Package ‘gmp’
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Description Multiple Precision Arithmetic (big integers and rationals,
prime number tests, matrix computation), "arithmetic without limitations"
using the C library GMP (GNU Multiple Precision Arithmetic).
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R topics documented:

apply ................................................................. 2
asNumeric .......................................................... 3
Bigq ................................................................. 4
bigq ................................................................. 5
Bigq operators ..................................................... 6
bigz ................................................................. 8
bigz operators ..................................................... 10
cumsum ............................................................ 13
apply

**Apply Functions Over Matrix Margins (Rows or Columns)**

### Description

These are S3 methods for `apply()` which we re-export as S3 generic function. They “overload” the `apply()` function for big rationals ("bigq") and big integers ("bigz").

### Usage

```r
## S3 method for class 'bigz'
apply(X, MARGIN, FUN, ...)
## S3 method for class 'bigq'
apply(X, MARGIN, FUN, ...)
```

### Arguments

- `X` a matrix of class bigz or bigq, see e.g., `matrix.bigz`
- `MARGIN` 1: apply function to rows; 2: apply function to columns
- `FUN` function to be applied
- `...` (optional) extra arguments for `FUN()`, as e.g., in `lapply`.
Value

The `bigz` and `bigq` methods return a vector of class "bigz" or "bigq", respectively.

Author(s)

Antoine Lucas

See Also

`apply; lapply` is used by our `apply()` method.

Examples

```r
x <- as.bigz(matrix(1:12, 3))
apply(x, 1, min)
apply(x, 2, max)

x <- as.bigq(x^3, d = (x + 3)^2)
apply(x, 1, min)
apply(x, 2, sum)
## now use the "..." to pass na.rm=TRUE :
x[2,3] <- NA
apply(x, 1, sum)
apply(x, 1, sum, na.rm = TRUE)
```

asNumeric

Coerce to 'numeric', not Loosing Dimensions

Description

A number-like object is coerced to type (typeof) "numeric", keeping `dim` (and maybe `dimnames`) when present.

Usage

```r
asNumeric(x)
```

Arguments

- `x` a “number-like” object, e.g., big integer (`bigz`), or `mpfr`, notably including matrices and arrays of such numbers.

Value

An R object of type (typeof) "numeric", a `matrix` or `array` if `x` had non-NULL dimension `dim()`.
Methods

signature(x = "ANY") the default method, which is the identity for numeric array.
signature(x = "bigq") the method for big rationals.
signature(x = "bigq") the method for big integers.
Note that package Rmpfr provides methods for its own number-like objects.

Author(s)

Martin Maechler

See Also

as.numeric coerces to both "numeric" and to a vector, whereas asNumeric() should keep dim (and other) attributes.

Examples

m <- matrix(1:6, 2,3)
stopifnot(identical(m, asNumeric(m)))# remains matrix

(M <- as.bigz(m) / 5) ##-> "bigq" matrix
asNumeric(M) # numeric matrix
stopifnot(all.equal(asNumeric(M), m/5))

Bigq

Relational Operators

Description

Binary operators which allow the comparison of values in atomic vectors.

Usage

## S3 method for class 'bigq'
sign(x)

## S3 method for class 'bigq'
e1 < e2
## S3 method for class 'bigq'
e1 <= e2
## S3 method for class 'bigq'
e1 == e2
## S3 method for class 'bigq'
e1 >= e2
## S3 method for class 'bigq'
e1 > e2
## S3 method for class 'bigq'
e1 != e2
Arguments

- `x, e1, e2`: Object or vector of class `bigq`

Examples

```r
x <- as.bigq(8000, 21)
x < 2 * x
```

---

**bigq**  
**Large sized rationals**

Description

Class "bigq" encodes rationals encoded as ratios of arbitrary large integers (via GMP). A simple S3 class (internally a `raw` vector), it has been registered as formal (S4) class (via `setOldClass`), too.

Usage

```r
as.bigq(n, d = 1)  # S3 method for class 'bigq'
as.character(x, b=10,...)  # S3 method for class 'bigq'
as.double(x,...)  # S3 method for class 'bigq'
as.bigz.bigq(a, mod=NA)  # S3 method for class 'bigq'
is.bigq(x)  # S3 method for class 'bigq'
is.na(x)  # S3 method for class 'bigq'
print(x, quote=FALSE, initLine = TRUE, ...)
denominator(x)
numerator(x)
```

Arguments

- `n, d`: either integer, numeric or string value (String value: either starting with `0x` for hexadecimal, `0b` for binary or without prefix for decimal values. Any format error results in `0`). `n` stands for numerator, `d` for denominator
- `a`: an element of class "bigq"
- `mod`: optional modulus to convert into biginteger
- `x`: a “rational number” (vector), of class "bigq".
- `b`: base: from 2 to 36
- `...`: additional arguments passed to methods
- `quote`: (for printing:) logical indicating if the numbers should be quoted (as characters are); the default used to be `TRUE` (implicitly) till 2011.
initLine (for printing:) logical indicating if an initial line (with the class and length or dimension) should be printed.

Details

as.bigz.bigq() returns the smallest integers not less than the corresponding rationals bigq.

Value

An R object of (S3) class "bigq" representing the parameter value.

Author(s)

Antoine Lucas

Examples

x <- as.bigq(21,6)
x
# 7 / 2
# Wow ! result is simplified.
y <- as.bigq(5,3)
# addition works !
x + y
# You can even try multiplication, division...
x * y / 13
# and, since May 2012,
x ^ 20
stopifnot(is.bigq(x), is.bigq(x + y),
x ^ 20 == as.bigz(7)^20 / 2^20)
# convert to string, double
as.character(x)
as.double(x)
# Depict the "S4-class" bigq, i.e., the formal (S4) methods:
if(require("Rmpfr")) # mostly interesting there
showMethods(class="bigq")
**Bigq operators**

**Usage**

```r
add.bigq(e1, e2)
## S3 method for class 'bigq'
e1 + e2

sub.bigq(e1, e2=NULL)
## S3 method for class 'bigq'
e1 - e2

mul.bigq(e1, e2)
## S3 method for class 'bigq'
e1 * e2

div.bigq(e1, e2)
## S3 method for class 'bigq'
e1 / e2

## S3 method for class 'bigq'
e1 ^ e2

abs(x)
```

**Arguments**

e1,e2, x of class "bigq", or (e1 and e2) integer or string from an integer

**Details**

Operators can be used directly when the objects are of class "bigq": `a + b`, `a * b`, etc, and `a ^ n`, where `n` must be coercable to a biginteger ("bigz").

**Value**

A bigq class representing the result of the arithmetic operation.

**Author(s)**

Immanuel Scholz and Antoine Lucas

**Examples**

```r
## 1/3 + 1 = 4/3 :
as.bigq(1,3) + 1

r <- as.bigq(12, 47)
stopifnot(r ^ 3 == r*r*r)
```
bigz

Large Sized Integer Values

Description

Class "bigz" encodes arbitrarily large integers (via GMP). A simple S3 class (internally a raw vector), it has been registered as formal (S4) class (via setOldClass), too.

Usage

as.bigz(a, mod = NA)
## S3 method for class 'bigz'
as.character(x, b = 10, ...)

is.bigz(x)
## S3 method for class 'bigz'
is.na(x)
## S3 method for class 'bigz'
print(x, quote=FALSE, initLine = is.null(modulus(x)), ...)

Arguments

a
either integer, numeric (i.e., double) or character vector.

b
to 36

x
a "big integer number" (vector), of class "bigz".

...additional arguments passed to methods

mod
an integer, numeric, string or bigz of the internal modulus, see below.

quote
(for printing:) logical indicating if the numbers should be quoted (as characters are); the default used to be TRUE (implicitly) till 2011.

initLine
(for printing:) logical indicating if an initial line (with the class and length or dimension) should be printed. The default prints it for those cases where the class is not easily discernable from the print output.

Details

Bigz’s are integers of arbitrary, but given length (means: only restricted by the host memory). Basic arithmetic operations can be performed on bigzs as addition, subtraction, multiplication, division, modulation (remainder of division), power, multiplicative inverse, calculating of the greatest common divisor, test whether the integer is prime and other operations needed when performing standard cryptographic operations.

For a review of basic arithmetics, see add.bigz.

Comparison are supported, i.e., "==", "!=", "<", "<=", ">", and ">=".
Objects of class "bigz" may have a “modulus”, accessible via `modulus()`. currently as an attribute `mod`. When the object has such a modulus $m$, arithmetic is performed “modulo $m$”, mathematically “within the ring $\mathbb{Z}/m\mathbb{Z}$”. For many operations, this means

```r
result <- mod.bigz(result, m) ## == result %% m
```

is called after performing the arithmetic operation and the result will have the attribute `mod` set accordingly. This however does not apply, e.g., for `/`, where $a/b := ab^{-1}$ and $b^{-1}$ is the *multiplicative inverse* of $b$ with respect to ring arithmetic, or `NA` with a warning when the inverse does not exist. The warning can be turned off via `options("gmp:warnModMismatch" = FALSE)`

Powers of `bigz` can only be performed, if either a modulus is going to be applied to the result `bigz` or if the exponent fits into an integer value. So, if you want to calculate a power in a finite group ("modulo $c$"), for large $c$ do not use `a ^ b %% c`, but rather `as.bigz(a, c) ^ b`.

The following rules for the result’s modulus apply when performing arithmetic operations on `bigz`:

- If none of the operand has a modulus set, the result will not have a modulus.
- If both operands have a different modulus, the result will not have a modulus, except in case of `mod.bigz`, where the second operand’s value is used.
- If only one of the operands has a modulus or both have a common (the same), it is set and used for the arithmetic operations, except in case of `mod.bigz`, where the second operand’s value is used.

**Value**

An R object of (S3) class “bigz”, representing the argument (x or a).

**Note**

```r
x <- as.bigz(1234567890123456789012345678901234567890)
```

will not work as `R` converts the number to a double, losing precision and only then convert to a "bigz" object.

Instead, use the syntax

```r
x <- as.bigz("1234567890123456789012345678901234567890")
```

**Author(s)**

Immanuel Scholz

**References**

The GNU MP Library, see [http://gmplib.org](http://gmplib.org)
Examples

## 1+1=2
a <- as.bigz(1)
a + a

## Two non-small Mersenne primes:
two <- as.bigz(2)
p1 <- two^107 - 1 ; isprime(p1); p1
p2 <- two^127 - 1 ; isprime(p2); p2

## Calculate \( c = x^e \mod n \)
## -----------------------------------------------
x <- as.bigz("0x123456789abcdef") # my secret message
e <- as.bigz(3) # something smelling like a dangerous public RSA exponent
(n <- p1 * p2) # a product of two primes
as.character(n, b=16)# as both primes were Mersenne's.

## recreate the three numbers above [for demo below]:
n. <- n; x. <- x; e. <- e # save
Rev <- function() { n <<- n.; x <<- x.; e <<- e.}

# first way to do it right
modulus(x) <- n
c <- x ^ e ; c ; Rev()

# similar second way (makes more sense if you reuse e) to do it right
modulus(e) <- n
c2 <- x ^ e
stopifnot(identical(c2, c), is.bigz(c2)); Rev()

# third way to do it right
c3 <- x ^ as.bigz(e, n); stopifnot(identical(c3, c))

# fourth way to do it right
c4 <- as.bigz(x, n) ^ e ; stopifnot(identical(c4, c))

# WRONG! (although very beautiful. Ok only for very small 'e' as here)
cc <- x ^ e %% n
cc == c

# Return result in hexa
as.character(c, b=16)

# Depict the "S4-class" bigz, i.e., the formal (S4) methods:
if(require("Rmpfr")) # mostly interesting there
  showMethods(class="bigz")
Description

Addition, subtraction, multiplication, (integer) division, remainder of division, multiplicative inverse, power and logarithm functions.

Usage

```r
add.bigz(e1, e2)
sub.bigz(e1, e2 = NULL)
mul.bigz(e1, e2)
div.bigz(e1, e2)
divq.bigz(e1, e2) ## == e1 %/% e2
mod.bigz(e1, e2) ## == e1 %% e2
## S3 method for class 'bigz'
abs(x)
inv.bigz(a, b,...)## == (1 / a) (modulo b)
pow.bigz(e1, e2,...)## == e1 ^ e2
## S3 method for class 'bigz'
log(x, base=exp(1))
## S3 method for class 'bigz'
log2(x)
## S3 method for class 'bigz'
log10(x)
```

Arguments

- `x` : bigz, integer or string from an integer
- `e1, e2, a,b` : bigz, integer or string from an integer
- `base` : base of the logarithm; base e as default
- `...` : Additional parameters

Details

Operators can be used directly when objects are of class bigz: `a + b`, `log(a)`, etc.

For details about the internal modulus state, and the rules applied for arithmetic operations on big integers with a modulus, see the `bigz` help page.

- `a / b = div(a, b)` returns a rational number unless the operands have a (matching) modulus where `a * b^-1` results.
- `a %% b` (or, equivalently, `divq(a, b)`) returns the quotient of simple integer division (with truncation towards zero), possibly re-adding a modulus at the end (but not using a modulus like in `a / b`).
- `r <- inv.bigz(a, m)`, the multiplicative inverse of a modulo `m`, corresponds to `1/a` or `a^-1` from above when `a` has modulus `m`. Note that `a` not always has an inverse modulo `m`, in which case `r` will be `NA` with a warning that can be turned off via

```r
```

```r
```
options("gmp:warnNoInv" = FALSE)
.

Value

Apart from / (or \texttt{div}), where rational numbers (\texttt{bigq}) may result, these functions return an object of class "\texttt{bigz}" representing the result of the arithmetic operation.

Author(s)

Immanuel Scholz and Antoine Lucas

References

The GNU MP Library, see \url{http://gmplib.org}

Examples

# 1+1=2
as.bigz(1) + 1
as.bigz(2)^10
as.bigz(2)^200

# if \texttt{my.large.num.string} is set to a number, this returns the least byte
\texttt{(my.large.num.string} <- paste(sample(0:9, 200, replace=TRUE), collapse="")
\texttt{mod.bigz(as.bigz(my.large.num.string), }"\texttt{0xff")}

# power exponents can be up to MAX\_INT in size, or unlimited if a
# \texttt{bigz}'s modulus is set.
\texttt{pow.bigz(10,10000)}

## Modulo 11, 7 and 8 are inverses:
\texttt{as.bigz(7, mod = 11) * 8} # \texttt{=} 1 (mod 11)
\texttt{inv.bigz(7, 11)}## hence, 8
\texttt{a} <- \texttt{1:10}
\texttt{(i.a <- inv.bigz(a, 11))
\texttt{d <- as.bigz(7))
\texttt{a /%/% d} # = \texttt{divq(a, d)}
\texttt{a /%/% d} # = \texttt{mod.bigz (a, d)}

(ii <- inv.bigz(1:10, 16))
## with 5 warnings (one for each NA)
opt <- options("gmp:warnNoInv" = FALSE)
i2 <- inv.bigz(1:10, 16) # no warnings
(i3 <- 1 / as.bigz(1:10, 16))
i4 <- as.bigz(1:10, 16) ^ -1
stopifnot(identical(ii, i2),
identical(as.bigz(i2, 16), i3),
identical(i3, i4))
options(op)# revert previous options' settings
stopifnot(inv.bigz(7, 11) == 8,
          all(as.bigz(1:a, 11) * a == 1),
          identical(a %/% d, divq.bigz(1:10, 7)),
          identical(a %% d, mod.bigz(a, d))
)

---

**cumsum**

(Cumulative) Sums, Products of Large Integers and Rationals

**Description**

These are methods to ‘overload’ the `sum()`, `cumsum()` and `prod()` functions for big rationals and big integers.

**Usage**

```r
## S3 method for class 'bigz'
cumsum(x)
## S3 method for class 'bigq'
cumsum(x)
## S3 method for class 'bigz'
sum(..., na.rm = FALSE)
## S3 method for class 'bigq'
sum(..., na.rm = FALSE)
## S3 method for class 'bigz'
prod(..., na.rm = FALSE)
## S3 method for class 'bigq'
prod(..., na.rm = FALSE)
```

**Arguments**

- `x, ...`  
  
  R objects of class bigz or bigq or ‘simple’ numbers.

- `na.rm`  
  
  logical indicating if missing values (NA) should be removed before the computation.

**Value**

return an element of class bigz or bigq.

**Author(s)**

Antoine Lucas

**See Also**

`apply`
Examples

```
x <- as.bigz(1:12)
cumsum(x)
prod(x)
sum(x)

x <- as.bigq(1:12)
cumsum(x)
prod(x)
sum(x)
```

extract

Extract or Replace Parts of an Object

Description

Operators acting on vectors, arrays and lists to extract or replace subsets.

Usage

```
## S3 method for class 'bigz'
[.bigz

## S3 method for class 'bigq'
[.bigq

## In the following, only the bigq method is mentioned: ______

## S3 method for class 'bigq'
c(..., recursive = FALSE)

## S3 method for class 'bigq'
rep(x, times=1, length.out=NA, each=1, ...)
```

Arguments

- `x` R object of class "bigz" or "bigq", respectively.
- `...` further arguments, notably for `c()`.
- `i, j` indices, see standard R subsetting and subassignment.
- `drop` logical, unused here.
- `times, length.out, each` integer; typically only `one` is specified; for more see `rep` (standard R, package base).
- `recursive` unused here
Note

Unlike standard matrices, x[i] and x[,i] do the same.

Examples

```r
a <- as.bigz(123)
## indexing "outside" --> extends the vectors (filling with NA)
a[4] <- -4

## create a vector of 3 a
c(a,a,a)

## repeat a 5 times
rep(a,5)

## with matrix
m <- matrix.bigz(1:6,3)

## these do the same:
m[1,]
m[1]
m[-c(2,3),]
m[-c(2,3)]
m[c(TRUE, FALSE, FALSE)]

## modification on matrix
m[2,-1] <- 11
```

Extremes

Extrema (Maxima and Minima)

Description

We provide S3 methods for `min` and `max` for big rationals (`bigq`) and big integers (`biqz`); consequently, `range()` works as well.

Similarly, S4 methods are provided for `which.min()` and `which.max()`.

Usage

```r
## S3 method for class 'bigz'
max(..., na.rm=FALSE)
```

```r
## S3 method for class 'bigq'
max(..., na.rm=FALSE)
```
## S4 method for signature 'bigz'
which.min(x)

## S4 method for signature 'bigq'
which.max(x)

### Arguments

- **x**: a “big integer” (bigz) or “big rational” (bigq) vector.
- **...**: numeric arguments
- **na.rm**: a logical indicating whether missing values should be removed.

### Value

return an element of class bigz or bigq.

### Author(s)

Antoine Lucas

### See Also

max

### Examples

```r
x <- as.bigz(1:10)
max(x)
min(x)
range(x) # works correctly via default method
x <- x[c(7:10, 6:3, 1:2)]
which.min(x) ## 9
which.max(x) ## 4

Q <- as.bigq(1:10, 3)
max(Q)
min(Q)
(Q <- Q[c(6:3, 7:10, 1:2)])
stopifnot(which.min(Q) == which.min(asNumeric(Q)),
          which.max(Q) == which.max(asNumeric(Q)))
stopifnot(range(x) == c(1,10), 3*range(Q) == c(1,10))
```
factorialZ

Factorial and Binomial Coefficient as Big Integer

Description

Efficiently compute the factorial $n!$ or a binomial coefficient \( \binom{n}{k} \) as big integer (class bigz).

Usage

factorialZ(n)
chooseZ(n, k)

Arguments

n  non-negative integer (vector), for factorialZ. For chooseZ, may be a bigz big integer, also negative.
k  non-negative integer vector.

Value

a vector of big integers, i.e., of class bigz.

See Also

factorial and gamma in base R;

Examples

factorialZ(0:10)# 1 1 2 6 ... 3628800
factorialZ(0:40)# larger
factorialZ(200)

n <- 1000
f1000 <- factorialZ(n)
stopifnot(1e-15 > abs(as.numeric(1 - lfactorial(n)/log(f1000))))

system.time(replicate(8, f1e4 <<- factorialZ(10000)))
nchar(as.character(f1e4))# 35660 ... (too many to even look at ..)

chooseZ(1000, 100:102)# vectorizes
chooseZ(as.bigz(2)^120, 10)

n <- c(50,80,100)
k <- c(20,30,40)
## currently with an undesirable warning: % from methods/src/eval.c _FIXME_
stopifnot(chooseZ(n,k) == factorialZ(n) / (factorialZ(k)*factorialZ(n-k)))
factorization

Factorize a number

Description

Give all primes numbers to factor the number

Usage

factorize(n)

Arguments

n

Either integer, numeric or string value (String value: either starting with 0x for hexadecimal, 0b for binary or without prefix for decimal values.) Or an element of class bigz.

Details

The factorization function uses the Pollard Rho algorithm.

Value

Vector of class bigz.

Author(s)

Antoine Lucas

References

The GNU MP Library, see http://gmplib.org

Examples

factorize(34455342)
Description

Format (generalized) numbers in a way that their classes are distinguishable. Contrary to `format()` which uses a common format for all elements of `x`, here, each entry is formatted individually.

Usage

```r
formatN(x, ...)
## Default S3 method:
formatN(x, ...)
## S3 method for class 'integer'
formatN(x, ...)
## S3 method for class 'double'
formatN(x, ...)
## S3 method for class 'bigz'
formatN(x, ...)
## S3 method for class 'bigq'
formatN(x, ...)
```

Arguments

- `x`: any R object, typically “number-like”.
- `...`: potentially further arguments passed to methods.

Value

A character vector of the same length as `x`, each entry a representation of the corresponding entry in `x`.

Author(s)

Martin Maechler

See Also

`format`, including its (sophisticated) default method; `as.character`.

Examples

```r
## Note that each class is uniquely recognizable from its output:
formatN(-2:5)# integer
formatN(0 + -2:5)# double precision
formatN(as.bigz(-2:5))
formatN(as.bigq(-2:5, 4))
```
**frexpZ**  
*Split Number into Fractional and Exponent of 2 Parts*

**Description**

Breaks the number \( x \) into its binary significand ("fraction") \( d \in [0.5, 1) \) and \( ex \), the integral exponent for 2, such that \( x = d \cdot 2^{ex} \).

If \( x \) is zero, both parts (significand and exponent) are zero.

**Usage**

\[
\text{frexpZ}(x)
\]

**Arguments**

- \( x \) integer or big integer (\texttt{bigz}).

**Value**

- a list with the two components
  - \( d \) a numeric vector whose absolute values are either zero, or in \([\frac{1}{2}, 1)\).
  - \( \text{exp} \) an integer vector of the same length; note that \( \exp = 1 + \text{floor}(\log_2(x)) \), and hence always \( \exp > \log_2(x) \).

**Author(s)**

Martin Maechler

**See Also**

\texttt{log2}, etc; for \texttt{bigz} objects built on (the C++ equivalent of) \texttt{frexp()}, actually GMP’s ‘\texttt{mpz_get_d_2exp()}’.

**Examples**

\[
\begin{align*}
\text{frexpZ}(1:10) \\
\text{# and confirm :}
\text{with(frexpZ(1:10), d * 2^exp)} \\
x <- \text{rpois(1000, lambda=100)} * (1 + \text{rpois(1000, lambda=16)}) \\
X <- \text{as.bigz(x)} \\
\text{stopifnot(all.equal(x, with(frexpZ(x), d* 2^exp)),} \\
\quad 1+\text{floor}(\log_2(x)) == (fx <- \text{frexpZ(x)$exp}), \\
\quad fx == \text{frexpZ(X)$exp,} \\
\quad 1+\text{floor}(\log_2(X)) == fx
\end{align*}
\]
gcd.bigz

Greatest Common Divisor (GCD) and Least Common Multiple (LCM)

Description

Compute the greatest common divisor (GCD) and least common multiple (LCM) of two (big) integers.

Usage

```r
## S3 method for class 'bigz'
gcd(a, b)
lcm.bigz(a, b)
```

Arguments

- `a, b` Either integer, numeric, `bigz` or a string value; if a string, either starting with `0x` for hexadecimal, `0b` for binary or without prefix for decimal values.

Value

An element of class `bigz`

Author(s)

Antoine Lucas

References

The GNU MP Library, see [http://gmplib.org](http://gmplib.org)

See Also

gcdex

Examples

```r
gcd.bigz(210, 342) # or also
lcm.bigz(210, 342)
a <- 210; b <- 342
stopifnot(gcd.bigz(a, b) * lcm.bigz(a, b) == a * b)
```

```r
## or
(a <- as.bigz("82696155787249022588"))
(b <- as.bigz("65175989479756205392"))
gcd(a, b) # 4
stopifnot(gcd(a, b) * lcm.bigz(a, b) == a * b)
```
**Compute Bezout Coefficient**

**Description**

Compute \( a s + bt = g = \gcd(a, b) \). \( s \) and \( t \) are also known as Bezout coefficients.

**Usage**

\[ \text{gcdex}(a, b) \]

**Arguments**

\( a, b \)

either integer, numeric, character string, or of class "bigz": If a string, either starting with "0x" for hexadecimal, "0b" for binary or without prefix for decimal values.

**Value**

a class "bigz" vector of length 3 with (long integer) values \( g, s, t \).

**Author(s)**

Antoine Lucas

**References**

The GNU MP Library, see http://gmplib.org

**See Also**

gcd.bigz

**Examples**

\[ \text{gcdex}(342, 654) \]
**gmp-ifiworkarounds**

**Base Functions in 'gmp'-ified Versions**

**Description**

Functions from base etc which need a copy in the gmp namespace so they correctly dispatch.

**Usage**

outer(X, Y, FUN = "*", 

**Arguments**

X, Y, FUN, ... See base package help: outer.

**See Also**

outer.

**Examples**

twop <- as.bigz(2)^(99:103)
(mtwt <- outer(twop, 0:2))
stopifnot(
  identical(dim(mtw), c(5,3))
  ,
  mtw[,1] == 0
  ,
  identical(as.vector(mtw[,2]), twop)
)

**gmp.utils**

**GMP Number Utilities**

**Description**

gmpVersion() returns the version of the GMP library which gmp is currently linked to.

**Usage**

gmpVersion()

**References**

The GNU MP Library, see http://gmplib.org

**Examples**

gmpVersion()
Description

Check which elements of $x[]$ are integer valued aka “whole” numbers.

Usage

```r
is.whole(x)
## Default S3 method:
is.whole(x)
## S3 method for class 'bigz'
is.whole(x)
## S3 method for class 'bigq'
is.whole(x)
```

Arguments

- `x`: any R vector

Value

logical vector of the same length as `x`, indicating where $x[.]$ is integer valued.

Author(s)

Martin Maechler

See Also

`is.integer(x)` (base package) checks for the internal mode or class; not if $x[i]$ are integer valued.

The `is.whole()` method for "mpfr" numbers.

Examples

```r
is.integer(3) # FALSE, it's internally a double
is.whole(3)  # TRUE
## integer valued complex numbers (two FALSE):
is.whole(c(7, 1 + 1i, 1.2, 3.4i, 7i))
is.whole(factorialZ(20)^(10:12)) ## "bigz" are *always* whole numbers
q <- c(as.bigz(36)^50 / as.bigz(30)^40, 3, factorialZ(30:31), 12.25)
is.whole(q) # F T T F
```
isprime

Determine if number is (very probably) prime

Description

Determine whether the number $n$ is prime or not, with three possible answers:

2: $n$ is prime,
1: $n$ is probably prime (without being certain),
0: $n$ is composite.

Usage

isprime(n, reps = 40)

Arguments

- **n**: integer number, to be tested.
- **reps**: integer number of primality testing repeats.

Details

This function does some trial divisions, then some Miller-Rabin probabilistic primary tests. The `reps` controls how many such tests are done, 5 to 10 is already a reasonable number. More will reduce the chances of a composite being returned as “probably prime”.

Value

- 0: $n$ is not prime
- 1: $n$ is probably prime
- 2: $n$ is prime

Author(s)

Antoine Lucas

References

The GNU MP Library, see http://gmplib.org

See Also

`nextprime`, `factorize`.

Note that for “small” $n$, which means something like $n < 10^{000}$, non-probabilistic methods (such as `factorize()`) are fast enough. For example, `primes` in package `sfsmisc`. 
Examples

```r
isprime(210)
isprime(71)

# All primes numbers from 1 to 100
t <- isprime(1:99)
(1:99)[t > 0]
table(isprime(1:10000))# 0 and 2 : surely prime or not prime

primes <- function(n) {
  ## all primes <= n
  stopifnot(length(n) == 1, n <= 1e7) # be reasonable
  p <- c(2L, as.integer(seq(3, n, by=2)))
p[isprime(p) > 0]
}

## quite quickly, but for these small numbers
## still slower than e.g., sfsmisc::primes()
system.time(p100k <- primes(100000))

## The first couple of Mersenne primes:
p.exp <- primes(1000)
Mers <- as.bigz(2)^p.exp - 1
isp.M <- sapply(seq_along(Mers), function(i) isprime(Mers[i], reps=256))
cbind(p.exp, isp.M)[isp.M > 0,]
Mers[isp.M > 0]
```

**Description**

`fibnum` compute n-th Fibonacci number. `fibnum2` compute (n-1)-th and n-th Fibonacci number. `lucnum` compute n-th lucas number. `lucnum2` compute (n-1)-th and n-th lucas number.

Fibonacci numbers are define by: $F_n = F_{n-1} + F_{n-2}$ Lucas numbers are define by: $L_n = F_n + 2F_{n-1}$

**Usage**

```r
fibnum(n)
fibnum2(n)
lucnum(n)
lucnum2(n)
```

**Arguments**

`n` Integer
matrix

Value

Fibonacci numbers and Lucas number.

Author(s)

Antoine Lucas

References

The GNU MP Library, see http://gmplib.org

Examples

fibnum(10)
fibnum2(10)
lucnum(10)
lucnum2(10)

matrix

Matrix manipulation with gmp

Description

Overload of “all” standard tools useful for matrix manipulation adapted to large numbers.

Usage

## S3 method for class 'bigz'
matrix(data = NA, nrow = 1, ncol = 1, byrow = FALSE, dimnames = NULL, mod = NA,...)
is.matrixZQ(x)

## S3 method for class 'bigz'
x %*% y
## S3 method for class 'bigq'
x %*% y
## S3 method for class 'bigq'
crossprod(x, y=NULL)
## S3 method for class 'bigz'
tcrossprod(x, y=NULL)
## ..... etc
Arguments

data      an optional data vector
nrow      the desired number of rows
ncol      the desired number of columns
byrow     logical. If FALSE (the default), the matrix is filled by columns, otherwise the matrix is filled by rows.
dimnames  not implemented for "bigz" or "bigq" matrices.
mod       optional modulus (when data is "bigz").
...       Not used
x, y      numeric, bigz, or bigq matrices or vectors.

Details

The extract function ("[" is the same use for vector or matrix. Hence, x[i] returns the same values as x[i,]. This is not considered a feature and may be changed in the future (with warnings).

All matrix multiplications should work as with numeric matrices.

Special features concerning the "bigz" class: the modulus can be

Unset: Just play with large numbers

Set with a vector of size 1: Example: matrix.bigz(1:6,nrow=2,ncol=3,mod=7) This means you work in \( \mathbb{Z}/n\mathbb{Z} \), for the whole matrix. It is the only case where the \%\% and solve functions will work in \( \mathbb{Z}/n\mathbb{Z} \).

Set with a vector smaller than data: Example: matrix.bigz(1:6,nrow=2,ncol=3,mod=1:5). Then, the modulus is repeated to the end of data. This can be used to define a matrix with a different modulus at each row.

Set with same size as data: Modulus is defined for each cell

Value

matrix(): A matrix of class "bigz" or "bigq".
is.matrixZQ(): TRUE or FALSE.
dim(), ncol(), etc: integer or NULL, as for simple matrices.

Author(s)

Antoine Lucas

See Also

Solving a linear system: solve.bigz.matrix
Examples

V <- as.bigz(v <- 3:7)
crossprod(V)# scalar product
(C <- t(V))
stopifnot(dim(C) == dim(t(v)), C == v,
        dim(t(C)) == c(length(v), 1),
crossprod(V) == sum(V * V),
tcrossprod(V) == outer(v,v),
        identical(C, t(t(C))),
        is.matrixZQ(C), !is.matrixZQ(V), !is.matrixZQ(5)
)

## a matrix
x <- diag(1:4)
## invert this matrix
(xI <- solve(x))

## matrix in Z/7Z
y <- as.bigz(x,7)
## invert this matrix (result is *different* from solve(x)):
(yI <- solve(y))
stopifnot(yI %*% y == diag(4),
         y %*% yI == diag(4))

## matrix in Q
z <- as.bigq(x)
## invert this matrix (result is the same as solve(x))
(zI <- solve(z))

stopifnot(abs(zI - xI) <= 1e-13,
        z %*% zI == diag(4),
        identical(crossprod(zI), zI %*% t(zI))
)

A <- matrix(2^as.bigz(1:12), 3,4)
for(a in list(A, as.bigq(A, 16), factorialZ(20), as.bigq(2:9, 3:4))) {
a.a <- crossprod(a)
aa. <- tcrossprod(a)
stopifnot(identical(a.a, crossprod(a,a)),
          identical(a.a, t(a) %*% a),
          identical(aa., tcrossprod(a,a)),
          identical(aa., a %*% t(a))
)
}

modulus

Modulus of a Big Integer
Description
The modulus of a bigz number \( a \) is “unset” when \( a \) is a regular integer, \( a \in \mathbb{Z} \). Or the modulus can be set to \( m \) which means \( a \in \mathbb{Z}/m \cdot \mathbb{Z} \), i.e., all arithmetic with \( a \) is performed ‘modulo \( m \)’.

Usage
modulus(a)
modulus(a) <- value

Arguments

\begin{itemize}
\item \texttt{a} \quad \texttt{R} object of class "\texttt{bigz}"
\item \texttt{value} \quad integer number or object of class "\texttt{bigz}".
\end{itemize}

Examples
\begin{verbatim}
x <- as.bigz(24)
modulus(x) # NULL, i.e. none

# x element of \( \mathbb{Z}/31\mathbb{Z} \):
modulus(x) <- 31
x+x # 48 |-> (17 \%\% 31)
10*x # 240 |-> (23 \%\% 31)
x31 <- x

# reset modulus to “none”:
modulus(x) <- NA; x; x. <- x
x <- x31
modulus(x) <- NULL; x

stopifnot(identical(x, as.bigz(24)), identical(x, x.), identical(modulus(x31), as.bigz(31)))
\end{verbatim}

nextprime

\textit{Next Prime Number}

Description
Return the next prime number, say \( p \), with \( p > n \).

Usage
nextprime(n)

Arguments

\begin{itemize}
\item \texttt{n} \quad \texttt{Integer}
\end{itemize}
Details

This function uses probabilistic algorithm to identify primes. For practical purposes, it is adequate, the chance of a composite passing will be extremely small.

Value

A (probably) prime number

Author(s)

Antoine Lucas

References

The GNU MP Library, see http://gmplib.org

See Also

isprime and its references and examples.

Examples

nextprime(14)
## still very fast:
(p <- nextprime(1e7))
## to be really sure { isprime() gives "probably prime" }:
stopifnot(identical(p, factorize(p)))

Oakley

RFC 2409 Oakley Groups - Parameters for Diffie-Hellman Key Exchange

Description

RFC 2409 standardizes global unique prime numbers and generators for the purpose of secure asymmetric key exchange on the Internet.

Usage

data(Oakley1)
data(Oakley2)

Value

Oakley1 returns an object of class bigz for a 768 bit Diffie-Hellman group. The generator is stored as value with the respective prime number as modulus attribute.

Oakley2 returns an object of class bigz for a 1024 bit Diffie-Hellman group. The generator is stored as value with the respective prime number as modulus attribute.
References

The Internet Key Exchange (RFC 2409), Nov. 1998

Examples

packageDescription("gmp") # {possibly useful for debugging}

data(Oakley1)
(M1 <- modulus(Oakley1))
isprime(M1)# '1' : "probably prime"
sizeinbase(M1)# 232 digits (was 309 in older version)

---

**powm**

*Exponentiation function*

Description

This function return \(x^y \mod n\).

This function return \(x^y \mod n\) pow.bigz do the same when modulus is set.

Usage

\[ \text{powm}(x, y, n) \]

Arguments

- **x**: Integer or big integer - possibly a vector
- **y**: Integer or big integer - possibly a vector
- **n**: Integer or big integer - possibly a vector

Value

A bigz class representing the parameter value.

Author(s)

A. L.

See Also

pow.bigz
Examples

powm(4, 7, 9)

x = as.bigz(4, 9)
x ^ 7

Random

Generate a random number

Description

Generate a uniformly distributed random number in the range 0 to \(2^{\text{size}} - 1\), inclusive.

Usage

urand.bigz(nb=1, size=200, seed = 0)

Arguments

nb  
Integer: number of random numbers to be generated (size of vector returned)

size  
Integer: number will be generated in the range 0 to \(2^{\text{size}} - 1\)

seed  
Bigz: random seed initialisation

Value

A biginteger of class bigz.

Author(s)

Antoine Lucas

References

‘mpz_urandomb’ from the GMP Library, see http://gmplib.org

Examples

# Integers are differents
urand.bigz()
urand.bigz()
urand.bigz()

# Integers are the same
urand.bigz(seed="234234234234234")
urand.bigz(seed="234234234234234")

# Vector
urand.bigz(nb=50, size=30)
Relational Operator  \hspace{1cm} \textit{Relational Operators}

\textbf{Description}

Binary operators which allow the comparison of values in atomic vectors.

\textbf{Usage}

\begin{verbatim}
## S3 method for class 'bigz'
sign(x)
## S3 method for class 'bigz'
e1 == e2
## S3 method for class 'bigz'
e1 < e2
## S3 method for class 'bigz'
e1 >= e2
\end{verbatim}

\textbf{Arguments}

\begin{itemize}
  \item \texttt{x}, \texttt{e1}, \texttt{e2} \hspace{1cm} R object (vector or matrix-like) of class "\texttt{bigz}".
\end{itemize}

\textbf{See Also}

\texttt{mod.bigz} for arithmetic operators.

\textbf{Examples}

\begin{verbatim}
x <- as.bigz(8000)
x ^ 300 < 2 ^x
sign(as.bigz(-3:3))
sign(as.bigq(-2:2, 7))
\end{verbatim}

\textbf{sizeinbase} \hspace{1cm} \textit{Compute size of a bigz in a base}

\textbf{Description}

Return an approximation to the number of character the integer \texttt{X} would have printed in base \texttt{b}. The approximation is never too small.

In case of powers of 2, function gives exact result.

\textbf{Usage}

\begin{verbatim}
sizeinbase(a, b=10)
\end{verbatim}
solve.bigz

Arguments

  a  big integer, i.e. "bigz"
  b  base

Value

  integer of the same length as a: the size, i.e. number of digits, of each a[i].

Author(s)

  Antoine Lucas

References

  The GNU MP Library, see http://gmplib.org

Examples

  sizeinbase(342434, 10)# 6 obviously

  Iv <- as.bigz(2:7)^500
  sizeinbase(Iv)
  stopifnot(sizeinbase(Iv) == nchar(as.character(Iv)),
            sizeinbase(Iv, b=16) == nchar(as.character(Iv, b=16)))

solve.bigz  Solve a system of equation

Description

  This generic function solves the equation \(a \times x = b\) for \(x\), where \(b\) can be either a vector or a matrix.

  If \(a\) and \(b\) are rational, return is a rational matrix.

  If \(a\) and \(b\) are big integers (of class bigz) solution is in \(\mathbb{Z}/n\mathbb{Z}\) if there is a common modulus, or a rational matrix if not.

Usage

  ## S3 method for class 'bigz'
  solve(a, b, ...)

  ## S3 method for class 'bigq'
  solve(a, b, ...)

Arguments

  a,b  A element of class bigz or bigq
  ...  Unused
Details

It uses the Gauss and trucmuch algo… (to be detailed).

Value

If a and b are rational, return is a rational matrix.

If a and b are big integers (of class bigz) solution is in Z/nZ if there is a common modulus, of a rational matrix if not.

Author(s)

Antoine Lucas

See Also

solve

Examples

```r
x <- matrix(1:4, 2, 2) ## standard solve :
solve(x)

q <- as.bigq(x) ## solve with rational
solve(q)

z <- as.bigz(x)
modulus(z) <- 7 ## solve in Z/7Z :
solve(z)

b <- c(1,3)
solve(q,b)
solve(z,b)

## Inversion of ("non-trivial") rational matrices :
A <- rbind(c(10, 1, 3),
          c( 4, 2, 10),
          c( 1, 8,  2))
(IA.q <- solve(as.bigq(A))) # fractions..
stopifnot(diag(3) == A %*% IA.q)# perfect

set.seed(5); B <- matrix(round(9*runif(5*2, -1,1)), 5)
B
(IB.q <- solve(as.bigq(B)))
stopifnot(diag(5) == B %*% IB.q, diag(5) == IB.q %*% B,
         identical(B, asNumeric(solve(IB.q))))
```
Eulerian and Stirling Numbers of First and Second Kind

Description

Compute Eulerian numbers and Stirling numbers of the first and second kind, possibly vectorized for all \( k \) “at once”.

Usage

\[
\begin{align*}
Eulerian(n, k, \text{method} = c(\text{"lookup.or.store", "direct"})) \\
Stirling(n, k, \text{method} = c(\text{"lookup.or.store", "direct"})) \\
Stirling1(n, k) \\
Stirling2(n, k) \\
Eulerian.all(n) \\
Stirling1.all(n) \\
Stirling2.all(n)
\end{align*}
\]

Arguments

\begin{itemize}
\item \( n \) positive integer (0 is allowed for Eulerian()).
\item \( k \) integer in 0:n.
\item \text{method} for Eulerian() and Stirling2(), string specifying the method to be used. "direct" uses the explicit formula (which may suffer from some cancelation for “large” \( n \)).
\end{itemize}

Details

Eulerian numbers:
\[
A(n, k) = \text{the number of permutations of 1,2,...,n with exactly } k \text{ ascents (or exactly } k \text{ descents).}
\]

Stirling numbers of the first kind:
\[
s(n,k) = (-1)^{n-k} \times \text{the number of permutations of 1,2,...,n with exactly } k \text{ cycles.}
\]

Stirling numbers of the second kind:
\[
S_{n}^{(k)} = \text{the number of ways of partitioning a set of } n \text{ elements into } k \text{ non-empty subsets.}
\]

Value

\[
A(n, k), s(n, k) \text{ or } S(n, k) = S_{n}^{(k)}, \text{respectively.}
\]

Eulerian.all(n) is the same as sapply(0:(n-1), Eulerian, n=n) (for \( n > 0 \)),
Stirling1.all(n) is the same as sapply(1:n, Stirling1, n=n), and
Stirling2.all(n) is the same as sapply(1:n, Stirling2, n=n), but more efficient.
Note

For typical double precision arithmetic,
Eulerian*(n,*) overflow (to Inf) for \( n \geq 172 \),
Stirling1*(n,*) overflow (to ±Inf) for \( n \geq 171 \), and
Stirling2*(n,*) overflow (to Inf) for \( n \geq 220 \).

Author(s)

Martin Maechler ("direct": May 1992)

References

Eulerians:

Stirling numbers:
Abramowitz and Stegun 24.1,4 (p. 824-5 ; Table 24.4, p.835); Closed Form : p.824 "C."
NIST Digital Library of Mathematical Functions, 26.8: http://dlmf.nist.gov/26.8

See Also

chooseZ for the binomial coefficients.

Examples

\[
\begin{align*}
\text{Stirling1}(7,2) \\
\text{Stirling2}(7,3)
\end{align*}
\]

stopifnot(
    Stirling1.all(9) == c(40320, -109584, 118124, -67284, 22449, -4536, 546, -36, 1)
    ,
    Stirling2.all(9) == c(1, 255, 3025, -7770, 6951, 2646, 462, 36, 1)
    ,
    Eulerian.all(7) == c(1, 120, 1191, 2416, 1191, 120, 1)
)
}
Index

!=.bigq (Bigq), 4
!=.bigz (Relational Operator), 34
*Topic arithmetic
  Stirling, 37
*Topic arith
  apply, 2
  asNumeric, 3
  Bigq, 4
  bigq, 5
  Bigq operators, 6
  bigz, 8
  bigz operators, 10
  cumsum, 13
  extract, 14
  Extremes, 15
  factorialZ, 17
  factorization, 18
  frexpZ, 20
  gcd.bigz, 21
  gcdex, 22
  gmp.utils, 23
  isprime, 25
  lucnum, 26
  matrix, 27
  modulus, 29
  nextprime, 30
  powm, 32
  Random, 33
  Relational Operator, 34
  sizeinbase, 34
  solve.bigz, 35
*Topic character
  formatN, 19
*Topic data
  Oakley, 31
*Topic math
  is.whole, 24
*Topic methods
  asNumeric, 3
*Topic misc
  gmp-ifiworkarounds, 23
*Topic print
  formatN, 19
  *.bigq (Bigq operators), 6
  *.bigz (bigz operators), 10
  +.bigq (Bigq operators), 6
  +.bigz (bigz operators), 10
  -.bigq (Bigq operators), 6
  -.bigz (bigz operators), 10
  /.bigq (Bigq operators), 6
  /.bigz (bigz operators), 10
  <.bigq (Bigq), 4
  <.bigz (Relational Operator), 34
  <=.bigq (Bigq), 4
  <=.bigz (Relational Operator), 34
  ==.bigq (Bigq), 4
  ==.bigz (Relational Operator), 34
  >.bigq (Bigq), 4
  >.bigz (Relational Operator), 34
  >=.bigq (Bigq), 4
  >=.bigz (Relational Operator), 34
  [.bigq (extract), 14
  [.bigz (extract), 14
  [<=.bigq (extract), 14
  [<=.bigz (extract), 14
  [[[.bigq (extract), 14
  [[[.bigz (extract), 14
  [<-.bigq (extract), 14
  [<-.bigz (extract), 14
  %%% (matrix), 27
  %/%.bigz (bigz operators), 10
  %%.bigz (bigz operators), 10
  ^+.bigq (Bigq operators), 6
  ^+.bigz (bigz operators), 10
  abs.bigq (Bigq operators), 6
  abs.bigz (bigz operators), 10
  add.bigq (Bigq operators), 6
  add.bigz, 8
add.bigz (bigz operators), 10
apply, 2, 2, 3, 13
array, 3
as.bigq (bigq), 5
as.bigz (bigz), 8
as.bigz.bigq (bigq), 5
as.character, 19
as.character.bigq (bigq), 5
as.character.bigz (bigz), 8
as.double.bigq (bigq), 5
as.double.bigz (bigz), 8
as.matrix.bigq (matrix), 27
as.matrix.bigz (matrix), 27
as.numeric, 4
as.vector.bigq (matrix), 27
as.vector.bigz (matrix), 27
asNumeric, 3
asNumeric,ANY-method (asNumeric), 3
asNumeric,bigq-method (asNumeric), 3
asNumeric,bigz-method (asNumeric), 3
asNumeric-methods (asNumeric), 3
biginteger_as (bigz), 8
biginteger_as_character (bigz), 8
Bigq, 4
bigq, 5, 5, 6, 7, 12
Bigq operators, 6
bigq-class (bigq), 5
bigz, 3, 7, 8, 11, 17, 20, 21, 28, 30, 31, 34, 35
bigz operators, 10
bigz-class (bigz), 8
c.bigq (extract), 14
c.bigz (extract), 14
cbind.bigq (matrix), 27
cbind.bigz (matrix), 27
character, 8
chooseZ, 38
chooseZ (factorialZ), 17
class, 19
crossprod (matrix), 27
cumsum, 13, 13
denominator (bigq), 5
denominator<-(bigq), 5
dim, 3, 4
dim.bigq (matrix), 27
dim.bigz (matrix), 27
dim<-.bigq (matrix), 27
dim<-.bigz (matrix), 27
dimnames, 3
div.bigq (Bigq operators), 6
div.bigz (bigz operators), 10
divq.bigz (bigz operators), 10
double, 8
Eulerian (Stirling), 37
extract, 14
Extremes, 15
factorial, 17
factorialZ, 17
factorization, 18
factorize, 25
factorize (factorization), 18
fibnum (lucnum), 26
fibnum2 (lucnum), 26
format, 19
formatN, 19
frexp (frexpZ), 20
frexpZ, 20
function, 2
gamma, 17
gcd (gcd.bigz), 21
gcd.bigz, 21, 22
gcdex, 21, 22
gmp-ifisworkarounds, 23
gmp.utils, 23
gmpVersion (gmp.utils), 23
integer, 8
inv (bigz operators), 10
is.bigq (bigq), 5
is.bigz (bigz), 8
is.integer, 24
is.matrixZQ (matrix), 27
is.na.bigq (bigq), 5
is.na.bigz (bigz), 8
is.whole, 24, 24
isprime, 25, 31
lapply, 2, 3
lcm.bigz (gcd.bigz), 21
lcm.default (gcd.bigz), 21
length, 19
length.bigq (extract), 14
length.bigz (extract), 14
INDEX

41

length<-.bigq(extract), 14
length<-.bigz(extract), 14
list, 20
log.bigz(bigz operators), 10
log10.bigz(bigz operators), 10
log2, 20
log2.bigz(bigz operators), 10
lucnum, 26
lucnum2(lucnum), 26
matrix, 3, 27, 28
matrix.bigz, 2
max, 15, 16
max.bigq(Extremes), 15
max.bigz(Extremes), 15
methods, 2, 15
min, 15
min.bigq(Extremes), 15
min.bigz(Extremes), 15
mod.bigz, 9, 34
mod.bigz(bigz operators), 10
modulus, 9, 29
modulus<-(modulus), 29
mpfr, 3
mul.bigq(Bigq operators), 6
mul.bigz(bigz operators), 10
NA, 9, 11, 13
ncol.bigq(matrix), 27
ncol.bigz(matrix), 27
nextprime, 25, 30
nrow.bigq(matrix), 27
nrow.bigz(matrix), 27
numerator(bigq), 5
numerator<-(bigq), 5
numeric, 4, 8
Oakley, 31
Oakley1(Oakley), 31
Oakley2(Oakley), 31
outer, 23
outer(gmp-ifiworkarounds), 23
pow(bigz operators), 10
pow.bigq(Bigq operators), 6
pow.bigz, 32
powm, 32
primes, 25
print.bigq(bigq), 5
print.bigz(bigz), 8
prod, 13
prod.bigq(cumsum), 13
prod.bigz(cumsum), 13
Random, 33
range, 15
raw, 5, 8
rbind.bigq(matrix), 27
rbind.bigz(matrix), 27
Relational Operator, 34
rep, 14
rep.bigq(extract), 14
rep.bigz(extract), 14
setOldClass, 5, 8
sign.bigq(Bigq), 4
sign.bigz(Relational Operator), 34
sizeinbase, 34
solve, 28, 36
solve.bigq(solve.bigz), 35
solve.bigz, 28, 35
Stirling, 37
Stirling1(Stirling), 37
Stirling2(Stirling), 37
stop, 8
sub.bigq(Bigq operators), 6
sub.bigz(bigz operators), 10
sum, 13
sum.bigq(cumsum), 13
sum.bigz(cumsum), 13
t.bigq(matrix), 27
t.bigz(matrix), 27
tcrossprod(matrix), 27
TRUE, 28
typeof, 3
urand.bigz(Random), 33
vector, 4
which.max, 15
which.max.bigq-method(Extremes), 15
which.max.bigz-method(Extremes), 15
which.min, 15
which.min.bigq-method(Extremes), 15
which.min.bigz-method(Extremes), 15