Package ‘freqdom’

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Description Implementation of dynamic principal component analysis (DPCA), simulation of VAR and VMA processes and frequency domain tools. These frequency domain methods for dimensionality reduction of multivariate time series were introduced by David Brillinger in his book Time Series (1974). We follow implementation guidelines as described in Hormann, Kidzinski and Hallin (2016), Dynamic Functional Principal Component <doi:10.1111/rssb.12076>.
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freqdom-package ................................................. 2
cov.structure ............................................... 3
dpca .......................................................... 4
dpca.filters .................................................. 5
dpca.KLexpansion .......................................... 6
dpca.scores ............................................... 7
dpca.var .................................................... 8
filter.process ............................................. 9
freqdom-package

Description

Implementation of dynamic principle component analysis (DPCA), simulation of VAR and VMA processes and frequency domain tools. The package also provides a toolset for developers simplifying construction of new frequency domain based methods for multivariate signals.

Details

freqdom package allows you to manipulate time series objects in both time and frequency domains. We implement dynamic principal component analysis methods, enabling spectral decomposition of a stationary vector time series into uncorrelated components.

Dynamic principal component analysis enables estimation of temporal filters which transform a vector time series into another vector time series with uncorrelated components, maximizing the long run variance explained. There are two key differences between classical PCA and dynamic PCA:

- Components returned by the dynamic procedure are uncorrelated in time, i.e. for any \( i \neq j \) and \( l \in Z, Y_i(t) \) and \( Y_j(t_l) \) are uncorrelated,
- The mapping maximizes the long run variance, which, in case of stationary vector time series, means that the process reconstructed from and \( d > 0 \) first dynamic principal components better approximates your vector time series process than the first \( d \) classic principal components.

For details, please refer to literature below and to help pages of functions \( \text{dpca} \) for estimating the components, \( \text{dpca.scores} \) for estimating scores and \( \text{dpca.Kexpansion} \) for retrieving the signal from components.

Apart from frequency domain techniques for stationary vector time series, freqdom provides a toolset of operators such as the vector Fourier Transform (\( \text{fourier.transform} \)) or a vector spectral density operator (\( \text{spectral.density} \)) as well as simulation of vector time series models \( \text{rar} \), \( \text{rma} \) generating vector autoregressive and moving average respectively. These functions enable developing new techniques based on the Frequency domain analysis.
cov.structure

Estimate cross-covariances of two stationary multivariate time series

Description

This function computes the empirical cross-covariance of two stationary multivariate time series. If only one time series is provided it determines the empirical autocovariance function.

Usage

cov.structure(X, Y = X, lags = 0)

Arguments

X vector or matrix. If matrix, then each row corresponds to a timepoint of a vector time series.

Y vector or matrix. If matrix, then each row corresponds to a timepoint of a vector time series.

lags an integer-valued vector \((\ell_1, \ldots, \ell_K)\) containing the lags for which covariances are calculated.

Details

Let \([X_1, \ldots, X_T]'\) be a \(T \times d_1\) matrix and \([Y_1, \ldots, Y_T]'\) be a \(T \times d_2\) matrix. We stack the vectors and assume that \((X_t', Y_t')'\) is a stationary multivariate time series of dimension \(d_1 + d_2\). This function determines empirical lagged covariances between the series \((X_t)\) and \((Y_t)\). More precisely it determines \(\hat{C}^{XY}(h)\) for \(h \in \text{lags}\), where \(\hat{C}^{XY}(h)\) is the empirical version of \(\text{Cov}(X_h, Y_0)\). For a sample of size \(T\) we set \(\hat{\mu}^X = \frac{1}{T} \sum_{t=1}^{T} X_t\) and \(\hat{\mu}^Y = \frac{1}{T} \sum_{t=1}^{T} Y_t\) and

\[
\hat{C}^{XY}(h) = \frac{1}{T} \sum_{t=1}^{T-h} (X_{t+h} - \hat{\mu}^X)(Y_t - \hat{\mu}^Y)' 
\]

and for \(h < 0\)

\[
\hat{C}^{XY}(h) = \frac{1}{T} \sum_{t=|h|+1}^{T} (X_{t+h} - \hat{\mu}^X)(Y_t - \hat{\mu}^Y)'.
\]
dpca

Compute Dynamic Principal Components and dynamic Karhunen-Loeve expansion

Description

Dynamic principal component analysis (DPCA) decomposes multivariate time series into uncorrelated components. Compared to classical principal components, DPCA decomposition outputs components which are uncorrelated in time, allowing simpler modeling of the processes and maximizing long run variance of the projection.

Usage

dpca(x, q = 30, freq = (-1000:1000/1000) * pi, Ndpc = dim(X)[2])

Arguments

x a vector time series given as a \((T \times d)\)-matrix. Each row corresponds to a time-point.
q window size for the kernel estimator, i.e. a positive integer.
freq a vector containing frequencies in \([-\pi, \pi]\) on which the spectral density should be evaluated.
Ndpc is the number of principal component filters to compute as in dpca.filters

Details

This convenience function applies the DPCA methodology and returns filters \(dpca.filters\), scores \(dpca.scores\), the spectral density \(spectral.density\), variances \(dpca.var\) and Karhunen-Loeve expansion \(dpca.KExpansion\).

See the example for understanding usage, and help pages for details on individual functions.

Value

A list containing

- scores DPCA scores \(dpca.scores\)
- filters DPCA filters \(dpca.filters\)
- spec.density spectral density of \(X\) \(spectral.density\)
- var amount of variance explained by dynamic principal components \(dpca.var\)
- Xhat Karhunen-Loeve expansion using \(Ndpc\) dynamic principal components \(dpca.KExpansion\)
dpca.filters

References


Examples

\[
X = \text{rarr}(100, 3)
\]

# Compute DPCA with only one component
res.dpca = dpca(X, q = 5, Ndpc = 1)

# Compute PCA with only one component
res.pca = prcomp(X, center = TRUE)
res.pca$x[, -1] = 0

# Reconstruct the data
var.dpca = (1 - sum((res.dpca$xhat - X)**2) / sum(X**2)) * 100
var.pca = (1 - sum((res.pca$x %*% t(res.pca$rotation) - X)**2) / sum(X**2)) * 100

cat("Variance explained by DPCA:\n", var.dpca, "\n")
cat("Variance explained by PCA:\n", var.pca, "\n")

dpca.filters

*Compute DPCA filter coefficients*

Description

For a given spectral density matrix dynamic principal component filter sequences are computed.

Usage

dpca.filters(F, Ndpc = dim(F$operators)[1], q = 30)

Arguments

- **F**: \((d \times d)\) spectral density matrix, provided as an object of class *freqdom*.
- **Ndpc**: an integer \(\in \{1, \ldots, d\}\). It is the number of dynamic principal components to be computed. By default it is set equal to \(d\).
- **q**: a non-negative integer. DPCA filter coefficients at lags \(|h| \leq q\) will be computed.
Details

Dynamic principal components are linear filters \( (\phi_{\ell k}; k \in \mathbb{Z}) \), \( 1 \leq \ell \leq d \). They are defined as the Fourier coefficients of the dynamic eigenvector \( \varphi_\ell(\omega) \) of a spectral density matrix \( \mathcal{F}_\omega \):

\[
\phi_{\ell k} := \frac{1}{2\pi} \int_{-\pi}^{\pi} \varphi_\ell(\omega) \exp(-ik\omega) d\omega.
\]

The index \( \ell \) is referring to the \( \ell \)-th largest dynamic eigenvalue. Since the \( \phi_{\ell k} \) are real, we have

\[
\phi_{\ell k}^* = \phi_{\ell k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \varphi_\ell^*(\omega) \exp(ik\omega) d\omega.
\]

For a given spectral density (provided as an object of class \texttt{freqdom}) the function \texttt{dpca.filters()} computes \( (\phi_{\ell k}) \) for \( |k| \leq q \) and \( 1 \leq \ell \leq Ndpc \).

For more details we refer to Chapter 9 in Brillinger (2001), Chapter 7.8 in Shumway and Stoffer (2006) and to Hormann et al. (2015).

Value

An object of class \texttt{timedom}. The list has the following components:

- \texttt{operators} an array. Each matrix in this array has dimension \( Ndpc \times d \) and is assigned to a certain lag. For a given lag \( k \), the rows of the matrix correspond to \( \phi_{\ell k} \).
- \texttt{lags} a vector with the lags of the filter coefficients.

References


See Also

\texttt{dpca.var}, \texttt{dpca.scores}, \texttt{dpca.KLexpansion}

dpca.KLexpansion

Dynamic KL expansion

Description

Computes the dynamic Karhunen-Loeve expansion of a vector time series up to a given order.

Usage

\texttt{dpca.KLexpansion(X, dpcs)}
Arguments

X  a vector time series given as a \((T \times d)\)-matix. Each row corresponds to a time-point.

dpcs an object of class timedom, representing the dpca filters obtained from the sample \(X\). If \(dpcs = NULL\), then \(dpcs = dpca.filter(spectral.density(X))\) is used.

Details

We obtain the dynamic Karhunen-Loeve expansion of order \(L\), \(1 \leq L \leq d\). It is defined as

\[
\sum_{\ell=1}^{L} \sum_{k \in \mathbb{Z}} Y_{\ell,t+k} \phi_{\ell k},
\]

where \(\phi_{\ell k}\) are the dynamic PC filters as explained in \(dpca.filters\) and \(Y_{\ell k}\) are dynamic scores as explained in \(dpca.scores\). For the sample version the sum in \(k\) extends over the range of lags for which the \(\phi_{\ell k}\) are defined.

For more details we refer to Chapter 9 in Brillinger (2001), Chapter 7.8 in Shumway and Stoffer (2006) and to Hormann et al. (2015).

Value

A \((T \times d)\)-matix. The \(\ell\)-th column contains the \(\ell\)-th data point.

References


See Also

\(dpca.filters, filter.process, dpca.scores\)

dpca.scores Obtain dynamic principal components scores

Description

Computes dynamic principal component score vectors of a vector time series.

Usage

\(dpca.scores(X, dpcs = dpca.filters(spectral.density(X)))\)
Arguments

- **x**: a vector time series given as a \((T \times d)\)-matix. Each row corresponds to a time-point.
- **dpcs**: an object of class `timedom`, representing the dpca filters obtained from the sample \(X\). If \(dpcs = NULL\), then \(dpcs = dpca.filter(spectralDensity(x))\) is used.

Details

The \(\ell\)-th dynamic principal components score sequence is defined by

\[
Y_{\ell t} := \sum_{k \in \mathbb{Z}} \phi_{\ell k}^t X_{t-k}, \quad 1 \leq \ell \leq d,
\]

where \(\phi_{\ell k}\) are the dynamic PC filters as explained in `dpca.filters`. For the sample version the sum extends over the range of lags for which the \(\phi_{\ell k}\) are defined. The actual operation carried out is `filter.process(x, A = dpcs)`.

We refer for more details to Chapter 9 in Brillinger (2001), Chapter 7.8 in Shumway and Stoffer (2006) and to Hormann et al. (2015).

Value

A \(T \times \text{Ndpc}-\text{matix with Ndpc} = \text{dim}(\text{dpcs}\$\text{operators})[1]\). The \(\ell\)-th column contains the \(\ell\)-th dynamic principal component score sequence.

References


See Also

- `dpca.filters`, `dpca.KExpansion`, `dpca.var`
Arguments

F

$(d \times d)$ spectral density matrix, provided as an object of class `freqdom`. To guarantee accuracy of numerical integration it is important that $F_{freq}$ is a dense grid of frequencies in $[-\pi, \pi]$.

Details

Consider a spectral density matrix $F_\omega$ and let $\lambda_\ell(\omega)$ by the $\ell$-th dynamic eigenvalue. The proportion of variance described by the $\ell$-th dynamic principal component is given as

$$v_\ell := \frac{\int_{-\pi}^{\pi} \lambda_\ell(\omega) \, d\omega}{\int_{-\pi}^{\pi} \text{tr}(F_\omega) \, d\omega}.$$

This function numerically computes the vectors $(v_\ell; 1 \leq \ell \leq d)$.

For more details we refer to Chapter 9 in Brillinger (2001), Chapter 7.8 in Shumway and Stoffer (2006) and to Hormann et al. (2015).

Value

A $d$-dimensional vector containing the $v_\ell$.

References


See Also

dpca.filters, dpca.KLexpansion, dpca.scores

filter.process

Convolute (filter) a multivariate time series using a time-domain filter

Description

This function applies a linear filter to some vector time series.

Usage

filter.process(X, A)

X %c% A
Arguments

\(X\) vector time series given in matrix form. Each row corresponds to a timepoint.

\(A\) an object of class `timedom`.

Details

Let \( [X_1, \ldots, X_T]' \) be a \( T \times d \) matrix corresponding to a vector series \( X_1, \ldots, X_T \). This time series is transformed to the series \( Y_1, \ldots, Y_T \), where

\[
Y_t = \sum_{k=-q}^{p} A_k X_{t-k}, \quad t \in \{p + 1, \ldots, T - q\}.
\]

The index \( k \) of \( A_k \) is determined by the lags defined for the time domain object. When index \( t - k \) falls outside the domain \( \{1, \ldots, T\} \) we set \( X_{t-k} = \frac{1}{T} \sum_{k=1}^{T} X_k \).

Value

A matrix. Row \( t \) corresponds to \( Y_t \).

Functions

- `filterNprocess`: Multivariate convolution (filter) in the time domain
- `%c%`: Convenience operator for `filterNprocess` function

See Also

`timedom`

---

`fourier.inverse` Coefficients of a discrete Fourier transform

Description

Computes Fourier coefficients of some functional represented by an object of class `freqdom`.

Usage

`fourier.inverse(F, lags = 0)`

Arguments

\(F\) an object of class `freqdom` which is corresponding to a function with values in \( \mathbb{C}^{d_1 \times d_2} \). To guarantee accuracy of inversion it is important that \( f \) is a dense grid of frequencies in \([-\pi, \pi]\).

\(lags\) lags of the Fourier coefficients to be computed.
Details

Consider a function $F: [-\pi, \pi] \to \mathbb{C}^{d_1 \times d_2}$. Its $k$-th Fourier coefficient is given as

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} F(\omega) \exp(ik\omega) d\omega.$$ 

We represent the function $F$ by an object of class `freqdom` and approximate the integral via

$$\frac{1}{|F_{freq}|} \sum_{\omega \in F_{freq}} F(\omega) \exp(i k \omega),$$

for $k \in \text{lags}$.

Value

An object of class `timedom`. The list has the following components:

- `operators` an array. The $k$-th matrix in this array corresponds to the $k$-th Fourier coefficient.
- `lags` the lags of the corresponding Fourier coefficients.

See Also

`fourier.transform`, `freqdom`

Examples

```r
Y = rar(100)
grid = c(pi*(1:2000)/1000 - pi) # a dense grid on -pi, pi
fourier.inverse(spectral.density(Y, q=2, freq=grid))
```

# compare this to
cov.structure(Y)

---

`fourier.transform` Computes the Fourier transformation of a filter given as `timedom` object

Description

Computes the frequency response function of a linear filter and returns it as a `freqdom` object.

Usage

`fourier.transform(A, freq = pi * ~100:100/100)`
freqdom

Arguments

- **A**: an object of class `timedom`.
- **freq**: a vector of frequencies $\in [-\pi, \pi]$.

Details

Consider a filter (a sequence of vectors or matrices) $(A_k)_{k \in A \setminus \text{lags}}$. Then this function computes

$$\sum_{k \in A \setminus \text{lags}} A_k e^{-ik\omega}$$

for all frequencies $\omega$ listed in the vector `freq`.

Value

An object of class `freqdom`.

See Also

`fourier.inverse`

Examples

```r
# We compute the discrete Fourier transform (DFT) of a time series X_1, ..., X_T.

X = rar(100)
T = dim(X)[1]
tdX = timedom(X/sqrt(T), lags=1:T)
DFT = fourier.transform(tdX, freq = pi*1:1000:1000/1000)
```

Description

Creates an object corresponding to a frequency domain functional.

Usage

`freqdom(F, freq)`

Arguments

- **F**: a vector, a matrix or an array. For vectors $F[k], 1 \leq k \leq K$ are complex numbers. For matrices $F[k,]$ are complex vectors. For arrays the elements $F[,, k]$, are complex valued $(d_1 \times d_2)$ matrices (all of same dimension).
- **freq**: a vector of dimension $K$ containing frequencies in $[-\pi, \pi]$. 
Details

This class is used to describe a frequency domain functional (like a spectral density matrix, a discrete Fourier transform, an impulse response function, etc.) on selected frequencies. Formally we consider a collection \([F_1, \ldots, F_K]\) of complex-valued matrices \(F_k\), all of which have the same dimension \(d_1 \times d_2\). Moreover, we consider frequencies \(\{\omega_1, \ldots, \omega_K\} \subset [-\pi, \pi]\). The object this function creates corresponds to the mapping \(f : \text{freq} \rightarrow \mathbb{C}^{d_1 \times d_2}\), where \(\omega_k \mapsto F_k\).

Consider, for example, the discrete Fourier transform of a vector time series \(X_1, \ldots, X_T\). It is defined as

\[
D_T(\omega) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} X_t e^{-it\omega}, \quad \omega \in [-\pi, \pi].
\]

We may choose \(\omega_k = \frac{2\pi k}{K} - \pi\) and \(F_k = D_T(\omega_k)\). Then, the object freqdom creates, is corresponding to the function which associates \(\omega_k\) and \(D_T(\omega_k)\).

Value

Returns an object of class freqdom. An object of class freqdom is a list containing the following components:

- operators the array \(F\) as given in the argument.
- freq the vector \(\text{freq}\) as given in the argument.

See Also

fourier.transform

Examples

```r
i = complex(imaginary=1)
OP = array(0, c(2, 2, 3))
OP[,1] = diag(2) * exp(i)/2
OP[,2] = diag(2)
OP[,3] = diag(2) * exp(-i)/2
freq = c(-pi/3, 0, pi/3)
A = freqdom(OP, freq)
```

freqdom.eigen

Eigendecompose a frequency domain operator at each frequency

Description

Gives the eigendecomposition of objects of class freqdom.

Usage

freqdom.eigen(F)
is.freqdom

Arguments

F an object of class freqdom. The matrices F$\text{operator}[,k]$ are required to be square matrices, say $d \times d$.

Details

This function makes an eigendecomposition for each of the matrices F$\text{operator}[,k]$.

Value

Returns a list. The list is containing the following components:

- vectors an array containing $d$ matrices. The $i$-th matrix contains in its $k$-th row the conjugate transpose eigenvector belonging to the $k$-th largest eigenvalue of F$\text{operator}[,i]$.
- values matrix containing in $k$-th column the eigenvalues of F$\text{operator}[,k]$.
- freq vector of frequencies defining the object F.

See Also

freqdom

is.freqdom Checks if an object belongs to the class freqdom

Description

Checks if an object belongs to the class freqdom.

Usage

is.freqdom(X)

Arguments

X some object

Value

TRUE if X is of type freqdom, FALSE otherwise

See Also

freqdom, timedom, is.timedom
is.timedom

Checks if an object belongs to the class timedom

Description

Checks if an object belongs to the class `timedom`.

Usage

```r
is.timedom(X)
```

Arguments

- `X`: some object

Value

`TRUE` if `X` is of type `timedom`, `FALSE` otherwise

See Also

- `freqdom`, `timedom`, `is.freqdom`

rar

Simulate a multivariate autoregressive time series

Description

Generates a zero mean vector autoregressive process of a given order.

Usage

```r
rar(n, d = 2, Psi = NULL, burnin = 10, noise = c("mnormal", "mt"),
    sigma = NULL, df = 4)
```

Arguments

- `n`: number of observations to generate.
- `d`: dimension of the time series.
- `Psi`: array of $p \geq 1$ coefficient matrices. $\Psi[,k]$ is the $k$-th coefficient. If no value is set then we generate a vector autoregressive process of order 1. Then, $\Psi[,1]$ is proportional to $\exp(-(i + j):1 \leq i,j \leq d)$ and such that the spectral radius of $\Psi[,1]$ is 1/2.
- `burnin`: an integer $\geq 0$. It specifies a number of initial observations to be trashed to achieve stationarity.
noise mnormal for multivariate normal noise or mt for multivariate student t noise. If not specified mnormal is chosen.
sigma covariance or scale matrix of the innovations. By default the identity matrix.
df degrees of freedom if noise = "mt".

Details

We simulate a vector autoregressive process

\[ X_t = \sum_{k=1}^{p} \Psi_k X_{t-k} + \varepsilon_t, \quad 1 \leq t \leq n. \]

The innovation process \( \varepsilon_t \) is either multivariate normal or multivariate t with a predefined covariance/scale matrix sigma and zero mean. The noise is generated with the package mvtnorm. For Gaussian noise we use rmvnorm. For Student-t noise we use rmvt. The parameters sigma and df are imported as arguments, otherwise we use default settings. To initialise the process we set \( [X_{1-p}, \ldots, X_0] = [\varepsilon_{1-p}, \ldots, \varepsilon_0] \). When burnin is set equal to \( K \) then, \( n+K \) observations are generated and the first \( K \) will be trashed.

Value

A matrix with \( d \) columns and \( n \) rows. Each row corresponds to one time point.

See Also

rma

rma

Moving average process

Description

Generates a zero mean vector moving average process.

Usage

\[
\text{rma}(n, d = 2, \Psi = \text{NULL}, \text{noise} = \text{c("mnormal", "mt")}, \sigma = \text{NULL}, \\
\text{df} = 4)
\]

Arguments

\( n \) number of observations to generate.
\( d \) dimension of the time series.
\( \Psi \) a timedom object with operators \( \text{Psi}\$\text{operators}[,,k] \) is the operator on the lag \( \text{lags}[k] \). If no value is set then we generate a vector moving average process of order 1. Then, \( \text{Psi}\$\text{lags} = \text{c}(1) \) and \( \text{Psi}\$\text{operators}[,,1] \) is proportional to \( \exp(-(i+j):1 \leq i, j \leq d) \) and such that the spectral radius of \( \text{Psi}[,,1] \) is \( 1/2 \).
spectral-density

noise

mnormal for multivariate normal noise or mt for multivariate t noise. If not specified mnormal is chosen.

sigma

covariance or scale matrix of the innovations. If NULL then the identity matrix is used.

df

degrees of freedom if noise = "mt".

Details

This simulates a vector moving average process

\[ X_t = \varepsilon_t + \sum_{k \in lags} \Psi_k \varepsilon_{t-k}, \quad 1 \leq t \leq n. \]

The innovation process \( \varepsilon_t \) is either multivariate normal or multivariate t with a predefined covariance/scale matrix sigma and zero mean. The noise is generated with the package mvtnorm. For Gaussian noise we use rmvnorm. For Student-t noise we use rmvt. The parameters sigma and df are imported as arguments, otherwise we use default settings.

Value

A matrix with d columns and n rows. Each row corresponds to one time point.

See Also

t

rar

spectral-density

Compute empirical spectral density

Description

Estimates the spectral density and cross spectral density of vector time series.

Usage

```r
spectral-density(X, Y = X, freq = (-1000:1000/1000) * pi, q = max(1, floor(dim(X)[1]^(1/3))), weights = c("Bartlett", "trunc", "Tukey", "Parzen", "Bohman", "Daniell", "ParzenCoburnDavis"))
```

Arguments

- **X**: a vector or a vector time series given in matrix form. Each row corresponds to a timepoint.
- **Y**: a vector or vector time series given in matrix form. Each row corresponds to a timepoint.
- **freq**: a vector containing frequencies in \([-\pi, \pi]\) on which the spectral density should be evaluated.
- **q**: window size for the kernel estimator, i.e. a positive integer.
- **weights**: kernel used in the spectral smoothing. By default the Bartlett kernel is chosen.
Details

Let \([X_1, \ldots, X_T]'\) be a \(T \times d_1\) matrix and \([Y_1, \ldots, Y_T]'\) be a \(T \times d_2\) matrix. We stack the vectors and assume that \((X'_t, Y'_t)'\) is a stationary multivariate time series of dimension \(d_1 + d_2\). The cross-spectral density between the two time series \((X_t)\) and \((Y_t)\) is defined as

\[
\sum_{h \in \mathbb{Z}} \text{Cov}(X_h, Y_0)e^{-ih\omega}.
\]

The function \code{spectralDensity} determines the empirical cross-spectral density between the two time series \((X_t)\) and \((Y_t)\). The estimator is of form

\[
\hat{F}_{XY}(\omega) = \sum_{|k| \leq q} w(|k|/q) \hat{C}_{XY}(h)e^{-ih\omega},
\]

with \(\hat{C}_{XY}(h)\) defined in \code{cov.structure}. Here \(w\) is a kernel of the specified type and \(q\) is the window size. By default the Bartlett kernel \(w(x) = 1 - |x|\) is used.

See, e.g., Chapter 10 and 11 in Brockwell and Davis (1991) for details.

Value

Returns an object of class \code{freqdom}. The list is containing the following components:

- \code{operators} an array. The \(k\)-th matrix in this array corresponds to the spectral density matrix evaluated at the \(k\)-th frequency listed in \code{freq}.
- \code{freq} returns argument vector \code{freq}.

References


Description

Defines a linear filter

Usage

timedom(A, lags)

Arguments

- \code{A} a vector, matrix or array. If array, the elements \(A[, , k], 1 \leq k \leq K\), are real valued \((d_1 \times d_2)\) matrices (all of same dimension). If \(A\) is a matrix, the \(k\)-th row is treated as \(A[, , k]\). Same for the \(k\)-th element of a vector. These matrices, vectors or scalars define a linear filter.
- \code{lags} a vector of increasing integers. It corresponds to the time lags of the filter.
Details

This class is used to describe a linear filter, i.e. a sequence of matrices, each of which correspond to a certain lag. Filters can, for example, be used to transform a sequence \((X_t)\) into a new sequence \((Y_t)\) by defining

\[ Y_t = \sum_k A_k X_{t-k}. \]

See `filter.process()`. Formally we consider a collection \([A_1, \ldots, A_K]\) of complex-valued matrices \(A_k\), all of which have the same dimension \(d_1 \times d_2\). Moreover, we consider lags \(\ell_1 < \ell_2 < \cdots < \ell_K\). The object this function creates corresponds to the mapping \(f : \text{lags} \to \mathbb{R}^{d_1 \times d_2}\), where \(\ell_k \mapsto A_k\).

Value

Returns an object of class `timedom`. An object of class `timedom` is a list containing the following components:

- `operators` returns the array \(A\) as given in the argument.
- `lags` returns the vector `lags` as given in the argument.

See Also

`freqdom`, `is.timedom`

Examples

```r
# In this example we apply the difference operator: Delta X_t = X_t - X_{t-1} to a time series
X = rar(20)
OP = array(0, c(2,2,2))
OP[,1] = diag(2)
OP[,2] = -diag(2)
A = timedom(OP, lags = c(0,1))
filter.process(X, A)
```

### timedom.norms

**Compute operator norms of elements of a filter**

Description

This function determines the norms of the matrices defining some linear filter.

Usage

```
timedom.norms(A, type = c("2", "O", "I", "F", "M"))
```

Arguments

- `A` an object of class `timedom`
- `type` matrix norm to be used as in `norm`
Details

Computes $\|A_h\|$ for $h$ in the set of lags belonging to the object $A$. When type is 2 then $\|A\|$ is the spectral radius of $A$. When type is F then $\|A\|$ is the Frobenius norm (or the Hilbert-Schmidt norm, or Schatten 2-norm) of $A$. Same options as for the function norm as in base package.

Value

A list which contains the following components:

- lags a vector containing the lags of $A$.
- norms a vector containing the norms of the matrices defining $A$.

Examples

d = 2

A = array(0,c(d,d,2))
A[1,,] = 2 * diag(d:1)/d
A[2,,] = 1.5 * diag(d:1)/d
OP = timedom(A,c(-2,1))
timedom.norms(OP)

Description

This function removes lags from a linear filter.

Usage

timedom.trunc(A, lags)

Arguments

A an object of class timedom.

lags a vector which contains a set of lags. These lags must be a subset of the lags defined for timedom object $A$. Only those lags will be kept, the other lags are removed.

Value

An object of class timedom.
Index

*Topic **DPCA**
  dpca, 4
  dpca.filters, 5
  dpca.Kexpansion, 6
  dpca.scores, 7
  dpca.var, 8

*Topic **classes**
  freqdom, 12
  is.freqdom, 14
  is.timedom, 15
  timedom, 18

*Topic **dpca**
  spectral.density, 17

*Topic **frequency.domain**
  fourier.inverse, 10
  fourier.transform, 11
  freqdom.eigen, 13

*Topic **simulations**
  rar, 15
  rma, 16

*Topic **time.domain**
  cov.structure, 3
  filter.process, 9
  fourier.inverse, 10
  fourier.transform, 11
  timedom.npms, 19
  timedom.trunc, 20
  %$(\text{filter.process})\$, 9
  _PACKAGEN_{freqdom-package}, 2

  cov.structure, 3

  dpca, 2, 4
  dpca.filters, 4, 5, 7–9
  dpca.Kexpansion, 2, 4, 6, 8, 9
  dpca.scores, 2, 4, 6, 7, 7, 9
  dpca.var, 4, 6, 8, 8

  filter.process, 7, 9
  fourier.inverse, 10, 12

  fourier.transform, 2, 11, 11, 13
  freqdom, 10, 11, 12, 13–15, 18, 19
  freqdom-package, 2
  freqdom.eigen, 13

  is.freqdom, 14, 15
  is.timedom, 14, 15, 19

  rar, 2, 15, 17
  rma, 2, 16, 16
  rmvnorm, 16, 17
  rmvt, 16, 17

  spectral.density, 2, 4, 17

  timedom, 4, 10, 11, 14–16, 18
  timedom.npms, 19
  timedom.trunc, 20