(1i,1i),pos.def.matrix=diag(2))

V <- latin.hypercube(7,2,complex=TRUE)

cm <- c(1,1+1i)  # "complex mean"

V <- latin.hypercube(7,2,complex=TRUE)

cs <- matrix(c(2,1i,-1i,1,2,2))  # "complex scales"

tb <- c(1,1i,1-1i)  # "true beta"

A <- corr_complex(V,means=cm,pos.def.matrix=cs)

Ainv <- solve(A)

z <- drop(rcmvnorm(n=1,mean=regressor.multi(V) %*% tb, sigma=A))

betahat.fun(V,Ainv,z)  # should be close to 'tb'

#scales.likelihood.complex(cs,cm,V,z)  # log-likelihood evaluated true parameters

interpolant.quick.complex(x=0.1i+V[1:3,],d=z,zold=V,Ainv=Ainv,pos.def.matrix=cs,means=cm)
```

---

**isHermitian**  

Is a Matrix Hermitian?

**Description**

Returns TRUE if a matrix is Hermitian or Hermitian positive-definite

**Usage**

```r
isHermitian(x, tol = 100 * .Machine$double.eps)

ishpd(x,tol= 100 * .Machine$double.eps)

zapim(x,tol= 100 * .Machine$double.eps)
```

**Arguments**

- `x` A square matrix
- `tol` Tolerance for numerical scruff

**Details**

Functions `isHermitian()` and `ishpd()` return a Boolean, indicating whether the argument is Hermitian or Hermitian positive definite respectively. Function `zapim()` zaps small imaginary parts of a vector, returning real if all elements are so zapped.
Author(s)
Robin K. S. Hankin

Examples

\[
v <- 2^{1:30} \\
zapim(v+1i*exp(-v))
\]

\[
\text{ishpd(matrix(c(1,0.1i,-0.1i,1),2,2))} \quad \# \text{ should be TRUE} \\
\text{isHermitian(matrix(c(1,3i,-3i,1),2,2))} \quad \# \text{ should be TRUE} \\
\text{ishpd(rcwis(6,2))} \quad \# \text{ should be TRUE}
\]

Mvnorm
Multivariate complex Gaussian density and random deviates

Description
Density function and a random number generator for the multivariate complex Gaussian distribution.

Usage

\[
\begin{align*}
\text{rcnorm}(n) \\
\text{dcmvnorm}(z, \text{mean}, \text{sigma}, \text{log} = \text{FALSE}) \\
\text{rcmvnorm}(n, \text{mean} = \text{rep}(0, nrow(\text{sigma})), \text{sigma} = \text{diag}(\text{length}(\text{mean})), \\
\text{method} = \text{c("svd", "eigen", "chol")}, \\
\text{tol} = 100 * \text{.Machine}$\text{double.eps})
\end{align*}
\]

Arguments

- \text{z} \quad \text{Complex vector or matrix of quantiles. If a matrix, each row is taken to be a quantile}
- \text{n} \quad \text{Number of observations}
- \text{mean} \quad \text{Mean vector}
- \text{sigma} \quad \text{Covariance matrix, Hermitian positive-definite}
- \text{tol} \quad \text{numerical tolerance term for verifying positive definiteness}
- \text{log} \quad \text{In dcmvnorm(), Boolean with default FALSE meaning to return the Gaussian density function, and TRUE meaning to return the logarithm}
- \text{method} \quad \text{Specifies the decomposition used to determine the positive-definite matrix square root of sigma. Possible methods are eigenvalue decomposition ("eigen", default), and singular value decomposition ("svd")}
Details

Function `dcmvnorm()` is the density function of the complex multivariate normal (Gaussian) distribution:

\[ p(z) = \frac{\exp(-z^* \Gamma z)}{|\pi \Gamma|} \]

Function `rcnrm()` is a low-level function designed to generate observations drawn from a standard complex Gaussian. Function `rcmvnorm()` is a user-friendly wrapper for this.

Author(s)

Robin K. S. Hankin

References


Examples

```r
S <- emulator::cprod(rcmvnorm(3, mean=c(1,1i), sigma=diag(2)))
rcmvnorm(10, sigma=S)
rcmvnorm(10, mean=c(0,1+10i), sigma=S)

# Now try and estimate the mean (viz 1,1i) and variance (S) from a
# random sample:

n <- 101
z <- rcmvnorm(n, mean=c(0,1+10i), sigma=S)
xbar <- colMeans(z)
Sbar <- cprod(sweep(z, 2, xbar))/n
```

**setreal**

**Manipulate real or imaginary components of an object**

Description

Manipulate real or imaginary components of an object
Usage

Im(x) <- value
Re(x) <- value

Arguments

x Complex-valued object
value Real-valued object

Author(s)

Robin K. S. Hankin

Examples

A <- matrix(c(1,0.1i,-0.1i,1),2,2)
Im(A) <- Im(A)*3
Re(A) <- matrix(c(5,2,2,5),2,2)

---

var Variance and standard deviation of complex vectors

Description

Complex generalizations of stats::sd() and stats::var()

Usage

var(x, y=NULL, na.rm=FALSE, use)
sd(x, na.rm=FALSE)

Arguments

x, y Complex vector or matrix
na.rm Boolean with default FALSE meaning to leave NA values present and TRUE meaning to remove them
use Ignored
Details

Intended to be broadly compatible with stats::sd() and stats::var().

If given real values, var() and sd() return the variance and standard deviation as per ordinary real analysis. If given complex values, returns the complex generalization in which Hermitian transposes are used.

If $z$ is a complex matrix, var($z$) returns the variance of the rows.

These functions use $n - 1$ on the denominator purely for consistency with stats::var() (for the record, I disagree with the rationale for $n - 1$).

Author(s)

Robin K. S. Hankin

Examples

sd(rcnorm(10)) # imaginary component suppressed by zapim()

var(rcmvnorm(1e5,mean=c(0,0)))

wishart

The complex Wishart distribution

Description

Returns an observation drawn from the complex Wishart distribution. To sample from the inverse complex Wishart distribution (or indeed the complex inverse Wishart distribution), use solve(rcwis(...)).

Usage

rcwis(n, S)

Arguments

n Integer; degrees of freedom
S Variance matrix. If an integer, use diag(nrow=S)

Value

Returns a (semi-) positive definite Hermitian matrix the same size as argument S

Note

The first argument of rcwis() is n, by universal statistics convention. But in the R world, functions returning random observations (such as runif()) generally reserve argument n for the number of observations to return. Although rchisq() uses df for the number of degrees of freedom.
Author(s)

Robin K. S. Hankin

Examples

```r
rcwis(10,2)
eigen(rcwis(7,3),TRUE,TRUE)  # all positive
eigen(rcwis(3,7),TRUE,TRUE)  # 4 positive, 3 zero
rcwis(10,rcwis(10,3))
```
Index

* complex
  *isHermitian, 5
  Mvnorn, 6
  setreal, 7
* distribution
  Mvnorn, 6
* math
  setreal, 7
* multivariate
  Mvnorn, 6
* package
  cmvnorm-package, 2

cmvnorm (cmvnorm-package), 2
cmvnorm-package, 2
complex_CF (corr_complex), 3
corr_complex, 3
dcmvnorm (Mvnorn), 6

Im<-(setreal), 7
interpolant.quick.complex
  (corr_complex), 3
isHermitian, 5
ishpd (isHermitian), 5

Mvnorn, 6

rcmvnorm (Mvnorn), 6
rcnorm (Mvnorn), 6
rcwis (wishart), 9
Re<-(setreal), 7
rwis (wishart), 9

scales.likelihood.complex
  (corr_complex), 3
sd (var), 8
setreal, 7

var, 8

wishart, 9
zapim (isHermitian), 5