Package ‘graphicalExtremes’

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Title Statistical Methodology for Graphical Extreme Value Models
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R topics documented:
censor ................................................................. 2
chi2Gamma ............................................................ 3
complete_Gamma ...................................................... 4
data2mpareto ......................................................... 5
dim_Gamma ............................................................ 6
emp_chi ................................................................. 6
Censor dataset

Description

Censors each row of matrix x with vector p.

Usage

censor(x, p)
chi2Gamma

Arguments

- x: Numeric matrix \( n \times d \).
- p: Numeric vector with \( d \) elements.

Value

Numeric matrix \( n \times d \).

Description

Transforms the extremal correlation \( \chi \) into the Gamma matrix from the definition of a Huesler–Reiss distribution.

Usage

\texttt{chi2Gamma(chi)}

Arguments

- chi: Numeric or matrix, with entries between 0 and 1.

Details

The formula for transformation from \( \chi \) to \( \Gamma \) that is applied element-wise is

\[
\Gamma = (2\Phi^{-1}(1 - 0.5\chi))^2,
\]

where \( \Phi^{-1} \) is the inverse of the standard normal distribution function. This is the inverse of \texttt{Gamma2chi}.

Value

Numeric or matrix. The \( \Gamma \) parameters in the Huesler–Reiss distribution.
Description

Given a block graph and Gamma matrix with entries only specified on edges within the cliques of the graph, it returns the full Gamma matrix implied by the conditional independencies.

Usage

complete_Gamma(Gamma, graph)

Arguments

Gamma  
Numeric $d \times d$ variogram matrix with entries only specified within the cliques of the graph. Alternatively, can be a vector containing the Gamma entries for each edge in the same order as in the graph object.

graph  
Graph object from igraph package. The graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.

Details

For a block graph it suffices to specify the dependence parameters of the Huesler–Reiss distribution within the cliques of the graph, the remaining entries are implied by the conditional independence properties. For details see Engelke and Hitz (2018).

Value

Completed $d \times d$ Gamma matrix. s

References


Examples

## Complete a 4-dimensional HR distribution

my_graph <- igraph::graph_from_adjacency_matrix(rbind(
c(0, 1, 0, 0),
c(1, 0, 1, 1),
c(0, 1, 0, 1),
c(0, 1, 1, 0)),
mode = "undirected")

Gamma <- rbind(
c(0, .5, NA, NA),
data2mpareto

```r
c(.5, 0, 1, 1.5),
c(NA, 1, 0, .8),
c(NA, 1.5, .8, 0))

complete_Gamma(Gamma, my_graph)

## Alternative
Gamma_vec <- c(.5, 1, 1.5, .8)
complete_Gamma(Gamma_vec, my_graph)
```

---

**data2mpareto**  
*Data standardization to multivariate Pareto scale*

---

**Description**

Transforms the data matrix empirically to the multivariate Pareto scale.

**Usage**

```r
data2mpareto(data, p)
```

**Arguments**

- **data**: Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.
- **p**: Numeric between 0 and 1. Probability used for the quantile to threshold the data.

**Details**

The columns of the data matrix are first transformed empirically to standard Pareto distributions. Then, only the observations where at least one component exceeds the $p$-quantile of the standard Pareto distribution are kept. Those observations are finally divided by the $p$-quantile of the standard Pareto distribution to standardize them to the multivariate Pareto scale.

**Value**

Numeric matrix $m \times d$, where $m$ is the number of rows in the original data matrix that are above the threshold.

**Examples**

```r
n <- 20
d <- 4
p <- .8
G <- cbind(c(0, 1.5, 1.5, 2),
           c(1.5, 0, 2, 1.5),
           c(NA, 1.5, .8, 0))
```
```r
set.seed(123)
my_data = rmstable(n, "HR", d = d, par = G)
data2mpareto(my_data, p)
```

---

### `dim_Gamma`

**Is Gamma square matrix?**

**Description**

Check if Gamma matrix is square matrix. If so, return the dimension. Else, raise an error.

**Usage**

```r
dim_Gamma(Gamma)
```

**Arguments**

- **Gamma**
  
  Numeric matrix. Matrix representing the variogram of an HR distribution.

**Value**

Numeric. The dimension of the matrix (number of rows and columns, if the matrix is symmetric). Else, raises an error.

---

### `emp_chi`

**Empirical estimation of extremal correlation χ**

**Description**

Estimates the $d$-dimensional extremal correlation coefficient $\chi$ empirically.

**Usage**

```r
dim_Gamma(Gamma)
```

**Arguments**

- **data**
  
  Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.

- **p**
  
  Numeric between 0 and 1. Probability used for the quantile to compute the $\chi$ coefficient.
emp_chi_mat

Value

Numeric. The empirical $d$-dimensional extremal correlation coefficient $\chi$ for the data.

Examples

```r
n <- 100
d <- 2
p <- .8
G <- cbind(c(0, 1.5),
           c(1.5, 0))

set.seed(123)
my_data = rmstable(n, "HR", d = d, par = G)
emp_chi(my_data, p)
```

emp_chi_mat

Empirical estimation of extremal correlation matrix $\chi$

Description

Estimates empirically the matrix of bivariate extremal correlation coefficients $\chi$.

Usage

```r
emp_chi_mat(data, p)
```

Arguments

data

Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.

p

Numeric between 0 and 1. Probability used for the quantile to compute the $\chi$ coefficient.

Value

Numeric matrix $d \times d$. The matrix contains the bivariate extremal coefficients $\chi_{ij}$, for $i, j = 1, \ldots, d$.

Examples

```r
n <- 100
d <- 4
p <- .8
Gamma <- cbind(c(0, 1.5, 1.5, 2),
                c(1.5, 0, 2, 1.5),
                c(1.5, 2, 0, 1.5),
                c(2, 1.5, 1.5, 0))
```
set.seed(123)
my_data = rmstable(n, "HR", d = d, par = Gamma)
emp_chi_mat(my_data, p)

emp_vario

Estimation of the variogram matrix $\Gamma$ of the Huesler–Reiss distribution

Description

Estimates the variogram of the Huesler–Reiss distribution empirically.

Usage

emp_vario(data, k = NULL, p = NULL)

Arguments

data
Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.

k
Integer between 1 and $d$. Component of the multivariate observations that is conditioned to be larger than the threshold $p$. If NULL (default), then an average over all $k$ is returned.

p
Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, $p$ is used as the probability in the function data2mpareto to standardize the data.

Value

Numeric matrix $d \times d$. The estimated variogram of the Huesler–Reiss distribution.

fmpareto_graph_HR

Parameter fitting for multivariate Huesler–Reiss Pareto distributions on block graphs

Description

Fits the parameters of a multivariate Huesler–Reiss Pareto distribution using (censored) likelihood estimation. Fitting is done separately on the cliques of the block graph. If edges_to_add are provided, then these edges are added in a greedy search to the original graph, such that in each step the likelihood is improved maximally and the new graph stays in the class of block graphs. See Engelke and Hitz (2018) for details.

Usage

fmpareto_graph_HR(data, graph, p = NULL, cens = FALSE, edges_to_add = NULL)
Arguments

data  Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.

graph  Graph object from igraph package. The graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.

p  Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, $p$ is used as the probability in the function data2mpareto to standardize the data.

cens  Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, cens = FALSE.

edges_to_add  Numeric matrix $m \times 2$, where $m$ is the number of edges that are tried to be added in the greedy search. By default, edges_to_add = NULL.

Value

List consisting of:

- graph: Graph object from igraph package. If edges_to_add are provided, then this is a list of the resulting graphs in each step of the greedy search.
- Gamma: Numeric $d \times d$ estimated variogram matrix $\Gamma$. If edges_to_add are provided, then this is a list of the estimated variogram matrices in each step of the greedy search.
- AIC: (only if edges_to_add are provided) List of AIC values of the fitted models in each step of the greedy search.
- edges_added: (only if edges_to_add are provided) Numeric matrix $m' \times 2$, where the $m'$ rows contain the edges that were added in the greedy search.

References


Examples

```r
## Fitting a 4-dimensional HR distribution

my_graph <- igraph::graph_from_adjacency_matrix(
  rbind(c(0, 1, 0, 0),
        c(1, 0, 1, 1),
        c(0, 1, 0, 0),
        c(0, 1, 0, 0)),
  mode = "undirected")

n <- 100
Gamma_vec <- c(.5,1.4,.8)
complete_Gamma(Gamma = Gamma_vec, graph = my_graph)  ## full Gamma matrix
edges_to_add <- rbind(c(1,3), c(1,4), c(3,4))

set.seed(123)
my_data <- rmpareto_tree(n, "HR", tree = my_graph, par = Gamma_vec)
```
my_fit <- fmpareto_graph_HR(my_data, graph = my_graph, 
p = NULL, cens = FALSE, edges_to_add = edges_to_add)

fmpareto_HR

Parameter fitting for multivariate Huesler–Reiss Pareto distribution

Description

Fits the parameters of a multivariate Huesler–Reiss Pareto distribution using (censored) likelihood estimation.

Usage

fmpareto_HR(data, p = NULL, cens = FALSE, init, maxit = 100, 
graph = NULL, method = "BFGS")

Arguments

data   Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.
p     Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, $p$ is used as the probability in the function data2mpareto to standardize the data.
cens   Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, cens = FALSE.
init   Numeric vector. Initial parameter values in the optimization. If graph is given, then the entries should correspond to the edges of the graph.
maxit  Positive integer. The maximum number of iterations in the optimization.
graph  Graph object from igraph package or NULL. If provided, the graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.
method String. A valid optimization method used by the function optim. By default, method = "BFGS".

Details

If graph = NULL, then the parameters of a $d \times d$ parameter matrix $\Gamma$ of a Huesler–Reiss Pareto distribution are fitted. If graph is provided, then the conditional independence structure of this graph is assumed and the parameters on the edges are fitted. In both cases the full likelihood is used and therefore this function should only be used for small dimensions, say, $d < 5$. For models in higher dimensions fitting can be done separately on the cliques; see fmpareto_graph_HR.
Value

List consisting of:

- convergence: Logical. Indicates whether the optimization converged or not.
- Gamma: Numeric matrix \(d \times d\). Fitted variogram matrix.

\[
\text{Gamma2chi} \quad \text{Transformation of the Huesler–Reiss variogram } \Gamma \text{ to extremal correlation } \chi
\]

Description

Transforms the Gamma matrix from the definition of a Huesler–Reiss distribution into the corresponding extremal correlation \(\chi\).

Usage

\[\text{Gamma2chi}(\text{Gamma})\]

Arguments

\[\text{Gamma} \quad \text{Numeric or matrix, with positive entries.}\]

Details

The formula for transformation from Gamma to \(\chi\) that is applied element-wise is

\[
\chi = 2 - 2\Phi(\sqrt{\Gamma}/2),
\]

where \(\Phi\) is the standard normal distribution function. This is the inverse of \text{chi2Gamma}.

Value

Numeric or matrix. The extremal correlation coefficient.
Gamma2chi_3D

*Compute theoretical $\chi$ in 3D*

**Description**

Computes the theoretical $\chi$ coefficient in 3 dimensions.

**Usage**

```r
Gamma2chi_3D(Gamma)
```

**Arguments**

- **Gamma**
  - Numeric matrix $3 \times 3$.

**Value**

The 3-dimensional $\chi$ coefficient, i.e., the extremal correlation coefficient for the HR distribution. Note that $0 \leq \chi \leq 1$.

---

Gamma2graph

*Transformation of $\Gamma$ matrix to graph object*

**Description**

Transforms Gamma matrix to an igraph object for the corresponding Huesler–Reiss extremal graphical model, and plots it (optionally).

**Usage**

```r
Gamma2graph(Gamma, to_plot = TRUE, ...)
```

**Arguments**

- **Gamma**
  - Numeric $d \times d$ variogram matrix.
- **to_plot**
  - Logical. If TRUE (default), it plots the resulting graph.
- **...**
  - Graphical parameters for the `plot.igraph` function of the package igraph.

**Details**

The variogram uniquely determines the extremal graph structure of the corresponding Huesler–Reiss distribution. The conditional independencies can be identified from the inverses of the matrices $\Sigma^{(k)}$ defined in equation (10) in Engelke and Hitz (2018).
Gamma2par

Value

Graph object from igraph package. An undirected graph.

References


Examples

```r
Gamma <- cbind(c(0, 1.5, 1.5, 2),
               c(1.5, 0, 2, 1.5),
               c(1.5, 2, 0, 1.5),
               c(2, 1.5, 1.5, 0))

Gamma2graph(Gamma, to_plot = TRUE)
```

Description

This function returns a vector containing the upper triangular part of the matrix Gamma. If Gamma is already a vector, it returns it as it is.

Usage

```r
Gamma2par(Gamma)
```

Arguments

- Gamma
  
  Numeric $d \times d$ variogram matrix.

Value

Numeric vector with $d$ elements. The upper triangular part of the given Gamma matrix.
Gamma2Sigma

*Transformation of $\Gamma$ matrix to $\Sigma^{(k)}$ matrix*

**Description**

Transforms the Gamma matrix from the definition of a Huesler–Reiss distribution to the corresponding $\Sigma^{(k)}$ matrix.

**Usage**

```r
Gamma2Sigma(Gamma, k = 1, full = FALSE)
```

**Arguments**

- **Gamma**
  - Numeric $d \times d$ variogram matrix.
- **k**
  - Integer between 1 (the default value) and $d$. Indicates which matrix $\Sigma^{(k)}$ should be produced.
- **full**
  - Logical. If true, then the $k$th row and column in $\Sigma^{(k)}$ are included and the function returns a $d \times d$ matrix. By default, `full = FALSE`.

**Details**

Every $d \times d$ Gamma matrix in the definition of a Huesler–Reiss distribution can be transformed into a $(d - 1) \times (d - 1)$ $\Sigma^{(k)}$ matrix, for any $k$ from 1 to $d$. The inverse of $\Sigma^{(k)}$ contains the graph structure corresponding to the Huesler–Reiss distribution with parameter matrix Gamma. If `full = TRUE`, then $\Sigma^{(k)}$ is returned as a $d \times d$ matrix with additional $k$th row and column that contain zeros. For details see Engelke and Hitz (2018). This is the inverse of function of `Sigma2Gamma`.

**Value**

Numeric $\Sigma^{(k)}$ matrix of size $(d - 1) \times (d - 1)$ if `full = FALSE`, and of size $d \times d$ if `full = TRUE`.

**References**


**Examples**

```r
Gamma <- cbind(c(0, 1.5, 1.5, 2),
               c(1.5, 0, 2, 1.5),
               c(1.5, 2, 0, 1.5),
               c(2, 1.5, 1.5, 0))
Gamma2Sigma(Gamma, k = 1, full = FALSE)
```
Description

The graphicalExtremes package provides three categories of functions: simulation, estimation and transformation.

Simulation functions

- rmpareto
- rmpareto_tree
- rmstable
- rmstable_tree

Estimation functions

- fmpareto_graph_HR
- mst_HR
- emp_chi
- emp_chi_mat

Transformation functions

- Gamma2graph
- Gamma2Sigma
- Sigma2Gamma
- Gamma2chi
- chi2Gamma
- complete_Gamma
- data2mpareto

References

\textbf{logdVK\_HR} \hspace{1cm} Compute censored exponent measure

\textbf{Description}
Computes the censored exponent measure density of HR distribution.

\textbf{Usage}
\begin{verbatim}
logdVK\_HR(x, K, par)
\end{verbatim}

\textbf{Arguments}
\begin{itemize}
\item \textbf{x} \hspace{1cm} Numeric vector with $d$ positive elements where the censored exponent measure is to be evaluated.
\item \textbf{K} \hspace{1cm} Integer vector, subset of \{1, \ldots, d\}. The index set that is not censored.
\item \textbf{par} \hspace{1cm} Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix $\Gamma$.
\end{itemize}

\textbf{Value}
Numeric. The censored exponent measure of the HR distribution.

\hline
\textbf{logdV\_HR} \hspace{1cm} Compute the exponent measure density of HR distribution
\hline
\textbf{Description}
Computes the exponent measure density of HR distribution.

\textbf{Usage}
\begin{verbatim}
logdV\_HR(x, par)
\end{verbatim}

\textbf{Arguments}
\begin{itemize}
\item \textbf{x} \hspace{1cm} Numeric matrix $n \times d$ or vector with $d$ elements.
\item \textbf{par} \hspace{1cm} Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix $\Gamma$.
\end{itemize}

\textbf{Value}
Numeric. The censored exponent measure of the HR distribution.
logLH_HR

Full censored log-likelihood of HR model

Description

Computes the full (censored) log-likelihood of HR model.

Usage

logLH_HR(data, Gamma, cens = FALSE)

Arguments

data: Numeric matrix \( n \times d \). It contains observations following a multivariate HR Pareto distribution.
Gamma: Numeric matrix \( n \times d \). It represents a variogram matrix \( \Gamma \).
cens: Boolean. If true, then censored log-likelihood is computed. By default, cens = FALSE.

Value

Numeric. The full censored log-likelihood of HR model.

mparetomargins

Marginalize multivariate Pareto dataset

Description

Marginalize a multivariate Pareto dataset data with respect to the variables in set_indices.

Usage

mparetomargins(data, set_indices)

Arguments

data: Numeric matrix \( n \times d \). A dataset containing observations following a multivariate Pareto distribution.
set_indices: Numeric vector with at most \( d \) different elements in 1, ..., \( d \). The variables with respect to which to marginalize the multivariate distribution.

Value

Numeric matrix \( n \times m \), where \( m \) is the length of set_indices. Marginalized multivariate Pareto data.
Description

Fits the Huesler–Reiss minimum spanning tree, where the edge weights are are the negative max-imized log-likelihoods of the bivariate Huesler–Reiss distributions. See Engelke and Hitz (2018) for details.

Usage

```r
mst_HR(data, p = NULL, cens = FALSE)
```

Arguments

- `data` Numeric matrix of size $n \times d$, where $n$ is the number of observations and $d$ is the dimension.
- `p` Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, $p$ is used as the probability in the function `data2mpareto` to standardize the data.
- `cens` Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, `cens = FALSE`.

Value

List consisting of:

- `tree`: Graph object from `igraph` package. The fitted minimum spanning tree.
- `Gamma`: Numeric $d \times d$ estimated variogram matrix $\Gamma$ corresponding to the fitted minimum spanning tree.

References


Examples

```r
## Fitting a 4-dimensional HR MST tree

my_graph <- igraph::graph_from_adjacency_matrix(
  rbind(c(0, 1, 0, 0),
        c(1, 0, 1, 1),
        c(0, 1, 0, 0),
        c(0, 1, 0, 0)),
  mode = "undirected")

n <- 100
Gamma_vec <- c(.5,1.4,.8)
```
complete_Gamma(Gamma = Gamma_vec, graph = my_graph) ## full Gamma matrix
set.seed(123)
my_data <- rmpareto_tree(n, "HR", tree = my_graph, par = Gamma_vec)
my_fit <- mst_HR(my_data, p = NULL, cens = FALSE)

par2Gamma

Create $\Gamma$ from vector

Description
This function takes the parameters in the vector par (upper triangular Gamma matrix) and returns full Gamma.

Usage
par2Gamma(par)

Arguments
par Numeric vector with $d$ elements. Upper triangular part of a Gamma matrix.

Value
Numeric matrix $d \times d$. Full Gamma matrix.

rmpareto

Sampling of a multivariate Pareto distribution

Description
Simulates exact samples of a multivariate Pareto distribution.

Usage
rmpareto(n, model = c("HR", "logistic", "neglogistic", "dirichlet")[[1]],
d, par)
Arguments

n  Number of simulations.
model  The parametric model type; one of:
   • HR (default),
   • logistic,
   • neglogistic,
   • dirichlet.

d  Dimension of the multivariate Pareto distribution.
par  Respective parameter for the given model, that is,
   • $\Gamma$, numeric $d \times d$ variogram matrix, if model = HR.
   • $\theta \in (0, 1)$, if model = logistic.
   • $\theta > 0$, if model = neglogistic.
   • $\alpha$, numeric vector of size $d$ with positive entries, if model = dirichlet.

Details

The simulation follows the algorithm in Engelke and Hitz (2018). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size $n \times d$ of simulations of the multivariate Pareto distribution.

References


Examples

```r
## A 4-dimensional HR distribution
n <- 10
d <- 4
G <- cbind(c(0, 1.5, 1.5, 2),
           c(1.5, 0, 2, 1.5),
           c(1.5, 2, 0, 1.5),
           c(2, 1.5, 1.5, 0))

rmpareto(n, "HR", d = d, par = G)
```
rmpareto_tree

### A 3-dimensional logistic distribution
n <- 10
d <- 3
theta <- .6
rmpareto(n, "logistic", d, par = theta)

### A 5-dimensional negative logistic distribution
n <- 10
d <- 5
theta <- 1.5
rmpareto(n, "neglogistic", d, par = theta)

### A 4-dimensional Dirichlet distribution
n <- 10
d <- 4
alpha <- c(.8, 1, .5, 2)
rmpareto(n, "dirichlet", d, par = alpha)

---

**rmpareto_tree**  
*Sampling of a multivariate Pareto distribution on a tree*

**Description**

Simulates exact samples of a multivariate Pareto distribution that is an extremal graphical model on a tree as defined in Engelke and Hitz (2018).

**Usage**

`rmpareto_tree(n, model = c("HR", "logistic", "dirichlet")[1], tree, par)`

**Arguments**

- `n`  
  Number of simulations.

- `model`  
  The parametric model type; one of:
  - HR (default),
  - logistic,
  - dirichlet.

- `tree`  
  Graph object from igraph package. This object must be a tree, i.e., an undirected graph that is connected and has no cycles.

- `par`  
  Respective parameter for the given `model`, that is,
  - Γ, numeric $d \times d$ variogram matrix, where only the entries corresponding to the edges of the `tree` are used, if `model = HR`. Alternatively, can be a vector of length $d - 1$ containing the entries of the variogram corresponding to the edges of the given `tree`.
  - $\theta \in (0, 1)$, vector of length $d - 1$ containing the logistic parameters corresponding to the edges of the given `tree`, if `model = logistic`. 
a matrix of size \((d - 1) \times 2\), where the rows contain the parameters vectors \(\alpha\) of size 2 with positive entries for each of the edges in \(\text{tree}\), if \(\text{model} = \text{dirichlet}\).

Details

The simulation follows the algorithm in Engelke and Hitz (2018). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size \(n \times d\) of simulations of the multivariate Pareto distribution.

References


Examples

```r
## A 4-dimensional HR tree model
my_tree <- igraph::graph_from_adjacency_matrix(rbind(
c(0, 1, 0, 0),
c(1, 0, 1, 1),
c(0, 1, 0, 0),
c(0, 1, 0, 0)),
mode = "undirected")
n <- 10
Gamma_vec <- c(.5, 1.4, .8)
set.seed(123)
rmpareto_tree(n, "HR", tree = my_tree, par = Gamma_vec)

## A 4-dimensional Dirichlet model with asymmetric edge distributions
alpha = cbind(c(.2, 1, .5), c(1.5, .6, .8))
rmpareto_tree(n, model = "dirichlet", tree = my_tree, par = alpha)
```
Sampling of a multivariate max-stable distribution

Description

Simulates exact samples of a multivariate max-stable distribution.

Usage

```r
rmstable(n, model = c("HR", "logistic", "neglogistic", "dirichlet")[1],
        d, par)
```

Arguments

- `n`: Number of simulations.
- `model`: The parametric model type; one of:
  - `HR` (default),
  - `logistic`,
  - `neglogistic`,
  - `dirichlet`.
- `d`: Dimension of the multivariate Pareto distribution.
- `par`: Respective parameter for the given `model`, that is,
  - `Γ`, numeric `d x d` variogram matrix, if `model = HR`.
  - `θ ∈ (0, 1)`, if `model = logistic`.
  - `θ > 0`, if `model = neglogistic`.
  - `α`, numeric vector of size `d` with positive entries, if `model = dirichlet`.

Details

The simulation follows the extremal function algorithm in Dombry et al. (2016). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size `n x d` of simulations of the multivariate max-stable distribution.

References


Examples

```r
## A 4-dimensional HR distribution
n <- 10
d <- 4
G <- cbind(c(0, 1.5, 1.5, 2),
           c(1.5, 0, 2, 1.5),
           c(1.5, 2, 0, 1.5),
           c(2, 1.5, 1.5, 0))
rmstable(n, "HR", d = d, par = G)

## A 3-dimensional logistic distribution
n <- 10
d <- 3
theta <- .6
rmstable(n, "logistic", d, par = theta)

## A 5-dimensional negative logistic distribution
n <- 10
d <- 5
theta <- 1.5
rmstable(n, "neglogistic", d, par = theta)

## A 4-dimensional Dirichlet distribution
n <- 10
d <- 4
alpha <- c(.8, 1, .5, 2)
rmstable(n, "dirichlet", d, par = alpha)
```

Description

Simulates exact samples of a multivariate max-stable distribution that is an extremal graphical model on a tree as defined in Engelke and Hitz (2018).

Usage

```r
rmstable_tree(n, model = c("HR", "logistic", "dirichlet")[[1]], tree, par)
```

Arguments

- **n**: Number of simulations.
- **model**: The parametric model type; one of:
  - HR (default),
  - logistic,
**rmstable_tree**

- **tree**
  
  Graph object from igraph package. This object must be a tree, i.e., an undirected graph that is connected and has no cycles.

- **par**
  
  Respective parameter for the given model, that is,
  
  - \( \Gamma \), numeric \( d \times d \) variogram matrix, where only the entries corresponding to the edges of the tree are used, if \( \text{model} = \text{HR} \). Alternatively, can be a vector of length \( d - 1 \) containing the entries of the variogram corresponding to the edges of the given tree.
  
  - \( \theta \in (0, 1) \), vector of length \( d - 1 \) containing the logistic parameters corresponding to the edges of the given tree, if \( \text{model} = \text{logistic} \).
  
  - a matrix of size \( (d - 1) \times 2 \), where the rows contain the parameter vectors \( \alpha \) of size 2 with positive entries for each of the edges in tree, if \( \text{model} = \text{dirichlet} \).

**Details**

The simulation follows a combination of the extremal function algorithm in Dombry et al. (2016) and the theory in Engelke and Hitz (2018) to sample from a single extremal function. For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

**Value**

Numeric matrix of size \( n \times d \) of simulations of the multivariate max-stable distribution.

**References**


**Examples**

```r
## A 4-dimensional HR tree model

my_tree <- igraph::graph_from_adjacency_matrix(rbind(  
c(0, 1, 0, 0),  
c(1, 0, 1, 1),  
c(0, 1, 0, 0),  
c(0, 1, 0, 0)),  
mode = "undirected")

n <- 10

Gamma_vec <- c(.5, 1.4, .8)

rmstable_tree(n, "HR", tree = my_tree, par = Gamma_vec)
```
## A 4-dimensional Dirichlet model with asymmetric edge distributions

```r
cbind(c(.2, 1, .5), c(1.5, .6, .8))
```

```r
rmstable_tree(n, model = "dirichlet", tree = my_tree, par = alpha)
```

---

### select_edges

**Select edges to add to a graph**

**Description**

This function selects all possible edges that can be added to the graph while still remaining in the class of block graphs.

**Usage**

```r
select_edges(graph)
```

**Arguments**

- `graph` Graph object from igraph package. The graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.

**Value**

Numeric vector.

---

### set_graph_parameters

**Set graphical parameters**

**Description**

Set graphical parameters to graph which is an object from the igraph package.

**Usage**

```r
set_graph_parameters(graph)
```

**Arguments**

- `graph` Graph object from igraph package.

**Value**

Graph object from igraph package.
Description

Transforms the $\Sigma^{(k)}$ matrix from the definition of a Huesler–Reiss distribution to the corresponding $\Gamma$ matrix.

Usage

Sigma2Gamma(S, k = 1, full = FALSE)

Arguments

- **S**: Numeric $(d - 1) \times (d - 1)$ covariance matrix $\Sigma^{(k)}$ from the definition of a Huesler–Reiss distribution. Numeric $d \times d$ covariance matrix if full = TRUE, see full parameter.
- **k**: Integer between 1 (the default value) and d. Indicates which matrix $\Sigma^{(k)}$ is represented by S.
- **full**: Logical. If true, then the kth row and column in $\Sigma^{(k)}$ are included and the function returns a $d \times d$ matrix. By default, full = FALSE.

Details

For any $k$ from 1 to $d$, the $\Sigma^{(k)}$ matrix of size $(d - 1) \times (d - 1)$ in the definition of a Huesler–Reiss distribution can be transformed into a the corresponding $d \times d$ $\Gamma$ matrix. If full = TRUE, then $\Sigma^{(k)}$ must be a $d \times d$ matrix with kth row and column containing zeros. For details see Engelke and Hitz (2018). This is the inverse of function of Gamma2Sigma.

Value

Numeric $d \times d$ $\Gamma$ matrix.

References


Examples

Sigma1 <- rbind(c(1.5, 0.5, 1),
                c(0.5, 1.5, 1),
                c(1, 1, 2))
Sigma2Gamma(Sigma1, k = 1, full = FALSE)
simu_px_dirichlet  
Simulate Dirichlet extremal functions

Description
Simulates Dirichlet extremal functions

Usage
simu_px_dirichlet(n, d, idx, alpha)

Arguments
- **n**: Number of simulations.
- **d**: Dimension of the multivariate Pareto distribution.
- **idx**: Integer or numeric vector with n elements. Indexes from 1 to d, that determine which extremal function to simulate.
- **alpha**: Numeric vector of size d.

Value
Numeric matrix \( n \times d \). Simulated data.

simu_px_HR  
Simulate HR extremal functions

Description
Simulates the Huessler–Reiss extremal functions

Usage
simu_px_HR(n, d, idx, trend, chol_mat)

Arguments
- **n**: Number of simulations.
- **d**: Dimension of the multivariate Pareto distribution.
- **idx**: Integer. Index corresponding to the variable over which the extremal function is simulated.
- **trend**: Numeric. Trend corresponding to the variable idx.
- **chol_mat**: Numeric matrix \( d \times d \). Cholesky decomposition of the desired covariance matrix.

Value
Numeric matrix \( n \times d \). Simulated data.
**simu.px_logistic**  
*Simulate logistic extremal functions*

**Description**
Simulates logistic extremal functions

**Usage**
simu.px_logistic(n, d, idx, theta)

**Arguments**
- **n**  
  Number of simulations.
- **d**  
  Dimension of the multivariate Pareto distribution.
- **idx**  
  Integer or numeric vector with n elements. Indexes from 1 to d, that determine which extremal function to simulate.
- **theta**  
  Numeric — assume 0 < θ < 1.

**Value**
Numeric matrix \( n \times d \). Simulated data.

---

**simu.px_neglogistic**  
*Simulate negative logistic extremal functions*

**Description**
Simulates negative logistic extremal functions

**Usage**
simu.px_neglogistic(n, d, idx, theta)

**Arguments**
- **n**  
  Number of simulations.
- **d**  
  Dimension of the multivariate Pareto distribution.
- **idx**  
  Integer or numeric vector with n elements. Indexes from 1 to d, that determine which extremal function to simulate.
- **theta**  
  Numeric — assume θ > 0.

**Value**
Numeric matrix \( n \times d \). Simulated data.
**simu_px_tree_dirichlet**  
*Simulate Dirichlet extremal functions on a tree*

**Description**
Simulates Dirichlet extremal functions on a tree

**Usage**
simu_px_tree_dirichlet(n, alpha.start, alpha.end, A)

**Arguments**
- **n**: Number of simulations.
- **alpha.start**: Numeric vector with \(d - 1\) elements, where \(d\) is the number of nodes in the tree (and \(d - 1\) is the number of edges).
- **alpha.end**: Numeric vector with \(d - 1\) elements, where \(d\) is the number of nodes in the tree (and \(d - 1\) is the number of edges).
- **A**: Numeric matrix \(d \times (d-1)\); the rows represent the nodes in the tree, the columns represent the edges. For a fixed node \(k = 1, \ldots, d\), each entry \((i,j)\) is equal to 1 if the edge in position \(j\) is on the directed path from node \(k\) to node \(i\) in the polytree rooted at node \(k\).

**Value**
Numeric matrix \(n \times d\). Simulated data.

---

**simu_px_tree_HR**  
*Simulate HR extremal functions on a tree*

**Description**
Simulates the Huessler–Reiss extremal functions on a tree

**Usage**
simu_px_tree_HR(n, Gamma_vec, A)
Arguments

\( n \)  
Number of simulations.

\( \text{Gamma_vec} \)  
Numeric vector with \( d - 1 \) elements, where \( d \) is the number of nodes in the tree (and \( d - 1 \) is the number of edges).

\( A \)  
Numeric matrix \( d \times (d-1) \); the rows represent the nodes in the tree, the columns represent the edges. For a fixed node \( k = 1, \ldots, d \), each entry \((i,j)\) is equal to 1 if the edge in position \( j \) is on the directed path from node \( k \) to node \( i \) in the polytree rooted at node \( k \).

Value

Numeric matrix \( n \times d \). Simulated data.

---

**simu_px_tree_logistic**  
*Simulate logistic extremal functions on a tree*

Description

Simulates logistic extremal functions on a tree.

Usage

\[
\text{simu_px_tree_logistic}(n, \theta, A)
\]

Arguments

\( n \)  
Number of simulations.

\( \theta \)  
Numeric vector with 1 or \( d - 1 \) elements. Assume that the entry are such that \( 0 < \theta < 1 \).

\( A \)  
Numeric matrix \( d \times (d-1) \); the rows represent the nodes in the tree, the columns represent the edges. For a fixed node \( k = 1, \ldots, d \), each entry \((i,j)\) is equal to 1 if the edge in position \( j \) is on the directed path from node \( k \) to node \( i \) in the polytree rooted at node \( k \).

Value

Numeric matrix \( n \times d \). Simulated data.
**unif**  
*Uniform margin*

**Description**  
Rescale the vector $x$ empirically to uniform margin.

**Usage**  
```
unif(x)
```

**Arguments**  
- `x`  
  Numeric vector.

**Value**  
Numeric vector with entries rescaled to uniform margins

**V_HR**  
*Compute exponent measure*

**Description**  
Computes the exponent measure of HR distribution.

**Usage**  
```
V_HR(x, par)
```

**Arguments**  
- `x`  
  Numeric vector with $d$ positive elements where the exponent measure is to be evaluated.
- `par`  
  Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix $\Gamma$.

**Value**  
Numeric. The exponent measure of the HR distribution.
Index

censor, 2
chi2Gamma, 3, 11, 15
complete_Gamma, 4, 15
data2mpareto, 5, 8–10, 15, 18
dim_Gamma, 6
emp_chi, 6, 15
data2mpareto
emp_chi_mat, 7, 15
data2mpareto
emp_vario, 8
Gamma2chi_3D, 12
Gamma2graph, 12, 15
Gamma2par, 13
data2mpareto
Gamma2Sigma, 14, 15, 27
graphicalExtremes, 15
data2mpareto
(graphicalExtremes-package
set_graph_parameters, 26
Gamma2Sigma, 14, 15, 27
data2mpareto
simu_px_dirichlet, 28
data2mpareto
simu_px_logistic, 29
data2mpareto
simu_px_neglogistic, 29
data2mpareto
simu_px_tree_dirichlet, 30
Gamma2graph
simu_px_tree_logistic, 31
Gamma2par
V_HR, 32
Gamma2Sigma
unif, 32
graphicalExtremes-package
logdV_HR, 16
logdV_HR
logLH_HR, 17
Gamma2graph
mparetomargins, 17
Gamma2par
mst_HR, 15, 18
Gamma2Sigma
optim, 10
Gamma2par
par2Gamma, 19
Gamma2Sigma
plot.igraph, 12
data2mpareto
rmpareto, 15, 19
data2mpareto
rmpareto_tree, 15, 21
Gamma2par
rmstable, 15, 23
Gamma2par
rmstable_tree, 15, 24
Gamma2par
select_edges, 26