Package ‘frailtyEM’

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**Type**  Package

**Title**  Fitting Frailty Models with the EM Algorithm

**Version**  1.0.1

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**URL**  https://github.com/tbalan/frailtyEM

**BugReports**  https://github.com/tbalan/frailtyEM/issues

**Description**  Contains functions for fitting shared frailty models with a semi-parametric baseline hazard with the Expectation-Maximization algorithm. Supported data formats include clustered failures with left truncation and recurrent events in gap-time or Andersen-Gill format. Several frailty distributions, such as the gamma, positive stable and the Power Variance Family are supported.

**License**  GPL (>= 2)

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**LazyData**  true

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**Imports**  Rcpp (>= 0.12.8), magrittr, msm, ggplot2, expint, tibble, Matrix, numDeriv

**LinkingTo**  Rcpp

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**Suggests**  dplyr, plotly, gridExtra, egg

**Collate**  'RcppExports.R' 'autoplot.emfrail.R' 'ca_test_cph.R'
            'ca_test_fit.R' 'em_fit.R' 'emfrail_aux.R' 'emfrail.R'
            'emfrail_arguments.R' 'emfrail_methods.R' 'fast_Estep.R'
            'frailtyEM.R' 'plot.emfrail.R' 'predict.emfrail.R'
            'print.emfrail.R' 'print.emfrail_summary.R' 'summary.emfrail.R'

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**autoplot**

Generic autoplot function

**Description**

The following is imported and then re-exported to avoid conflicts with ggplot2

**autoplot.emfrail**

Plots for emfrail objects using ggplot2

**Description**

Plots for emfrail objects using ggplot2

**Usage**

```r
## S3 method for class 'emfrail'
autoplot(object, type = c("hist", "hr", "pred", "frail"),
         newdata = NULL, lp = NULL, strata = NULL,
         quantity = "cumhaz", type_pred = c("conditional", "marginal"),
         conf_int = "adjusted", conf_level = 0.95, individual = FALSE, ...)
```
Arguments

object  emfrail object, typically result of emfrail()

type  One (or more) of hist for a histogram of the estimated frailty values, hr for a plot of the conditional and marginal hazard ratio between two cases, pred for the predicted conditional and marginal cumulative hazard or survival for one case, frail for a caterpillar plot of the ordered frailty estimates with confidence intervals, where available.

newdata  A data.frame with values of the covariates. For type == "hr" the hazard ratio between the first two rows of newdata is calculated. For type == "pred" the prediction for the first row of newdata is calculated.

lp  A numeric vector of values of the linear predictor, each corresponding to a case. For type == "hr" the hazard ratio between the first two values of lp is calculated. For type == "pred" the prediction for the first value of lp is calculated.

strata  The name of the strata (if applicable) for which the prediction should be made.

quantity  One of c("cumhaz", "survival") for type == "pred"; see quantity in predict.emfrail

type_pred  One of c("conditional", "marginal") for type == "pred"; see type in predict.emfrail

conf_int  One of c("regular", "adjusted") for type == "pred"; see conf_int in predict.emfrail

conf_level  The width of the confidence interval for type == "pred"; see conf_level in predict.emfrail

individual  Logical, for type == "pred" to be used for drawing a curve when the rows of newdata refer to the same individual; see individual in predict.emfrail

...  Further arguments to be passed on to ‘ggplot’ (ignored)

Value

A list of ggplot2 objects corresponding to the required plots, or one ggplot2 object if only one plot is selected

For the caterpillar plot, in the case of the gamma frailty model, the vertical lines represent the 0.025 and 0.975 quantiles of the posterior gamma distribution. For other distributions, this quantity is not easy to calculate (at least not in closed form) and only the frailty estimates are shown.

Note

It’s normal for autoplot to give a warning of the type Warning: Ignoring unknown aesthetics: id. This is because, in ggplot2 terms, the id aesthetic is not recognized. This is correct, and for any practical purpose this will not make a difference (you can safely ignore the warnings). However, this makes it easier to create an interactive plot out of the resulting object.

See Also

predict.emfrail, summary.emfrail, plot.emfrail.
Examples

```r
mod_rec <- emfrail(Surv(start, stop, status) ~ treatment + number + cluster(id), bladder1, 
control = emfrail.control(ca_test = FALSE, lik_ci = FALSE))

# Histogram of the estimated frailties
autoplot(mod_rec, type = "hist")

# Ordered estimated frailties (with confidence intervals, for gamma distribution)
autoplot(mod_rec, type = "frail")

# hazard ratio between placebo and pyridoxine
newdata1 <- data.frame(treatment = c("placebo", "pyridoxine"),
                        number = c(1, 3))

autoplot(mod_rec, type = "hr", newdata = newdata1)

# predicted cumulative hazard for placebo, and number = 1
autoplot(mod_rec, type = "pred", newdata = newdata1[1,])

# predicted survival for placebo, and number = 1
autoplot(mod_rec, type = "pred", quantity = "survival", newdata = newdata1[1,])

# predicted survival for an individual that switches from
# placebo to pyridoxine at time = 15
## Not run:
newdata2 <- data.frame(treatment = c("placebo", "pyridoxine"),
                        number = c(1, 3),
                        tstart = c(0, 15),
                        tstop = c(15, Inf))

autoplot(mod_rec, type = "pred", quantity = "survival", newdata = newdata2, individual = TRUE)
## End(Not run)
```

cal_test

Commenges-Andersen test for heterogeneity

Description

Commenges-Andersen test for heterogeneity

Usage

```r
cal_test(object, id = NULL)
```

Arguments

- **object**: A coxph object with a `cluster()` statement in the right-hand side of the formula.
- **id**: Optionally, a vector determining the grouping to be tested. See details.
Details

The Cox model with a `+cluster()` statement has the same point estimates as the one without that statement. The only difference is in the adjusted standard errors. In some cases, a model with `+cluster()` statements can't be fitted. For example, when there are no covariates. In that case, a vector may be passed on in the cluster argument.

Value

A named vector containing the test statistic, variance, and p-value

References


Examples

```r
mcox1 <- coxph(Surv(time, status) ~ rx + sex + cluster(litter),
    rats, model = TRUE, x = TRUE)
ca_test(mcox1)

mcox2 <- coxph(Surv(time, status) ~ 1, rats, x = TRUE)
ca_test(mcox2, rats$litter)
```

emfrail

Fitting semi-parametric shared frailty models with the EM algorithm

Description

Fitting semi-parametric shared frailty models with the EM algorithm

Usage

`emfrail(formula, data, distribution = emfrail_dist(),
          control = emfrail_control(), model = FALSE, model.matrix = FALSE,
          ...)

Arguments

formula A formula that contains on the left hand side an object of the type Surv and on the right hand side a `+cluster(id)` statement. Two special statements may also be used: `+strata()` for specifying a grouping column that will represent different strata and `+terminal()`
data A data.frame in which the formula argument can be evaluated
distribution An object as created by `emfrail_dist`
control An object as created by `emfrail_control`
model Logical. Should the model frame be returned?
model.matrix Logical. Should the model matrix be returned?
... Other arguments, currently used to warn about deprecated argument names

Details

The `emfrail` function fits shared frailty models for processes which have intensity
\[ \lambda(t) = z\lambda_0(t) \exp(\beta'x) \]
with a non-parametric (Breslow) baseline intensity \( \lambda_0(t) \). The outcome (left hand side of the formula) must be a `Surv` object.

If the object is `Surv(tstop, status)` then the usual failure time data is represented. Gap-times between recurrent events are represented in the same way. If the left hand side of the formula is created as `Surv(tstart, tstop, status)`, this may represent a number of things: (a) recurrent events episodes in calendar time where a recurrent event episode starts at `tstart` and ends at `tstop` (b) failure time data with time-dependent covariates where `tstop` is the time of a change in covariates or censoring (`status = 0`) or an event time (`status = 1`) or (c) clustered failure time with left truncation, where `tstart` is the individual’s left truncation time. Unlike regular Cox models, a major distinction is that in case (c) the distribution of the frailty must be considered conditional on survival up to the left truncation time.

The `+cluster()` statement specifies the column that determines the grouping (the observations that share the same frailty). The `+strata()` statement specifies a column that determines different strata, for which different baseline hazards are calculated. The `+terminal` specifies a column that contains an indicator for dependent censoring, and then performs a score test

The distribution argument must be generated by a call to `emfrail_dist`. This determines the frailty distribution, which may be one of gamma, positive stable or PVF (power-variance-function), and the starting value for the maximum likelihood estimation. The PVF family also includes a tuning parameter that differentiates between inverse Gaussian and compound Poisson distributions. Note that, with univariate data (at most one event per individual, no clusters), only distributions with finite expectation are identifiable. This means that the positive stable distribution should have a maximum likelihood on the edge of the parameter space (\( \theta = +\inf \), corresponding to a Cox model for independent observations).

The control argument must be generated by a call to `emfrail_control`. Several parameters may be adjusted that control the precision of the convergence criteria or suppress the calculation of different quantities.

Value

An object of class `emfrail` that contains the following fields:

- `coefficients` A named vector of the estimated regression coefficients
- `hazard` The breslow estimate of the baseline hazard at each event time point, in chronological order
- `var` The variance-covariance matrix corresponding to the coefficients and hazard, assuming \( \theta \) constant
emfrail

The variance-covariance matrix corresponding to the coefficients and hazard, adjusted for the estimation of theta

The logarithm of the point estimate of \( \theta \). For the gamma and PVF family of distributions, this is the inverse of the estimated frailty variance.

The variance of the estimated logarithm of \( \theta \)

The likelihood-based 95% confidence interval for the logarithm of \( \theta \)

The posterior (empirical Bayes) estimates of the frailty for each cluster

A list with two elements, cluster which is a vector that the sum of the cumulative hazards from each cluster for a frailty value of 1, and individual, which is a vector that contains the cumulative hazard corresponding to each row of the data, multiplied by the corresponding frailty estimate

The time points of the events in the data set, this is the same length as hazard

The number of events for each cluster

A vector of length two with the log-likelihood of the starting Cox model and the maximized log-likelihood

The results of the Commenges-Andersen test for heterogeneity

The results of the test for dependence between a recurrent event and a terminal event, if the \(+\text{terminal}()\) statement is specified and the frailty distribution is gamma

The result of \text{cox.zph} called on a model with the estimated log-frailties as offset

The original arguments

Number of observations and fitted values (i.e. \( z \exp(\beta^T x) \))

The model.frame, if model = TRUE

The model.matrix, if model.matrix = TRUE

Note

Several options in the control argument shorten the running time for \text{emfrail} significantly. These are disabling the adjustment of the standard errors (se_adj = FALSE), disabling the likelihood-based confidence intervals (lik_ci = FALSE) or disabling the score test for heterogeneity (ca_test = FALSE).

The algorithm is detailed in the package vignette. For the gamma frailty, the results should be identical with those from \text{coxph} with ties = "breslow".

Author(s)

Theodor Balan <hello@tbalan.com>

References

See Also

plot.emfrail and autoplot.emfrail for plot functions directly available, emfrail_pll for calculating $\hat{L}(\theta)$ at specific values of $\theta$, summary.emfrail for transforming the emfrail object into a more human-readable format and for visualizing the frailty (empirical Bayes) estimates, predict.emfrail for calculating and visualizing conditional and marginal survival and cumulative hazard curves, residuals.emfrail for extracting martingale residuals and logLik.emfrail for extracting the log-likelihood of the fitted model.

Examples

```r
m_gamma <- emfrail(formula = Surv(time, status) ~ rx + sex + cluster(litter),
                   data = rats)

# Inverse Gaussian distribution
m_ig <- emfrail(formula = Surv(time, status) ~ rx + sex + cluster(litter),
                 data = rats,
                 distribution = emfrail_dist(dist = "pvf"))

# for the PVF distribution with m = 0.75
m_pvf <- emfrail(formula = Surv(time, status) ~ rx + sex + cluster(litter),
                 data = rats,
                 distribution = emfrail_dist(dist = "pvf", pvfm = 0.75))

# for the positive stable distribution
m_ps <- emfrail(formula = Surv(time, status) ~ rx + sex + cluster(litter),
                data = rats,
                distribution = emfrail_dist(dist = "stable"))

## Not run:
# Compare marginal log-likelihoods
models <- list(m_gamma, m_ig, m_pvf, m_ps)
models
logliks <- lapply(models, logLik)

```
# Test for conditional proportional hazards (log-frailty as offset)
## Not run:
m_gamma <- emfrail(formula = Surv(time, status) ~ rx + sex + cluster(litter),
  data = rats, control = emfrail_control(zph = TRUE))
par(mfrow = c(1,2))
plot(m_gamma$zph)
## End(Not run)

# Draw the profile log-likelihood
## Not run:
fr_var <- seq(from = 0.01, to = 1.4, length.out = 20)
# For gamma the variance is 1/theta (see parametrizations)
pll_gamma <- emfrail_pll(formula = Surv(time, status) ~ rx + sex + cluster(litter),
  data = rats,
  values = 1/fr_var)
plot(fr_var, pll_gamma,
  type = "l",
  xlab = "Frailty variance",
  ylab = "Profile log-likelihood")

# Recurrent events
mod_rec <- emfrail(Surv(start, stop, status) ~ treatment + cluster(id), bladder1)
# The warnings appear from the Surv object, they also appear in coxph.
plot(mod_rec, type = "hist")
## End(Not run)

# Left truncation
## Not run:
# We simulate some data with truncation times
set.seed(2018)
nclus <- 300
nind <- 5
x <- sample(c(0,1), nind * nclus, TRUE)
u <- rep(rgamma(nclus,1,1), each = 3)
stime <- rexp(nind * nclus, rate = u * exp(0.5 * x))
status <- ifelse(stime > 5, 0, 1)
stime[status == 0] <- 5
# truncate uniform between 0 and 2
ltime <- runif(nind * nclus, min = 0, max = 2)
d <- data.frame(id = rep(1:nclus, each = nind),
  x = x,
  stime = stime,
emfrail_control

Control parameters for emfrail

Description
Control parameters for emfrail

Usage
emfrail_control(opt_fit = TRUE, se = TRUE, se_adj = TRUE,
ca_test = TRUE, lik_ci = TRUE, lik_interval = exp(c(-3, 20)),
lik_interval_stable = exp(c(0, 20)), nlm_control = list(stepmax = 1),
zph = FALSE, zph_transform = "km", em_control = list(eps = 1e-04,
maxit = Inf, fast_fit = TRUE, verbose = FALSE, upper_tol = exp(10),
lik_tol = 1))

Arguments

opt_fit Logical. Whether the outer optimization should be carried out. If FALSE, then
the frailty parameter is treated as fixed and the emfrail function returns only
log-likelihood. See details.

se Logical. Whether to calculate the variance / covariance matrix.
**emfrail_control**

- **se_adj** Logical. Whether to calculate the adjusted variance / covariance matrix (needs se == TRUE)
- **ca_test** Logical. Should the Commenges-Andersen test be calculated?
- **lik_ci** Logical. Should likelihood-based confidence interval be calculated for the frailty parameter?
- **lik_interval** The edges, on the scale of \( \theta \), of the parameter space in which to search for likelihood-based confidence interval
- **lik_interval_stable** (for dist = "stable") The edges, on the scale of \( \theta \), of the parameter space in which to search for likelihood-based confidence interval
- **nlm_control** A list of named arguments to be sent to nlm for the outer optimization.
- **zph** Logical. Should the cox.zph test be performed at the maximum likelihood estimate?
- **zph_transform** One of "km", "rank", "identity" or a function of one argument to be passed on to cox.zph.
- **em_control** A list of parameters for the inner optimization. See details.

**Details**

The nlm_control argument should not overlap with hessian, f or p.

The em_control argument should be a list with the following items:

- **eps** A criterion for convergence of the EM algorithm (difference between two consecutive values of the log-likelihood)
- **maxit** The maximum number of iterations between the E step and the M step
- **fast_fit** Logical, whether the closed form formulas should be used for the E step when available
- **verbose** Logical, whether details of the optimization should be printed
- **upper_tol** An upper bound for \( \theta \); after this threshold, the algorithm returns the limiting log-likelihood of the no-frailty model. That is because the no-frailty scenario corresponds to a \( \theta = \infty \), which could lead to some numerical issues
- **lik_tol** For values higher than this, the algorithm returns a warning when the log-likelihood decreases between EM steps. Technically, this should not happen, but if the parameter \( \theta \) is somewhere really far from the maximum, numerical problems might lead in very small likelihood decreases.

The fast_fit option make a difference when the distribution is gamma (with or without left truncation) or inverse Gaussian, i.e. pvf with \( m = -1/2 \) (without left truncation). For all the other scenarios, the fast_fit option will automatically be changed to FALSE. When the number of events in a cluster / individual is not very small, the cases for which fast fitting is available will show an improvement in performance.

The starting value of the outer optimization may be set in the distribution argument.

**Value**

An object of the type emfrail_control.
See Also

emfrail, emfrail_dist, emfrail_pll

Examples

emfrail_control()
emfrail_control(em_control = list(eps = 1e-7))

---

**emfrail_dist**

*Distribution parameters for emfrail*

---

**Description**

Distribution parameters for emfrail

**Usage**

emfrail_dist(dist = "gamma", theta = 2, pvfm = -1/2,
left_truncation = FALSE, basehaz = "breslow")

**Arguments**

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<td>dist</td>
<td>One of 'gamma', 'stable' or 'pvf'.</td>
</tr>
<tr>
<td>theta</td>
<td>A starting value for the 'outer' maximization with respect to the frailty parameter ( \theta ). Must be &gt;0.</td>
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<tr>
<td>pvfm</td>
<td>Only relevant if dist = 'pvf' is used. It determines which PVF distribution should be used. Must be larger than -1 and not equal to 0.</td>
</tr>
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<td>left_truncation</td>
<td>Logical. Whether the data set represents left truncated survival times.</td>
</tr>
<tr>
<td>basehaz</td>
<td>A character string which determines how the baseline hazard is calculated. The default is &quot;breslow&quot;, but other possible options are &quot;weibull&quot;, &quot;exponential&quot; &quot;gaussian&quot;, &quot;logistic&quot;, &quot;lognormal&quot; or &quot;loglogistic&quot;.</td>
</tr>
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</table>

**Details**

The theta argument must be positive. In the case of gamma or PVF, this is the inverse of the frailty variance, i.e. the larger the theta is, the closer the model is to a Cox model. When dist = "pvf" and pvfm = -0.5, the inverse Gaussian distribution is obtained. For the positive stable distribution, the \( \gamma \) parameter of the Laplace transform is \( \theta/(1 + \theta) \), with the alpha parameter fixed to 1.

**Value**

An object of the type `emfrail_dist`, which is mostly used to denote the supported frailty distributions in a consistent way.
emfrail_pll

See Also
emfrail, emfrail_control

Examples
emfrail_dist()
# Compound Poisson distribution:
emfrail_dist(dist = 'pvf', theta = 1.5, pvfm = 0.5)
# Inverse Gaussian distribution:
emfrail_dist(dist = 'pvf')

emfrail_pll
Profile log-likelihood calculation

Description
Profile log-likelihood calculation

Usage
emfrail_pll(formula, data, distribution = emfrail_dist(), values)

Arguments
formula Same as in emfrail
data Same as in emfrail
distribution Same as in emfrail
values A vector of values on where to calculate the profile likelihood. See details.

Details
This function can be used to calculate the profile log-likelihood for different values of \( \theta \). The scale is that of theta as defined in emfrail_dist(). For the gamma and pvf frailty, that is the inverse of the frailty variance.

Value
The profile log-likelihood at the specific value of the frailty parameter

Note
This function is just a simple wrapper for emfrail() with the control argument a call from emfrail_control with the option opt_fit = FALSE. More flexibility can be obtained by calling emfrail with this option, especially for setting other emfrail_control parameters.
Examples

```r
fr_var <- seq(from = 0.01, to = 1.4, length.out = 20)
pll_gamma <- emfrail_pll(formula = Surv(time, status) ~ rx + sex + cluster(litter),
data = rats,
values = 1/fr_var )
plot(fr_var, pll_gamma,
type = "l",
xlab = "Frailty variance",
ylab = "Profile log-likelihood")

# check with coxph;
# attention: theta is the the inverse frailty variance in emfrail,
# but theta is the frailty variance in coxph.

pll_cph <- sapply(fr_var, function(th)
  coxph(data = rats, formula = Surv(time, status) ~ rx + sex + frailty(litter, theta = th),
           method = "breslow")$history[[1]][[3]])
lines(fr_var, pll_cph, col = 2)

# Same for inverse gaussian
pll_if <- emfrail_pll(Surv(time, status) ~ rx + sex + cluster(litter),
rats,
distribution = emfrail_dist(dist = "pvf"),
values = 1/fr_var )

# Same for pvf with a positive pvfm parameter
pll_pvf <- emfrail_pll(Surv(time, status) ~ rx + sex + cluster(litter),
rats,
distribution = emfrail_dist(dist = "pvf", pvfm = 1.5),
values = 1/fr_var )

miny <- min(c(pll_gamma, pll_cph, pll_if, pll_pvf))
maxy <- max(c(pll_gamma, pll_cph, pll_if, pll_pvf))

plot(fr_var, pll_gamma,
type = "l",
xlab = "Frailty variance",
ylab = "Profile log-likelihood",
ylim = c(miny, maxy))
points(fr_var, pll_cph, col = 2)
lines(fr_var, pll_if, col = 3)
lines(fr_var, pll_pvf, col = 4)

legend(legend = c("gamma (emfrail)", "gamma (coxph)", "inverse gaussian", "pvf, m=1.5"),
        col = 1:4,
lty = 1,
x = 0,
y = (maxy + miny)/2)
```
Log-likelihood for `emfrail` fitted models

### Usage

```r
## S3 method for class 'emfrail'
logLik(object, ...)  
```

### Arguments

- `object` - An `emfrail` object
- `...` - Other arguments

### Details

The formula for the likelihood can be found in the manual which accompanies the package. Note that a constant is added. If we denote $\bar{n}$ the total number of events and $\bar{n}_i$ the total number of events at time point $i$, for each time point where events are observed, then this is equal to

$$\bar{n} - \sum_i \bar{n}_i \log \bar{n}_i.$$  

This is mostly because of compatibility, i.e. to match the log-likelihood given by the `survival` package.

The df attribute of this object is equal to the number of regression coefficients plus 1. In general, the number of degrees of freedom for a frailty model is an unclear concept. For the coxph frailty fits, and in general for the shared frailty models fitted by penalized likelihood, the degrees of freedom is a number that depends on the penalization. However, even in that case, there is no straightforward interpretation or use of this quantity. The decision made here is because this would keep the likelihood ratio test for a covariate effect valid.

### Value

An object of class `logLik` containing the marginal log-likelihood of the fitted model
plot.emfrail

Plots for emfrail objects

Description

Plots for emfrail objects

Usage

```
## S3 method for class 'emfrail'
plot(x, type = c("hist", "hr", "pred"),
     newdata = NULL, lp = NULL, strata = NULL, quantity = "cumhaz",
     type_pred = c("conditional", "marginal"), conf_int = "adjusted",
     conf_level = 0.95, individual = FALSE, ...)
```

Arguments

- **x**: `emfrail` object, typically result of `emfrail()`
- **type**: One (or more) of `hist` for a histogram of the estimated frailty values, `hr` for a plot of the conditional and marginal hazard ratio between two cases and `pred` for the predicted conditional and marginal cumulative hazard or survival for one case
- **newdata**: A `data.frame` with values of the covariates. For `type == "hr"` the hazard ratio between the first two rows of `newdata` is calculated. For `type == "pred"` the prediction for the first row of `newdata` is calculated.
- **lp**: A numeric vector of values of the linear predictor, each corresponding to a case. For `type == "hr"` the hazard ratio between the first two values of `lp` is calculated. For `type == "pred"` the prediction for the first value of `lp` is calculated.
- **strata**: The name of the strata (if applicable) for which the prediction should be made.
- **quantity**: For `type == "pred"` the predicted quantity; see `quantity` in `predict.emfrail`
- **type_pred**: For `type == "pred"` the type of predicted quantity; see `type` in `predict.emfrail`
- **conf_int**: For `type == "pred"` the type of confidence intervals; see `conf_int` in `predict.emfrail`
- **conf_level**: The width of the confidence interval for `type == "pred"`; see `conf_level` in `predict.emfrail`
- **individual**: For `type == "pred"` for drawing a curve when the rows of `newdata` refer to the same individual; see `individual` in `predict.emfrail`
- **...**: Further arguments to be passed to the plot function

Value

Nothing

See Also

`predict.emfrail`, `summary.emfrail`, `autoplot.emfrail`. 
Examples

mod_rec <- emfrail(Surv(start, stop, status) ~ treatment + number + cluster(id), bladder1,
control = emfrail_control(ca_test = FALSE, lik_ci = FALSE))

# Histogram of the estimated frailties
plot(mod_rec, type = "hist")

# hazard ratio between placebo and pyridoxine
newdata1 <- data.frame(treatment = c("placebo", "pyridoxine"),
number = c(1, 3))

plot(mod_rec, type = "hr", newdata = newdata1)

# predicted cumulative hazard for placebo, and number = 1
plot(mod_rec, type = "pred", newdata = newdata1[1,])

# predicted survival for placebo, and number = 1
plot(mod_rec, type = "pred", quantity = "survival", newdata = newdata1[1,])

# predicted survival for an individual that switches from
# placebo to pyridoxine at time = 15
newdata2 <- data.frame(treatment = c("placebo", "pyridoxine"),
number = c(1, 3),
tstart = c(0, 15),
tstop = c(15, Inf))

plot(mod_rec, type = "pred", quantity = "survival", newdata = newdata2, individual = TRUE)

---

**predict.emfrail**

*Predicted hazard and survival curves from an emfrail object*

Description

Predicted hazard and survival curves from an emfrail object

Usage

```r
## S3 method for class 'emfrail'
predict(object, newdata = NULL, lp = NULL,
strata = NULL, quantity = c("cumhaz", "survival"),
type = c("conditional", "marginal"), conf_int = NULL,
individual = FALSE, conf_level = 0.95, ...)
```

Arguments

- **object**: An emfrail fit object
- **newdata**: A data frame with the same variable names as those that appear in the emfrail formula, used to calculate the lp (optional).
predict.emfrail

lp
A vector of linear predictor values at which to calculate the curves. Default is 0 (baseline).

strata
The name of the strata (if applicable) for which the prediction should be made.

quantity
Can be "cumhaz" and/or "survival". The quantity to be calculated for the values of lp.

type
Can be "conditional" and/or "marginal". The type of the quantity to be calculated.

conf_int
Can be "regular" and/or "adjusted". The type of confidence interval to be calculated.

individual
Logical. Are the observations in newdata from the same individual? See details.

conf_level
The width of the confidence intervals. By default, 95% confidence intervals are calculated.

... Ignored

Details

The function calculates predicted cumulative hazard and survival curves for given covariate or linear predictor values; for the first, newdata must be specified and for the latter lp must be specified. Each row of newdata or element of lp is considered to be a different subject, and the desired predictions are produced for each of them separately.

In newdata two columns may be specified with the names tstart and tstop. In this case, each subject is assumed to be at risk only during the times specified by these two values. If the two are not specified, the predicted curves are produced for a subject that is at risk for the whole follow-up time.

A slightly different behaviour is observed if individual == TRUE. In this case, all the rows of newdata are assumed to come from the same individual, and tstart and tstop must be specified, and must not overlap. This may be used for describing subjects that are not at risk during certain periods or subjects with time-dependent covariate values.

The two "quantities" that can be returned are named cumhaz and survival. If we denote each quantity with q, then the columns with the marginal estimates are named q_m. The confidence intervals contain the name of the quantity (conditional or marginal) followed by _l or _r for the lower and upper bound. The bounds calculated with the adjusted standard errors have the name of the regular bounds followed by _a. For example, the adjusted lower bound for the marginal survival is in the column named survival_m_l_a.

The emfrail only gives the Breslow estimates of the baseline hazard λ_0(t) at the event time points, conditional on the frailty. Let λ(t) be the baseline hazard for a linear predictor of interest. The estimated conditional cumulative hazard is then Λ(t) = \sum_{s=0}^{t} λ(s). The variance of Λ(t) can be calculated from the (maybe adjusted) variance-covariance matrix.

The conditional survival is obtained by the usual expression \( S(t) = \exp(-\Lambda(t)) \). The marginal survival is given by

\[ \bar{S}(t) = E[\exp(-\Lambda(t))] = L(\Lambda(t)), \]

i.e. the Laplace transform of the frailty distribution calculated in Λ(t).

The marginal hazard is obtained as

\[ \bar{\Lambda}(t) = -\log \bar{S}(t). \]
The only standard errors that are available from emfrail are those for $\lambda_0(t)$. From this, standard errors of $\log \Lambda(t)$ may be calculated. On this scale, the symmetric confidence intervals are built, and then moved to the desired scale.

**Value**

The return value is a single data frame (if `lp` has length 1, `newdata` has 1 row or `individual == TRUE`) or a list of data frames corresponding to each value of `lp` or each row of `newdata` otherwise. The names of the columns in the returned data frames are as follows: `time` represents the unique event time points from the data set, `lp` is the value of the linear predictor (as specified in the input or as calculated from the lines of `newdata`). By default, for each `lp` a data frame will contain the following columns: `cumhaz`, `survival`, `cumhaz_m`, `survival_m` for the cumulative hazard and survival, conditional and marginal, with corresponding confidence bands. The naming of the columns is explained more in the Details section.

**Note**

The linear predictor is taken as fixed, so the variability in the estimation of the regression coefficient is not taken into account. Does not support left truncation (at the moment). That is because, if `individual == TRUE` and `tstart` and `tstop` are specified, for the marginal estimates the distribution of the frailty is used to calculate the integral, and not the distribution of the frailty given the truncation.

For performance reasons, consider running with `conf_int = NULL`; the reason is that the `deltamethod` function that is used to calculate the confidence intervals easily becomes slow when there is a large number of time points for the cumulative hazard.

**See Also**

`plot.emfrail`, `autoplot.emfrail`

**Examples**

```r
kidney$sex <- ifelse(kidney$sex == 1, "male", "female")
m1 <- emfrail(formula = Surv(time, status) ~ sex + age + cluster(id),
           data = kidney)

# get all the possible prediction for the value 0 of the linear predictor
predict(m1, lp = 0)

# get the cumulative hazards for two different values of the linear predictor
predict(m1, lp = c(0, 1), quantity = "cumhaz", conf_int = NULL)

# get the cumulative hazards for a female and for a male, both aged 30
newdata1 <- data.frame(sex = c("female", "male"),
                       age = c(30, 30))

predict(m1, newdata = newdata1, quantity = "cumhaz", conf_int = NULL)

# get the cumulative hazards for an individual that changes
# sex from female to male at time 40.
newdata2 <- data.frame(sex = c("female", "male"),
                       age = c(30, 30))

predict(m1, newdata = newdata2, quantity = "cumhaz", conf_int = NULL)
```

residuals.emfrail

Residuals for frailty models

Description

Residuals for frailty models

Usage

## S3 method for class 'emfrail'
residuals(object, type = "group", ...)

Arguments

- **object**: An emfrail object
- **type**: One of `cluster` or `individual`
- **...**: Other arguments

Details

For cluster \( i \), individual \( j \) and observation row \( k \), we write the cumulative hazard contribution as

\[
\Lambda_{ijk} = \exp(\beta^\top x_{ijk})\Lambda_{0,ijk}
\]

where \( \Lambda_{0,ijk} \) is the baseline cumulative hazard corresponding to the row \( (i, j, k) \).

When `type` == "individual", the returned residuals are equal to \( z_i \Lambda_{ijk} \) where \( z_i \) is the estimated frailty in cluster \( i \). When `type` == "cluster", the returned residuals are equal to \( \sum_{j,k} \Lambda_{ijk} \).

Value

A vector corresponding to the Martingale residuals, either for each cluster or for each individual (row of the data).
summary.emfrail

Summary for emfrail objects

Description

Summary for emfrail objects

Usage

## S3 method for class 'emfrail'
summary(object, lik_ci = TRUE, print_opts = list(coef = TRUE, dist = TRUE, fit = TRUE, frailty = TRUE, adj_se = TRUE, verbose_frailty = TRUE), ...)

Arguments

object An object of class emfrail
lik_ci Logical. Should the confidence intervals for the frailty parameter be calculated based on the likelihood? If not, they are calculated with the delta method.
print_opts A list with options for printing the summary object. These include coef, dist, fit, frailty, adj_se, verbose_frailty.
... Ignored

Details

Regardless of the fitted model, the following fields will be present in this object: est_dist (an object of class emfrail_distribution) with the estimated distribution, loglik (a named vector with the log-likelihoods of the no-frailty model, the frailty model, the likelihood ratio test statistic and the p-value of the one-sided likelihood ratio test), theta (a named vector with the estimated value of the parameter \( \theta \), the standard error, and the limits of a 95% confidence interval), and optional lower_q and upper_q as the 2.5 quantiles.

For the the PVF or gamma distributions, the field fr_var contains a transformation of theta to correspond to the frailty variance. The fields pvf_pars and stable_pars are for quantities that are calculated only when the distribution is PVF or stable. If the model contains covariates, the field coefmat contains the corresponding estimates. The p-values are based on the adjusted standard errors, if they have been calculated successfully (i.e. if they appear when printing the summary object). Otherwise, they are based on the regular standard errors.

Value

An object of class emfrail_summary, with some more human-readable results from an emfrail object.

See Also

predict.emfrail, plot.emfrail
Examples

```r
data("bladder")
mod_gamma <- emfrail(Surv(start, stop, status) ~ treatment + cluster(id), bladder1)
summary(mod_gamma)
summary(mod_gamma, print_opts = list(frailty_verbose = FALSE))

# plot the Empirical Bayes estimates of the frailty
# easy way:
plot(mod_gamma, type = "hist")

# a fancy graph:
sum_mod <- summary(mod_gamma)
library(dplyr)
library(ggplot2)

# Create a plot just with the points
pl1 <- sum_mod$frail %>%
  arrange(z) %>%
  mutate(x = 1:n()) %>%
  ggplot(aes(x = x, y = z)) +
  geom_point()

# If the quantiles of the posterior distribution are
# known, then error bars can be added:
if(!is.null(sum_mod$frail$lower_q))
  pl1 <- pl1 + geom_errorbar(aes(ymin = lower_q, ymax = upper_q), alpha = 0.5)

pl1

# The plot can be made interactive!
# ggplot2 gives a warning about the "id" aesthetic, just ignore it
pl2 <- sum_mod$frail %>%
  arrange(z) %>%
  mutate(x = 1:n()) %>%
  ggplot(aes(x = x, y = z)) +
  geom_point(aes(id = id))

if(!is.null(sum_mod$frail$lower_q))
  pl2 <- pl2 + geom_errorbar(aes(ymin = lower_q, ymax = upper_q, id = id), alpha = 0.5)

library(plotly)
ggplotly(pl2)

# Proportional hazards test
off_z <- log(sum_mod$frail$z)[match(bladder1$id, sum_mod$frail$id)]

zph1 <- cox.zph(coxph(Surv(start, stop, status) ~ treatment + cluster(id), data = bladder1))

# no sign of non-proportionality
zph2 <- cox.zph(coxph(Surv(start, stop, status) ~ treatment + offset(off_z), data = bladder1))
zph2
```
# the p-values are even larger; the frailty "corrects" for proportionality.
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