Package ‘flatness’

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Type Package

Title Indices and Tests for Assessing the Flatness of (Rank) Histograms

Version 0.1.4

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Description S3 classes, plotting functions, indices and tests to analyse the flatness of histograms.

It is specifically (but not only) aimed for assessing whether ``rank'' histograms (much used in weather forecasting) are flat.

Specifically functions are provided to use the Jolliffe-Primo flatness tests introduced in Jolliffe and Primo (2008, <doi:10.1175/2007MWR2219.1>).

Flatness indices described in Wilks (2019, <doi:10.1175/MWR-D-18-0369.1>) can be computed.

Finally a function to use the Benjamini-Hochberg procedure for multiple hypothesis testing is provided (Benjamini and Hochberg, 1995, <doi:10.1111/j.2517-6161.1995.tb02031.x>).

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R topics documented:

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Multiple statistical hypothesis testing with the Benjamini-Hochberg procedure

**Description**

This function applies the procedure described in Benjamini & Hochberg (1995) for controlling the False Discovery Rate in multiple statistical hypothesis testing.

**Usage**

```r
BH_procedure(pvalues, alpha = 0.01, ...)
```

**Arguments**

- `pvalues`: a vector of p-values, with length N
- `alpha`: a numeric between 0 and 1
- `...`: same arguments as in function base::sort, except "index.return" which is set to TRUE.
Details

The procedure works as follows. Let N p-values \( p_i \) (with \( i = 1, \ldots, N \)) and a significance level \( \alpha \). The decision threshold is \( p_k \) where \( k = \max(i; p_i \leq \alpha \frac{i}{N}) \) where \( p_i \) are the sorted p-values \( p_i \).

For every test with \( p_i \leq p_k \), the null hypothesis is rejected. By convention \( p(0) = 0 \).

Value

A named listed with entries:

- H0: a logical vector of length N. The nth entry has value TRUE if the null hypothesis associated with the nth p-value is not rejected and FALSE otherwise.
- pk: the decision threshold. A p-value under this threshold leads to rejection of the associated null hypothesis.
- alpha: the chosen significance level
- pvalues: the vector of p-values provided to apply the Benjamini-Hochberg procedure.

References


boxplot.rkhist

Methods for function boxplot of an S3 object of class rkhist

Description

For each rank in an rkhist object, draws a boxplot of the counts for all ensembles. A line at the count value for a perfectly flat rank histogram is also drawn.

Usage

```r
## S3 method for class 'rkhist'
boxplot(x, mini = min(0, min(x)), what = "counts", ...)
```

Arguments

- \( x \) an rkhist S3 object containing the rank histograms whose boxplot must be drawn. See function flatness::rkhist.
- mini a numeric. The minimum value in the ylim argument used to plot. Relevant only when plotting counts, ignored otherwise.
- what a character string taking its value in c("counts", "percents", "proportions"). What must be printed.
- ... further arguments passed to function graphics::boxplot.matrix

Value

a list as in graphics::boxplot.
deviate_ends  
Return a deviate vector with deviates at the extreme rank

Description
Based on the formula given in Jolliffe and Primo (2008) this function returns a vector with the right properties to test the presence of a deviate to flatness at both extreme ranks in a rank histogram.

Usage
deviate_ends(k)

Arguments
k  
an integer. The number of components.

Details
Although the deviate vector is not a rank histogram, this function returns an rkhist object for the sake of simplicity. This allows for instance to plot the vector.

Value
An rkhist object.

References

deviate_linear  
Return a deviate vector with a linear trend

Description
Based on the formula given in Jolliffe and Primo (2008) this function returns a vector with the right properties to test the presence of a linear deviate to flatness in a rank histogram.

Usage
deviate_linear(k)

Arguments
k  
an integer. The number of components.
deviate_U

Details

Although the deviate vector is not a rank histogram, this function returns an rkhist object for the sake of simplicity. This allows for instance to plot the vector.

Value

An rkhist object.

References


deviate_U

Return a deviate vector with a U-shape trend

Description

Based on the formula given in Jolliffe and Primo (2008) this function returns a vector with the right properties to test the presence of a U-shape deviate to flatness in a rank histogram.

Usage

deviate_U(k)

Arguments

k

an integer. The number of components.

Details

Although the deviate vector is not a rank histogram, this function returns an rkhist object for the sake of simplicity. This allows for instance to plot the vector.

Value

An rkhist object.

References

deviate_V

Return a deviate vector with a V-shape trend

Description

Based on the formula given in Jolliffe an Primo (2008) this function returns a vector with the right properties to test the presence of a V-shape deviate to flatness in a rank histogram.

Usage

deviate_V(k)

Arguments

k an integer. The number of components.

Details

Although the deviate vector is not a rank histogram, this function returns an rkhist object for the sake of simplicity. This allows for instance to plot the vector.

Value

An rkhist object.

References


deviate_wave

Return a deviate vector with a wave-shape trend

Description

This function returns a deviate vector with the right properties to test the presence of a deviate to flatness with a sine trend in a rank histogram, as introduced in (Zamo et al. 2021) or (Zamo, 2016).

Usage

deviate_wave(k)

Arguments

k an integer. The number of components.
**ensembles**

**Details**

Although the deviate vector is not a rank histogram, this function returns an `rkhist` object for the sake of simplicity. This allows for instance to plot the vector.

**Value**

An `rkhist` object.

**References**


---

**ensembles**

*Ensemble forecasts of temperature and associated observation*

---

**Description**

This is a data set containing the forecasts of five ensemble weather prediction models for two-meter temperature from March, 2019 to March, 2021.

**Usage**

`ensembles`

**Format**

A named list with five entries, each containing a `data.table` with 731 rows and a varying number of columns (depending on the number of members).

- **date_run** initial condition date (with format YYYY-MM-DD)
- **latitude** latitude of the forecast location (in degrees)
- **longitude** longitude of the forecast location (in degrees)
- **1 ... M** member forecast (M members) in Kelvins
- **T** the measured two-meter air temperature in Kelvins
Details

The five ensemble models are named CWAO (20 perturbed members, from ECCC), DEMS (11 perturbed members, from NCMRWF), ECMF (50 members, from ECMWF), EGRR (17 perturbed members, from UKMO) and RKSL (24 perturbed members, from KMA).

The forecasts are the ones at the nearest grid point to Toulouse-Blagnac station (France) in the TIGGE dataset. The observation is the two-meter height temperature measured at this same station, at 06UTC. The forecast initial time is 00UTC, with a 30 hour lead-time.

Source

https://apps.ecmwf.int/datasets/data/tigge/levtype=sfc/type=pf/
https://donneespubliques.meteofrance.fr/?fond=produit&id_produit=91&id_rubrique=32

flatness

flatness: a package to assess the flatness of (rank) histograms

Description

The flatness package offers tools (scores, tests, ...) to compute histograms and assess whether they are flat.

Details

The S3 generic function rkhist allows to compute one or several rank histograms from ensemble forecasts and corresponding scalar observations. (In Meteorology an ensemble forecast is a set of forecasts for the same variable, aimed at assessing the forecasting uncertainty). It creates an object with class rkhist that can then be plotted and printed.

Flatness of (rank) histograms may then be tested with function JP_test that implements the Jolliffe-Primo flatness tests. This test requires a set of deviance vectors, some of which can be provided with functions named deviate_XXX. The user can easily implement its own deviate-returning function (please see details in get_deviates on how to do this). Functions is_JP_ready and make_JP_ready are provided to ensure that a set of deviate vectors meet the requirements to be used in the Jolliffe-Primo tests. The result of the test is stored in an object with class JPtest that can be printed or drawn with the function lattice::levelplot.

Flatness indices can be computed with the S3 generic function flatness_indices.

See the vignette for further details and an illustration with the datasets ensembles and ppensembles provided with this package.
**Description**

S3 generic function that computes and returns indices of flatness of one or several (rank) histograms, presented in Wilks (2019).

**Usage**

```r
flatness_indices(rkhists, ...)
```

**Arguments**

- `rkhists`: an object containing (rank) histograms
- `...`: other arguments

**Details**

Currently the implemented flatness indices are the chi-square statistics, the reliability index and the entropy.

**Value**

The expected returned object is a matrix, with one flatness index in each column for each rank histogram (row-wise). The columns should be named, with "chisq" for the chi-square statistics, "RI" for the reliability index and "entropy" for the entropy.

**References**

Arguments

- `rkhists`: an object containing (rank) histograms
- `...`: other arguments

Value

No return value, called for side effects (generate an error message).

---

**flatness_indices.matrix**

Method for S3 generic function `flatness_indices` and objects with class `matrix`

Description

This function is the method called when using the S3 generic function `flatness_indices` with an object of class `matrix`.

Usage

```r
# S3 method for class 'matrix'
flatness_indices(rkhists, col_wise = FALSE, ...)
```

Arguments

- `rkhists`: an object containing (rank) histograms
- `col_wise`: a logical. Are the rank histograms stored column wise, that is one rank histogram in each column?
- `...`: other arguments

Details

The input matrices is supposed to contain one rank histograms in each row, unless `col_wise == TRUE`.

Value

See `Value` in function `flatness_indices`
**get_deviates**

Return a set of vectors with chosen shapes

---

**Description**

This function returns an *rkhist* object containing vectors with chosen shapes or trends. This is intended to be used to apply the Jolliffe-Primo flatness tests of rank histograms (see Jolliffe and Primo, 2008).

**Usage**

```r
get_deviates(k, shapes = c("linear", "U", "wave"), constrain = FALSE)
```

**Arguments**

- `k`: an integer. The number of possible ranks.
- `shapes`: a vector of character strings. The required shapes of the vectors.
- `constrain`: a logical. If TRUE the returned set of vectors is constrained to be orthonormal, with each vector having components summing to 0. This is required to use the vectors in the Jolliffe-Primo flatness test.

**Details**

The convention is that each row of the *rkhist* object contains a vector. It is not required that the set be a basis.

For each shape in `shapes` this function calls a function named 'deviate_shape' with one argument `k`. Some pre-coded functions already exist but the user can easily add its own by following this naming convention. The added function must have only one argument `k` and return an *rkhist* object. It is advised that the returned deviate vector’s components should sum to 0 and have a unit module. But this can be imposed by setting the argument `constrain` to TRUE.

If `constrain == TRUE` the vector set is modified to have the right properties to be used in the Jolliffe-Primo test, through the Grahm-Schmidt method. It is strongly advised to plot the resulting set with function `flatness::plot`, since this transformation may greatly change the shape of the original vectors.

**Value**

An *rkhist* object with each row representing a vector of deviation from flatness.

**References**

is_JP_ready

Check whether a set of vectors obeys the constraints necessary to be used in the Jolliffe-Primo flatness test

Description

This function checks whether a set of vectors has the following two properties:

- the set is orthonormal
- each vector has components summing to zero (that is, it is a deviation)

Usage

is_JP_ready(x, verbose = TRUE, tol = 1e-04)

Arguments

x      an rkhist or matrix object containing the vectors to check (by row).
verbose a logical. Should the result of the check be displayed?
tol     a positive numeric. The accepted tolerance for the constraints.

Value

A list with entries

isOk   TRUE if the set obeys the constraints, FALSE otherwise
tol     the tolerance allowed on the constraints
crossprod the cross product of the vectors
sums    the sum of the components of each vector
x        the checked vectors

Examples

de viates <- get_deviates(k = 36, shapes = c("linear", "U", "V", "ends", "wave"))
plot(deviates)
isJPOK <- is_JP_ready(deviates)
JPdeviates <- make_JP_ready(deviates)
plot(JPdeviates)
JPcheck <- is_JP_ready(JPdeviates)
is_orthonormal  

Check whether a vector set is orthonormal

Description

Check whether a vector set is orthonormal

Usage

is_orthonormal(set, tol = 1e-04)

Arguments

set  
a matrix. The convention used here is that each row of set contains a vector.

tol  
a positive numeric. The accepted tolerance for the conditions of orthonormality.

Value

TRUE if the set of vectors is orthonormal, FALSE otherwise.

JP_test  

Implementation of the Jolliffe-Primo flatness tests for rank histograms

Description

Given a matrix of rank histograms and an orthonormal set of deviate vector(s), this function computes the projection components, test statistics and p-values of the Jolliffe-Primo flatness test for each inputted rank histogram. See Jolliffe and Primo (2008) for details of the method.

Usage

JP_test(rkhists, deviates, ...)

Arguments

rkhists  
an rkhist, a matrix, or any other object that can be coerced to a matrix. It contains the rank histogram(s) whose flatness must be tested (one in each row).

deviates  
the matrix containing the deviate vectors used for testing. Each row contains a deviate vector: the vector set must be orthonormal, and each deviate vector must have its components summing to zero.

...

further arguments (currently not used).

Details

Note that the test statistics and p-values of the projections over the residual vector (after removing all the projection on the deviates) are also computed and returned.
Value

A list (with additional first class JPtest) with the following entries:

- **test** an array containing the result of the Jolliffe-Primo test(s). The first dimension is of length three (the projection over the deviate vectors, the test statistics and the p-values), the second and third dimensions correspond to the rank histogram and the test, respectively

- **deviates** the set of deviate vectors used in the test

- **rkhist** the tested rank histogram(s) (an rkhist object).

References


Examples

```r
require(lattice)
require(xtable)
M <- 15
N <- 100
n <- 20
fcsts <- vector("list", n)
names(fcsts) <- letters[1:n]
obs <- rnorm(N)
for (i in 1:n) {
  fcsts[[i]] <- matrix(rnorm(M*N), ncol = M)
}
rkhsts <- rkhist(fcsts, obs)
deviates <- get_deviates(M + 1)
test <- JP_test(rkhsts, deviates)
print(test)
for (what in c("projections", "statistics", "pvalues")){
  levelplot(test, what = what, main = what, rotate = what == "pvalues")
}
xtable(test$test["pvalues", ,])
xtable(t(test$test["pvalues", ,]))
```

---

levelplot.JPtest  
*Levelplot method for data in a JPtest object*

Description

Plot a chosen result matrix contained in the test entry of a JPtest object. The underlying function is the lattice::levelplot.matrix function.

Usage

```r
## S3 method for class 'JPtest'
levelplot(JPobj, what = "pvalues", rotate = TRUE, plot = TRUE, ...)
```
**make_JP_ready**

**Arguments**

- **JObj**: the JPtest object to plot.
- **what**: a character or an integer. Which component of the test entry in JObj should be plotted? Can be any of `projections`, `statistics`, `pvalues` or, respectively, 1, 2 or 3.
- **rotate**: a logical. Should the matrix containing the data be transposed before plotting? By default in `lattice::levelplot.matrix`, the rows of the plotted matrix correspond to the x-axis.
- **plot**: a logical. Should the lattice object be plotted?
- **...**: further arguments passed to the `lattice::levelplot` function.

**Value**

An object of class `trellis`, invisibly.

---

**Description**

The Jolliffe-Primo flatness test requires deviate vectors that are orthonormal and that each has components summing to zero. This function ensures this by using, if it is required, the Graham-Schmidt method to make the set orthonormal and also makes the sum of each vector's components equal to zero.

**Usage**

```r
make_JP_ready(x, verbose = TRUE)
```

**Arguments**

- **x**: an `rkhist` object.
- **verbose**: a logical. Should the result of the check be displayed?

**Details**

Note this procedure may greatly change the shape of vectors.

**Value**

An `rkhist` object, containing the modified vectors set obeying the requirements for the Jolliffe-Primo flatness test.
orthonormalize

Make a vector set orthonormal

**Description**

This function uses the Grahm-Schmidt method to make a set of vector orthonormal.

**Usage**

orthonormalize(set)

**Arguments**

set

a matrix. The convention used here is that each row of set contains a vector.

**Value**

A matrix with the same dimension as set containing an orthonormal set of vector. The vector are stored in each row.

---

plot.rkhist

Method for function plot an S3 object of class rkhist

**Description**

Plot a rank histogram stored in an object of S3 class rkhist, with an horizontal dashed line indicating the expected count for a perfectly flat rank histogram.

**Usage**

## S3 method for class 'rkhist'
plot(x, mini = min(0, min(x)), what = "counts", ...)

**Arguments**

x

an rkhist S3 object containing the rank histogram to plot. See function flatness::rkhist.

mini

a numeric. The minimum value in the ylim argument used to plot. Relevant only when plotting counts, ignored otherwise.

what

a character string taking its value in c("counts", "percents", "proportions"). What must be plotted.

... other arguments passed to function base::plot.default. Some arguments are already used in this function, and may not be changed.

**Value**

NULL, invisibly.
ppensembles

Post-processed ensemble forecasts of temperature and associated observation

Description

This is a dataset containing the post-processed forecasts of five ensemble weather prediction models for two-meter temperature from March, 2019 to March, 2021.

Usage

ppensembles

Format

A named list with five entries, each containing a data.table with 731 rows.

- **date_run**: initial condition date (with format YYYY-MM-DD)
- **latitude**: latitude of the forecast location (in degrees)
- **longitude**: longitude of the forecast location (in degrees)
- **1 ... 30**: forecast in Kelvins sampled from the gaussian distribution. The forecasts are sorted in increasing order.
- **T**: the measured two-meter air temperature in Kelvins

Details

Each ensemble has been post-processed with the non homogeneous regression technique, described in Gneiting et al.(2005). In a nutshell the true distribution is supposed to be gaussian, with mean and standard deviation being a linear function of the ensemble mean and standard deviation (respectively). The intercept and slope of each regression is determined by minimizing the CRPS over a 60-day sliding window. The forecast in the data set is a sample of 30 values from this gaussian distribution.

The five ensemble models are named based on the raw ensemble: CWAO (from ECCC), DEMS (from NCMRWF), ECMF (from ECMWF), EGRR (from UKMO) and RKSL (from KMA).

The raw forecasts are the ones at the nearest grid point to Toulouse-Blagnac station (France) in the TIGGE data set. The observation is the two-meter height temperature measured at this same station, at 06UTC. The forecast initial time is 00UTC, with a 30 hour lead-time.

Source

https://apps.ecmwf.int/datasets/data/tigge/levtype=sfc/type=pf/
https://donneespubliques.meteofrance.fr/?fond=produit&id_produit=91&id_rubrique=32
References


---

`print.JPtest`  
*Print method for a JPtest object*

**Description**

Print method for a JPtest object

**Usage**

```r
## S3 method for class 'JPtest'
print(x, what = c("projections", "statistics", "pvalues"), ...)
```

**Arguments**

- `x`: the JPtest object that must be printed.
- `what`: a character vector. What must be printed? Can contain any or several among c("projections", "statistics", "pvalues").
- `...`: further arguments. Passed to `print` method for matrix objects.

**Value**

No return value, called for side effects.

---

`print.rkhist`  
*Print an S3 object of class rkhist*

**Description**

Print an S3 object of class rkhist, using the same layout as `base::print.table`.

**Usage**

```r
## S3 method for class 'rkhist'
print(x, what = "counts", ...)
```

**Arguments**

- `x`: an rkhist S3 object containing the rank histogram to print. See function `flatness::rkhist`.
- `what`: a character string taking its value in c("counts", "percents", "proportions"). What must be printed.
- `...`: further arguments, passed to `print`.
**Value**

No return value, called for side effects.

---

**rbind_rkhists**

*Stack objects of class rkhist*

---

**Description**

Use `rbind` to stack several rank histograms (stored in S3 objects of class `rkhist`, or matrix, or vectors).

**Usage**

```
rbind_rkhists(..., names = NULL)
```

**Arguments**

- `...`: matrix, `rkhist` objects or vectors to bind by row.
- `names`: a character vector of names to give to each rank histogram (i.e. each row of the resulting matrix). Although its length must be equal to the number of row in the resulting matrix, NAs are allowed.

**Value**

An object of S3 class `rkhist`

---

**rkhist**

*Return the rank histogram of an observation in an ensemble forecast*

---

**Description**

This S3 generic function is intended to compute the rank of each observation when pooled with its associated ensemble forecast and return the count in each rank (i.e. the rank histogram).

**Usage**

```
rkhist(fcst, obs, ...)
```

**Arguments**

- `fcst`: an object containing the ensemble forecasts.
- `obs`: an object containing the observation associated to the forecast in `fcst`.
- `...`: additional arguments.
Details

For new methods, the output should be an object of class `c("rkhist", "matrix")`, with one rank histogram in each row. Rows may be named.

Value

An S3 object of class `rkhist` (indeed a matrix containing the count for each rank, with class "rkhist"). Each row of the matrix contains the counts for one rank histogram.

Examples

```r
set.seed(42)
N <- 1000
M <- 20
fcst <- matrix(rnorm(N*M), ncol = M)
fcest2 <- matrix(rnorm(N*M), ncol = M)
obs <- rnorm(N)
## Computation of one rank histogram
# Named
rkh <- rkhist(fcst, obs, names = "a")
print(rkh)
plot(rkh)
# Unnamed
rkh2 <- rkhist(fcest2, obs)
print(rkh2)
plot(rkh2)
## Computation of two rank histograms, from a list of forecasts, with the
## same observation vector
fcstsl <- list(fcst, fcst2)
rkhsl <- rkhist(fcstsl, obs, names = c("a", NA))
print(rkhsl)
plot(rkhsl)
## Concatenation of two rank histograms, with different names
rkhs <- rbind_rkhists(rkh, rkh2, names = letters[3:4])
rownames(rkhs)
print(rkhs)
plot(rkhs)
```

---

rkhist.data.frame  
Method of S3 generic function rkhist for data.frame objects

Description

This is the method called when the fcst argument in function rkhist is a data frame.
Usage

```r
## S3 method for class 'data.frame'
rkhist(fcst, obs, names = NULL, ...)
```

Arguments

- **fcst**: an object containing the ensemble forecasts. It must be a data frame, each row containing a forecast.
- **obs**: an object containing the observation associated to the forecast in `fcst`. It can be a vector or a matrix, whose length is the same as the number of row in `fcst`
- **names**: a character vector. The row names in the returned `rkhist` object.
- **...**: additional arguments, passed to function `base::rank`.

Value

An S3 object of class `rkhist` (indeed just 1-row matrix containing the count for each rank, with class == "rkhist").

---

rkhist.default  

Default method for S3 generic function `rkhist`

Description

Just generate an error.

Usage

```r
## Default S3 method:
rkhist(fcst, obs, ...)
```

Arguments

- **fcst**: an object containing the ensemble forecasts.
- **obs**: an object containing the observation associated to the forecast in `fcst`.
- **...**: additional arguments.

Value

No return value, called for side effects (generate an error message).
rkhist.list  Method of S3 generic function rkhist for list objects

Description

This is the method called when the fcst argument in function rkhist is a list.

Usage

## S3 method for class 'list'
rkhist(fcst, obs, names = NULL, ...)

Arguments

fcst  an object containing the ensemble forecasts. It must be a list, each entry containing the forecast of one ensemble. The forecast of each ensemble is a matrix whose rows contain one forecast.

obs  an object containing the observation associated to the forecast in fcst. It can be a vector or a list, whose length is the same as the number of row in fcst. If a vector, it is used as the observation vector for all the ensemble in fcst. If it is a list, each entry is associated to the same entry in fcst.

names  a character vector. The row names in the returned rkhist object. If missing, the row names are the names of the list fcst (if any).

...  additional arguments, passed to function base::rank.

Value

An S3 object of class rkhist (indeed an N-row matrix containing the count for each rank, with class 'rkhist'). N is the length of fcst, i.e. the number of ensembles.

rkhist.matrix  Method of S3 generic function rkhist for matrix objects

Description

This is the method called when the fcst argument in function rkhist is a matrix.

Usage

## S3 method for class 'matrix'
rkhist(fcst, obs, names = NULL, ...)

...
Arguments

fcst an object containing the ensemble forecasts. It must be a matrix, each row containing a forecast.

obs an object containing the observation associated to the forecast in fcst. It can be a vector or a matrix, whose length is the same as the number of row in fcst

names a character vector. The row names in the returned rkhist object.

... additional arguments, passed to function base::rank.

Value

An S3 object of class rkhist (indeed just 1-row matrix containing the count for each rank, with class == "rkhist").
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