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Description Implementation of the Interval Testing Procedure for functional data in different frameworks (i.e., one or two-population frameworks, functional linear models) by means of different basis expansions (i.e., B-spline, Fourier, and phase-amplitude Fourier). The current version of the package requires functional data evaluated on a uniform grid; it automatically projects each function on a chosen functional basis; it performs the entire family of multivariate tests; and, finally, it provides the matrix of the p-values of the previous tests and the vector of the corrected p-values. The functional basis, the coupled or uncoupled scenario, and the kind of test can be chosen by the user. The package provides also a plotting function creating a graphical output of the procedure: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

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Interval Testing Procedure for Functional Data

Implementation of the Interval Testing Procedure for functional data in different frameworks (i.e., one or two-population frameworks, functional linear models) by means of different basis expansions (i.e., B-spline, Fourier, and phase-amplitude Fourier). The current version of the package requires functional data evaluated on a uniform grid; it automatically projects each function on a chosen functional basis; it performs the entire family of multivariate tests; and, finally, it provides the matrix of the p-values of the previous tests and the vector of the corrected p-values. The functional basis, the coupled or uncoupled scenario, and the kind of test can be chosen by the user. The package provides also a plotting function creating a graphical output of the procedure: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

Details

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Author(s)

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References


See Also

See also ITP1bspline, ITP1fourier, ITP2bspline, ITP2fourier, ITP2pafourier, ITP1mbspline, ITPaovbspline, and ITPimage.

Examples

```r
# Importing the NASA temperatures data set
data(NASAtemp)

# Example 1:
# Performing the ITP for one population with the Fourier basis
ITP.result <- ITP1fourier(NASAtemp$milan, maxfrequency=10, B=1000)
# Plotting the results of the ITP
plot(ITP.result)

# Plotting the p-value heatmap
ITPimage(ITP.result)

# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)

# Example 2:
# Performing the ITP for two populations with the B-spline basis
ITP.result <- ITP2bspline(NASAtemp$milan, NASAtemp$paris, nknots=20, B=1000)
# Plotting the results of the ITP
plot(ITP.result)

# Plotting the p-values heatmap
ITPimage(ITP.result)

# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)

# Example 3:
# Fitting and testing a functional-on-scalar linear model
# Defining data and covariates
temperature <- rbind(NASAtemp$milan, NASAtemp$paris)
groups <- c(rep(0,22), rep(1,22))

# Performing the ITP
ITP.result <- ITP1mbspline(temperature ~ groups, B=1000, nknots=20, order=3)
# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
layout(1)
```
plot(ITP.result, main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))

# All graphics on the same device
layout(matrix(1:6, nrow=3, byrow=FALSE))
plot(ITP.result, main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))

---

**ITP1bspline**  
*One population Interval Testing Procedure with B-spline basis*

---

**Description**

The function implements the Interval Testing Procedure for testing the center of symmetry of a functional population evaluated on a uniform grid. Data are represented by means of the B-spline expansion and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

**Usage**

ITP1bspline(data, mu = 0, order = 2, nknots = dim(data)[2], B = 10000)

**Arguments**

data: Pointwise evaluations of the functional data set on a uniform grid. data is a matrix of dimensions c(n,J), with J evaluations on columns and n units on rows.

mu: The center of symmetry under the null hypothesis: either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.

order: Order of the B-spline basis expansion. The default is order=2.

nknots: Number of knots of the B-spline basis expansion. The default is nknots=dim(data)[2].

B: The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.

**Value**

ITP1bspline returns an object of class "ITP1".

An object of class "ITP" is a list containing at least the following components:

basis: String vector indicating the basis used for the first phase of the algorithm. In this case equal to "B-spline".

test: String vector indicating the type of test performed. In this case equal to "1pop".

mu: Center of symmetry under the null hypothesis (as entered by the user).
**ITP1bspline**

- `coeff`: Matrix of dimensions \(c(n,p)\) of the \(p\) coefficients of the B-spline basis expansion. Rows are associated to units and columns to the basis index.
- `pval`: Uncorrected \(p\)-values for each basis coefficient.
- `pval.matrix`: Matrix of dimensions \(c(p,p)\) of the \(p\)-values of the multivariate tests. The element \((i,j)\) of matrix `pval.matrix` contains the \(p\)-value of the joint NPC test of the components \((j,j+1,...,j+(p-i))\).
- `corrected.pval`: Corrected \(p\)-values for each basis coefficient.
- `labels`: Labels indicating the population membership of each data (in this case always equal to 1).
- `data.eval`: Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.
- `heatmap.matrix`: Heatmap matrix of \(p\)-values (used only for plots).

**Author(s)**

Alessia Pini, Simone Vantini

**References**


**See Also**

See also `ITP1fourier`, `ITP2bspline`, `ITP2fourier`, `ITP2pafourier`, and `ITPimage`.

**Examples**

```r
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP for two populations with the B-spline basis
ITP.result <- itp1bspline(NASAtemp$paris,mu=4,nknots=50,B=1000)
# Plotting the results of the ITP
plot(ITP.result,xrange=c(0,12),main='Paris temperatures')

# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(0,12))

# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)
```
Description

The function implements the Interval Testing Procedure for testing the center of symmetry of a functional population evaluated on a uniform grid. Data are represented by means of the Fourier expansion and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate.

Usage

ITP1fourier(data, mu = 0, maxfrequency=floor(dim(data)[2]/2), B = 10000)

Arguments

data Pointwise evaluations of the functional data set on a uniform grid. data is a matrix of dimensions c(n,J), with J evaluations on columns and n units on rows.

mu The center of symmetry under the null hypothesis: either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.

maxfrequency The maximum frequency to be used in the Fourier basis expansion of data. The default is floor(dim(data)[2]/2), leading to an interpolating expansion.

B The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.

Value

ITP1fourier returns an object of class "ITP1". An object of class "ITP1" is a list containing at least the following components:

basis String vector indicating the basis used for the first phase of the algorithm. In this case equal to "Fourier".

test String vector indicating the type of test performed. In this case equal to "1pop".

mu Center of symmetry under the null hypothesis (as entered by the user).

coeff Matrix of dimensions c(n,p) of the p coefficients of the Fourier basis expansion. Rows are associated to units and columns to the basis index: the first column is a0, the following (p-1)/2 columns are the ak coefficients (sine coefficients) and the last (p-1)/2 columns the bk coefficients (cosine coefficients).

pval Uncorrected p-values for each frequency.

pval.matrix Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The element (i,j) of matrix pval.matrix contains the p-value of the joint NPC test of the components (j,j+1,...,j+(p-1)).
corrected.pval  Corrected p-values for each frequency.
labels   Labels indicating the population membership of each data (in this case always
equal to 1).
data.eval  Evaluation on a fine uniform grid of the functional data obtained through the
basis expansion.
heatmap.matrix  Heatmap matrix of p-values (used only for plots).

Author(s)
Alessia Pini, Simone Vantini

References
A. Pini and S. Vantini (2013). The Interval Testing Procedure: Inference for Functional Data Con-
trolling the Family Wise Error Rate on Intervals. MOX-report 13/2013, Politecnico di Milano.

See Also
See also ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier, and ITPimage.

Examples
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP1fourier(NASAtemp$milan,maxfrequency=20,B=1000)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')

# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(1,365))

# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)

ITP2bspline  Two populations Interval Testing Procedure with B-spline basis

Description
The function implements the Interval Testing Procedure for testing the difference between two
functional populations evaluated on a uniform grid. Data are represented by means of the B-spline
basis and the significance of each basis coefficient is tested with an interval-wise control of the
Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise
interpolating function.
Usage

ITP2bspline(data1, data2, mu = 0,
order = 2, nknots = dim(data1)[2], B = 10000, paired = FALSE)

Arguments

data1  Pointwise evaluations of the first population’s functional data set on a uniform grid. data1 is a matrix of dimensions c(n1, J), with J evaluations on columns and n1 units on rows.
data2  Pointwise evaluations of the second population’s functional data set on a uniform grid. data2 is a matrix of dimensions c(n2, J), with J evaluations on columns and n2 units on rows.
mu     The difference between the first functional population and the second functional population under the null hypothesis. Either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.
order  Order of the B-spline basis expansion. The default is order=2.
nknots Number of knots of the B-spline basis expansion. The default is nknots=dim(data1)[2].
B      The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.
paired A logical indicating whether the test is paired. The default is FALSE.

Value

ITP2bspline returns an object of class "ITP2".
An object of class "ITP2" is a list containing at least the following components:
basis  String vector indicating the basis used for the first phase of the algorithm. In this case equal to "B-spline".
test   String vector indicating the type of test performed. In this case equal to "2pop".
mu     Difference between the first functional population and the second functional population under the null hypothesis (as entered by the user).
paired  Logical indicating whether the test is paired (as entered by the user).
coeff  Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expansion, with n=n1+n2. Rows are associated to units and columns to the basis index. The first n1 rows report the coefficients of the first population units and the following n2 rows report the coefficients of the second population units.
pval   Uncorrected p-values for each basis coefficient.
pval.matrix  Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The element (i,j) of matrix pval.matrix contains the p-value of the joint NPC test of the components (j,j+1,...,j+(p-i)).
corrected.pval  Corrected p-values for each basis coefficient.
labels  Labels indicating the population membership of each data.
data.eval  Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.
heatmap.matrix  Heatmap matrix of p-values (used only for plots).
Author(s)

Alessia Pini, Simone Vantini

References


See Also

For tests of comparison between two populations, see *ITP2fourier, ITP2pafourier*. For different types of ITP-based tests, see *ITP1bspline, ITP1fourier, ITPlmbspline, ITPaovbspline* and *ITPimage*.

Examples

```r
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=50,B=1000)

# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')

# Plotting the p-values heatmap
ITPimage(ITP.result,abscissa.range=c(0,12))

# Selecting the significant components at 5% level
which(ITP.result$corrected.pval < 0.05)
```

ITP2fourier  Two populations Interval Testing Procedure with Fourier basis

Description

The function implements the Interval Testing Procedure for testing the difference between two functional populations evaluated on a uniform grid. Data are represented by means of the Fourier basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate.

Usage

```r
ITP2fourier(data1, data2, mu = 0,
             maxfrequency=floor(dim(data1)[2]/2), B = 10000, paired = FALSE)
```
Arguments

data1  Pointwise evaluations of the first population’s functional data set on a uniform grid. data1 is a matrix of dimensions c(n1,J), with J evaluations on columns and n1 units on rows.

data2  Pointwise evaluations of the second population’s functional data set on a uniform grid. data2 is a matrix of dimensions c(n2,J), with J evaluations on columns and n2 units on rows.

mu    The difference between the first functional population and the second functional population under the null hypothesis. Either a constant (in this case, a constant function is used) or a J-dimensional vector containing the evaluations on the same grid which data are evaluated. The default is mu=0.

maxfrequency  The maximum frequency to be used in the Fourier basis expansion of data. The default is floor(dim(data1)/2), leading to an interpolating expansion.

B     The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.

paired  A logical indicating whether the test is paired. The default is FALSE.

Value

ITP2fourier returns an object of class "ITP2". An object of class "ITP2" is a list containing at least the following components:

basis     String vector indicating the basis used for the first phase of the algorithm. In this case equal to "Fourier".

test      String vector indicating the type of test performed. In this case equal to "2pop".

mu        Difference between the first functional population and the second functional population under the null hypothesis (as entered by the user).

paired    Logical indicating whether the test is paired (as entered by the user).

coeff     Matrix of dimensions c(n,p) of the p coefficients of the Fourier basis expansion. Rows are associated to units and columns to the basis index: the first n1 rows report the coefficients of the first population units and the following n2 rows report the coefficients of the second population units; the first column is a0, the following (p-1)/2 columns are the ak coefficients (sine coefficients) and the last (p-1)/2 columns the bk coefficients (cosine coefficients).

pval      Uncorrected p-values for each frequency.

pval.matrix  Matrix of dimensions c(p,p) of the p-values of the multivariate tests. The element (i,j) of matrix pval.matrix contains the p-value of the joint NPC test of the frequencies (j,j+1,...,j+(p-i)).

corrected.pval  Corrected p-values for each frequency.

labels    Labels indicating the population membership of each data.

data.eval  Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.

heatmap.matrix  Heatmap matrix of p-values (used only for plots).
**ITP2pafourier**

**Author(s)**

Alessia Pini, Simone Vantini

**References**


**See Also**

See also `ITP2pafourier`, `ITP2bspline`, `ITP1fourier`, `ITP1bspline`, and `ITPimage`.

**Examples**

```r
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- itpRfourier(NASAtemp$milan, NASAtemp$paris, maxfrequency=20, B=1000, paired=TRUE)

# Plotting the results of the ITP
plot(ITP.result, main='NASA data', xrange=c(1, 365), xlab='Day')

# Plotting the p-value heatmap
ITPimage(ITP.result, abscissa.range=c(1, 365))

# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)
```

---

**ITP2pafourier**

Two populations Interval Testing Procedure with Fourier basis (phase-amplitude decomposition)

**Description**

The function implements the Interval Testing Procedure for testing the difference between two functional populations evaluated on a uniform grid. Data are represented by means of the Fourier basis expansion with the phase-amplitude decomposition and the significance of the amplitude and phase of each frequency is tested with an interval-wise control of the Family Wise Error Rate.

**Usage**

```r
ITP2pafourier(data1, data2,
               maxfrequency=floor(dim(data1)[2]/2), B = 10000, paired = FALSE)
```
Arguments

data1 Pointwise evaluations of the first population’s functional data set on a uniform grid. data1 is a matrix of dimensions c(n1,J), with J evaluations on columns and n1 units on rows.
data2 Pointwise evaluations of the second population’s functional data set on a uniform grid. data2 is a matrix of dimensions c(n2,J), with J evaluations on columns and n2 units on rows.
maxfrequency The maximum frequency to be used in the Fourier basis expansion of data. The default is floor(dim(data1)[2]/2), leading to an interpolating expansion.
B The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.
paired A logical indicating whether the test is paired. The default is FALSE.

Value

ITP2pafourier returns an object of class "ITP2pafourier".
An object of class "ITP2pafourier" is a list containing at least the following components:
basis String vector indicating the basis used for the first phase of the algorithm. Equal to "paFourier".
test String vector indicating the type of test performed. Equal to "2pop".
paired Logical indicating whether the test is paired (as entered by the user).
coeff_phase Matrix of dimensions c(n,p) of the p phases of the Fourier basis expansion. Rows are associated to units and columns to frequencies: the first n1 rows report the coefficients of the first population units and the following n2 rows report the coefficients of the second population units.
coeff_amplitude Matrix of dimensions c(n,p) of the p amplitudes of the Fourier basis expansion. Rows are associated to units and columns to frequencies: the first n1 rows report the coefficients of the first population units and the following n2 rows report the coefficients of the second population units.
pval_phase Uncorrected p-values of the phase tests for each frequency.
pval_amplitude Uncorrected p-values of the amplitude tests for each frequency.
pval.matrix_phase Matrix of dimensions c(p,p) of the p-values of the multivariate tests on phase. The element (i,j) of matrix pval.matrix_phase contains the p-value of the joint NPC test of the frequencies (j,j+1,...,j+(p-i)).
pval.matrix_amplitude Matrix of dimensions c(p,p) of the p-values of the multivariate tests on amplitude. The element (i,j) of matrix pval.matrix_amplitude contains the p-value of the joint NPC test of the frequencies (j,j+1,...,j+(p-i)).
corrected.pval_phase Corrected p-values of the phase tests for each frequency.
corrected.pval_amplitude Corrected p-values of the amplitude tests for each frequency.
ITPaovbspline

labels  
Labels indicating the population membership of each data.

data.eval  
Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.

heatmap.matrix.phase  
Heatmap matrix of p-values for phase (used only for plots).

heatmap.matrix.amplitude  
Heatmap matrix of p-values for amplitude (used only for plots).

Author(s)
Alessia Pini, Simone Vantini

References

See Also
See also ITP2fourier, ITP2bspline, ITP1fourier, ITP1bspline, and ITPimage.

Examples

```r
# Importing the NASA temperatures data set
data(NASAtemp)
# Performing the ITP
ITP.result <- ITP2pafourier(NASAtemp$milan,NASAtemp$paris,maxfrequency=20,B=1000,paired=TRUE)
# Plotting the results of the ITP
plot(ITP.result,main='NASA data',xrange=c(1,365),xlab='Day')

# Plotting the p-value heatmap
ITPimage(ITP.result,abscissa.range=c(1,365))

# Selecting the significant coefficients
which(ITP.result$corrected.pval < 0.05)
```

ITPaovbspline  
Interval Testing Procedure for testing Functional analysis of variance with B-spline basis

Description

ITPaovbspline is used to fit and test functional analysis of variance. The function implements the Interval Testing Procedure for testing for significant differences between several functional population evaluated on a uniform grid. Data are represented by means of the B-spline basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.
Usage

ITPaovbspline(formula, order = 2,
    nknots = dim(model.response(model.frame(formula)))[2],
    B = 10000, method = "residuals")

Arguments

formula  An object of class "formula" (or one that can be coerced to that class): a symbolic description of the model to be fitted.
order    Order of the B-spline basis expansion. The default is order=2.
nknots   Number of knots of the B-spline basis expansion. The default is dim(model.response(model.frame(formula)))[2].
B        The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.
method   Permutation method used to calculate the p-value of permutation tests. Choose "residuals" for the permutations of residuals under the reduced model, according to the Freedman and Lane scheme, and "responses" for the permutation of the responses, according to the Manly scheme.

Value

ITPaovbspline returns an object of class "ITPaov".
The function summary is used to obtain and print a summary of the results.
An object of class "ITPlm" is a list containing at least the following components:
call     The matched call.
design.matrix The design matrix of the functional-on-scalar linear model.
 basis    String vector indicating the basis used for the first phase of the algorithm. In this case equal to "B-spline".
coeff    Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expansion. Rows are associated to units and columns to the basis index.
coeff.regr Matrix of dimensions c(L+1,p) of the p coefficients of the B-spline basis expansion of the intercept (first row) and the L effects of the covariates specified in formula. Columns are associated to the basis index.
pval.F   Uncorrected p-values of the functional F-test for each basis coefficient.
pval.matrix.F   Matrix of dimensions c(p,p) of the p-values of the multivariate F-tests. The element (i,j) of matrix pval.matrix contains the p-value of the joint NPC test of the components (j,j+1,...,j+(p-i)).
corrected.pval.F Corrected p-values of the functional F-test for each basis coefficient.
pval.factors Uncorrected p-values of the functional F-tests on each factor of the analysis of variance, separately (rows) and each basis coefficient (columns).
pval.matrix.factors
Array of dimensions c(l+1,p,p) of the p-values of the multivariate F-tests on factors. The element (1, i, j) of array pval.matrix contains the p-value of the joint NPC test on factor 1 of the components (j, j+1, ..., j+(p-i)).

corrected.pval.factors
Corrected p-values of the functional F-tests on each factor of the analysis of variance (rows) and each basis coefficient (columns).
data.eval
Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.
coeff.regr.eval
Evaluation on a fine uniform grid of the functional regression coefficients.
fitted.eval
Evaluation on a fine uniform grid of the fitted values of the functional regression.
residuals.eval
Evaluation on a fine uniform grid of the residuals of the functional regression.
R2.eval
Evaluation on a fine uniform grid of the functional R-squared of the regression.
heatmap.matrix.F
Heatmap matrix of p-values of functional F-test (used only for plots).
heatmap.matrix.factors
Heatmap matrix of p-values of functional F-tests on each factor of the analysis of variance (used only for plots).

Author(s)
Alessia Pini, Simone Vantini

References


See Also
See summary.ITPov for summaries and plot.ITPov for plotting the results.
See also ITPlmbspline to fit and test a functional-on-scalar linear model applying the ITP, and ITP1bspline, ITP2bspline, ITP2fourier, ITP2pafourier for one-population and two-population tests.
Examples

```r
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris) groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups,B=1000,nknots=20,order=3)

# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
layout(1)
plot(ITP.result)

# All graphics on the same device
layout(matrix(1:4,nrow=2,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))
```

---

**ITPimage**  
*Plot of the Interval Testing Procedure results*

**Description**

Plotting function creating a graphical output of the ITP: the p-value heat-map, the plot of the corrected p-values, and the plot of the functional data.

**Usage**

```
ITPimage(ITP.result, alpha = 0.05, abscissa.range = c(0, 1), nlevel = 20)
```

**Arguments**

- **ITP.result**  
  Results of the ITP, as created by `ITP1bspline`, `ITP1fourier`, `ITP2bspline`, `ITP2fourier`, and `ITP2pafourier`.

- **alpha**  
  Level of the hypothesis test. The default is alpha=0.05.

- **abscissa.range**  
  Range of the plot abscissa. The default is c(0,1).

- **nlevel**  
  Number of desired color levels for the p-value heatmap. The default is nlevel=20.

**Value**

No value returned. The function produces a graphical output of the ITP results: the p-value heat-map, a plot of the corrected p-values and the plot of the functional data. The basis components selected as significant by the test at level alpha are highlighted in the plot of the corrected p-values by a gray area.
**ITPlmbspline**

**Author(s)**
Alessia Pini, Simone Vantini

**References**

**See Also**
See `plot.ITP1`, `plot.ITP2`, `plot.ITPlm`, and `plot.ITPaoov` for the plot method applied to the ITP results of one- and two-population tests, linear models, and ANOVA, respectively.

See also `ITP1bspline`, `ITP1fourier`, `ITP2bspline`, `ITP2fourier`, and `ITP2pafourier` for applying the ITP.

**Examples**

```r
# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for two populations with the B-spline basis
ITP.result <- ITPlmbspline(NASAtemp$milan, NASAtemp$paris, nknots=20, B=1000)

# Plotting the results of the ITP
ITPlimage(ITP.result, abscissa.range=c(0,12))

# Selecting the significant components for the radius at 5% level
which(ITP.result$corrected.pval < 0.05)
```

---

**ITPlmbspline**  
*Interval Testing Procedure for testing Functional-on-Scalar Linear Models with B-spline basis*

**Description**

`ITPlmbspline` is used to fit and test functional linear models. It can be used to carry out regression, and analysis of variance. The function implements the Interval Testing Procedure for testing the significance of the effects of scalar covariates on a functional population evaluated on a uniform grid. Data are represented by means of the B-spline basis and the significance of each basis coefficient is tested with an interval-wise control of the Family Wise Error Rate. The default parameters of the basis expansion lead to the piece-wise interpolating function.

**Usage**

```r
ITPlmbspline(formula, order = 2,  
nknots = dim(model.response(model.frame(formula)))[2],  
B = 10000, method = "residuals")
```
Arguments

formula
An object of class "formula" (or one that can be coerced to that class): a symbolic description of the model to be fitted.

order
Order of the B-spline basis expansion. The default is order=2.

nknots
Number of knots of the B-spline basis expansion. The default is `dim(model.response(model.frame(formula)))[2]`.

B
The number of iterations of the MC algorithm to evaluate the p-values of the permutation tests. The default is B=10000.

method
Permutation method used to calculate the p-value of permutation tests. Choose "residuals" for the permutations of residuals under the reduced model, according to the Freedman and Lane scheme, and "responses" for the permutation of the responses, according to the Manly scheme.

Value

ITPlmbspline returns an object of class "ITPlm".

The function summary is used to obtain and print a summary of the results.

An object of class "ITPlm" is a list containing at least the following components:

call
The matched call.

design.matrix
The design matrix of the functional-on-scalar linear model.

basis
String vector indicating the basis used for the first phase of the algorithm. In this case equal to "B-spline".

coeff
Matrix of dimensions c(n,p) of the p coefficients of the B-spline basis expansion. Rows are associated to units and columns to the basis index.

coeff.regr
Matrix of dimensions c(L+1,p) of the p coefficients of the B-spline basis expansion of the intercept (first row) and the L effects of the covariates specified in formula. Columns are associated to the basis index.

pval.F
Uncorrected p-values of the functional F-test for each basis coefficient.

pval.matrix.F
Matrix of dimensions c(p,p) of the p-values of the multivariate F-tests. The element (i,j) of matrix pval.matrix contains the p-value of the joint NPC test of the components (j,j+1,...,j+(p-i)).

corrected.pval.F
Corrected p-values of the functional F-test for each basis coefficient.

pval.t
Uncorrected p-values of the functional t-tests for each partial regression coefficient including the intercept (rows) and each basis coefficient (columns).

pval.matrix.t
Array of dimensions c(L+1,p,p) of the p-values of the multivariate t-tests. The element (l,i,j) of array pval.matrix contains the p-value of the joint NPC test on covariate l of the components (j,j+1,...,j+(p-i)).

corrected.pval.t
Corrected p-values of the functional t-tests for each partial regression coefficient including the intercept (rows) and each basis coefficient (columns).

data.eval
Evaluation on a fine uniform grid of the functional data obtained through the basis expansion.
coeff.regr.eval Evaluation on a fine uniform grid of the functional regression coefficients.
fitted.eval Evaluation on a fine uniform grid of the fitted values of the functional regression.
residuals.eval Evaluation on a fine uniform grid of the residuals of the functional regression.
R2.eval Evaluation on a fine uniform grid of the functional R-squared of the regression.
heatmap.matrix.F Heatmap matrix of p-values of functional F-test (used only for plots).
heatmap.matrix.t Heatmap matrix of p-values of functional t-tests (used only for plots).

Author(s)
Alessia Pini, Simone Vantini

References

See Also
See summary.ITPlm for summaries and plot.ITPlm for plotting the results.
See also ITPaovbspline to fit and test a functional analysis of variance applying the ITP, and ITPlmbspline, ITP2bspline, ITP2fourier, ITP2pafourier for one-population and two-population tests.

Examples
# Importing the NASA temperatures data set
data(NASAtemp)

# Defining the covariates
temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlmbspline(temperature ~ groups,B=1000,nknots=20)
# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
data(NASAtemp)

## not run:
matplot(t(NASAtemp$milan), type='l')
matplot(t(NASAtemp$paris), type='l')
plot.ITP1

Plotting ITP results for one-population tests

Description

plot method for class "ITP1". Plotting function creating a graphical output of the ITP for the test of the mean of one population: functional data and ITP-adjusted p-values are plotted.

Usage

## S3 method for class 'ITP1'
plot(x, xrange = c(0, 1), alpha1 = 0.05, alpha2 = 0.01,
     ylab = "functional data", main = NULL, lwd = 1, col = 1,
     pch = 16, ylim = range(object$eval), ...)

Arguments

x The object to be plotted. An object of class "ITP1", that is, a result of an ITP for comparison between two populations. Usually a call to ITP1bspline or ITP1fourier.
xrange Range of the x-axis.
alpha1 First level of significance used to select and display significant differences. Default is alpha1 = 0.05.
alpha2 Second level of significance used to select and display significant differences. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
ylab Label of y-axis of the plot of functional data. Default is "functional Data".
main An overall title for the plots (it will be pasted to "Functional Data" for the first plot and "adjusted p-values" for the second plot).
lwd Line width for the plot of functional data.
col Color used to plot the functional data.
pch Point character for the plot of adjusted p-values.
ylim Range of the y-axis.
... Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data and the one of the adjusted p-values. The basis components selected as significant by the test at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data (in case the test is based on a local basis, such as B-splines) by gray areas (light and dark gray, respectively).
Author(s)

Alessia Pini, Simone Vantini

References


See Also

ITPimage for the plot of p-values heatmaps.

See also ITPbspline and ITPfourier to perform the ITP to test for the mean of a functional populations. See plot.ITP2 and plot.ITPlm for the plot method applied to the ITP results of two-population tests and linear models, respectively.

Examples

# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for one population with the B-spline basis
ITP.result.bspline <- itp1bspline(NASAtemp$paris, mu=4, nknots=50, B=1000)

# Plotting the results of the ITP
plot(ITP.result.bspline, xlab='Day', xrange=c(0, 365), main='NASA data')

# Selecting the significant components for the radius at 5% level
which(ITP.result.bspline$corrected.pval < 0.05)

plot.ITP2

Plotting ITP results for two-population tests

Description

plot method for class "ITP2". Plotting function creating a graphical output of the ITP for the test of comparison between two populations: functional data and ITP-adjusted p-values are plotted.

Usage

## S3 method for class 'ITP2'
plot(x, xrange = c(0, 1), alpha1 = 0.05, alpha2 = 0.01,
     ylab = "Functional Data", main = NULL, lwd = 1,
     col = c(1, 2), pch = 16, ylim = range(object$data.eval), ...)
plot.ITP2

Arguments

x                The object to be plotted. An object of class "ITP2", that is, a result of an ITP for comparison between two populations. Usually a call to ITP2bspline, ITP2fourier or ITP2pafourier.
xrange            Range of the x axis.
alpha1           First level of significance used to select and display significant differences. Default is alpha1 = 0.05.
alpha2           Second level of significance used to select and display significant differences. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
ylab             Label of y axis of the plot of functional data. Default is "Functional Data".
main             An overall title for the plots (it will be pasted to "Functional Data" for the first plot and "adjusted p-values" for the second plot).
lwd              Line width for the plot of functional data.
col              Color used to plot the functional data.
pch              Point character for the plot of adjusted p-values.
ylim             Range of the y axis.
...              Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data and the one of the adjusted p-values. The basis components selected as significant by the test at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data (in case the test is based on a local basis, such as B-splines) by gray areas (light and dark gray, respectively). In the case of a Fourier basis with amplitude and phase decomposition, two plots of adjusted p-values are done, one for phase and one for amplitude.

Author(s)

Alessia Pini, Simone Vantini

References


See Also

ITPimage for the plot of p-values heatmaps.

See also ITP2bspline, ITP2fourier, ITP2pafourier to perform the ITP to test for differences between two populations. See plot.ITP1 and plot.ITP1m for the plot method applied to the ITP results of one-population tests and a linear models, respectively.
Examples

# Importing the NASA temperatures data set
data(NASAtemp)

# Performing the ITP for two populations with the B-spline basis
ITP.result.bspline <- ITP2bspline(NASAtemp$milan,NASAtemp$paris,nknots=30,B=1000)

# Plotting the results of the ITP
plot(ITP.result.bspline,xlab='Day',xrange=c(1,365),main='NASA data')

# Selecting the significant components for the radius at 5% level
which(ITP.result.bspline$corrected.pval < 0.05)

plot.ITPaov

Plotting ITP results for functional analysis of variance testing

Description

plot method for class "ITPaov". Plotting function creating a graphical output of the ITP for the test on a functional analysis of variance: functional data, and ITP-adjusted p-values of the F-tests on the whole model and on each factor are plotted.

Usage

## S3 method for class 'ITPaov'
plot(x,xrange=c(0,1), alpha1=0.05, alpha2=0.01,
plot.adjpval=FALSE, ylim=range(x$data.eval), col=1,
ylab='Functional Data', main=NULL, lwd=1, pch=16,...)

Arguments

x The object to be plotted. An object of class "ITPaov", usually, a result of a call to ITPaovbspline.
xrange Range of the x axis.
alpha1 First level of significance used to select and display significant effects. Default is alpha1 = 0.05.
alpha2 Second level of significance used to select and display significant effects. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
plot.adjpval A logical indicating wether the plots of adjusted p-values have to be done. Default is plot.adjpval = FALSE.
col Colors for the plot of functional data. Default is col = 1.
ylim Range of the y axis. Default is ylim = range(x$data.eval).
ylab Label of y axis of the plot of functional data. Default is "Functional Data".
plot.ITPaov

main
An overall title for the plots (it will be pasted to "Functional Data and F-test" for the first plot and "factor" for the other plots).
lwd
Line width for the plot of functional data. Default is lwd=16.
pch
Point character for the plot of adjusted p-values. Default is pch=16.
... Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

Value
No value returned. The function produces a graphical output of the ITP results: the plot of the functional data, functional regression coefficients, and ITP-adjusted p-values of the F-tests on the whole model and on each factor. The basis components selected as significant by the tests at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data by gray areas (light and dark gray, respectively). The first plot reports the gray areas corresponding to a significant F-test on the whole model. The remaining plots report the gray areas corresponding to significant F-tests on each factor (with colors corresponding to the levels of the factor).

Author(s)
Alessia Pini, Simone Vantini

References

See Also
See also ITPaovbspline to fit and test a functional analysis of variance applying the ITP, and summary.ITPaov for summaries. See plot.ITP1m, plot.ITP1, and plot.ITP2 for the plot method applied to the ITP results of functional-on-scalar linear models, one-population and two-population, respectively.

Examples
# Importing the NASA temperatures data set
data(NASatemp)

temperature <- rbind(NASatemp$milan,NASatemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups,B=1000,nknots=20,order=3)

# Summary of the ITP results
summary(ITP.result)
# Plot of the ITP results
layout(1)
plot(ITP.result)

# All graphics on the same device
layout(matrix(1:4,nrow=2,byrow=FALSE))
plot(ITP.result,main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))

---

**plot.ITPlm**

*Plotting ITP results for functional-on-scalar linear model testing*

**Description**

plot method for class "ITPlm". Plotting function creating a graphical output of the ITP for the test on a functional-on-scalar linear model: functional data, functional coefficients and ITP-adjusted p-values for the F-test and t-tests are plotted.

**Usage**

```r
## S3 method for class 'ITPlm'
plot(x, xrange = c(0,1), alpha1 = 0.05, alpha2 = 0.01,
     plot.adjpval = FALSE, col = c(1, rainbow(dim(x$corrected.pval.t)[1])),
     ylim = range(x$data.eval), ylab = "Functional Data",
     main = NULL, lwd = 1, pch = 16, ...)
```

**Arguments**

- **x**: The object to be plotted. An object of class "ITPlm", usually, a result of a call to `ITPlmbspline`.
- **xrange**: Range of the x axis.
- **alpha1**: First level of significance used to select and display significant effects. Default is alpha1 = 0.05.
- **alpha2**: Second level of significance used to select and display significant effects. Default is alpha1 = 0.01. alpha1 and alpha2 are s.t. alpha2 < alpha1. Otherwise the two values are switched.
- **plot.adjpval**: A logical indicating weather the plots of adjusted p-values have to be done. Default is plot.adjpval = FALSE.
- **col**: Vector of colors for the plot of functional data (first element), and functional coefficients (following elements). Default is col = c(1, rainbow(dim(x$corrected.pval.t)[1])).
- **ylim**: Range of the y axis. Default is ylim = range(x$data.eval).
- **ylab**: Label of y axis of the plot of functional data. Default is "Functional Data".
- **main**: An overall title for the plots (it will be pasted to "Functional Data and F-test" for the first plot and "t-test" for the other plots).
plot.ITPlm

lwd  Line width for the plot of functional data. Default is lwd=16.
pch  Point character for the plot of adjusted p-values. Default is pch=16.
...  Additional plotting arguments that can be used with function plot, such as graphical parameters (see par).

Value

No value returned. The function produces a graphical output of the ITP results: the plot of the functional data, functional regression coefficients, and ITP-adjusted p-values for the F-test and t-tests. The basis components selected as significant by the tests at level alpha1 and alpha2 are highlighted in the plot of the corrected p-values and in the one of functional data by gray areas (light and dark gray, respectively). The plot of functional data reports the gray areas corresponding to a significant F-test. The plots of functional regression coefficients report the gray areas corresponding to significant t-tests for the corresponding covariate.

Author(s)

Alessia Pini, Simone Vantini

References


See Also

See also ITPlm.bspline to fit and test a functional-on-scalar linear model applying the ITP, and summary.ITPlm for summaries. See plot.ITPaov, plot.ITPlm and plot.ITHm for the plot method applied to the ITP results of functional analysis of variance, one-population and two-population, respectively.

Examples

# Importing the NASA temperatures data set
data(NASAtemp)

data <- rbind(NASAtemp$milan,NASAtemp$paris)
lab <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlm.bspline(data ~ lab,B=1000,nknots=20,order=3)
# Summary of the ITP results
summary(ITP.result)

# Plot of the ITP results
layout(1)
plot(ITP.result,main='NASA data',xlab='Day',xrange=c(1,365))
# Plots of the adjusted p-values
plot(ITP.result, main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))

# To have all plots in one device
layout(matrix(1:6, nrow=3, byrow=FALSE))
plot(ITP.result, main='NASA data', plot.adjpval = TRUE, xlab='Day', xrange=c(1,365))

---

**summary.ITPaov**  
*Summarizing Functional Analysis of Variance Fits*

**Description**

summary method for class "ITPaov".

**Usage**

```r
## S3 method for class 'ITPaov'
summary(object, ...)  
```

**Arguments**

- `object` An object of class "ITPaov", usually, a result of a call to `ITPaovbspline`
- `...` Further arguments passed to or from other methods.

**Value**

The function `summary.ITPaov` computes and returns a list of summary statistics of the fitted functional analysis of variance given in `object`, using the component "call" from its arguments, plus:

- `factors` A L x 1 matrix with columns for the factors of ANOVA, and corresponding (two-sided) ITP-adjusted minimum p-values of the corresponding tests of significance (i.e., the minimum p-value over all p basis components used to describe functional data).
- `R2` Range of the functional R-squared.
- `ftest` ITP-adjusted minimum p-value of functional F-test.

**Author(s)**

Alessia Pini, Simone Vantini

**References**


See Also

See `ITPaovbspline` for fitting and testing the functional ANOVA and `plot.ITPaov` for plots. See also `ITPlmbspline`, `ITPlbspline`, `ITP2bspline`, `ITP2fourier`, `ITP2pafourier`.

Examples

```r
# Importing the NASA temperatures data set
data(nasatemp)

temperature <- rbind(nasatemp$milan, nasatemp$paris)
groups <- c(rep(0, 22), rep(1, 22))

# Performing the ITP
ITP.result <- ITPaovbspline(temperature ~ groups, B=1000, nknots=20, order=3)

# Summary of the ITP results
summary(ITP.result)
```

### summary.ITPlm

*Summarizing Functional-on-Scalar Linear Model Fits*

**Description**

summary method for class "ITPlm".

**Usage**

```r
## S3 method for class 'ITPlm'
summary(object, ...)  
```

**Arguments**

- `object` An object of class "ITPlm", usually, a result of a call to `ITPlmbspline`.
- `...` Further arguments passed to or from other methods.

**Value**

The function `summary.ITPlm` computes and returns a list of summary statistics of the fitted functional-on-scalar linear model given in `object`, using the component "call" from its arguments, plus:

- `ttest` A \((L+1 \times 1)\) matrix with columns for the functional regression coefficients, and corresponding (two-sided) ITP-adjusted minimum p-values of t-tests (i.e., the minimum p-value over all \(p\) basis components used to describe functional data).
- `R2` Range of the functional R-squared.
- `ftest` ITP-adjusted minimum p-value of functional F-test.
Author(s)
Alessia Pini, Simone Vantini

References

See Also
See ITPlmbspline for fitting and testing the functional linear model and plot.ITPlm for plots. See also ITPaovbspline, ITPlbspline, ITP2bspline, ITP2fourier, ITP2pafourier.

Examples
# Importing the NASA temperatures data set
data(NASAtemp)

temperature <- rbind(NASAtemp$milan,NASAtemp$paris)
groups <- c(rep(0,22),rep(1,22))

# Performing the ITP
ITP.result <- ITPlmbspline(temperature ~ groups, B=1000,nknots=20)

# Summary of the ITP results
summary(ITP.result)
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