This document explains how to use the parameters in an object returned by `svm()` for own prediction functions.

1 Binary Classifier

For class prediction in the binary case, the class of a new data vector \( n \) is usually given by the *sign* of

\[
\sum_i a_i y_i K(x_i, n) + \rho \tag{1}
\]

where \( x_i \) is the \( i \)-th support vector, \( y_i \) the corresponding label, \( a_i \) the corresponding coefficient, and \( K \) is the kernel (for example the linear one, i.e. \( K(u,v) = u^T v \)).

Now, the `libsvm` library interfaced by the `svm()` function actually returns \( a_i y_i \) as \( i \)-th coefficient and the negative \( \rho \), so in fact uses the formula:

\[
\sum_i \text{coef}_i K(x_i, n) - \rho
\]

where the training examples (=training data) are labeled \{1,-1\} (!). A simplified R function for prediction with linear kernel would be:

```r
svmpred <- function (m, newdata, K=crossprod) {
  ## this guy does the computation:
  pred.one <- function (x)
    sign(sum(sapply(1:m$tot.nSV, function (j)
      K(m$SV[j,], x) * m$coefs[j]
    )) - m$rho)
  
  ## this is just for convenience:
  if (is.vector(newdata))
    newdata <- t(as.matrix(x))
  sapply (1:nrow(newdata),
          function (i) pred.one(newdata[i,]))
}
```
where \texttt{pred\_one()} does the actual prediction for one new data vector, the remainder is just a convenience for prediction of multiple new examples. It is easy to extend this to other kernels, just replace \texttt{K()} with the appropriate function (see the help page for the formulas used) and supply the additional constants.

As we will see in the next section, the multi-class prediction is more complicated, because the coefficients of the diverse binary SVMs are stored in a compressed format.

2 Multiclass-classifier

To handle \( k \) classes, \( k > 2 \), \texttt{svm()} trains all binary subclassifiers (one-against-one-method) and then uses a voting mechanism to determine the actual class. Now, this means \( k(k-1)/2 \) classifiers, hence in principle \( k(k-1)/2 \) sets of SVs, coefficients and rhos. These are stored in a compressed format:

1. Only one SV is stored in case it were used by several classifiers. The \texttt{model$SV-matrix} is ordered by classes, and you find the starting indices by using \texttt{nSV} (number of SVs):

\[
\text{start} \leftarrow c(1, \text{cumsum(model$nSV)})
\]
\[
\text{start} \leftarrow \text{start}[-\text{length(start)}]
\]

\texttt{sum(nSV)} equals the total number of (distinct) SVs.

2. The coefficients of the SVs are stored in the \texttt{model$coefs-matrix}, grouped by classes. Because the separating hyperplanes found by the SVM algorithm has SVs on both sides, you will have two sets of coefficients per binary classifier, and e.g., for 3 classes, you could build a block-matrix like this for the classifiers (\( i, j \)) (\( i, j = \)class numbers):

\[
\begin{array}{ccc}
1 & 0 & 1 & 2 \\
0 & \text{X} & \text{set (0, 1)} & \text{set (0, 2)} \\
1 & \text{set (1, 0)} & \text{X} & \text{set (1, 2)} \\
2 & \text{set (2, 0)} & \text{set (2, 1)} & \text{X} \\
\end{array}
\]

where set\((i, j)\) are the coefficients for the classifier \((i, j)\), lying on the side of class \( j \). Because there are no entries for \((i, i)\), we can save the diagonal and shift up the lower triangular matrix to get

\[
\begin{array}{ccc}
i & 0 & 1 & 2 \\
0 & \text{set (1, 0)} & \text{set (0, 1)} & \text{set (0, 2)} \\
1 & \text{set (2, 0)} & \text{set (2, 1)} & \text{set (1, 2)} \\
\end{array}
\]

Each set \((., j)\) has length \texttt{nSV}[j], so of course, there will be some filling 0s in some sets. \texttt{model$coefs} is the \textit{transposed} of such a matrix, therefore for a data set with, say, 6 classes, you get 6-1=5 columns.

The coefficients of \((i, j)\) start at \texttt{model$coefs[start[i],j]} and those of \((j, i)\) at \texttt{model$coefs[start[j],i-1]}.

3. The \( k(k-1)/2 \) rhos are just linearly stored in the vector \texttt{model$rho}.
The following code shows how to use this for prediction:

```r
## Linear Kernel function
K <- function(i,j) crossprod(i,j)
predsvm <- function(object, newdata)
{
## compute start-index
start <- c(1, cumsum(object$nSV)+1)
start <- start[-length(start)]

## compute kernel values
kernel <- sapply (1:object$tot.nSV,
function (x) K(object$SV[x,], newdata))

## compute raw prediction for classifier (i,j)
predone <- function (i,j)
{
## ranges for class i and j:
ri <- start[i] : (start[i] + object$nSV[i] - 1)
rj <- start[j] : (start[j] + object$nSV[j] - 1)

## coefs for (i,j):
coef1 <- object$coefs[ri, j-1]
coef2 <- object$coefs[rj, i]

## return raw values:
crossprod(coef1, kernel[ri]) + crossprod(coef2, kernel[rj])
}

## compute votes for all classifiers
votes <- rep(0,object$nclasses)
c <- 0 # rho counter
for (i in 1 : (object$nclasses - 1))
  for (j in (i + 1) : object$nclasses)
    if (predone(i,j) > object$rho[c <- c + 1])
      votes[i] <- votes[i] + 1
    else
      votes[j] <- votes[j] + 1

## return winner (index with max. votes)
object$levels[which(votes %in% max(votes))[1]]
}
```

In case data were scaled prior fitting the model (note that this is the default for `svm()`), the new data needs to be scaled as well before applying the prediciton functions, for example using the following code snipped (object is an object returned by `svm()`, `newdata` a data frame):

```r
if (any(object$scaled))
  newdata[,object$scaled] <-
  scale(newdata[,object$scaled, drop = FALSE],
        center = object$x.scale$"scaled:center",
        scale = object$x.scale$"scaled:scale"
)
```

For regression, the response needs to be scaled as well before training, and the predictions need to be scaled back accordingly.