During simulations seed production depends on the competition for resources within (sub)-communities. First strength of competition (\(\alpha\)) is calculated for each pair of co-occurring individuals from the trait values related to resource use by competition kernels specified in `competition.kernel` parameter of `comm.simul` function. Then the matrix of pairwise competition coefficient are used in function specified by `fSeedProduction` parameter. This vignette shows the available symmetric (where \(\alpha_{ij} = \alpha_{ji}\)) and asymmetric (where \(\alpha_{ij} \neq \alpha_{ji}\) if \(i \neq j\)) kernels, and `SeedProduction` function that recently the only available function in the package for this purpose.

### 1 Symmetric competition kernels

Recently the only available symmetric competition kernel is the Gaussian one:

\[
\alpha_{ij} = \exp \left( -\frac{(B_i - B_j)^2}{\sigma_b} \right)
\]

where \(B_i\) and \(B_j\) are the resource use related trait values of the two species, while \(\sigma_b\) determines how steeply decrease the strength of competition with increasing difference in resource use (Figure 1).

For \(B_i = B_j\), \(\alpha_{ij} = 1\) irrespectively to value of \(\sigma_b\). If \(\sigma_b = 0\) the strength of competition is zero any case of \(B_i \neq B_j\). If \(\sigma_b = \infty\), \(\alpha_{ij} = 1\) for all species pairs.

Gaussian competition kernel can be used by setting `competition.kernel="Gaussian.competition,kernel"` (which is the default value of this parameter). Value of \(\sigma_b\) has to be set by parameter `sigma.b`.

According to MacArthur & Levins (1967), this competition kernel can be deduced as overlap of Gaussian resource use curves. Their general formula for overlap is
\[ \alpha_{ij} = \frac{\int_{-\infty}^{\infty} U_i(x)U_j(x) \, dx}{\int_{-\infty}^{\infty} U_i^2(x) \, dx} \]  

(2)

where \( U \) is the resource use function, and \( x \) is the quality of the resource (e.g. seed size or rooting depth). Let both \( U_i \) and \( U_j \) be density function of normal (Gaussian) distribution with same standard deviation \( \sigma \), and let rescale \( x \) to be expected values equals to zero and \( d = B_i - B_j \), respectively:

\[ \alpha_{ij} = \frac{\int_{-\infty}^{\infty} \exp \left( -\frac{x^2}{\sigma^2} \right) \exp \left( -\frac{(x-d)^2}{\sigma^2} \right) \, dx}{\int_{-\infty}^{\infty} \exp \left( -\frac{x^2}{\sigma^2} \right) \, dx} = \frac{\int_{-\infty}^{\infty} \exp \left( -\frac{x^2 + (x-d)^2}{\sigma^2} \right) \, dx}{\int_{-\infty}^{\infty} \exp \left( -\frac{2x^2}{\sigma^2} \right) \, dx} \]  

(3)

Since \( x^2 + (x-d)^2 = 2x^2 + d^2 - 2xd = 2 \left( x - \frac{d}{2} \right)^2 + \frac{d^2}{2} \),

\[ \alpha_{ij} = \frac{\exp \left( \frac{d^2}{2\sigma^2} \right) \int_{-\infty}^{\infty} \exp \left( -\frac{2(x-d/2)^2}{\sigma^2} \right) \, dx}{\int_{-\infty}^{\infty} \exp \left( -\frac{2x^2}{\sigma^2} \right) \, dx} = \exp \left( \frac{d^2}{2\sigma^2} \right) \]  

(4)

Note that \( \sigma_b \) in equation (1) equals to \( 2\sigma^2 \) in equation (4)

2 **Asymmetric competition kernels**

Recently two types of asymmetric competition kernels are available via asymmetric.competition.kernel function:

- Kisdi’s convex-concave function
- smooth function suggested by Nattrass et al. (2012)
2.1 Kisdi’s convex-concave function

It is a function defined by equation (2) in Kisdi (1999), however the parametrization are slightly modified:

\[
\alpha_{ij} = C \left( 1 - \frac{1}{1 + v \exp \left( -\frac{B_i - B_j}{\sigma_b} \right)} \right)
\]

Contrary to the Gaussian competition kernel, it has three parameters \((C, v, \sigma_b)\) instead of the only one parameter of Gaussian competition kernel. Note that in the R function these parameters are called `ac.C`, `ac.v` and `sigma.b`, respectively. \(C\) and \(v\) have to be positive, while \(\sigma_b \neq 0\). Possible values of the function ranges from zero to \(C\). If \(\sigma_b > 0\) it is a decreasing sigmoid (convex-concave) function (Figure 2) of trait difference \((B_i - B_j)\) with inflection point at \(B_i - B_j = \sigma_b \ln v\), where the strength of competition is \(C/2\).

![Figure 2: Shape of Kisdi’s convex-concave function with different values of v (C=1, \(\sigma_b=0.1\)](image)

Strength of competition between functionally equivalent individuals (i.e. if \(B_i = B_j\)) is \(\alpha_{ij} = C \left( \frac{v}{1 + v} \right)\). If the other two parameter fixed, absolute value of parameter \(\sigma_b\) determines the steepness of the curve around its inflection point (Figure 3).

2.2 Smooth function proposed by Nattrass et al. (2012)

This is also a sigmoid function defined by a formula similar to Kisdi’s function:

\[
\alpha_{ij} = 1 + C - \frac{2C}{1 + \exp \left( -\frac{2(B_i - B_j)}{\sigma_b} \right)}
\]

It ranges from \(1 - C\) to \(1 + C\). Position of its inflection point is \(B_i - B_j = 0\), where its value is 1, irrespective to the parameter values. When its range (i.e. value of parameter \(C\)) is fixed, \(\sigma_b\) determines the steepness of the curve at the inflection point: lower \(\sigma_b\) results in steeper curve.
Figure 3: Shape of Kisdi’s convex-convave function with different values of $\sigma_b$ (C=1,v=1)

Figure 4: Shape of smooth function by Nattrass et al. with different values of $\sigma_b$ (C=1)
However equation (6) is not defined when $\sigma_{b_k} = 0$ and $B_j = B_i$, following suggestion of Nattrass et al. (2012) asymmetric.competition.kernel function set the strength of competition to 1 in this case.

3 SeedProduction function

This function calculates number of produced seeds for each individual. The number of seeds is random number from Bernoulli (zero or one seed) or Poisson distribution (unlimited number of seeds). The expected value of produced seeds (irrespectively to the distribution) for individual $i$ in local community $k$ depends on the competition for resources:

$$p_{ik} = b_0 \max \left( \frac{K - \sum_{i \in k} \alpha_{ij}}{K}, 0 \right)$$

(7)

Where: $b_0$ is the maximum probability of reproduction in competition free conditions; $K$ is level of competition above which probability of reproduction becomes zero; $\alpha_{ij} = $ competitive effect of individual $j$ on individual $i$, calculated from resource acquisition traits by the competition kernel functions.

References

