Package ‘flexmet’

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Type Package

Title Flexible Latent Trait Metrics using the Filtered Monotonic Polynomial Item Response Model

Version 1.0.0.0

Description Application of the filtered monotonic polynomial (FMP) item response model to flexibly fit item response models. The package includes tools that allow the item response model to be build on any monotonic transformation of the latent trait metric, as described by Feuerstahler (2016) <http://hdl.handle.net/11299/182267>.

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Find the Greek-Letter Parameterization corresponding to a b Vector of Item Parameters

Description

Convert the b vector of item parameters (polynomial coefficients) to the corresponding Greek-letter parameterization (used to ensure monotonicity).

Usage

b2greek(bvec, eps = 1e-08)

Arguments

bvec b vector of item parameters (i.e., polynomial coefficients).

eps Convergence tolerance.

Details

See greek2b for more information about the b (polynomial coefficient) and Greek-letter parameterizations of the FMP model.

Value

A vector of item parameters in the Greek-letter parameterization.

References


See Also

greek2b
fmp

Examples

(bvec <- greek2b(xi = 0, omega = 1, alpha = c(.1, .1), tau = c(-2, -2)))
## 0.0 0.0 0.0 0.0 2.71828183 -0.54365637 0.29961860 -0.03950623 0.01148330

(b2greek(bvec))
## 0.0 1.0 0.1 -2.0 0.1 -2.0

fmp

Estimate FMP Item Parameters

Description

Estimate FMP item parameters for a single item using user-specified theta values (fixed-effects) using fmp_1, or estimate FMP item parameters for multiple items using fixed-effects or random-effects with fmp.

Usage

fmp_1(dat, k, tsur, start_vals = NULL, method = "BFGS", ...)

fmp(dat, k, start_vals = NULL, em = TRUE, eps = 1e-04, n_quad = 49, method = "BFGS", max_em = 500, ...)

Arguments

dat Vector of 0/1 item responses for N (# subjects) examinees.
k Vector of item complexities for each item, see details.
tsur Vector of N (# subjects) surrogate theta values.
start_vals Start values. For fmp_1, a vector of length 2k+2 in the following order:
If k = 0: (xi, omega)
If k = 1: (xi, omega, alpha1, tau1)
If k = 2: (xi, omega, alpha1, tau1, alpha2, tau2)
and so forth. For fmp, add start values for item 1, followed by those for item 2, and so forth. For further help, first fit the model without start values, then inspect the outputted parmat data frame.
method Optimization method passed to optim.
em Logical, use random-effects estimation using the EM algorithm? If FALSE, fixed effects estimation is used with theta surrogates.
eps Convergence tolerance for the EM algorithm. The EM algorithm is said to converge is the maximum absolute difference between parameter estimates for successive iterations is less than eps. Ignored if em = FALSE.
n_quad Number of quadrature points for EM integration. Ignored if em = FALSE
max_em Maximum number of EM iterations.
... Additional arguments passed to optim.
Details

The FMP item response function for a single item is specified using the composite function,

\[ P(\theta) = [1 + \exp(-m(\theta))]^{-1}, \]

where \( m(\theta) \) is an unbounded and monotonically increasing polynomial function of the latent trait \( \theta \).

The item complexity parameter \( k \) controls the degree of the polynomial:

\[ m(\theta) = b_0 + b_1 \theta + b_2 \theta^2 + \ldots + b_{2k+1} \theta^{2k+1}, \]

where \( 2k + 1 \) equals the order of the polynomial, \( k \) is a nonnegative integer, and

\[ b = (b_0, b_1, \ldots, b(2k + 1))' \]

are item parameters that define the location and shape of the IRF. The vector \( b \) is called the b-vector parameterization of the FMP Model. When \( k = 0 \), the FMP IRF equals

\[ P(\theta) = [1 + \exp(-(b_0 + b_1 \theta))]^{-1}, \]

and is equivalent to the slope-threshold parameterization of the two-parameter item response model. For \( m(\theta) \) to be a monotonic function, the FMP IRF can also be expressed as a function of the vector

\[ \gamma = (\xi, \omega, \alpha_1, \tau_1, \alpha_2, \tau_2, \ldots, \alpha_k, \tau_k)' \]

The \( \gamma \) vector is called the Greek-letter parameterization of the FMP model. See Feuerstahler (2016) or Liang & Browne (2015) for details about the relationship between the b-vector and Greek-letter parameterizations.

Value

- **bmat**: Matrix of estimated b-matrix parameters, each row corresponds to an item, and contains \( b_0, b_1, \ldots, b(\max(k)) \).
- **parmat**: Data frame of parameter estimation information, including the Greek-letter parameterization, starting value, and parameter estimate.
- **k**: Vector of item complexities chosen for each item.
- **log_lik**: Model log likelihood.
- **mod**: Optimization information, including output from optim.
- **AIC**: Model AIC.
- **BIC**: Model BIC.

References


Examples

```r
set.seed(2342)
mat <- sim_bmat(n_items = 5, k = 2)$mat
theta <- rnorm(50)
dat <- sim_data(bmat = mat, theta = theta)

## fixed-effects estimation for item 1

tsur <- get_surrogates(dat)
# k = 0
fmp0_it_1 <- fmp_1(dat = dat[, 1], k = 0, tsur = tsur)
# k = 1
fmp1_it_1 <- fmp_1(dat = dat[, 1], k = 1, tsur = tsur)
# k = 2
fmp2_it_1 <- fmp_1(dat = dat[, 1], k = 2, tsur = tsur)

## fixed-effects estimation for all items
fmp0_fixed <- fmp(dat = dat, k = 0, em = FALSE)

## random-effects estimation for all items
fmp0_random <- fmp(dat = dat, k = 0, em = TRUE)
```
get_surrogates

Find Theta Surrogates

Description

Compute surrogate theta values as the set of normalized first principal component scores.

Usage

get_surrogates(dat)

Arguments

dat                Matrix of binary item responses.

Details

Compute surrogate theta values as the normalized first principal component scores.

Value

Vector of surrogate theta values.

References


Examples

```r
set.seed(2342)
bmat <- sim_bmat(n_items = 5, k = 2)$bmat
theta <- rnorm(50)
dat <- sim_data(bmat = bmat, theta = theta)
tsur <- get_surrogates(dat)
```
Find the $b$ Vector from a Greek-Letter Parameterization of Item Parameters.

**Description**

Convert the Greek-letter parameterization of item parameters (used to ensure monotonicity) to the $b$-vector parameterization (polynomial coefficients).

**Usage**

greek2b(xi, omega, alpha = NULL, tau = NULL)

**Arguments**

- `xi`: see details
- `omega`: see details
- `alpha`: see details, vector of length k, set to NULL if k = 0
- `tau`: see details, vector of length k, set to NULL if k = 0

**Details**

For

$$m(\theta) = b_0 + b_1 \theta + b_2 \theta^2 + \cdots + b_{2k} \theta^{2k+1}$$

to be a monotonic function, a necessary and sufficient condition is that its first derivative,

$$p(\theta) = a_0 + a_1 \theta + \cdots + a_{2k} \theta^{2k},$$

is nonnegative at all theta. Here, let

$$b_0 = \xi$$

be the constant of integration and

$$b_s = a_{s-1}/s$$

for $s = 1, 2, \ldots, 2k + 1$. Notice that $p(\theta)$ is a polynomial function of degree $2k$. A nonnegative polynomial of an even degree can be re-expressed as the product of $k$ quadratic functions.

If $k \geq 1$:

$$p(\theta) = \exp \omega \Pi_{s=1}^{k}[1 - 2\alpha_s \theta + (\alpha_s^2 + \exp(\tau_s))\theta^2]$$

If $k = 0$:

$$p(\theta) = 0.$$

**Value**

A vector of item parameters in the $b$ parameterization.
References


See Also

b2greek

Examples

```r
(bvec <- greek2b(xi = 0, omega = 1, alpha = .1, tau = -1))
## 0.0000000  2.7182818 -0.2718282  0.3423943

(b2greek(bvec))
## 0.0 1.0 0.1 -1.0
```

---

### iif_fmp  
*FMP Item Information Function*

**Description**

Find FMP item information for user-supplied item and person parameters.

**Usage**

```r
iif_fmp(theta, bmat, cvec = NULL, dvec = NULL)
```

**Arguments**

- `theta`  
  Vector of latent trait parameters.

- `bmat`  
  Items x parameters matrix of FMP item parameters (or a vector of FMP item parameters for a single item).

- `cvec`  
  Optional vector of lower asymptote parameters. If `cvec = NULL`, then all lower asymptotes set to 0.

- `dvec`  
  Optional vector of upper asymptote parameters. If `dvec = NULL`, then all upper asymptotes set to 1.

**Value**

Matrix of item information.
Examples

# plot the IIF for an item with k = 2

set.seed(2342)
bmat <- sim_bmat(n_items = 1, k = 2)$bmat
theta <- seq(-3, 3, by = .01)
information <- iif_fmp(theta = theta, bmat = bmat)
plot(theta, information, type = 'l')

---

**int_mat**

**Numerical Integration Matrix**

**Description**

Create a matrix for numerical integration.

**Usage**

int_mat(distr = dnorm, par1 = 0, par2 = 1, lb = -4, ub = 4,
npts = 10000)

**Arguments**

- **distr**: A density function with two user-specified parameters. Defaults to the normal distribution, but any density function is permitted.
- **par1**: First parameter passed to distr.
- **par2**: Second parameter passed to distr.
- **lb**: Lower bound of range over which to numerically integrate.
- **ub**: Upper bound of range over which to numerically integrate.
- **npts**: Number of integration points.

**Value**

Matrix of two columns. Column 1 is a sequence of x-coordinates, and column 2 is a sequence of y-coordinates from a normalized distribution.

**See Also**

rimse th_est_ml th_est_eap sl_link hb_link
**Polynomial Functions**

**inv_poly**

**Description**

Evaluate a forward or inverse (monotonic) polynomial function.

**Usage**

```plaintext
inv_poly(x, coefs, lb = -1000, ub = 1000)
fw_poly(y, coefs)
```

**Arguments**

- `x` Scalar polynomial function input.
- `coefs` Vector of coefficients that define a monotonic polynomial, see details.
- `lb` Lower bound of the search interval.
- `ub` Upper bound of the search interval.
- `y` Scalar polynomial function output.

**Details**

\[ x = t_0 + t_1 y + t_2 y^2 + ... \]

Then, for coefs = \((t_0, t_1, t_2, ...)^T\), this function finds the corresponding \(y\) value (inv_poly) or \(x\) value (fw_poly).

---

**FMP Item Response Function**

**irf_fmp**

**Description**

Find FMP item response probabilities for user-supplied item and person parameters.

**Usage**

```plaintext
irf_fmp(theta, bmat, cvec = NULL, dvec = NULL)
```
Arguments

- **theta**: Vector of latent trait parameters.
- **bmat**: Items x parameters matrix of FMP item parameters (or a vector of FMP item parameters for a single item).
- **cvec**: Optional vector of lower asymptote parameters. If cvec = NULL, then all lower asymptotes set to 0.
- **dvec**: Optional vector of upper asymptote parameters. If dvec = NULL, then all upper asymptotes set to 1.

Value

Matrix of item response probabilities.

Examples

```r
# plot the IRF for an item with k = 2
set.seed(2342)
bmat <- sim_bmat(n_items = 1, k = 2)$bmat
theta <- seq(-3, 3, by = .01)
probability <- irf_fmp(theta = theta, bmat = bmat)
plot(theta, probability, type = 'l')
```

Description

Link two sets of FMP item parameters using linear or nonlinear transformations of the latent trait.

Usage

```r
sl_link(bmat1, bmat2, cvec1 = NULL, cvec2 = NULL, dvec1 = NULL, dvec2 = NULL, k_theta, int = int_mat(), ...)
```

```r
hb_link(bmat1, bmat2, cvec1 = NULL, cvec2 = NULL, dvec1 = NULL, dvec2 = NULL, k_theta, int = int_mat(), ...)
```
Arguments

- **bmat1**: FMP item parameters on an anchor test.
- **bmat2**: FMP item parameters to be rescaled.
- **cvec1**: Vector of lower asymptote parameters for the anchor test.
- **cvec2**: Vector of lower asymptote parameters corresponding to the rescaled item parameters.
- **dvec1**: Vector of upper asymptote parameters for the anchor test.
- **dvec2**: Vector of upper asymptote parameters corresponding to the rescaled item parameters.
- **k_theta**: Complexity of the latent trait transformation (k_theta = 0 is linear, k_theta > 0 is nonlinear).
- **int**: Matrix with two columns, used for numerical integration. Column 1 is a grid of theta values, column 2 are normalized densities associated with the column 1 values.
- **...**: Additional arguments passed to optim.

Details

The goal of item parameter linking is to find a metric transformation such that the fitted parameters for one test can be transformed to the same metric as those for the other test. In the Haebara approach, the overall sum of squared differences between the original and transformed individual item response functions is minimized. In the Stocking-Lord approach, the sum of squared differences between the original and transformed test response functions is minimized. See Feuerstahler (2016) for details on linking with the FMP model.

Value

- **par**: (Greek-letter) parameters estimated by optim.
- **value**: Value of the minimized criterion function.
- **counts**: Number of function counts in optim.
- **convergence**: Convergence criterion given by optim.
- **message**: Message given by optim.
- **tvec**: Vector of theta transformation coefficients \((t = t_0, \ldots, t(2k_\theta + 1))\)
- **bmat**: Transformed bmat2 item parameters.

References


Examples

```r
dat1 <- sim_data(bmat = bmat, theta = theta1)
dat2 <- sim_data(bmat = bmat, theta = theta2)

# estimate each model with fixed-effects and k = 0
fmp0_1 <- fmp(dat = dat1, k = 0, em = FALSE)
fmp0_2 <- fmp(dat = dat2, k = 0, em = FALSE)

# Stocking-Lord linking
sl_res <- sl_link(bmat1 = fmp0_1$bmat[, 1:5],
                 bmat2 = fmp0_2$bmat[, 1:5],
                 k_theta = 0)

## Not run:
hb_res <- hb_link(bmat1 = fmp0_1$bmat[, 1:5],
                  bmat2 = fmp0_2$bmat[, 1:5],
                  k_theta = 0)

## End(Not run)
```

rimse

**Root Integrated Mean Squared Difference Between FMP IRFs**

Description

Compute the root integrated mean squared error (RIMSE) between two FMP IRFs.

Usage

```r
rimse(bvec1, bvec2, c1 = 0, d1 = 1, c2 = 0, d2 = 1, int = int_mat())
```

Arguments

- `bvec1` Either a vector of FMP item parameters or a function corresponding to a non-FMP IRF. Functions should have exactly one argument, corresponding to the latent trait.
either a vector of FMP item parameters or a function corresponding to a non-
FMP IRF. Functions should have exactly one argument, corresponding to the
latent trait.

c1        Lower asymptote parameter for bvec1. Ignored if bvec1 is a function.
d1        Upper asymptote parameter for bvec1. Ignored if bvec1 is a function.
c2        Lower asymptote parameter for bvec2. Ignored if bvec2 is a function.
d2        Upper asymptote parameter for bvec2. Ignored if bvec2 is a function.
int       Matrix with two columns, used for numerical integration. Column 1 is a grid
of theta values, column 2 are normalized densities associated with the column 1
values

Value

Root integrated mean squared difference between two IRFs.

References


Examples

```r
set.seed(2342)
bmat <- sim_bmat(n_items = 1, k = 2)$bmat
theta <- rnorm(500)
dat <- sim_data(bmat = bmat, theta = theta)

# k = 0
fmp0 <- fmp_1(dat = dat, k = 0, tsur = theta)

# k = 1
fmp1 <- fmp_1(dat = dat, k = 1, tsur = theta)

## compare estimated curves to the data-generating curve
rimse(fmp0$bmat, bmat)
rimse(fmp1$bmat, bmat)
```

---

**sim_bmat**

*Randomly Generate FMP Parameters*

**Description**

Generate monotonic polynomial coefficients for user-specified item complexities and prior distributions.
### Usage

```r
sim_bmat(n_items, k, xi_dist = c(-1, 1), omega_dist = c(-1, 1),
alpha_dist = c(-1, 0.5), tau_dist = c(-7, -1))
```

### Arguments

- **n_items**
  - Number of items for which to simulate item parameters.

- **k**
  - Either a scalar for the item complexity of all items or a vector of length `n_items` if different items have different item complexities.

- **xi_dist**
  - Vector of two elements indicating the lower and upper bounds of the uniform distribution from which to draw xsi parameters.

- **omega_dist**
  - Vector of two elements indicating the lower and upper bounds of the uniform distribution from which to draw omega parameters.

- **alpha_dist**
  - Vector of two elements indicating the lower and upper bounds of the uniform distribution from which to draw alpha parameters. Ignored if all `k = 0`.

- **tau_dist**
  - Vector of two elements indicating the lower and upper bounds of the uniform distribution from which to draw tau parameters. Ignored if all `k = 0`.

### Details

Randomly generate FMP item parameters for a given `k` value.

### Value

- **bmat**
  - Item parameters in the b parameterization (polynomial coefficients).

- **greekmat**
  - Item parameters in the Greek-letter parameterization

### Examples

```r
## generate FMP item parameters for 5 items all with k = 2
set.seed(2342)
pars <- sim_bmat(n_items = 5, k = 2)
pars$bmat

## generate FMP item parameters for 5 items with varying k values
set.seed(2432)
pars <- sim_bmat(n_items = 5, k = c(1, 2, 0, 0, 2))
pars$bmat
```
Simulate FMP Data

Description

Simulate data according to user-specified FMP item parameters and latent trait parameters.

Usage

```
sim_data(bmat, theta, cvec = NULL, dvec = NULL)
```

Arguments

- `bmat`: Matrix of FMP item parameters.
- `theta`: Vector of latent trait values.
- `cvec`: Optional vector of lower asymptote parameters. If `cvec = NULL`, then all lower asymptotes set to 0.
- `dvec`: Optional vector of upper asymptote parameters. If `dvec = NULL`, then all upper asymptotes set to 1.

Value

Matrix of randomly generated binary item responses.

Examples

```r
# generate binary item responses for normally distributed theta
# and 5 items with k = 2
set.seed(2342)
bmat <- sim_bmat(n_items = 5, k = 2)$bmat
theta <- rnorm(50)
dat <- sim_data(bmat = bmat, theta = theta)
```

Latent Trait Estimation

Description

Compute latent trait estimates using either maximum likelihood (ML) or expected a posteriori (EAP) trait estimation.
th_est_ml

Usage

th_est_ml(dat, bmat, cvec = NULL, dvec = NULL, lb = -4, ub = 4)

th_est_eap(dat, bmat, cvec = NULL, dvec = NULL, int = int_mat(npts = 33))

Arguments

| dat        | Data matrix of binary item responses with one column for each item. Alternatively, a vector of binary item responses for one person. |
| bmat       | Matrix of FMP item parameters, one row for each item. |
| cvec       | Vector of lower asymptote parameters, one element for each item. |
| dvec       | Vector of upper asymptote parameters, one element for each item. |
| lb         | Lower bound at which to truncate ML estimates. |
| ub         | Upper bound at which to truncate ML estimates. |
| int        | Matrix with two columns used for numerical integration in EAP. Column 1 contains the x coordinates and Column 2 contains the densities. |

Value

Matrix with two columns: est and either sem or psd

| est | Latent trait estimate |
| sem | Standard error of measurement (mle estimates) |
| psd | Posterior standard deviation (eap estimates) |

Examples

```r
set.seed(3453)
bmat <- sim_bmat(n_items = 20, k = 0)$bmat
theta <- rnorm(10)
dat <- sim_data(bmat = bmat, theta = theta)

## mle estimates
mles <- th_est_ml(dat = dat, bmat = bmat)

## eap estimates
eaps <- th_est_eap(dat = dat, bmat = bmat)
cor(mles[,1], eaps[,1])
# 0.9967317
```
**transform_b**

**Transform FMP Item Parameters**

**Description**

Given FMP item parameters for a single item and the polynomial coefficients defining a latent trait transformation, find the transformed FMP item parameters.

**Usage**

```
transform_b(bvec, tvec)
```

```
inv_transform_b(bstarvec, tvec)
```

**Arguments**

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>bvec</td>
<td>Vector of item parameters on the $\theta$ metric: $(b_0, b_1, b_2, b_3, ...)$</td>
</tr>
<tr>
<td>tvec</td>
<td>Vector of theta transformation polynomial coefficients: $(t_0, t_1, t_2, t_3, ...)$</td>
</tr>
<tr>
<td>bstarvec</td>
<td>Vector of item parameters on the $\theta^<em>$ metric: $(b^</em>_0, b^<em>_1, b^</em>_2, b^*_3, ...)$</td>
</tr>
</tbody>
</table>

**Details**

Equivalent item response models can be written

\[ P(\theta) = b_0 + b_1 \theta + b_2 \theta^2 + \cdots + b_{2k+1} \theta^{2k+1} \]

and

\[ P(\theta^*) = b^*_0 + b^*_1 \theta^* + b^*_2 \theta^{*2} + \cdots + b^*_{2k+1} \theta^{*2k+1} \]

where

\[ \theta = t_0 + t_1 \theta^* + t_2 \theta^{*2} + \cdots + t_{2k+1} \theta^{*2k+1} \]

**Value**

Vector of transformed FMP item parameters.

**Examples**

```r
## example parameters from Table 7 of Reise & Waller (2003)
## goal: transform IRT model to sum score metric

a <- c(0.57, 0.68, 0.76, 0.72, 0.69, 0.57, 0.53, 0.64,
       0.45, 1.01, 1.05, 0.50, 0.58, 0.58, 0.60, 0.59,
       1.03, 0.52, 0.59, 0.99, 0.95, 0.39, 0.50)
```
### convert from difficulties and discriminations to FMP parameters

\[ b \leftarrow c(0.87, 1.02, 0.87, 0.81, 0.75, -0.22, 0.14, 0.56, \\
1.69, 0.37, 0.68, 0.56, 1.70, 1.20, 1.04, 1.69, \\
0.76, 1.51, 1.89, 1.77, 0.39, 0.08, 2.02) \]

# theta transformation vector (k_theta = 3)

# see vignette for details about how to find tvec

\[ tvec \leftarrow c(-3.80789e+00, 2.14164e+00, -6.47773e-01, 1.17182e-01, \\
-1.20807e-02, 7.02295e-04, -2.13809e-05, 2.65177e-07) \]

# transform bmat

\[ bstarmat \leftarrow t(apply(bmat, 1, \text{transform}_b, tvec = tvec)) \]

# inspect transformed parameters

\[ \text{signif(head(bstarmat), 2)} \]

# plot test response function

# should be a straight line if transformation worked

\[ \text{curve(rowSums(irf_fmp(x, bmat = bstarmat)), xlim = c(0, 23),}
\]
\[ ylim = c(0, 23), xlab = expression(paste(theta,"*")),
\]
\[ ylab = "Expected Sum Score") \]

\[ \text{abline(0, 1, col = 2)} \]
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