Package ‘fellow’

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feasible_overlap  Find Smallest Feasible Ellipse Overlap

Description

**feasible_overlap** will find the smallest radius such that the ellipses have a non-empty intersection.

Usage

**feasible_overlap**(ell, ...)

Arguments

- **ell**: a list of at least two (non degenerate) ellipses; see `wrangle_ellipse`
- ... additional arguments to be passed to internal functions.

Details

Given a list of ellipses `ell` the function `feasible_overlap` will find the smallest radius such that the ellipses from `ell` overlap. This is done by solving the following quadratically constrained problem

\[
\min_{(x,s)} s \\
\text{s.t. } (x - c_i)^T P_i (x - c_i) - r_i \leq s \quad \text{for all } i = 1, ..., d
\]

To solve this convex problem the logarithmic barrier method is used.

Note that it is not necessary to specify ellipse radii in `ell`.

Value

`feasible_overlap` returns an object of class "feasible_overlap". This object is a list with the following entries:

- **radii**: the smallest ellipse radii resulting in a non-empty intersection.
- **x**: The limiting common point.
- **distance**: The ellipse specific distances.
- **call**: The matched call.

See Also

`wrangle_ellipse` for detailed on ellipse parameterization.
feasible_point

Examples

## two dimensional ellipses

e1 <- list("c" = c(1,1), "P" = matrix(c(2,0,0,0.5), ncol = 2))
e2 <- list("c" = c(0,0), "S" = matrix(c(1, 0.2, 0.2, 2), ncol = 2), "r" = 1)
# note: it is not necessary to specify an ellipse radius "r"
feasible_overlap(list(e1, e2))

## regression example

# generate data
n <- 100
E <- rbinom(n, 1, 0.5)
X <- rnorm(n, E * 3, 1)
Y <- rnorm(n, 2 + 1.5 * X, 1)

# create confidence region ellipses
m0 <- lm(Y ~ X, data = data.frame(Y, X), subset = (E == 0))
m1 <- lm(Y ~ X, data = data.frame(Y, X), subset = (E == 1))
ConfRegion0 <- list(c = coefficients(m0), S = vcov(m0))
ConfRegion1 <- list(c = coefficients(m0), S = vcov(m0))

# find smallest radius
res <- feasible_overlap(list(ConfRegion0, ConfRegion1))
# this radius now corresponds to the chisq quantile at which
# the confidence regions intersect non-emptily.
# In other words the (1 - alpha)-confidence intervals intersect for alpha:
alpha <- pchisq(res$radii, 2)

feasible_point

Find Feasible Point in Ellipse Overlap

Description

feasible_point will find a point in the interior of the intersection of two or more fully specified ellipses. If the intersections is empty NA is returned.

Usage

feasible_point(ell, ...)

Arguments

e1

a list of at least two (non degenerate) ellipses; see wrangle_ellipse.

... additional arguments to be passed to internal functions.
Details

feasible_point will find a point in the interior of the intersection of two or more fully specified ellipses ell. If the intersections is empty NA is returned.

Value

feasible_point returns an object of class "feasible_point" with the following entries

- **x**: An interior point.
- **distance**: A data.frame with the ellipse specific distances.
- **optim**: The final internal optimization value.
- **call**: The matched call.

See Also

wrangle_ellipse for detailed on ellipse parameterization.

Examples

```r
# two dimensional ellipses
e1 <- list("c" = c(1,2), "P" = matrix(c(2,0,0,1), ncol = 2), "r" = 3)
e2 <- list("c" = c(0,0), "S" = matrix(c(1, 0.2, 0.2, 2), ncol = 2), "r" = 1)

# find point in intersection
feasible_point(list(e1, e2))

# make new ellipse
e3 <- list("c" = c(2,2), "P" = matrix(c(1,0,0,1), ncol = 2), "r" = 0.5)

# now there is no overlap
feasible_point(list(e1, e2, e3))
```

is_feasible_point

Determine If A Point Is In Ellipse Overlap

Description

is_feasible_point will determine if a given point is in the interior of the intersection of one or more fully specified ellipses.

Usage

is_feasible_point(point, ell)

Arguments

- **point**: a numeric of length equal to the dimensions of the ellipses in ell.
- **ell**: a list of at least one ellipse; see wrangle_ellipse.
Details

Given a point `is_feasible_point` will check if this point is in the intersection of the list of ellipses `ell`. Note that this function will not check if the intersection is non-empty.

Value

`is_feasible_point` returns an object of class "is_feasible_point". This object is a list containing the following components:

- `point`: the inputted point.
- `feasible`: logical; is TRUE when the point `x` is in the interior of all ellipses.
- `distance`: a data.frame with the distance from `x` to the center of each ellipse, the radius of each ellipse and a logical indicator, which is TRUE when `x` is an element in the ellipse.
- `call`: the match call.

See Also

`wrangle_ellipse` for detailed on ellipse parameterization.

Examples

e1 <- list("c" = c(1,1), "P" = matrix(c(3,1,2), ncol = 2), "r" = 2)
e2 <- list("c" = c(0,2), "S" = matrix(c(4,1,1,1), ncol = 2), "r" = 3)

is_feasible_point(c(1.1,0.9), e1)
is_feasible_point(c(1,0), list(e1, e2))

marginal_overlap

Feasibility of all Marginal Ellipse Overlaps

Description

Determine if the projections of ellipses onto each margin overlap.

Usage

`marginal_overlap(ell, margins = "all")`

Arguments

- `ell`: a list of at least two (non degenerate) ellipses; see `wrangle_ellipse`.
- `margins`: either "all" or a vector indicating the margins to project the ellipses onto and take intersections.
Details

The ellipses are projected onto the specified margins. For each margin the intersection of the projected ellipses is found. The Lower and Upper endpoints of the intersection interval is reported. If the intersection along a margin is empty then Lower and Upper is reported as NA.

Note that if the ellipses overlap when projected onto each margin this does not imply that the ellipses themselves intersect non-emptily. The example below is constructed to illustrate this.

Value

marginal_overlap returns an object of class "marginal_overlap" which contains a data.frame where the columns describe the following:

<table>
<thead>
<tr>
<th>Margin</th>
<th>Inputted margins.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlap</td>
<td>Whether the ellipses overlap when projected onto corresponding margin.</td>
</tr>
<tr>
<td>Lower</td>
<td>Lower endpoint of intersection interval. NA if the intersection is empty.</td>
</tr>
<tr>
<td>Upper</td>
<td>Upper endpoint of intersection interval. NA if the intersection is empty.</td>
</tr>
</tbody>
</table>

See Also

wrangle_ellipse for detailed on ellipse parameterization.

Examples

## two dimensional ellipses

e1 <- list(c = c(0.1, 0), P = matrix(c(3, 0, 0, 1), ncol = 2), r = 1)
e2 <- list(c = c(1, 1), P = matrix(c(3, 1.2, 1.2, 1), ncol = 2), r = 0.8)
e3 <- list(c = c(2, 1.5), P = matrix(c(1, 0.6, 0.6, 1), ncol = 2), r = 0.4)

# Note: These three ellipses have been chosen so (some of) the marginal
# projections intersect, but the actual ellipses do not intersect.

# Ellipses e1 and e2 overlap when projected onto margin 1 and 2 respectively.
marginal_overlap(list(e1, e2))

# Adding ellipse e3:
# Then there is no overlap when projecting onto margin 1
marginal_overlap(list(e1, e2, e3), margins = c(1))

## regression example

n <- 100
E <- rbinom(n, 1, 0.5)
X <- rnorm(n, E * 3, 1)
Y <- rnorm(n, 2 + X, 1)

lm_E0 <- lm(Y ~ X, data = data.frame(Y, X), subset = (E == 0))
lm_E1 <- lm(Y ~ X, data = data.frame(Y, X), subset = (E == 1))

# create 95% confidence ellipses and check marginal overlap
q <- qchisq(0.95, 2) # df = 2, as there are two covariates (1, X)
ell0 <- list(c = coefficients(lm_E0), S = vcov(lm_E0), r = q)
ell1 <- list(c = coefficients(lm_E1), S = vcov(lm_E1), r = q)
marginal_overlap(list(e10, e11))
Description

Determine if pairs of ellipses intersect non-emptily.

Usage

pairwise_overlap(ell, ...)

Arguments

ell a list of at least two (non degenerate) ellipses; see wrangle_ellipse.
... additional arguments to be passed to the low level function.

Details

The pairwise_overlap function goes through all pairs of ellipses from ell and checks if their intersection is non-empty.

Note that if all pairs of ellipses intersect this does not mean that the intersection of all the ellipses is non-empty. The example below is constructed to illustrate this.

Value

The pairwise_overlap function returns an object of class "pairwise_overlap" with the following components:

intersection a data.frame where the two first columns specify the two ellipses intersected and the last column indicate if they have a non-empty intersection.
call the matched call.

See Also

wrangle_ellipse for detailed on ellipse parameterization.

Examples

## three different two dimensional ellipses
e1 <- list(c = c(0, 0.7), P = matrix(c(0.2, 0, 0, 3), ncol = 2), r = 0.5)
e2 <- list(c = c(0, 1), P = matrix(c(3, -1.5, -1.5, 1), ncol = 2), r = 1)
e3 <- list(c = c(1.5, 1), P = matrix(c(3, 1.2, 1.2, 1), ncol = 2), r = 1.2)
# Note: These ellipses have been chosen so all pairs intersect,
# but the intersection of all three is empty.

# test pairwise overlaps
pairwise_overlap(list(e1, e2, e3))
## regression example

# generate data

```r
n <- 100
E <- rbinom(n, 2, 0.5)
X <- rnorm(n, 3 * E, 1)
Y <- rnorm(n, 2 + 1.5 * E, 1)

m0 <- lm(Y ~ X, data = data.frame(Y,X), subset = (E == 0))
m1 <- lm(Y ~ X, data = data.frame(Y,X), subset = (E == 1))
m2 <- lm(Y ~ X, data = data.frame(Y,X), subset = (E == 2))
```

# create 95% confidence ellipses and check pairwise intersection

```r
q <- qchisq(0.95, 2) # df = 2, as there are two covariates (1, X)
E0 <- list(c = coefficients(m0), S = vcov(m0), r = q)
E1 <- list(c = coefficients(m1), S = vcov(m1), r = q)
E2 <- list(c = coefficients(m2), S = vcov(m2), r = q)
pairwise_overlap(list("model 0" = E0, "model 1" = E1, "model 2" = E2))
```

---

**wrangle_ellipse**

**Ellipse Wrangler**

**Description**

wrangle_ellipse is used to wrangle one or more ellipses from one parametrization to another.

**Usage**

```r
wrangle_ellipse(ell, out_params = c("c", "P", "r"))
```

**Arguments**

- **ell**
  - a list of (non degenerate) ellipses to be wrangled. An ellipse is a named list and each entry corresponds to a parameter. To ensure all out_params can be calculated one of the parametrizations listed below in the description must be specified. Some out_params do not require a fully parametrized ellipse and so partially specified ellipses can be used.

- **out_params**
  - a vector of names of the output parameters. A list of possible parameters is given below in the details.

**Details**

Takes ellipse parameters and and calculates the wanted out_params. A parameterization is a named list, where each named entry is a parameter. The following parameters are accepted both input and output:

- **n**: dimension of ellipse; an integer.
- **c**: center of the ellipse; a vector.
- **P**: precision matrix - inverse of S; a positive definit, symmetric matrix.
• $S$: deviation matrix - inverse of $P$; a positive definite, symmetric matrix.
• $r$: radius; a positive number.
• $q$: cross term $-Pc$; a vector.
• $L$: Cholesky decomposition of $P$
• $e$: eigen values of $P$; a vector of eigenvalues.
• $U$: eigen vectors of $P$; a matrix, where each column is an eigen vector.
• $D$: diagonal matrix with $\sqrt{e}$ as diagonal entries.

An ellipse $E$ may be fully parameterized using the above parameters in the following ways:

\[
E = \{ x \in \mathbb{R}^p : (x - c)^T P (x - c) \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : (x - c)^T S^{-1} (x - c) \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : (x - c)^T LL^T (x - c) \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : (x - c)^T U D^2 U^T (x - c) \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : \|L^T (x - c)\|_2 \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : \|DU^T (x - c)\|_2 \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : c + L^{-T} w, \|w\| \leq r \}
\]

\[
E = \{ x \in \mathbb{R}^p : c + UD^{-1} w, \|w\| \leq r \}
\]

To ensure that all of the above parameters can be calculated it is advised (but in some cases not needed) that the input ellipses are fully parameterized.

Value

A list of wrangled ellipses. The wrangled ellipses are now given by the out_params.

Examples

# two dimensional unite ball
e2d <- list(c = c(0,0), S = matrix(c(1,0,0,1), ncol = 2), r = 1)

# three dimensional ellipse
e3d <- list(c = c(3,2,1), P = matrix(c(3,1,2,1,5,0,2,0,2), ncol = 3))

f1 <- wrangle_ellipse(e2d) # (c,P,r) parameterization
f2 <- wrangle_ellipse(e2d, out_params = c("c", "e", "U", "r"))
f3 <- wrangle_ellipse(list("ellipse1" = e2d, "ellipse2" = e3d),
c("n", "c", "U", "D"))
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