

# Package ‘deldir’

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**Title** Delaunay Triangulation and Dirichlet (Voronoi) Tessellation

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**Depends** R (>= 0.99)

**Suggests** polyclip

**Imports** graphics, grDevices

**Description** Calculates the Delaunay triangulation and the Dirichlet or Voronoi tessellation (with respect to the entire plane) of a planar point set. Plots triangulations and tessellations in various ways. Clips tessellations to sub-windows. Calculates perimeters of tessellations. Summarises information about the tiles of the tessellation.

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deldir	<i>Delaunay triangulation and Dirichlet tessellation</i>
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**Description**

This function computes the Delaunay triangulation (and hence the Dirichlet or Voronoi tessellation) of a planar point set according to the second (iterative) algorithm of Lee and Schacter — see REFERENCES. The triangulation is made to be with respect to the whole plane by suspending it from so-called ideal points  $(-\infty,-\infty)$ ,  $(\infty,-\infty)$ ,  $(\infty,\infty)$ , and  $(-\infty,\infty)$ . The triangulation is also enclosed in a finite rectangular window. A set of dummy points may be added, in various ways, to the set of data points being triangulated.

**Usage**

```
deldir(x, y, dpl=NULL, rw=NULL, eps=1e-09, sort=TRUE, plot=FALSE,
       round=TRUE,digits=6, z=NULL, zdum=NULL, suppressMsge=FALSE, ...)
```

**Arguments**

**x,y**                    These arguments specify the coordinates of the point set being triangulated or tessellated. These can be given by two separate arguments x and y which are vectors or by a single argument x which is either a data frame or a generic list, possibly one of class ppp. (See package spatstat.)

If x is a data frame then the x coordinates of the points to be triangulated or tessellated are taken to be the column of this data frame which is named “x” if there is one, else the first column of the data frame which is not named either “y” or “z”. The y coordinates are taken to be the column of this data frame which is named “y” if there is one. If there is no column named “y” but there are columns named “x” and “z” then the y coordinates are taken to be the first

“other” column. If there no columns named either “x” or “y”, then the x coordinates are taken to be the first column not named “z” and the y coordinates are taken to be the *second* column not named “z”.

If there is a column named “z” and if the argument z (see below) is NULL, then this the column named “z” is taken to be the value of z.

If x is a list (but not a data frame) then it must have components named x and y, and possibly a component named z. The x and y components give the x and y coordinates respectively of the points to be triangulated or tessellated. If x is *not* of class ppp, if it has a component z and if argument z is NULL, then the z argument is set equal to this component z. If x is of class “ppp”, if the argument z is NULL, if x is “marked” (see package spatstat) and if the marks of x are a vector or a factor (as opposed to a data frame) then the z argument is set equal to these marks. In this case x should *not* have a component z, and at any rate such a component would be ignored.

dpl	<p>A list describing the structure of the dummy points to be added to the data being triangulated. The addition of these dummy points is effected by the auxiliary function <code>dumps()</code>. The list may have components:</p> <ul style="list-style-type: none"> <li>• <code>ndx</code>: The x-dimension of a rectangular grid; if either <code>ndx</code> or <code>ndy</code> is null, no grid is constructed.</li> <li>• <code>ndy</code>: The y-dimension of the aforementioned rectangular grid.</li> <li>• <code>nrad</code>: The number of radii or “spokes”, emanating from each data point, along which dummy points are to be added.</li> <li>• <code>nper</code>: The number of dummy points per spoke.</li> <li>• <code>fcctr</code>: A numeric “multiplicative factor” determining the length of each spoke; each spoke is of length equal to <code>fcctr</code> times the mean nearest neighbour distance of the data. (This distance is calculated by the auxiliary function <code>mnnd()</code>.)</li> <li>• <code>x</code>: A vector of x-coordinates of “ad hoc” dummy points</li> <li>• <code>y</code>: A vector of the corresponding y-coordinates of “ad hoc” dummy points</li> </ul>
rw	<p>The coordinates of the corners of the rectangular window enclosing the triangulation, in the order (xmin, xmax, ymin, ymax). Any data points (including dummy points) outside this window are discarded. If this argument is omitted, it defaults to values given by the range of the data, plus and minus 10 percent.</p>
eps	<p>A value of epsilon used in testing whether a quantity is zero, mainly in the context of whether points are collinear. If anomalous errors arise, it is possible that these may averted by adjusting the value of <code>eps</code> upward or downward.</p>
sort	<p>Logical argument; if TRUE (the default) the data (including dummy points) are sorted into a sequence of “bins” prior to triangulation; this makes the algorithm slightly more efficient. Normally one would set <code>sort</code> equal to FALSE only if one wished to observe some of the fine detail of the way in which adding a point to a data set affected the triangulation, and therefore wished to make sure that the point in question was added last. Essentially this argument would get used only in a de-bugging process.</p>
plot	<p>Logical argument; if TRUE a plot is produced. The nature of the plot may be controlled by using the <code>...</code> argument to pass appropriate arguments to <code>plot.deldir()</code>.</p>

	Without “further instruction” a plot of the points being triangulated and of both the triangulation and the tessellation is produced;
round	Logical scalar. Should the data stored in the returned value be rounded to digits decimal digits? This is essentially for cosmetic purposes. This argument is a “new addition” to <code>deldir()</code> , as of version 0.1-26. Previously rounding was done willy-nilly. The change was undertaken when Kodi Arfer pointed out that the rounding might have unwanted effects upon “downstream” operations.
digits	The number of decimal places to which all numeric values in the returned list should be rounded. Defaults to 6. Ignored if round (see above) is set to FALSE.
z	An optional vector of “auxiliary” values or “weights” associated with the respective points. ( <b>NOTE:</b> These “weights” are values associated with the points and hence with the tiles of the tessellation produced. They <b>DO NOT</b> affect the tessellation, i.e. the tessellation produced is the same as it would be if there were no weights. The <code>deldir</code> package <b>DOES NOT</b> do weighted tessellation. The so-called weights in fact need not be numeric.) If z is left NULL then it is taken to be the third column of x, if x is a data frame or to be the z component of x if x is a generic list. If z is left NULL and if x is of class “ppp” and is “marked” (see package <code>spatstat</code> ) and if in addition the marks are atomic (i.e. <i>not</i> a data frame) then z is taken to be the marks of x.
zdum	Values of z to be associated with any dummy points that are created. See <b>Warnings</b> .
suppressMsge	Logical scalar indicating whether a message (alerting the user to changes from previous versions of <code>deldir</code> ) should be suppressed.
...	Auxiliary arguments <code>add</code> , <code>wlines</code> , <code>wpoints</code> , <code>number</code> , <code>nex</code> , <code>col</code> , <code>lty</code> , <code>pch</code> , <code>xlim</code> , and <code>ylim</code> (and possibly other plotting parameters) may be passed to <code>plot.deldir()</code> through ... if <code>plot=TRUE</code> .

## Details

This package is a (straightforward) adaptation of the Splus library section “delaunay” to R. That library section is an implementation of the Lee-Schacter algorithm, which was originally written as a stand-alone Fortran program in 1987/88 by Rolf Turner, while with the Division of Mathematics and Statistics, CSIRO, Sydney, Australia. It was re-written as an Splus function (using dynamically loaded Fortran code), by Rolf Turner while visiting the University of Western Australia, May, 1995.

Further revisions were made December 1996. The author gratefully acknowledges the contributions, assistance, and guidance of Mark Berman, of D.M.S., CSIRO, in collaboration with whom this project was originally undertaken. The author also acknowledges much useful advice from Adrian Baddeley, formerly of D.M.S., CSIRO (now Professor of Statistics at Curtin University). Daryl Tingley of the Department of Mathematics and Statistics, University of New Brunswick provided some helpful insight. Special thanks are extended to Alan Johnson, of the Alaska Fisheries Science Centre, who supplied two data sets which were extremely valuable in tracking down some errors in the code.

Don MacQueen, of Lawrence Livermore National Lab, wrote an Splus driver function for the old stand-alone version of this software. That driver, which was available on Statlib, is now deprecated in favour of the current package “delaunay” package. Don also collaborated in the preparation of that package.

See the ChangeLog for information about further revisions and bug-fixes.

**Value**

A list (of class `deldir`), invisible if `plot=TRUE`, with components:

<code>delsgs</code>	A data frame with 6 columns. The first 4 entries of each row are the coordinates of the points joined by an edge of a Delaunay triangle, in the order $(x_1, y_1, x_2, y_2)$ . The last two entries are the indices of the two points which are joined.
<code>dirsgs</code>	<p>A data frame with 10 columns. The first 4 entries of each row are the coordinates of the endpoints of one the edges of a Dirichlet tile, in the order <math>(x_1, y_1, x_2, y_2)</math>. The fifth and sixth entries, in the columns named <code>ind1</code> and <code>ind2</code>, are the indices of the two points, in the set being triangulated, which are separated by that edge. The seventh and eighth entries, in the columns named <code>bp1</code> and <code>bp2</code> are logical values. The entry in column <code>bp1</code> indicates whether the first endpoint of the corresponding edge of a Dirichlet tile is a boundary point (a point on the boundary of the rectangular window). Likewise for the entry in column <code>bp2</code> and the second endpoint of the edge.</p> <p>The ninth and tenth entries, in columns named <code>thirdv1</code> and <code>thirdv2</code> are the indices of the respective third vertices of the Delaunay triangle whose circumcentres constitute the corresponding endpoints of the edge under consideration. (The other two vertices of the triangle in question are indexed by the entries of columns <code>ind1</code> and <code>ind2</code>.)</p> <p>The entries of columns <code>thirdv1</code> and <code>thirdv2</code> may (also) take the values <math>-1</math>, <math>-2</math>, <math>-3</math>, and <math>-4</math>. This will be the case if the circumcentre in question lies outside of the rectangular window <code>rw</code>. In these circumstances the corresponding endpoint of the tile edge is the intersection of the line joining the two circumcentres with the boundary of <code>rw</code>, and the numeric value of the entry of column “<code>thirdv1</code>” (respectively “<code>thirdv2</code>”) indicates which side. The numbering follows the convention for numbering the sides of a plot region in R: 1 for the bottom side, 2 for the left hand side, 3 for the top side and 4 for the right hand side.</p> <p>Note that the entry in column <code>thirdv1</code> will be negative if and only if the corresponding entry in column <code>bp1</code> is <code>TRUE</code>. Similarly for columns <code>thirdv2</code> and <code>bp2</code>.</p>
<code>summary</code>	<p>a data frame with 9, 10 or 11 columns and <code>n.data + n.dum</code> rows (see below). The rows correspond to the points in the set being triangulated. Note that the row names are the indices of the points in the original sequence of points being triangulated/tessellated. Usually these will be the sequence 1, 2, ..., <code>npd</code> (“<code>n</code> plus dummy”). However if there were <i>duplicated</i> points then the row name corresponding to a point is the <i>first</i> of the indices of the set of duplicated points in which the given point appears. The columns are:</p> <ul style="list-style-type: none"> <li>• <code>x</code> (the <math>x</math>-coordinate of the point)</li> <li>• <code>y</code> (the <math>y</math>-coordinate of the point)</li> <li>• <code>pt.type</code> (a character vector with entries “data” and “dummy”; present only if <code>n.dum &gt; 0</code>)</li> <li>• <code>z</code> (the auxiliary values or “weights”; present only if these were specified)</li> <li>• <code>n.tri</code> (the number of Delaunay triangles emanating from the point)</li> <li>• <code>del.area</code> (<math>1/3</math> of the total area of all the Delaunay triangles emanating from the point)</li> </ul>

- `del.wts` (the corresponding entry of the `del.area` column divided by the sum of this column)
- `n.tside` (the number of sides — within the rectangular window — of the Dirichlet tile surrounding the point)
- `nbpt` (the number of points in which the Dirichlet tile intersects the boundary of the rectangular window)
- `dir.area` (the area of the Dirichlet tile surrounding the point)
- `dir.wts` (the corresponding entry of the `dir.area` column divided by the sum of this column).

Note that the factor of 1/3 associated with the `del.area` column arises because each triangle occurs three times — once for each corner.

<code>n.data</code>	the number of real (as opposed to dummy) points in the set which was triangulated, with any duplicate points eliminated. The first <code>n.data</code> rows of summary correspond to real points.
<code>n.dum</code>	the number of dummy points which were added to the set being triangulated, with any duplicate points (including any which duplicate real points) eliminated. The last <code>n.dum</code> rows of summary correspond to dummy points.
<code>del.area</code>	the area of the convex hull of the set of points being triangulated, as formed by summing the <code>del.area</code> column of summary.
<code>dir.area</code>	the area of the rectangular window enclosing the points being triangulated, as formed by summing the <code>dir.area</code> column of summary.
<code>rw</code>	the specification of the corners of the rectangular window enclosing the data, in the order (xmin, xmax, ymin, ymax).
<code>ind.orig</code>	A vector of the indices of the points (x,y) in the set of coordinates initially supplied (as data points or as dummy points) to <code>deldir()</code> before duplicate points (if any) were removed. These indices are used by <a href="#">triang.list()</a> .

#### Remark:

If `ndx >= 2` and `ndy >= 2`, then the rectangular window IS the convex hull, and so the values of `del.area` and `dir.area` (if the latter is not NULL) are identical.

#### Side Effects

If `plot=TRUE` a plot of the triangulation and/or tessellation is produced or added to an existing plot.

#### Notes on error messages

In the underlying Fortran code, error traps have been set for 17 different errors, which are identified by an error number `nerror`. When one of these traps detects an error, the value of `nerror` is passed back along the call stack to the R function `deldir()` that calls the Fortran subroutines. (I.e. to *this* function, the documentation of which you are currently reading.) The `deldir()` function then prints out a message and returns (invisibly) a NULL value. The message consists only of the value of `nerror`. A glossary of the meanings of the values of `nerror` is to be found in the file `err.list`, located in the top level of the package directory (“folder” if you are a Windoze weenie).

Note that the values 4, 14 and 15 of `nerror` do not cause `deldir()` to return a `NULL` value but rather cause a message to be printed, storage (memory) to be re-allocated (increased) and `deldir()` to be re-started so as to take advantage of the increased amount of storage.

In version 0.1-16 of `deldir` a new error trap was introduced, and this new trap triggers a genuine error and does so in a direct and perspicuous manner.

This new error trap relates to “triangle problems”. It was drawn to my attention by Adam Dadvar (on 18 December, 2018) that in some data sets collinearity problems may cause the “triangle finding” procedure, used by the algorithm to successively add new points to a tessellation, to go into an infinite loop. A symptom of the collinearity is that the vertices of a putative triangle appear *not* to be in anticlockwise order irrespective of whether they are presented in the order  $i, j, k$  or  $k, j, i$ . The result of this anomaly is that the procedure keeps alternating between moving to “triangle”  $i, j, k$  and moving to “triangle”  $k, j, i$ , forever.

The new error trap, set in `trifnd`, the triangle finding subroutine, detects such occurrences of “clockwise in either orientation” vertices. The trap causes the `deldir()` function to throw an error rather than disappearing into a black hole. The error is thrown “directly” rather than via passing a `nerror` number back up the call stack. The facility for triggering an error in this manner was not available when the `deldir` package was originally written. In the reasonably near future the `deldir` package will be adjusted so that all error traps throw errors in the “direct” manner, and use of the `nerror` numbers will be eliminated.

When an error of the “triangle problems” nature occurs, a *possible* remedy is to increase the value of the `eps` argument of `deldir()`. (See the **Examples**.) There may conceivably be other problems that lead to infinite loops and so I have put in another error trap to detect whether the procedure has inspected more triangles than actually exist, and if so to throw an error.

Note that the strategy of increasing the value of `eps` is *probably* the appropriate one in most (if not all) of the cases where errors of this nature arise. (Similarly this strategy is *probably* the appropriate response to errors with `nerror` equal to 3, 12 and 13.) However it is impossible to be sure. The intricacy and numerical delicacy of triangulations is too great for anyone to be able to foresee all the possibilities that could arise.

If there is any doubt as the appropriateness of the “increase `eps`” strategy, the user is advised to do his or her best to explore the data set, graphically or by other means, and thereby determine what is actually going on and why problems are occurring.

## Warnings

1. The process for determining if points are duplicated changed between versions 0.1-9 and 0.1-10. Previously there was an argument `frac` for this function, which defaulted to 0.0001. Points were deemed to be duplicates if the difference in x-coordinates was less than `frac` times the width of `rw` and y-coordinates was less than `frac` times the height of `rw`. This process has been changed to one which uses `duplicated()` on the data frame whose columns are `x` and `y`. As a result it may happen that points which were previously eliminated as duplicates will no longer be eliminated. (And possibly vice-versa.)
2. The components `delsgs` and summary of the value returned by `deldir()` are now *data frames* rather than matrices. The component `summary` was changed to allow the “auxiliary” values `z` to be of arbitrary mode (i.e. not necessarily numeric). The component `delsgs` was then changed for consistency. Note that the other “matrix-like” component `dirsgs` has been a data frame since time immemorial.

A message alerting the user to the foregoing two items is printed out the first time that `deldir()` is called with `suppressMsge=FALSE` in a given session. In succeeding calls to `deldir()` in the same session, no message is printed. (I.e. the “alerting” message is printed *at most once* in any given session.)

The “alerting” message is *not* produced via the `warning()` function, so `suppressWarnings()` will *not* suppress its appearance. To effect such suppression (necessary only on the first call to `deldir()` in a given session) one must set the `suppressMsge` argument of `deldir` equal to `TRUE`.

3. If any dummy points are created, and if a vector `z`, of “auxiliary” values or “weights” associated with the points being triangulated, is supplied, then it is up to the user to supply the corresponding auxiliary values or weights associated with the dummy points. These values should be supplied as `zdum`. If `zdum` is not supplied then the auxiliary values or weights associated with the dummy points are all taken to be missing values (i.e. `NA`).

### Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

### References

Lee, D. T., and Schacter, B. J. Two algorithms for constructing a Delaunay triangulation, *Int. J. Computer and Information Sciences*, Vol. 9, No. 3, 1980, pp. 219 – 242.

Ahuja, N. and Schacter, B. J. (1983). *Pattern Models*. New York: Wiley.

### See Also

[plot.deldir\(\)](#), [tile.list\(\)](#), [triang.list\(\)](#)

### Examples

```
x <- c(2.3,3.0,7.0,1.0,3.0,8.0)
y <- c(2.3,3.0,2.0,5.0,8.0,9.0)

# Let deldir() choose the rectangular window.
dxy1 <- deldir(x,y)

# User chooses the rectangular window.
dxy2 <- deldir(x,y,rw=c(0,10,0,10))

# Put dummy points at the corners of the rectangular
# window, i.e. at (0,0), (10,0), (10,10), and (0,10)
dxy3 <- deldir(x,y,dpl=list(ndx=2,ndy=2),rw=c(0,10,0,10))

# Plot the triangulation created (but not the tessellation).
## Not run:
dxy2 <- deldir(x,y,rw=c(0,10,0,10),plot=TRUE,wl='tr')

## End(Not run)

# Auxiliary values associated with points; 4 dummy points to be
```



```
# added so 4 dummy "z-values" provided.
z    <- c(1.63,0.79,2.84,1.56,0.22,1.07)
zdum <- rep(42,4)
dxy4 <- deldir(x,y,dpl=list(ndx=2,ndy=2),rw=c(0,10,0,10),z=z,zdum=zdum)

# Example of collinearity error.
## Not run:
  dniP <- deldir(niProperties) # Throws an error

## End(Not run)
  dniP <- deldir(niProperties,eps=1e-8) # No error.
```

---

divchain	<i>Dividing chain; generic.</i>
----------	---------------------------------

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## Description

Generic function for creating the “dividing chain” of a Dirichlet tessellation. The tessellation must have been created from a set of points having associated categorical “weights”. The dividing chain consists of those edges of Dirichlet tiles which separate points having different values of the given weights.

## Usage

```
divchain(x, ...)
```

## Arguments

<code>x</code>	Either an object specifying coordinates or an object of class “deldir”.
<code>...</code>	Arguments to be passed to the appropriate method for this generic function.

## Details

If `x` is a (numeric) vector it will be taken to be the “x” coordinates of the points being tessellated. In this case the `...` arguments must contain a vector `y` specifying the “y” coordinates. The `...` arguments must also contain a *factor* `z` specifying the relevant “weights” argument. The argument `x` may also be a data frame or list from which the coordinates and the weights will be extracted. See [divchain.default\(\)](#) for details.

If `x` is an object of class “deldir” then it must have been created with an appropriate (factor) “weights” argument, otherwise an error is given.

## Value

An object of class “divchain”. See [divchain.deldir\(\)](#) for details.

## Note

This function was created in response to a question asked on [stackoverflow.com](#) by a user named “Dan”.

**Author(s)**

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**See Also**

[divchain.default\(\)](#) [divchain.deldir\(\)](#) [deldir\(\)](#) [plot.divchain\(\)](#)

**Examples**

```
set.seed(42)
x  <- runif(50)
y  <- runif(50)
z  <- factor(kmeans(cbind(x,y),centers=4)$cluster)
dc1 <- divchain(x,y,z,rw=c(0,1,0,1))
dxy <- deldir(x,y,z=rw=c(0,1,0,1))
dc2 <- divchain(dxy)
```

---

divchain.default	<i>Dividing chain; default method.</i>
------------------	--

---

**Description**

Create the “dividing chain” of the Dirichlet tessellation of a given set of points having distinguishing (categorical) “weights”. This dividing chain consists of those edges of Dirichlet tiles which separate points having different values of the given weights.

**Usage**

```
## Default S3 method:
divchain(x, y, z, ...)
```

**Arguments**

<code>x,y</code>	These provide the coordinates of the set of points being tessellated. Argument <code>x</code> may be a data frame or a list, in particular one of class <code>ppp</code> . (See the <code>spatstat</code> package.) For a full description see the discussion of these arguments in the help for <a href="#">deldir()</a> .
<code>z</code>	A <i>factor</i> specifying “auxiliary” values or “weights” If this argument is left <code>NULL</code> then it is extracted, if possible, from the components of <code>x</code> . See <a href="#">deldir()</a> for further details.
<code>...</code>	Other arguments to be passed to <code>deldir</code> .

**Value**

An object of class `divchain`. See [divchain.deldir\(\)](#) for details.

**Note**

This function was created in response to a question asked on [stackoverflow.com](https://stackoverflow.com) by a user named “Dan”.

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**See Also**

[divchain.deldir\(\)](#) [deldir\(\)](#) [plot.divchain\(\)](#)

**Examples**

```
set.seed(42)
x  <- runif(50)
y  <- runif(50)
z  <- factor(kmeans(cbind(x,y),centers=4)$cluster)
dcxy <- divchain(x,y,z,rw=c(0,1,0,1))
```

---

divchain.deldir

**Description**

Create the “dividing chain” of a Dirchlet tessellation. The tessellation must have been created from a set of points having associated categorical “weights”. The dividing chain consists of those edges of Dirichlet tiles which separate points having different values of the given weights.

**Usage**

```
## S3 method for class 'deldir'
divchain(x, ...)
```

**Arguments**

x	An object of class “deldir”. This object must have been created in such a way that the points of the set being tessellated have associate categorical “weights”. That is, <a href="#">deldir()</a> must have been called with a <i>factor</i> valued z argument or the x argument to <a href="#">deldir()</a> must have had an appropriate component which could be taken to be z.
...	Not used.

**Value**

An object of class “divchain” consisting of a data frame with columns named “x0”, “y0”, “x1”, “y1”, “v01”, “v02”, “v03”, “v11”, “v12” and “v13”.

The columns named “x0” and “y0” consist of the coordinates of one endpoint of an edge of a Dirichlet tile and the columns named “x1” and “y1” consist of the coordinates of the other endpoint.

The columns named “vij”,  $i = 0, 1, j = 1, 2, 3$ , consist of the indices of the vertices of the Delaunay triangles whose circumcentres constitute the respective endpoints of the corresponding edge of a Dirichlet tile. The entries of column “vi3” may (also) take the values  $-1, -2, -3$ , and  $-4$ . This will be the case if the circumcentre in question lay outside of the rectangular window `rw` (see `deldir()`) enclosing the points being tessellated. In these circumstances the corresponding endpoint of the tile edge is the intersection of the line joining the two circumcentres with the boundary of `rw`, and the numeric value of the entry of column “vi3” indicates which side. The numbering follows the convention for numbering the sides of a plot region in R: 1 for the bottom side, 2 for the left side, 3 for the top side and 4 for the right side.

Note that the triple of vertices uniquely identify the endpoint of the tile edge.

The object has an attribute `rw` which is equal to the specification of the rectangular window within which the class “deldir” object `x` was constructed. (See `deldir()`.)

**Note**

This function was created in response to a question asked on [stackoverflow.com](https://stackoverflow.com) by a user named “Dan”.

**Author(s)**

Rolf Turner <[r.turner@auckland.ac.nz](mailto:r.turner@auckland.ac.nz)>

**See Also**

`divchain.default()` `deldir()` `plot.divchain()`

**Examples**

```
set.seed(42)
x  <- runif(50)
y  <- runif(50)
z  <- factor(kmeans(cbind(x,y),centers=4)$cluster)
dxy <- deldir(x,y,z=z,rw=c(0,1,0,1))
dc  <- divchain(dxy)
```

---

duplicatedxy	<i>Determine duplicated points.</i>
--------------	-------------------------------------

---

## Description

Find which points among a given set are duplicates of others.

## Usage

```
duplicatedxy(x, y)
```

## Arguments

x	Either a vector of x coordinates of a set of (two dimensional) points, or a list (or data frame) with columns x and y giving the coordinates of a set of such points.
y	A vector of y coordinates of a set of (two dimensional) points. Ignored if x is a list or data frame.

## Details

Often it is of interest to associate each Dirichlet tile in a tessellation of a planar point set with the point determining the tile. This becomes problematic if there are *duplicate* points in the set being tessellated/triangulated. Duplicated points are automatically eliminated “internally” by `deldir()`. The association between tiles and the indices of the original set of points is now preserved by the component `ind.orig` of the object returned by `deldir()`. However confusion could still arise.

If it is of interest to associate Dirichlet tiles with the points determining them, then it is better to proceed by eliminating duplicate points to start with. This function (`duplicatedxy()`) provides a convenient way of doing so.

## Value

A logical vector of length equal to the (original) number of points being considered, with entries TRUE if the corresponding point is a duplicate of a point with a smaller index, and FALSE otherwise.

## Warning

Which indices will be considered to be indices of duplicated points (i.e. get TRUE values) will of course depend on the order in which the points are presented.

## Note

The real work is done by the base **R** function `duplicated()`.

## Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

`duplicated()`, `deldir()`

**Examples**

```
set.seed(42)
xy <- data.frame(x=runif(20),y=runif(20))
# Lots of duplicated points.
xy <- rbind(xy,xy[sample(1:20,20,TRUE),])
# Scramble.
ii <- sample(1:40,40)
x <- xy$x[ii]
y <- xy$y[ii]
# Unduplicate!
iii <- !duplicatedxy(x,y)
xu <- x[iii]
yu <- y[iii]
# The i-th tile is determined by (xu[i],yu[i]):
dxy <- deldir(xu,yu)
```

---

grapherXmpl

*grapherXmpl*


---

**Description**

A data set taken from an example in the grapherator package. This data set demonstrates handling a data set with duplicated points.

**Usage**

```
grapherXmpl
```

**Format**

A data frame with 250 observations on the following 2 variables.

`x` a numeric vector

`y` a numeric vector

**Details**

There are 25 duplicated points, so the net number of observations is 225. These data constitute a structure (named `coordinates`) generated internally in the function `addEdgesDelaunay`. The call is to be found in the examples in the help file for the `plot.grapherator()` in the `grapherator` package. The relevant example initially threw an error, revealing a bug in `deldir()` that was triggered when there were duplicated points in the data.

**Source**

The grapherator package, <https://CRAN.R-project.org/package=grapherator>

**Examples**

```
dgX <- deldir(grapherXmpl) # Now works!!!`
```

---

lawSummary

*Produce a Lewis-Aboav-Weaire summary of a tessellation.*

---

**Description**

Produce a summary of a Dirichlet (Voronoi) tessellation in terms of parameters relevant to Lewis's law and Aboav-Weaire's law. Note that "law" in the function name corresponds to "Lewis-Aboav-Weaire".

**Usage**

```
lawSummary(object)
```

**Arguments**

**object**                      An object of class "deldir" as returned by the function deldir().

**Details**

Tiles are stripped away from the tessellation in "layers". Layer 1 consists of "boundary" tiles, i.e. tiles having at least one vertex on the enclosing rectangle (determined by the `rw` argument of `deldir()`). Layer 2 consists of tiles which are neighbours of tiles in layer 1 (i.e. tiles determined by points that are Delaunay neighbours of points determining the tiles in layer 1). Layer 3 consists of tiles which are neighbours of tiles in layer 2.

The parameters of interest in respect of the Lewis-Aboav-Weaire summary are then calculated in terms of the tiles that remain after the three layers have been stripped away, which will be referred to as "interior" tiles. These parameters are:

- the areas of each of the interior tiles
- the number of edges of each of the interior tiles
- the number of edges of all neighbouring tiles of each of the interior tiles.

Note that the neighbouring tiles of the interior tiles may include tiles which are *not themselves* interior tiles (i.e. tiles which are in layer 3).

This function was created at the request of Kai Xu (Fisheries College, Jimei University, Xiamen, Fujian, China 361021).

**Value**

If no tiles remain after the three layers have been stripped away, then the returned value is NULL. Otherwise the returned value is a list with components calculated in terms of the remaining (“interior”) tiles. These components are:

- `tile.vertices` A list whose entries are data frames giving the coordinates of the vertices of the interior tiles.
- `tile.areas` A vector of the areas of the interior tiles in the tessellation in question.
- `num.edges` A vector of the number of edges of each such tile.
- `num.nbr.edges` A list with a component for each point, in the set being tessellated, whose corresponding tile is an interior tile. Each component of this list is the vector of the number of edges of the interior tiles determined by points which are Delaunay neighbours of the point corresponding to the list component in question.
- `totnum.nbr.edges` A vector whose entries consist of the sums of the vectors in the foregoing list.

The returned list also has attributes as follows:

- `i1` An integer vector whose entries are in the indices of the tiles in layer 1.
- `i2` An integer vector whose entries are in the indices of the tiles in layer 2.
- `i3` An integer vector whose entries are in the indices of the tiles in layer 3.
- `i.kept` An integer vector whose entries are in the indices of the tiles that are kept, i.e. those that remain after the three layers have been stripped away.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

`tile.list()` `tile.centroids()`

**Examples**

```
# A random pattern:
set.seed(42)
xy1 <- data.frame(x=runif(400,0,20),y=runif(400,0,20))
dxy1 <- deldir(xy1)
ldxy1 <- lawSummary(dxy1)
tl1 <- tile.list(dxy1)
plot(0,0,type="n",xlim=c(-2,35),ylim=c(0,20),asp=1,xlab="x",ylab="y",bty="l")
plot(tl1,showpoints=FALSE,add=TRUE)
points(xy1[attr(ldxy1,"i1"),],pch=20,col="yellow")
points(xy1[attr(ldxy1,"i2"),],pch=20,col="blue")
points(xy1[attr(ldxy1,"i3"),],pch=20,col="green")
points(xy1[attr(ldxy1,"i.kept"),],pch=20,col="red")
legend("right",pch=20,col=c("yellow","blue","green","red"),
      legend=c("layer 1","layer 2","layer 3","interior"))
```



```
# A highly structured pattern (example due to Kai Xu):
set.seed(115)
x <- c(rep(1:20,10),rep((1:20)+0.5,10))
y <- c(rep(1:10,each=20),rep((1:10)+0.5,each=20))*sqrt(3)
a <- runif(400,0,2*pi)
b <- runif(400,-1,1)
x <- x+0.1*cos(a)*b
y <- y+0.1*sin(a)*b
xy2 <- data.frame(x,y)
dxy2 <- deldir(xy2)
ldxy2 <- lawSummary(dxy2)
tl2 <- tile.list(dxy2)
plot(0,0,type="n",xlim=c(-2,35),ylim=c(0,20),asp=1,xlab="x",ylab="y",bty="l")
plot(tl2,showpoints=FALSE,add=TRUE)
points(xy2[attr(ldxy2,"i1"),],pch=20,col="yellow")
points(xy2[attr(ldxy2,"i2"),],pch=20,col="blue")
points(xy2[attr(ldxy2,"i3"),],pch=20,col="green")
points(xy2[attr(ldxy2,"i.kept"),],pch=20,col="red")
legend("right",pch=20,col=c("yellow","blue","green","red"),
      legend=c("layer 1","layer 2","layer 3","interior"))
```

---

niProperties

*Northern Ireland properties.*


---

## Description

The locations (in longitude and latitude) of a number of properties (land holdings) in Northern Ireland.

## Usage

```
data("niProperties")
```

## Format

A data frame with 240 observations on the following 2 variables.

x A numeric vector of longitudes.

y A numeric vector of latitudes.

## Source

These data were kindly provided by Adam Dadvar of the *Cartesian Limited* consulting service.  
 URL: <http://www.cartesian.com>.

## Examples

```
# data(niProperties)
# It is unnecessary to use \code{data} since \code{niProperties} is
# a "first class object". It is "lazily loaded".
plot(niProperties)
```

---

plot.deldir

*Plot objects produced by deldir*

---

## Description

This is a method for plot.

## Usage

```
## S3 method for class 'deldir'
plot(x, add=FALSE, wlines=c('both', 'triang', 'tess'),
      wpoints=c('both', 'real', 'dummy', 'none'),
      number=FALSE, cex=1, nex=1, cmpnt_col=NULL, cmpnt_lty=NULL,
      pch=NULL, xlim=NULL, ylim=NULL, axes=FALSE,
      xlab=if(axes) 'x' else '', ylab=if(axes) 'y' else '',
      showrect=FALSE, asp=1, ...)
```

## Arguments

x	An object of class "deldir" as constructed by the function deldir.
add	logical argument; should the plot be added to an existing plot?
wlines	"which lines?". I.e. should the Delaunay triangulation be plotted (wlines='triang'), should the Dirichlet tessellation be plotted (wlines='tess'), or should both be plotted (wlines='both', the default) ?
wpoints	"Which points to plot?". I.e. should the real points be plotted (wpoints='real'), should the dummy points be plotted (wpoints='dummy'), should both be plotted (wpoints='both', the default) or should no points be plotted (wpoints='none')?
number	Logical argument, defaulting to FALSE; if TRUE then the points plotted will be labelled with their index numbers (corresponding to the row numbers of the matrix "summary" in the output of deldir).
cex	The value of the character expansion argument cex to be used with the plotting symbols for plotting the points.
nex	The value of the character expansion argument cex to be used by the text function when numbering the points with their indices. Used only if number=TRUE.
cmpnt_col	A vector or list specifying the colours to be used for plotting the (up to six) different components of the graphic, explicitly the triangulation, the tessellation, the data points, the dummy points, the point numbers and the enclosing rectangle (x\$rw) in that order. The components of this vector or list may be named, with the names chosen from "tri", "tess", "data", "dummy", "num", "rect". The

default is `c(tri=1, tess=1, data=1, dummy=1, num=1, rect=1)`. If the vector or list is not named, the component names are assumed to be from the default list of names in the given order.

Colours may be specified as positive integers, corresponding to the entries of the current `palette()`, or as names of colours (from the list given by `link{colors}()`), or in terms of their RGB components with strings of the form `"#RRGGBB"`. (See **Color Specification** in `par()`.)

If fewer than six colours are given, the missing components are filled in with 1 or `palette()[1]` as is appropriate. If `cmpnt_col` does not have names then the components are simply recycled. If more than six colours are given, the redundant ones are ignored (i.e. only the first six are used).

<code>cmpnt_lty</code>	A vector or list (of length two) of line type numbers for plotting the two components of the graphic that consist of lines, i.e. the triangulation and the tessellation. The components of this vector or list may have names ( <code>"tri"</code> , <code>"tess"</code> ). The default is <code>c(tri=1, tess=1)</code> . If a single value is given the missing one is filled in as 1. If more than two numbers are given, the redundant ones are ignored, i.e. only the first two are used. See <code>par()</code> for the meaning of these numbers.
<code>pch</code>	The plotting symbols for plotting the data points and the dummy points, in that order; may be either integer or character; defaults to 1:2. If only one value is given it is repeated. (If more than two values are given, the redundant ones are ignored.)
<code>xlim</code>	The limits on the x-axis. Defaults to <code>rw[1:2]</code> where <code>rw</code> is the rectangular window specification returned by <code>deldir()</code> .
<code>ylim</code>	The limits on the y-axis. Defaults to <code>rw[3:4]</code> where <code>rw</code> is the rectangular window specification returned by <code>deldir()</code> .
<code>axes</code>	Logical scalar. Should axes be drawn on the plot?
<code>xlab</code>	Label for the x-axis. Defaults to <code>x</code> if <code>axes</code> is TRUE and to the empty string if <code>axes</code> is FALSE. Ignored if <code>add=TRUE</code> .
<code>ylab</code>	Label for the y-axis. Defaults to <code>y</code> if <code>axes</code> is TRUE and to the empty string if <code>axes</code> is FALSE. Ignored if <code>add=TRUE</code> .
<code>showrect</code>	Logical scalar; show the enclosing rectangle <code>rw</code> (see <code>deldir()</code> ) be plotted?
<code>asp</code>	The aspect ratio of the plot; integer scalar or NA. Set this argument equal to NA to allow the data to determine the aspect ratio and hence to make the plot occupy the complete plotting region in both x and y directions. This is inadvisable; see the <b>Warning</b> . The <code>asp</code> argument is ignored if <code>add</code> is TRUE.
<code>...</code>	Further plotting parameters (e.g. <code>lw</code> , <code>col</code> ) to be passed to <code>segments()</code> . (Conceivably one might also use <code>...</code> to supply additional arguments to be passed to <code>points()</code> or <code>text()</code> but this is unlikely.) Note that if <code>col</code> is specified as one of the <code>...</code> arguments then this is used as the <code>col</code> argument of <code>segments</code> (and the values of <code>cmpnt_col[1:2]</code> are ignored).

## Details

The points in the set being triangulated are plotted with distinguishing symbols. By default the real points are plotted as circles (`pch=1`) and the dummy points are plotted as triangles (`pch=2`).

## Side Effects

A plot of the edges of the Delaunay triangles and/or of the Dirichlet tiles is produced or added to an existing plot. By default the triangles are plotted with solid lines (lty=1) and the tiles with dotted lines (lty=2). In addition the (data and dummy) points being triangulated may be plotted.

## Warning

In previous versions of the deldir package, the aspect ratio was not set. Instead, the shape of the plotting region was set to be square (pty="s"). This practice was suboptimal. To reproduce previous behaviour set asp=NA (and possibly pty="s") in the call to plot.deldir(). Users, unless they *really* understand what they are doing and why they are doing it, are now *strongly advised* not to set the value of asp but rather to leave asp equal to its default value of 1. Any other value may distort the tessellation and destroy the perpendicular appearance of lines which are indeed perpendicular. (And conversely may cause lines which are not perpendicular to appear as if they are.)

## Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

## See Also

[deldir\(\)](#)

## Examples

```
x <- c(2.3,3.0,7.0,1.0,3.0,8.0) + 0.5
y <- c(2.3,3.0,2.0,5.0,8.0,9.0) + 0.5
dxy <- deldir(x,y,dpl=list(x=c(1,10,10,1),y=c(1,1,10,10)),rw=c(0,11,0,11))
plot(dxy)

# Plots the tessellation, but does not save the results:
deldir(x,y,dpl=list(x=c(1,10,10,1),y=c(1,1,10,10)),rw=c(0,11,0,11)),plot=TRUE,
      wl='tess',cmpnt_col=c(1,1,2,3,4,5),num=TRUE)

# Plots the triangulation, but not the tessellation or the points,
# and saves the returned structure:

dxy <- deldir(x,y,dpl=list(x=c(1,10,10,1),y=c(1,1,10,10)),rw=c(0,11,0,11),plot=TRUE,
      wl='triang',wp="none")

# Plot everything:
plot(dxy,cmpnt_col=c("orange","green","red","blue","black","black"),cmpnt_lty=1,
      number=TRUE,nex=1.5,pch=c(19,9),showrect=TRUE,axes=TRUE)

# Complicated example from He Huang:
# "Linguistic distances".
vldm <- c(308.298557,592.555483,284.256926,141.421356,449.719913,
          733.976839,591.141269,282.842712,1.414214,732.562625)
ldm <- matrix(nrow=5,ncol=5)
ldm[row(ldm) > col(ldm)] <- vldm
```

```

ldm[row(ldm) <= col(ldm)] <- 0
ldm <- (ldm + t(ldm))/2
rownames(ldm) <- LETTERS[1:5]
colnames(ldm) <- LETTERS[1:5]

# Data to be triangulated.
id <- c("A","B","C","D","E")
x <- c(0.5,1,1,1.5,2)
y <- c(5.5,3,7,6.5,5)
dat_Huang <- data.frame(id = id, x = x, y = y)

# Form the triangulation/tessellation.
dH <- deldir(dat_Huang)

# Plot the triangulation with line widths proportional
# to "linguistic distances".
all_col <- rainbow(100)
i <- pmax(1,round(100*vldm/max(vldm)))
use_col <- all_col[i]
ind <- as.matrix(dH$delsgs[,c("ind1","ind2")])
lwv <- ldm[ind]
lwv <- 10*lwv/max(lwv)
plot(dH,wlines="triang",col=use_col,wpoints="none",
     lw=lwv,asp=NA)
with(dH,text(x,y,id,cex=1.5))

```

---

plot.divchain	<i>Plot a dividing chain.</i>
---------------	-------------------------------

---

## Description

Plot the dividing chain of a Dirichlet tessellation. The tessellation must have been created from a set of points having associated categorical “weights”. The dividing chain consists of those edges of Dirichlet tiles which separate points having different values of the given weights.

## Usage

```

## S3 method for class 'divchain'
plot(x, add = FALSE, ...)

```

## Arguments

x	An object of class “divchain”. See <a href="#">divchain.deldir()</a> for details.
add	Logical scalar. If add=TRUE the plot of the dividing chain is added to an existing plot.
...	Graphical parameters such as main, xlab, col.main, col.lab. In particular if bty is supplied (as a value other than n) a “box” will be drawn around the plot that is formed when add=FALSE. Also a non-standard graphical parameter

boxcol may be supplied which will be taken to be the colour with which the box is drawn. If a col argument is supplied, this determines the colour for plotting the segments constituting the dividing chain.

### Value

None.

### Note

This function was created in response to a question asked on [stackoverflow.com](https://stackoverflow.com) by a user named “Dan”.

### Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

### See Also

`divchain()` `divchain.default()` `divchain.deldir()` `deldir()`

### Examples

```
set.seed(42)
x  <- runif(50)
y  <- runif(50)
z  <- factor(kmeans(cbind(x,y),centers=4)$cluster)
dc <- divchain(x,y,z,rw=c(0,1,0,1))
plot(dc,lwd=2,col="blue",bty="o")
```

---

plot.tile.list

*Plot Dirchlet (Voronoi) tiles*

---

### Description

A method for plot. Plots (sequentially) the tiles associated with each point in the set being tessellated.

### Usage

```
## S3 method for class 'tile.list'
plot(x, verbose = FALSE, close = FALSE, pch = 1,
      fillcol = getCol(x,warn=warn), col.pts=NULL,
      col.num=NULL,border=NULL, showpoints = !number,
      add = FALSE, asp = 1, clipp=NULL, xlab = "x",
      ylab = "y", main = "", warn=FALSE,
      number=FALSE,adj=NULL,...)
```

**Arguments**

x	A list of the tiles in a tessellation, as produced the function <code>tile.list()</code> .
verbose	Logical scalar; if TRUE the tiles are plotted one at a time (with a “Go?” prompt after each) so that the process can be watched.
close	Logical scalar; if TRUE the outer edges of of the tiles (i.e. the edges which are contained in the enclosing rectangle) are drawn. Otherwise tiles on the periphery of the tessellation are left “open”.
pch	The plotting character (or vector of plotting characters) with which to plot the points of the pattern which was tessellated. Ignored if <code>showpoints</code> is FALSE.
fillcol	Optional vector (possibly of length 1, i.e. a scalar) whose entries can be interpreted as colours by <code>col2rgb()</code> . The <i>i</i> -th entry indicates with which colour to fill the <i>i</i> -th tile. Note that an NA entry cause the tile to be left unfilled. This argument will be replicated to have length equal to the number of tiles.
col.pts	Optional vector like unto <code>fillcol</code> whose entries can be interpreted as colours by <code>col2rgb()</code> . The <i>i</i> -th entry indicates with which colour to plot the <i>i</i> -th point. This argument will be replicated to have length equal to the number of tiles. Ignored if <code>showpoints</code> is FALSE.
col.num	Optional vector like unto <code>col.pts</code> . Determines the colours in which the point numbers (see <code>number</code> below). This argument will be replicated to have length equal to the number of tiles. Ignored if <code>number</code> is FALSE.
border	A <i>scalar</i> indicating the colour with which to plot the tile boundaries. Defaults to black unless all of the fill colours specified by <code>fillcol</code> are black, in which case it defaults to white. If <code>length(border) &gt; 1</code> then an error is given.
showpoints	Logical scalar; if TRUE the points of the pattern which was tesselated are plotted.
add	Logical scalar; should the plot of the tiles be added to an existing plot?
asp	The aspect ratio of the plot; integer scalar or NA. Set this argument equal to NA to allow the data to determine the aspect ratio and hence to make the plot occupy the complete plotting region in both x and y directions. This is inadvisable; see the <b>Warnings</b> .
clipp	<p>An object specifying a polygon to which the tessellation being plotted should be clipped. It should consist either of:</p> <ul style="list-style-type: none"> <li>• a list containing two components x and y giving the coordinates of the vertices of a single polygon. The last vertex should not repeat the first vertex. Or;</li> <li>• a list of <code>list(x,y)</code> structures giving the coordinates of the vertices of several polygons.</li> </ul> <p>If this argument is provided then the plot of the tessellation is “clipped” to the polygon specified by <code>clipp</code>.</p>
xlab	Label for the x-axis (used only if <code>add</code> is FALSE).
ylab	Label for the y-axis (used only if <code>add</code> is FALSE).
main	A title for the plot (used only if <code>add</code> is FALSE).
warn	Logical scalar passed to the internal function <code>getCol()</code> . Should a warning be issued if the z components of the entries of x cannot all be interpreted as colours. See <b>Notes</b> .

number	Logical scalar; if TRUE the numbers of the points determining the tiles are plotted in the tiles. Note that if number is TRUE then showpoints defaults to FALSE
adj	The “adjustment” argument to text(). If number and showpoints are both TRUE it defaults to -1 (so that the numbers and point symbols are not superimposed). If number is TRUE and showpoints is FALSE it defaults to 0. If number is FALSE it is ignored.
...	Optional arguments; may be passed to points() and text().

### Value

NULL; side effect is a plot.

### Warnings

- The behaviour of this function with respect to “clipping” has changed substantially since the previous release of deldir, i.e. 1.1-0. The argument clipwin has been re-named clipp (“p” for “polygon”). Clipping is now effected via the new package polyclip. The spatstat package is no longer used. The argument use.gpclip has been eliminated, since gpclip (which used to be called upon by spatstat has been superceded by polyclip which has an unrestrictive license.
- As of release 0.1-1 of the deldir package, the argument fillcol to this function *replaces* the old argument polycol, but behaves somewhat differently.
- The argument showrect which was present in versions of this function prior to release 0.1-1 has been eliminated. It was redundant.
- As of release 0.1-1 the col.pts argument *might* behave somewhat differently from how it behaved in the past.
- The arguments border, clipp, and warn are new as of release 0.1-1.
- Users, unless they *really* understand what they are doing and why they are doing it, are *strongly advised* not to set the value of asp but rather to leave asp equal to its default value of 1. Any other value distorts the tessellation and destroys the perpendicular appearance of lines which are indeed perpendicular. (And conversely can cause lines which are not perpendicular to appear as if they are.)

### Notes

- If clipp is not NULL and showpoints is TRUE then it is possible that some of the points “shown” will not fall inside any of the plotted tiles. (This will happen if the parts of the tiles in which they fall have been “clipped” out.) If a tile is clipped out *completely* then the point which determines that tile is *not* plotted irrespective of the value of showpoints.
- If the z components of the entries of x cannot all be interpreted as colours (e.g. if there *aren’t* any z components, which will be the case if no such values were supplied in the call to deldir()) then the internal function getCol() returns NA. This value of fillcol results (as is indicated by the argument list entry for fillcol) in (all of) the tiles being left unfilled.
- The new behaviour in respect of the colours with which to fill the plotted tiles, and the argument clipp were added at the request of Chris Triggs.
- The argument asp was added at the request of Zubin Dowlaty.



**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

[deldir\(\)](#), [tile.list\(\)](#), [triang.list\(\)](#), [plot.triang.list\(\)](#)

**Examples**

```
set.seed(42)
x <- runif(20)
y <- runif(20)
z <- deldir(x,y,rw=c(0,1,0,1))
w <- tile.list(z)
plot(w)
ccc <- heat.colors(20) # Or topo.colors(20), or terrain.colors(20)
                        # or cm.colors(20), or rainbow(20).
plot(w,fillcol=ccc,close=TRUE)
if(require(polyclip)) {
  CP <- list(x=c(0.49,0.35,0.15,0.20,0.35,0.42,
               0.43,0.62,0.46,0.63,0.82,0.79),
            y=c(0.78,0.86,0.79,0.54,0.58,0.70,
               0.51,0.46,0.31,0.20,0.37,0.54))
  plot(w,clip=CP,showpoints=FALSE,fillcol=topo.colors(20))
}
plot(w,number=TRUE,col.num="red")
plot(w,number=TRUE,col.num="red",cex=0.5)
plot(w,showpoints=TRUE,number=TRUE,col.pts="green",col.num="red")
```

---

plot.triang.list	<i>Plot Delaunay triangles</i>
------------------	--------------------------------

---

**Description**

A method for plot. Plots the triangles of a Delaunay triangulation of a set of points in the plane.

**Usage**

```
## S3 method for class 'triang.list'
plot(x, showrect = FALSE, add = FALSE,
     xlab = "x", ylab = "y", main = "", asp = 1, ...)
```

**Arguments**

x	An object of class “triang.list” as produced by <a href="#">triang.list()</a> .
showrect	Logical scalar; show the enclosing rectangle rw (see <a href="#">deldir()</a> ) be plotted?
add	Logical scalar; should the plot of the triangles be added to an existing plot?
xlab	Label for the x-axis.

ylab	Label for the y-axis.
main	A title for the plot (used only if add is FALSE).
asp	The aspect ratio of the plot; integer scalar or NA. Set this argument equal to NA to allow the data to determine the aspect ratio and hence to make the plot occupy the complete plotting region in both x and y directions. This is inadvisable; see the <b>Warnings</b> .
...	Arguments passed to <code>polygon()</code> which does the actual plotting of the triangles.

**Value**

None. This function has the side effect of producing (or adding to) a plot.

**Warnings**

The user is *strongly advised* not to set the value of `asp` but rather to leave `asp` equal to its default value of 1. Any other value distorts the tessellation and destroys the perpendicular appearance of lines which are indeed perpendicular. (And conversely can cause lines which are not perpendicular to appear as if they are.)

The argument `asp` was added at the request of Zubin Dowlaty.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

`deldir()`, `plot.triang.list()`, `tile.list()`, `plot.tile.list()`

**Examples**

```
set.seed(42)
x <- runif(20)
y <- runif(20)
d <- deldir(x,y)
ttt <- triang.list(d)
plot(ttt,border="red",showrect=TRUE)
sss <- tile.list(d)
plot(sss)
plot(ttt,add=TRUE,border="blue")
```

---

print.deldir	<i>Print some information about a tessellation/triangulation.</i>
--------------	---

---

## Description

Prints a very brief description of an object of class "deldir" as returned by [deldir\(\)](#).

## Usage

```
## S3 method for class 'deldir'  
print(x,digits=NULL,...)
```

## Arguments

x	A Delaunay triangulation and Dirichlet (Voronoi) tessellation of a set of points (object of class "deldir").
digits	Integer scalar. The number of digits to which to round the numeric information before printing. Note this may be give negative values. (See <a href="#">round()</a> .)
...	Not used.

## Details

This is a method for the generic [print\(\)](#) function.

## Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

## See Also

[print\(\)](#)

## Examples

```
set.seed(42)  
x <- rnorm(200,0,4)  
y <- rnorm(200,0,4)  
dxy1 <- deldir(x,y)  
dxy2 <- deldir(x,y,rw=c(-12,12,-11,11))  
dxy1  
dxy2  
print(dxy1,digits=4)
```

---

print.tileInfo	<i>Print a summary of tile information.</i>
----------------	---

---

## Description

Print a reasonably readable summary of an object of class `tileInfo` as produced by the `tileInfo()` function.

## Usage

```
## S3 method for class 'tileInfo'
print(x, digits = 4, ...)
```

## Arguments

<code>x</code>	An object of class <code>tileInfo</code> as produced by the <code>tileInfo()</code> function.
<code>digits</code>	The (maximum) number of decimal digits to which the output is to be printed.
<code>...</code>	Not used. Present for compatibility with the generic <code>print()</code> function.

## Details

The list produced by `tileInfo()` is a bit messy and hard to comprehend, especially if there is a large number of tiles. This print method produces a screen display which is somewhat more perspicuous.

There are three components to the display:

- A matrix, each row of which is the vector of edge lengths of the tile. The number of columns is the *maximum* of the lengths of the edge length vectors. Rows corresponding to shorter vectors are filled in with blanks. The row names of the matrix indicate the number of the point corresponding to the tile. Note that this number is the index of the point in the original sequence of points that is being tessellated.
- A table of the edge counts of the tiles.
- A simple print out of the areas of the tiles (rounded to a maximum of `digits` decimal digits).

This screen display is for “looking at” only. In order to do further calculations on the output of `tileInfo` it is necessary to delve into the bowels of `x` and extract the relevant bits.

In order to get a decent looking display you may (if there are tiles with a large number of edges) need to widen the window in which you are displaying the output and increase the value of the `width` option. E.g. use `options(width=120)`.

## Value

None.

## Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**[tileInfo\(\)](#)**Examples**

```

set.seed(179)
x  <- runif(100)
y  <- runif(100)
dxy <- deldir(x,y,rw=c(0,1,0,1))
ixy1 <- tileInfo(dxy)
print(ixy1)
ixy2 <- tileInfo(dxy,bndry=TRUE)
print(ixy2)

```

seaweed

*seaweed***Description**

A data frame whose columns are the coordinates of the centroids of the cells in a seaweed frond. The points are estimates-by-eye of where the centroids of the cells occur.

**Usage**

```
data("seaweed")
```

**Format**

A data frame with 266 observations on the following 2 variables.

- x The *x*-coordinates of the cell centroids.
- y The *y*-coordinates of the cell centroids.

**Source**

These data were kindly supplied by Dr. John Bothwell of the Department of Biosciences, Durham University. The data were collected by Kevin Yun and Georgia Campbell, members of Dr. Bothwell's research group.

**Examples**

```

# data(seaweed)
# It is unnecessary to use \code{data} since \code{seaweed} is
# a "first class object". It is "lazily loaded".

dsw <- deldir(seaweed)
isw <- tileInfo(dsw)
# Expand the width of the terminal window.
options(width=120)

```

```
isw
tsw <- tile.list(dsw)
plot(tsw,number=TRUE,col.num="red",cex=0.5,adj=0.5)
```

---

tile.centroids	<i>Compute centroids of Dirichlet (Voronoi) tiles</i>
----------------	---

---

## Description

Given a list of Dirichlet tiles, as produced by `tile.list()`, produces a data frame consisting of the centroids of those tiles.

## Usage

```
tile.centroids(xxx)
```

## Arguments

xxx	A list of the tiles (produced by <code>tile.list()</code> ) in a Dirichlet tessellation of a set of planar points.
-----	--

## Value

A data frame with two columns named `x` and `y`. Each row of this data frame consitutes the centroid of one of the Dirichlet tiles.

## Author(s)

Rolf Turner <[r.turner@auckland.ac.nz](mailto:r.turner@auckland.ac.nz)>

## References

URL <http://en.wikipedia.org/wiki/Centroid>

## See Also

[tile.list\(\)](#)

## Examples

```
set.seed(42)
x <- runif(20)
y <- runif(20)
d <- deldir(x,y)
l <- tile.list(d)
g <- tile.centroids(l)
## Not run:
plot(l,close=TRUE)
points(g,pch=20,col="red")

## End(Not run)
```

---

tile.list	Create a list of tiles in a tessellation
-----------	--

---

### Description

For each point in the set being tessellated produces a list entry describing the Dirichlet/Voronoi tile containing that point.

### Usage

```
tile.list(object,minEdgeLength=NULL)
```

### Arguments

object	An object of class <code>deldir</code> as produced by the function <code>deldir()</code> .
minEdgeLength	Positive numeric scalar specifying the minimum length that an edge of a tile may have. It is used to eliminate edges that are effectively of zero length, which can cause tiles to be “invalid”. This argument defaults to <code>sqrt(.Machine\$double.eps)</code> time the diameter (length of the diagonal) of the “rectangular window” associated with the tessellation. This rectangular window is available as the <code>rw</code> component of object.

### Value

A list with one entry for each of the points in the set being tessellated. Each entry is in turn a list with components

ptNum	The index of the point in the original sequence of points that is being tessellated. Note that if a point is one of a set of <i>duplicated</i> points then <code>ptNum</code> is the <i>first</i> of the indices of the points in this set.
pt	The coordinates of the point whose tile is being described.
ptType	The “type” of the <code>pt</code> , either “data” or “dummy”. Present only if any dummy points were specified in the call to <code>deldir()</code> .
x	The x coordinates of the vertices of the tile, in anticlockwise order.
y	The y coordinates of the vertices of the tile, in anticlockwise order.
bp	Vector of logicals indicating whether the tile vertex is a “real” vertex, or a <i>boundary point</i> , i.e. a point where the tile edge intersects the boundary of the enclosing rectangle.
z	The “auxiliary value” or “weight” associated with the <code>pt</code> ; present only if such values were supplied in the call to <code>deldir()</code> .
area	The area of the tile.

### Acknowledgement

The author expresses sincere thanks to Majid Yazdani who found and pointed out a serious bug in `tile.list` in a previous version (0.0-5) of the `deldir` package.

**Warning**

The set of vertices of each tile may be “incomplete”. Only vertices which lie within the enclosing rectangle, and “boundary points” are listed.

Note that the enclosing rectangle may be specified by the user in the call to `deldir()`.

In contrast to some earlier versions of `deldir`, the corners of the enclosing rectangle are now include as vertices of tiles. I.e. a tile which in fact extends beyond the rectangular window and contains a corner of that window will have that corner added to its list of vertices. Thus the corresponding polygon is the intersection of the tile with the enclosing rectangle.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

`deldir()`, `plot.tile.list()` `triang.list()` `plot.triang.list()`

**Examples**

```
x <- runif(20)
y <- runif(20)
z <- deldir(x,y)
w <- tile.list(z)

z <- deldir(x,y,rw=c(0,1,0,1))
w <- tile.list(z)

z <- deldir(x,y,rw=c(0,1,0,1),dpl=list(ndx=2,ndy=2))
w <- tile.list(z)
```

---

tileInfo

*Extract information from a tile list.*

---

**Description**

Produces a summary of information about the tiles in an object of class `deldir` as produced by the function `deldir()`.

**Usage**

```
tileInfo(object, bndry = FALSE)
```

**Arguments**

<code>object</code>	An object of class <code>deldir</code> as produced by the function <code>deldir()</code> .
<code>bndry</code>	Logical scalar. If TRUE then the “boundary” tiles (those tiles having edges forming part of the “rectangular window” enclosing the tessellation) are included in the summary. Otherwise they are not included.



**Value**

A list with components:

indivTiles	This is itself a list with one entry for each tile in “object”. It is in fact a <i>named</i> list, the names being of form <code>tile.n</code> , where <code>n</code> is equal to the value of <code>ptNum</code> (see below) corresponding to the tile. The entries of <code>indivTiles</code> are themselves in turn lists with entries <code>edgeLengths</code> (a vector of the lengths of the edges of the tiles), <code>numEdges</code> (an integer equal to the number of edges of the tile), <code>area</code> (a positive number equal to the area of the tile) and <code>ptNum</code> (an integer equal to the number of the point determining the tile). Note that <code>ptNum</code> is the number of the point in the <i>original</i> sequence of points that were tessellated.
allEdgeCounts	An integer vector of the edge counts (given by <code>numEdge</code> for each tile) of the tiles.
tabEdgeCounts	A table of <code>numEdge</code> .
allEdgeLengths	A vector of all of the tile edge lengths; a catenation of the <code>edgeLengths</code> components of the entries of <code>indivTiles</code> . Note that there will be many duplicate lengths since each tile edge is, in general, an edge of <i>two</i> tiles.
Areas	A vector of the areas of the tiles.
uniqueEdgeLengths	A vector of the lengths of the tiles edges with the duplicates (which occur in <code>allEdgeLengths</code> ) being eliminated. Each tile edge is represented only once.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

`deldir()` `tile.list()` `print.tileInfo()`

**Examples**

```
set.seed(42)
x <- runif(20)
y <- runif(20)
dxy <- deldir(x,y,rw=c(0,1,0,1))
ixy1 <- tileInfo(dxy)
ixy2 <- tileInfo(dxy,bndry=TRUE)
```

---

tilePerim

*Calculate tile perimeters.*

---

**Description**

Calculates the perimeters of all of the Dirichlet (Voronoi) tiles in a tessellation of a set of planar points. Also calculates the sum and the mean of these perimeters.



---

triang.list

---

Create a list of Delaunay triangles

---

## Description

From an object of class “deldir” produces a list of the Delaunay triangles in the triangulation of a set of points in the plane.

## Usage

```
triang.list(object)
```

## Arguments

object            An object of class “deldir” as produced by `deldir()`.

## Value

A list each of whose components is a  $3 \times 3$ ,  $3 \times 4$  or  $3 \times 5$  data frame corresponding to one of the Delaunay triangles specified by “object”. The rows of each such data frame correspond to the vertices of the corresponding Delaunay triangle. The columns are:

- ptNum (the index of the point in the original sequence of points that is being triangulated. Note that if a point is one of a set of *duplicated* points then ptNum is the *first* of the indices of the points in this set.)
- ptType (the type of the vertex; “data” or “dummy”; present only if there were any dummy points specified)
- x (the  $x$ -coordinate of the vertex)
- y (the  $y$ -coordinate of the vertex)
- z (the “auxiliary value” or “weight”  $z$  associated with the vertex; present only if such values were supplied in the call to `deldir()`)

The returned value has an attribute “rw” consisting of the enclosing rectangle of the triangulation.

## Note

The code of this function was taken more-or-less directly from code written by Adrian Baddeley for the “delaunay()” function in the “spatstat” package.

## Author(s)

Rolf Turner <r.turner@auckland.ac.nz>

## See Also

`deldir()`, `plot.triang.list()`, `tile.list()`, `plot.tile.list()`

## Examples

```
set.seed(42)
x <- runif(20)
y <- runif(20)
z <- sample(1:100,20)
d <- deldir(x,y,z=z)
ttt <- triang.list(d)
```

---

triMat

---

*Produce matrix of triangle vertex indices.*


---

## Description

Lists the indices of the vertices of each Delaunay triangle in the triangulation of a planar point set. The indices are listed (in increasing numeric order) as the rows of an  $n \times 3$  matrix where  $n$  is the number of Delaunay triangles in the triangulation.

## Usage

```
triMat(object)
```

## Arguments

**object**                      An object of class `deldir` (as produced by the function `deldir()`) specifying the Delaunay triangulation and Dirichlet (Voronoi) tessellation of a planar point set.

## Details

This function was suggested by Robin Hankin of the School of Mathematical and Computing Sciences at Auckland University of Technology.

## Value

An  $n \times 3$  matrix where  $n$  is the number of Delaunay triangles in the triangulation specified by `object`. The  $i^{th}$  row consists of the indices (in the original list of points being triangulated) of vertices of the  $i^{th}$  Delaunay triangle. The indices are listed in increasing numeric order in each row.

## Note

Earlier versions of this function (prior to release 0.1-14 of **deldir**) could sometimes give incorrect results. This happened if the union of three contiguous Delaunay triangles happened to constitute another triangle. This latter triangle would appear in the list of triangles produced by `triMat()` but is *not* itself a Delaunay triangle. The updated version of `triMat()` now checks for this possibility and gives (*I think!*) correct results.

Many thanks to Jay Call, who pointed out this bug to me.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

[deldir\(\)](#) [triang.list\(\)](#) [plot.triang.list\(\)](#)

**Examples**

```
# These are the data used by Jay Call to illustrate the bug
# that appeared in a previous incarnation of triMat.
xy <- data.frame(
  x = c(0.048,0.412,0.174,0.472,0.607,0.565,0.005,0.237,0.810,0.023),
  y = c(0.512,0.928,0.955,0.739,0.946,0.134,0.468,0.965,0.631,0.782)
)
dxy <- deldir(xy)
M <- triMat(dxy)
plot(dxy,wlines="triang",num=TRUE,axes=FALSE,cmpnt_col=c(1,1,1,1,2,1))
# The triangle with vertices {4,5,8} was listed in the output of
# the previous (buggy) version of triMat(). It is NOT a Delaunay
# triangle and hence should NOT be listed.
```

---

which.tile

*Determine the tile containing a given point.*

---

**Description**

Finds the Dirchlet/Voronoi tile of a tessellation produced by [deldir](#) that contains a given point.

**Usage**

```
which.tile(x, y, tl)
```

**Arguments**

x	The x coordinate of the point in question.
y	The y coordinate of the point in question.
tl	A tile list, as produced by the function <a href="#">tile.list()</a> from a tessellation produced by <a href="#">deldir()</a> .

**Details**

Just minimises the distance from the point in question to the points of the pattern determining the tiles.

**Value**

An integer equal to the index of the tile in which the given point lies.

**Author(s)**

Rolf Turner <r.turner@auckland.ac.nz>

**See Also**

[tile.list\(\)](#) [deldir\(\)](#).

**Examples**

```
set.seed(42)
x  <- runif(20,0,100)
y  <- runif(20,0,100)
dxy <- deldir(x,y)
txy <- tile.list(dxy)
i   <- which.tile(30,50,txy) # The value of i here is 14.
plot(txy,showpoints=FALSE)
text(x,y,labels=1:length(txy),col="red")
points(30,50,pch=20,col="blue")
```

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