Package ‘hierband’

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Description Implementation of the convex banding procedure (using a
hierarchical group lasso penalty) for covariance estimation that is
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hierband-package

Convex banding of the covariance matrix using

Description

hierband is the R package implementing the convex banding approach to covariance estimation of
Bien, Bunea, & Xiao (see full reference below).

Details

The package is designed for situations in which there is a large number of variables that have a
known ordering and in which it is believed that variables far apart in this ordering have little to no
dependence.

It is called hierband (pronounced "hair band") because it makes use of a hierarchical group lasso
penalty and provides a banded estimate of the covariance matrix.

Its main functions are hierband, hierband.path, hierband.cv.

Author(s)

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References

Bien, J., Bunea, F. Xiao, L. (2014) "Convex banding of the covariance matrix." Accepted for publi-
cation in JASA.

banded

Generates a banded covariance matrix and matrix squareroot sig: value of kth band (starting with
diagonal) size of band is length(sig)

Description

Generates a banded covariance matrix and matrix squareroot sig: value of kth band (starting with
diagonal) size of band is length(sig)

Usage

banded(p, sig)

Arguments

p dimension of covariance matrix

sig vector of values of Toeplitz matrix
formw

Form the "general weights" matrix

Description

Form the "general weights" matrix

Usage

formw(p)

Arguments

p  dimension of covariance matrix

gpband

Groupwise soft-thresholds subdiagonals by lam * w

Description

Groupwise soft-thresholds subdiagonals by lam * w

Usage

gpband(R, lam, w = NULL)

Arguments

R  p-by-p symmetric matrix
lam  Non-negative penalty parameter. Controls sparsity level.
w  (p-1) vector of weights. Default, w[1]=sqrt(2 * l)
**hierband**

*Solves main optimization problem for fixed lambda value*

**Description**

Solves the main optimization problem appearing in Bien, Bunea, & Xiao (2015):

\[
\min_P \| \text{Sighat} - P \|_F^2 + \lambda m \sum_l \| (W_l * P)_g \|_2^2
\]

where \( g_l \) are the outermost \((l+1)\) elements of a square matrix. And \( \| (W_l * P)_g \|_2^2 = \sum_{m=l}^\infty w_{lm}^2 \| P_s_{m} \|_2 \). If a non-NULL \( \delta \) is provided, then a constraint of the form \( SP \geq \delta I_p \) is included. Problem is solved by performing blockwise coordinate descent on the dual problem. See paper for more explanation.

**Usage**

```r
hierband(Sighat, lam, w = NULL, delta = NULL, maxiter = 100, tol = 1e-07)
```

**Arguments**

- **Sighat**: The sample covariance matrix
- **lam**: Non-negative penalty parameter. Controls sparsity level.
- **w**: \((p-1)\)-by-\((p-1)\) lower-triangular matrix (above diagonal ignored). \( w[l,1] \) gives the \( l \) weights for \( g_l \). Defaults to \( w[l,m] = \sqrt{2 * M + 1} \) for \( m < l \)
- **delta**: Lower bound on eigenvalues. If this is NULL (which is default), then no eigenvalue constraint is included.
- **maxiter**: Number of iterations of blockwise coordinate descent to perform.
- **tol**: Only used when \( \delta \) is non-NULL. When no eigenvalue changes by more than \( \text{tol} \) in \( \text{BCD} \), convergence is assumed.

**Value**

Returns the convex banded estimate of covariance.

**See Also**

- `hierband.path`
- `hierband.cv`

**Examples**

```r
set.seed(123)
p <- 100
n <- 50
K <- 10
ture <- ma(p, K)
```
hierband.cv

x <- matrix(rnorm(n*p), n, p)  # true A
Sighat <- cov(x)
fit <- hierband(Sighat, lam=0.4)
min(eigen(fit)$values)
fit2 <- hierband(Sighat, lam=0.4, delta=0.2)
min(eigen(fit2)$values)
# Use cross validation to select lambda:
path <- hierband.path(Sighat)
cv <- hierband.cv(path, x)
fit <- hierband(Sighat, lam=cv$lam.best)

hierband.cv Performs nfolds-cross validation

Description
This function can be used to select a value of lam that performs well according to a user-specified measure of error.

Usage
hierband.cv(pathObj, x, errfun = function(est, true) sum((est - true)^2),
nfolds = 5)

Arguments
pathObj output of hierband.path
x n by p matrix
errfun a user-specified function measuring the loss incurred by estimating est (first argument) when the true covariance matrix is true (second argument). Default: Squared Frobenius norm.
nfolds number of folds (default: 5)

Value
errs: A nlam-by-nfolds matrix of errors. errs[i,j] is error incurred in using lamlist[i] on fold j
m: CV error error for each value of lambda.
se: Standard error (estimated over folds) for each value of lambda
lam.best: Value of lamlist minimizing CV error.
ibest: Index of lamlist minimizing CV error.
lam.1se.rule: Selected value of lambda using the one-standard-error rule, a common heuristic that favors a sparser model when there isn’t strong evidence against it.
ilse.rule: Index of lamlist of one-standard-error rule.
See Also

hierband hierband.path

Examples

```r
set.seed(123)
p <- 100
n <- 50
K <- 10
ture <- ma(p, K)
x <- matrix(rnorm(n*p), n, p) %*% true
Sighat <- cov(x)
path <- hierband.path(Sighat)
cv <- hierband.cv(path, x)
fit <- hierband(Sighat, lam=cv$lam.best)
```  

```
# Not run:
plot(cv$m, main="CV Frob Error", type="b")
lines(cv$m+cv$se, main="CV Frob Error")
lines(cv$m-cv$se, main="CV Frob Error")
abline(v=(cv$ibest,cv$i.1se.rule), lty=2)
```

```
# End(Not run)
```

---

hierband.path  
Solves main optimization problem over a grid of lambda values

**Description**

See hierband for the problem this is solving. If lamlist not provided, then grid will be constructed starting at lambda_max, the smallest value of lam for which the solution (with delta=NULL) is diagonal.

**Usage**

```r
hierband.path(Sighat, nlam = 20, flmin = 0.01, lamlist = NULL, w = NULL, 
delta = NULL, maxiter = 100, tol = 1e-07)
```

**Arguments**

- **Sighat**  
The sample covariance matrix
- **nlam**  
The number of lambda values to include in grid.
- **flmin**  
The ratio between the smallest lambda and largest lambda in grid. (Default: 0.01)  
  Decreasing this gives less sparse solutions.
- **lamlist**  
A grid of lambda values to use. If this is non-NULL, then nlam and flmin are ignored.
- **w**  
A (p-1)-by-(p-1) lower-triangular matrix (above diagonal ignored). w[1,] gives the l weights for g_l.  
  Defaults to w[1,m]=sqrt(2 * l)/(l - m + 1) for m <= l
**lam.max.hierband**

delta Lower bound on eigenvalues. If this is NULL (which is default), then no eigenvalue constraint is included.

maxiter Number of iterations of blockwise coordinate descent to perform.

tol Only used when delta is non-NULL. When no eigenvalue changes by more than tol in BCD, convergence is assumed.

**Value**

Returns a sequence of convex banded estimates of the covariance matrix.

**P:** A $\text{nrrow}$(Sighat)-by-$\text{nrrow}$(Sighat)-by-$nlam$ array where $P[,,i]$ gives the $i$th estimate of the covariance matrix.

**lamlist:** Grid of lambda values used.

**w:** Value of $w$ used.

**delta:** Value of $\delta$ used.

**See Also**

hierband hierband.cv

**Examples**

```r
set.seed(123)
p <- 100
n <- 50
K <- 10
true <- ma(p, K)
x <- matrix(rnorm(n*p), n, p) %*% true$A
Sighat <- cov(x)
path <- hierband.path(Sighat)
cv <- hierband.cv(path, x)
fit <- hierband(Sighat, lam=cv$lam$best)
```

**Description**

Computes $\lambda_{\text{max}}$, which is the smallest value of $\lambda$ for which hierband (with $\delta$=NULL) gives a diagonal covariance matrix.

**Usage**

`lam.max.hierband(Sighat, ww)`

**Arguments**

Sighat empirical covariance matrix

ww the diagonal of $w$
Covariance of an equal-weighted moving-average process

Description

Here, \( \sigma[j,k] = 0 \) if \( |j-k| > K \) and \( \sigma[j,k] = 1 - \frac{|j-k|}{K} \) otherwise.

Usage

\[ \text{ma}(p, K) \]

Arguments

- \( p \)  
  dimension of covariance matrix
- \( K \)  
  moving-average bandwidth

Value

Returns the covariance matrix, \( \sigma \), and the symmetric square root, \( A \), of this matrix.

Make folds for cross validation

Description

Divides the indices 1:n into \( n \text{folds} \) random folds of about the same size.

Usage

\[ \text{MakeFolds}(n, nfolds) \]

Arguments

- \( n \)  
  sample size
- \( nfolds \)  
  number of folds
subdiag.thresh

Performs a single pass of BCD on a matrix R.

Description

To solve the unconstrained problem, R is $\Sigma$. To solve constrained problem, R is the current partial residual (excluding A terms).

Usage

subdiag.thresh(R, lam, w = NULL)

Arguments

- R: p-by-p symmetric matrix
- lam: Non-negative penalty parameter. Controls sparsity level.
- w: (p-1)-by-(p-1) lower-triangular matrix (above diagonal ignored). $w[l,]$ gives the $l$ weights for $g_l$. Defaults to $w[1, m] = \sqrt{R * l \over l + m + 1}$ for $m <= l$

subdiagonal.12norms

Compute the L2 norm of each subdiagonal of a symmetric matrix R.

Description

Compute the L2 norm of each subdiagonal of a symmetric matrix R.

Usage

subdiagonal.12norms(R)

Arguments

- R: a symmetric matrix
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