

Package ‘fwb’

October 2, 2023

Type Package

Title Fractional Weighted Bootstrap

Version 0.1.2

Description

An implementation of the fractional weighted bootstrap to be used as a drop-in for functions in the 'boot' package. The fractional weighted bootstrap (also known as the Bayesian bootstrap) involves drawing weights randomly that are applied to the data rather than resampling units from the data. See Xu et al. (2020) <[doi:10.1080/00031305.2020.1731599](https://doi.org/10.1080/00031305.2020.1731599)> for details.

Depends R (>= 3.0.0)

Imports rlang,
chk,
pbapply,
stats, utils, graphics

Suggests survival,
boot,
sandwich (>= 2.4-0),
lmtest,
parallel

License GPL (>=2)

Encoding UTF-8

URL <https://github.com/ngreifer/fw>,
<https://ngreifer.github.io/fw/>

BugReports <https://github.com/ngreifer/fw/issues>

RoxygenNote 7.2.3

Roxygen list(markdown = TRUE)

LazyData true

R topics documented:

bearingcage	2
fwb	2
fwb.ci	5

get_ci	8
plot.fwb	9
summary.fwb	10
vcovFWB	11

Index	14
--------------	-----------

bearingcage	<i>Bearing Cage field failure data</i>
-------------	--

Description

The data consist of 1703 aircraft engines put into service over time. There were 6 failures and 1697 right-censored observations. These data were originally given in Abernethy et al. (1983) and were reanalyzed in Meeker and Escobar (1998, chap.8). The dataset used here specifically comes from Xu et al. (2020) and are used in a Weibull analysis of failure times.

Usage

```
data("bearingcage")
```

Format

A data frame with 1703 rows and 2 variables:

hours integer; the number of hours until failure or censoring

failure logical; whether a failure occurred

References

Abernethy, R. B., Breneman, J. E., Medlin, C. H., and Reinman, G. L. (1983), "Weibull Analysis Handbook," Technical Report, Air Force Wright Aeronautical Laboratories, available at <https://apps.dtic.mil/sti/citations/ADA143100>.

Meeker, W. Q., and Escobar, L. A. (1998), *Statistical Methods for Reliability Data*, New York: Wiley.

Xu, L., Gotwalt, C., Hong, Y., King, C. B., & Meeker, W. Q. (2020). Applications of the Fractional-Random-Weight Bootstrap. *The American Statistician*, 74(4), 345–358. doi:10.1080/00031305.2020.1731599

fwb	<i>Fractional Weighted Bootstrap</i>
-----	--------------------------------------

Description

fwb() implements the fractional (random) weighted bootstrap, also known as the Bayesian bootstrap. Rather than resampling units to include in bootstrap samples, weights are drawn to be applied to a weighted estimator.

Usage

```
fwb(
  data,
  statistic,
  R = 999,
  cluster = NULL,
  simple = FALSE,
  verbose = TRUE,
  cl = NULL,
  ...
)

## S3 method for class 'fwb'
print(x, digits = getOption("digits"), index = 1L:ncol(x$t), ...)
```

Arguments

<code>data</code>	the dataset used to compute the statistic
<code>statistic</code>	a function, which, when applied to <code>data</code> , returns a vector containing the statistic(s) of interest. The function should take at least two arguments; the first argument should correspond to the dataset and the second argument should correspond to a vector of weights. Any further arguments can be passed to <code>statistic</code> through the <code>...</code> argument.
<code>R</code>	the number of bootstrap replicates. Default is 999 but more is always better. For the percentile bootstrap confidence interval to be exact, it can be beneficial to use one less than a multiple of 100.
<code>cluster</code>	optional; a vector containing cluster membership. If supplied, will run the cluster bootstrap. See Details. Evaluated first in <code>data</code> and then in the global environment.
<code>simple</code>	logical; if <code>TRUE</code> , weights will be computed on-the-fly in each bootstrap replication rather than all at once. This can save memory at the cost of some speed.
<code>verbose</code>	logical; whether to display a progress bar.
<code>cl</code>	a cluster object created by <code>parallel::makeCluster()</code> , or an integer to indicate the number of child-processes (integer values are ignored on Windows) for parallel evaluations. See <code>pbapply::pblapply()</code> for details. If <code>NULL</code> , no parallelization will take place.
<code>...</code>	other arguments passed to <code>statistic</code> .
<code>x</code>	an <code>fwb</code> object; the output of a call to <code>fwb()</code> .
<code>digits</code>	the number of significant digits to print
<code>index</code>	the index or indices of the position of the quantity of interest in <code>x\$t0</code> if more than one was specified in <code>fwb()</code> . Default is to print all quantities.

Details

`fwb()` implements the fractional weighted bootstrap and is meant to function as a drop-in for `boot::boot(., stype = "f")` (i.e., the usual bootstrap but with frequency weights representing the number of times each unit is drawn). In each bootstrap replication, the weights are sampled from independent exponential distributions with rate parameter 1 and then normalized to have a mean of 1, equivalent to drawing the weights from a Dirichlet distribution. The function supplied to

statistic must incorporate the weights to compute a weighted statistic. For example, if the output is a regression coefficient, the weights supplied to the `w` argument of `statistic` should be supplied to the `weights` argument of `lm()`. These weights should be used any time frequency weights would be, since they are meant to function like frequency weights (which, in the case of the traditional bootstrap, would be integers). Unfortunately, there is no way for `fwb()` to know whether you are using the weights correctly, so care should be taken to ensure weights are correctly incorporated into the estimator.

When fitting binomial regression models (e.g., logistic) using `glm()`, it may be useful to change the family to a "quasi" variety (e.g., `quasibinomial()`) to avoid a spurious warning about "non-integer #successes".

The cluster bootstrap can be requested by supplying a vector of cluster membership to `cluster`. Rather than generating a weight for each unit, a weight is generated for each cluster and then applied to all units in that cluster.

Ideally, `statistic` should not involve a random element, or else it will not be straightforward to replicate the bootstrap results using the seed included in the output object. Setting a seed using `set.seed()` is always advised.

The `print()` method displays the value of the statistics, the bias (the difference between the statistic and the mean of its bootstrap distribution), and the standard error (the standard deviation of the bootstrap distribution).

Value

A `fwb` object, which also inherits from `boot`, with the following components:

<code>t0</code>	The observed value of <code>statistic</code> applied to data with uniform weights.
<code>t</code>	A matrix with <code>R</code> rows, each of which is a bootstrap replicate of the result of calling <code>statistic</code> .
<code>R</code>	The value of <code>R</code> as passed to <code>fwb()</code> .
<code>data</code>	The data as passed to <code>fwb()</code> .
<code>seed</code>	The value of <code>.Random.seed</code> just prior to generating the weights (after the first call to <code>statistic</code> with uniform weights).
<code>statistic</code>	The function <code>statistic</code> as passed to <code>fwb()</code> .
<code>call</code>	The original call to <code>fwb()</code> .
<code>cluster</code>	The vector passed to <code>cluster</code> , if any.

Methods (by generic)

- `print(fwb)`: Print an `fwb` object

See Also

`fwb.ci()` for calculating confidence intervals; `summary.fwb()` for displaying output in a clean way; `plot.fwb()` for plotting the bootstrap distributions; `vcovFWB()` for estimating the covariance matrix of estimates using the FWB; `boot::boot()` for the traditional bootstrap.

Examples

```
# Performing a Weibull analysis of the Bearing Cage
# failure data as done in Xu et al. (2020)
set.seed(123)
data("bearingcage")
```

```

weibull_est <- function(data, w) {
  fit <- survival::survreg(survival::Surv(hours, failure) ~ 1,
    data = data, weights = w,
    dist = "weibull")

  c(eta = unname(exp(coef(fit))), beta = 1/fit$scale)
}

boot_est <- fwb(bearingcage, statistic = weibull_est,
  R = 199, verbose = FALSE)
boot_est

#Get standard errors and CIs; uses bias-corrected
#percentile CI by default
summary(boot_est, ci.type = "bc")

#Plot statistic distributions
plot(boot_est, index = "beta", type = "hist")

```

fwb.ci

Fractional Weighted Bootstrap Confidence Intervals

Description

`fwb.ci()` generates several types of equi-tailed two-sided nonparametric confidence intervals. These include the normal approximation, the basic bootstrap interval, the percentile bootstrap interval, the bias-corrected percentile bootstrap interval, and the bias-correct and accelerated (BCa) bootstrap interval.

Usage

```

fwb.ci(
  fwb.out,
  conf = 0.95,
  type = "bc",
  index = 1L,
  h = base::identity,
  hinv = base::identity,
  ...
)

## S3 method for class 'fwbci'
print(x, hinv = NULL, ...)

```

Arguments

`fwb.out` an fwb object; the output of a call to `fwb()`.

`conf` the desired confidence level. Default is .95 for 95% confidence intervals.

type	the type of confidence interval desired. Allowable options include "norm" (normal approximation), "basic" (basic interval), "perc" (percentile interval), "bc" (bias-correct percentile interval), and "bca" (BCa interval). More than one is allowed. Can also be "all" to request all of them. BCa intervals require that the number of bootstrap replications is larger than the sample size.
index	the index of the position of the quantity of interest in <code>fwb.out\$t0</code> if more than one was specified in <code>fwb()</code> . Only one value is allowed at a time. By default the first statistic is used.
h	a function defining a transformation. The intervals are calculated on the scale of $h(t)$ and the inverse function <code>hinv</code> applied to the resulting intervals. It must be a function of one variable only and for a vector argument, it must return a vector of the same length. Default is the identity function.
hinv	a function, like <code>h</code> , which returns the inverse of <code>h</code> . It is used to transform the intervals calculated on the scale of $h(t)$ back to the original scale. The default is the identity function. If <code>h</code> is supplied but <code>hinv</code> is not, then the intervals returned will be on the transformed scale.
...	ignored
x	an <code>fwbc</code> object; the output of a call to <code>fwb.ci()</code> .

Details

`fwb.ci()` functions similarly to `boot::boot.ci()` in that it takes in a bootstrapped object and computes confidence intervals. This interface is a bit old-fashioned, but was designed to mimic that of `boot.ci()`. For a more modern interface, see `summary.fwb()`.

The bootstrap intervals are defined as follows, with $\alpha = 1 - \text{conf}$, t_0 the estimate in the original sample, \hat{t} the average of the bootstrap estimates, s_t the standard deviation of the bootstrap estimates, $t^{(i)}$ the set of ordered estimates with i corresponding to their quantile, and $z_{\frac{\alpha}{2}}$ and $z_{1-\frac{\alpha}{2}}$ the upper and lower critical z scores.

- "norm" (normal approximation): $[2t_0 - \hat{t} + s_t z_{\frac{\alpha}{2}}, 2t_0 - \hat{t} + s_t z_{1-\frac{\alpha}{2}}]$

This involves subtracting the "bias" ($\hat{t} - t_0$) from the estimate t_0 and using a standard Wald-type confidence interval. This method is valid when the statistic is normally distributed.

- "basic": $[2t_0 - t^{(1-\frac{\alpha}{2})}, 2t_0 - t^{(\frac{\alpha}{2})}]$
- "perc" (percentile confidence interval): $[t^{(\frac{\alpha}{2})}, t^{(1-\frac{\alpha}{2})}]$
- "bc" (bias-corrected percentile confidence interval): $[t^{(l)}, t^{(u)}]$

$l = \Phi(2z_0 + z_{\frac{\alpha}{2}})$, $u = \Phi(2z_0 + z_{1-\frac{\alpha}{2}})$, where $\Phi(\cdot)$ is the normal cumulative density function (i.e., `pnorm()`) and $z_0 = \Phi^{-1}(q)$ where q is the proportion of bootstrap estimates less than the original estimate t_0 . This is similar to the percentile confidence interval but changes the specific quantiles of the bootstrap estimates to use, correcting for bias in the original estimate. It is described in Xu et al. (2020). When t^0 is the median of the bootstrap distribution, the "perc" and "bc" intervals coincide.

- "bca" (bias-corrected and accelerated confidence interval): $[t^{(l)}, t^{(u)}]$

$l = \Phi\left(z_0 + \frac{z_0 + z_{\frac{\alpha}{2}}}{1 - a(z_0 + z_{\frac{\alpha}{2}})}\right)$, $u = \Phi\left(z_0 + \frac{z_0 + z_{1-\frac{\alpha}{2}}}{1 - a(z_0 + z_{1-\frac{\alpha}{2}})}\right)$, using the same definitions as above,

but with the additional acceleration parameter a , where $a = \frac{1}{6} \frac{\sum L^3}{(\sum L^2)^{3/2}}$. L is the empirical influence value of each unit, which is computed using the regression method described in `boot::empinf()`. The acceleration parameter corrects for bias and skewness in the statistic. It can only be used when

clusters are absent and the number of bootstrap replications is larger than the sample size. When $a = 0$, the "bca" and "bc" intervals coincide.

Interpolation on the normal quantile scale is used when a non-integer order statistic is required, as in `boot::boot.ci()`. Note that unlike with `boot::boot.ci()`, studentized confidence intervals (`type = "stud"`) are not allowed.

Value

An `fwbci` object, which inherits from `bootci` and has the following components:

<code>R</code>	the number of bootstrap replications in the original call to <code>fwb()</code> .
<code>t0</code>	the observed value of the statistic on the same scale as the intervals (i.e., after applying <code>h</code> and then <code>hinv</code>).
<code>call</code>	the call to <code>fwb.ci()</code> .

There will be additional components named after each confidence interval type requested. For "norm", this is a matrix with one row containing the confidence level and the two confidence interval limits. For the others, this is a matrix with one row containing the confidence level, the indices of the two order statistics used in the calculations, and the confidence interval limits.

Functions

- `print(fwbci)`: Print a bootstrap confidence interval

See Also

`fwb()` for performing the fractional weighted bootstrap; `get_ci()` for extracting confidence intervals from an `fwbci` object; `summary.fwb()` for producing clean output from `fwb()` that includes confidence intervals calculated by `fwb.ci()`; `boot::boot.ci()` for computing confidence intervals from the traditional bootstrap; `vcovFWB()` for computing parameter estimate covariance matrices using the fractional weighted bootstrap

Examples

```
set.seed(123)
data("infert")

fit_fun <- function(data, w) {
  fit <- glm(case ~ spontaneous + induced, data = data,
            family = "quasibinomial", weights = w)
  coef(fit)
}

fwb_out <- fwb(infert, fit_fun, R = 199, verbose = FALSE)

# Bias corrected percentile interval
bccci <- fwb.ci(fwb_out, index = "spontaneous", type = "bc")
bccci

# Using `get_ci()` to extract confidence limits
get_ci(bccci)

# Interval calculated on original (log odds) scale,
# then transformed by exponentiation to be on OR
```

```
fwb.ci(fwb_out, index = "induced", type = "norm",
       hinv = exp)
```

get_ci

Extract Confidence Intervals from a bootci Object

Description

`get_ci()` extracts the confidence intervals from the output of a call to `boot::boot.ci()` or `fwb.ci()` in a clean way. Normally the confidence intervals can be a bit challenging to extract because of the unusual structure of the object.

Usage

```
get_ci(x, type = "all")
```

Arguments

`x` a `bootci` object; the output of a call to `boot::boot.ci()` or `fwb.ci()`.

`type` the type of confidence intervals to extract. Only those available in `x` are allowed. Should be a given as a subset of the types passed to `type` in `boot.ci()` or `fwb.ci()`. The default, "all", extracts all confidence intervals in `x`.

Value

A list with an entry for each confidence interval type; each entry is a numeric vector of length 2 with names "L" and "U" for the lower and upper interval bounds, respectively. The "conf" attribute contains the confidence level.

See Also

`fwb.ci()`, `boot::boot.ci()`

Examples

```
#See example at help("fwb.ci")
```


plot.fwb

*Plots of the Output of a Fractional Weighted Bootstrap***Description**

plot.fwb() takes an fwb object and produces plots for the bootstrap replicates of the statistic of interest.

Usage

```
## S3 method for class 'fwb'
plot(
  x,
  index = 1,
  qdist = "norm",
  nclass = NULL,
  df,
  type = c("hist", "qq"),
  ...
)
```

Arguments

x	an fwb object; the output of a call to <code>fwb()</code> .
index	the index of the position of the quantity of interest in <code>x</code> if more than one was specified in <code>fwb()</code> . Only one value is allowed at a time. By default the first statistic is used.
qdist	character; when a Q-Q plot is requested (as it is by default; see <code>type</code> argument below), the distribution against which the Q-Q plot should be drawn. Allowable options include "norm" (normal distribution - the default) and "chisq" (chi-squared distribution).
nclass	when a histogram is requested (as it is by default; see <code>type</code> argument below), the number of classes to be used. The default is the integer between 10 and 100 closest to <code>ceiling(length(R)/25)</code> where R is the number of bootstrap replicates.
df	if <code>qdist</code> is "chisq", the degrees of freedom for the chi-squared distribution to be used. If not supplied, the degrees of freedom will be estimated using maximum likelihood.
type	the type of plot to display. Allowable options include "hist" for a histogram of the bootstrap estimates and "qq" for a Q-Q plot of the estimates against the distribution supplied to <code>qdist</code> .
...	ignored.

Details

This function can produce two side-by-side plots: a histogram of the bootstrap replicates and a Q-Q plot of the bootstrap replicates against theoretical quantiles of a supplied distribution (normal or chi-squared). For the histogram, a vertical dotted line indicates the position of the estimate computed in the original sample. For the Q-Q plot, the expected line is plotted.

Value

x is returned invisibly.

See Also

[fwb\(\)](#), [summary.fwb\(\)](#), [boot::plot.boot\(\)](#), [hist\(\)](#), [qqplot\(\)](#)

Examples

```
# See examples at help("fwb")
```

```
summary.fwb
```

```
Summarize fwb Output
```

Description

`summary()` creates a regression summary-like table that displays the bootstrap estimates, their empirical standard errors, and their confidence intervals, which are computed using [fwb.ci\(\)](#).

Usage

```
## S3 method for class 'fwb'
summary(
  object,
  conf = 0.95,
  ci.type = "bc",
  p.value = FALSE,
  index = 1L:ncol(object$t),
  ...
)
```

Arguments

<code>object</code>	an <code>fwb</code> object; the output of a call to fwb() .
<code>conf</code>	the desired confidence level. Default is .95 for 95% confidence intervals.
<code>ci.type</code>	the type of confidence interval desired. Allowable options include "norm" (normal approximation), "basic" (basic interval), "perc" (percentile interval), "bc" (bias-correct percentile interval), and "bca" (bias-corrected and accelerated [BCa] interval). Only one is allowed. BCa intervals require that the number of bootstrap replications is larger than the sample size. See fwb.ci() for details. The default is "bc".
<code>p.value</code>	logical; whether to display p-values for the test that each parameter is equal to 0. The p-value is computed using a Z-test with the test statistic computed as the ratio of the estimate to its bootstrap standard error. This test is only valid when the bootstrap distribution is normally distributed around 0 and is not guaranteed to agree with any of the confidence intervals. Default is FALSE.
<code>index</code>	the index or indices of the position of the quantity of interest in <code>x\$t0</code> if more than one was specified in fwb() . Default is to display all quantities.
<code>...</code>	ignored.

Value

A summary .fwb object, which is a matrix with the following columns:

- Estimate: the statistic estimated in the original sample
- Std. Error: the standard deviation of the bootstrap estimates
- CI {L}% and CI {U}%, the upper and lower confidence interval bounds computed using the argument to ci.type.

When p.value = TRUE, two additional columns, z value and Pr(>|z|) are included containing the z-statistic and p-value for each computed statistic.

See Also

`fwb()` for performing the fractional weighted bootstrap; `fwb.ci()` for computing multiple confidence intervals for a single bootstrapped quantity

Examples

```
set.seed(123)
data("infert")

fit_fun <- function(data, w) {
  fit <- glm(case ~ spontaneous + induced, data = data,
            family = "quasibinomial", weights = w)
  coef(fit)
}

fwb_out <- fwb(infert, fit_fun, R = 199, verbose = FALSE)

# Basic confidence interval for both estimates
summary(fwb_out, ci.type = "basic")

# Just for "induced" coefficient; p-values requested
summary(fwb_out, index = "induced", p.value = TRUE)
```

vcovFWB

Fractional Weighted Bootstrap Covariance Matrix Estimation

Description

`vcovFWB()` estimates the covariance matrix of model coefficient estimates using the fractional weighted bootstrap. It serves as a drop-in for `stats::vcov()` or `sandwich::vcovBS()`. Clustered covariances are can be requested.

Usage

```
vcovFWB(
  x,
  cluster = NULL,
  R = 1000,
  start = FALSE,
  ...,
```

```

  fix = FALSE,
  use = "pairwise.complete.obs",
  verbose = FALSE,
  cl = NULL
)

```

Arguments

<code>x</code>	a fitted model object, such as the output of a call to <code>lm()</code> or <code>glm()</code> . The model object must result from a function that can be updated using <code>update()</code> and has a <code>weights</code> argument to input non-integer case weights.
<code>cluster</code>	a variable indicating the clustering of observations, a <code>list</code> (or <code>data.frame</code>) thereof, or a formula specifying which variables from the fitted model should be used (see examples). By default (<code>cluster = NULL</code>), either <code>attr(x, "cluster")</code> is used (if any) or otherwise every observation is assumed to be its own cluster.
<code>R</code>	the number of bootstrap replications.
<code>start</code>	logical; should <code>coef(x)</code> be passed as <code>start</code> to the <code>update(x, weights = ...)</code> call? In case the model <code>x</code> is computed by some numeric iteration, this may speed up the bootstrapping.
<code>...</code>	ignored.
<code>fix</code>	logical; if <code>TRUE</code> , the covariance matrix is fixed to be positive semi-definite in case it is not.
<code>use</code>	character; specification passed to <code>stats::cov()</code> for handling missing coefficients/parameters.
<code>verbose</code>	logical; whether to display a progress bar.
<code>cl</code>	a cluster object created by <code>parallel::makeCluster()</code> , or an integer to indicate the number of child-processes (integer values are ignored on Windows) for parallel evaluations. See <code>pbapply::pblapply()</code> for details. If <code>NULL</code> , no parallelization will take place.

Details

`vcovFWB()` functions like other `vcov()`-like functions, such as those in the `sandwich` package, in particular, `sandwich::vcovBS()`, which implements the traditional bootstrap (and a few other bootstrap varieties for linear models). Sets of weights are generated as described in the documentation for `fwb()`, and the supplied model is re-fit using those weights. When the fitted model already has weights, these are multiplied by the bootstrap weights.

For `lm` objects, the model is re-fit using `.lm.fit()` for speed, and, similarly, `glm` objects are re-fit using `glm.fit()` (or whichever fitting method was originally used). For other objects, `update()` is used to populate the weights and re-fit the model (this assumes the fitting function accepts non-integer case weights through a `weights` argument). If a model accepts weights in some other way, `fwb()` should be used instead; `vcovFWB()` is inherently limited in its ability to handle all possible models. It is important that the original model was not fit using frequency weights (i.e., weights that allow one row of data to represent multiple full, identical, individual units).

See `sandwich::vcovBS()` and `sandwich::vcovCL()` for more information on clustering covariance matrices, and see `fwb()` for more information on how clusters work with the fractional weighted bootstrap. When clusters are specified, each cluster is given a bootstrap weight, and all members of the cluster are given that weight; estimation then proceeds as normal. By default, when `cluster` is unspecified, each unit is considered its own cluster.

Value

A matrix containing the covariance matrix estimate.

See Also

`fwb()` for performing the fractional weighted bootstrap on an arbitrary quantity; `fwb.ci()` for computing nonparametric confidence intervals for `fwb` objects; `summary.fwb()` for producing standard errors and confidence intervals for `fwb` objects; `sandwich::vcovBS()` for computing covariance matrices using the traditional bootstrap

Examples

```
set.seed(123)
data("infert")
fit <- glm(case ~ spontaneous + induced, data = infert,
           family = "binomial")
lmtest::coefstest(fit, vcov. = vcovFWB, R = 200)

# Example from help("vcovBS", package = "sandwich")
data("PetersenCL", package = "sandwich")
m <- lm(y ~ x, data = PetersenCL)

# Note: this is not to compare performance, just to
# demonstrate the syntax
cbind(
  "BS" = sqrt(diag(sandwich::vcovBS(m))),
  "FWB" = sqrt(diag(vcovFWB(m))),
  "BS-cluster" = sqrt(diag(sandwich::vcovBS(m, cluster = ~firm))),
  "FWB-cluster" = sqrt(diag(vcovFWB(m, cluster = ~firm)))
)
```

Index

* datasets

- bearingcage, [2](#)
- .lm.fit(), [12](#)

- bearingcage, [2](#)
- boot::boot(), [4](#)
- boot::boot.ci(), [6–8](#)
- boot::empinf(), [6](#)
- boot::plot.boot(), [10](#)

- fwb, [2](#)
- fwb(), [5, 7, 9–13](#)
- fwb.ci, [5](#)
- fwb.ci(), [4, 8, 10, 11, 13](#)

- get_ci, [8](#)
- get_ci(), [7](#)
- glm(), [4](#)
- glm.fit(), [12](#)

- hist(), [10](#)

- parallel::makeCluster(), [3, 12](#)
- pbapply::pblapply(), [3, 12](#)
- plot.fwb, [9](#)
- plot.fwb(), [4](#)
- pnorm(), [6](#)
- print.fwb(fwb), [2](#)
- print.fwbci(fwb.ci), [5](#)

- qqplot(), [10](#)
- quasibinomial(), [4](#)

- sandwich::vcovBS(), [12, 13](#)
- sandwich::vcovCL(), [12](#)
- set.seed(), [4](#)
- stats::cov(), [12](#)
- summary.fwb, [10](#)
- summary.fwb(), [4, 6, 7, 10, 13](#)

- update(), [12](#)

- vcovFWB, [11](#)
- vcovFWB(), [4, 7](#)