Jubilee: Forecasting Long-Term Growth of S&P 500 Index

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Abstract This tutorial shows how to use the jubilee package in R to forecast the 10-year and 20-year growth trajectory of S&P 500 Index. R programs are provided to run the regressions, make predictions, and plot the charts. Additionally, in order to capture the short-horizon volatility (less than 10 years), we derive a new quantity that shows the deviation between the realized index level and the forecast. Interestingly, this quantity is mean-reverting in the same frequencies as the commonly-known business cycles. We attempt to locate potential leading indicator(s) for the next bear market and recession. A few factors are explored, among which yield spread inversion is considered most important.

Introduction

In Lihn (2018), we enhanced the CAPE model (Campbell and Shiller (1988); Campbell and Shiller (1998)) and provided a regression framework called the “Jubilee-Tectonic Model” to forecast the long-term growth of S&P 500 (SPX) index. The $R^2$ of the regression is above 80%, indicating very high consistency with historical data, and therefore, explanatory power. The data and utilities are packaged into the jubilee package in R. This tutorial provides an introduction how to use the package and generate the forecasts.

The main objective of long-term forecast is to regress the 10-year and 20-year forward nominal log-returns of SPX: $r_{10}^\text{nom}(t)$ and $r_{20}^\text{nom}(t)$, from which we can derive the predicted log-index $X_{\text{pred}}(t)$ from the realized total-return log-index $X(t)$. The 10-year forecast is popular in asset management, since the time horizon is reasonable for long-term asset allocators, such as pensions and endowments. However, as we found in Lihn (2018), the 20-year period matches the natural frequency of the long-term mean reversion cycles. Thus the 20-year regression requires the least tectonic adjustments.

The forecast horizon is called the “look-forward period”, denoted as $\Delta T_f = 10$ or 20. We construct the 5-factor forecast model as follows:

$$f_{t,T_f} \sim \beta_0 + \beta_1 Y(t) + \beta_2 R(t) + \beta_3 \text{CPI}_{10}(t) + \beta_4 \text{CPI}_{20}(t) + \beta_5 \log \text{CAPE}_{\Delta T_f}(t) + \epsilon; \quad (1)$$

$\text{CPI}_{10}(t)$ and $\text{CPI}_{20}(t)$ are the 10-year and 20-year log-returns of CPI (FRED symbol: CPIAUCSL). The channel return $R(t)$ and channel deviation $Y(t)$ are two important quantities derived from the trend-following channel in Section 2.3 of Lihn (2018). $Y(t)$ is the difference between $X(t)$ and the trend-following channel moving average $a(t)$, that is, $Y(t) = X(t) - a(t)$. $Y(t)$ is mean-reverting with an approximate 40-year cycle (See Figure 1 below).

We briefly review the concept of the trend-following channel and mean-reversion decomposition here. However, the reader can skip this paragraph since the detail here is encapsulated in the R package, and will not be used explicitly in this paper. Given a channel look-back period $\Delta T_b = 45$ years, at time $t$, we apply the causal regression $X(\tau) \sim a(t) + R(t)(\tau - t)$, where $\tau \in [t - \Delta T_b, t]$, to produce $a(t)$ and $R(t)$, and therefore $Y(t)$. Using the discrete notation on $X(\tau)$ and $\tau$, the input data points to the regression are $\{(X_i = X(\tau_i), \tau_i) | \forall i = 1 \cdots N, \tau_i \in [t - \Delta T_b, t] \}$. And the regression yields analytical solutions: $a(t) = \langle X_i \rangle + \sqrt{3 \left( \frac{N+1}{N} \right)} \text{Cor}(X_i, \tau_1) \text{Stdev}(X_j); R(t) = \text{Cor}(X_i, \tau_j) \text{Stdev}(X_i) / \text{Stdev}(\tau_j);$ and $Y(t) = X(t) - \langle X_i \rangle - \sqrt{3 \left( \frac{N+1}{N} \right)} \text{Cor}(X_i, \tau_j) \text{Stdev}(X_i).$ From the $\langle X_i \rangle$ term in $a(t)$, we can see why $a(t)$ resembles some kind of moving average.

We assume the in-sample window is $t \in [T_{\text{start}}, T_{\text{end}}]$ during which period all the factors are available. Due to the forward-looking nature of $r_{t,T_f}^\text{nom}(t)$, it is NaN after $t > T_{\text{end}} - \Delta T_f$. Thus the regression is performed for $t \in [T_{\text{start}}, T_{\text{end}} - \Delta T_f]$, and the factor loadings are used to make prediction, denoted as $r_{t,T_f}^\text{pred}(t)$, that extends to $t \in [T_{\text{end}} - \Delta T_f, T_{\text{end}}].$

The log-CAPE is tectonically adjusted by the fault lines $\{(t_i, \Delta, \log \text{CAPE}_{\Delta T_f}) | \forall i = 1 \cdots N \}$ with the linear “ramp-up” period $\Delta_{\text{ramp}}$ such that

$$r_{t_i}^\text{adj}(t) = \log \text{CAPE}_{\Delta T_f}(t) + \left( t \right. \left. - t_i \right) \Delta \log \text{CAPE}_{\Delta T_f}.$$
We begin with loading the jubilee package and setting up several essential data tables:

```r
> library(jubilee)
> repo <- jubilee.repo(online=FALSE)
```

[1] "Maximum date for unrate is 2019-12-16 and for GDP, 2019-08-16"

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> ju <- jubilee(repo@ie, lookback.channel=45, fwd.rtn.duration=10)
> dt <- ju@dtb
> dj <- ju@reg.dtb

The repo object is an instance of the jubilee.repo class. It stores the raw data, mainly in the repo@ie data table slot, which is named after “irrational exuberance” to honor the main data source of Shiller (2018). The ju object is an instance of the jubilee class that most functions are based on. Inside the ju object, the dt=ju@dtb data table contains simple time series, many are directly copied from the repo@ie data table. The dj=ju@reg.dtb data table contains derived time series required for regression. The separation is to prevent each data table from getting too bulky. We will work with the dj data table most of the time.

The lookback.channel parameter is set to 45 years, according to Section 2.4 of Lihn (2018). The user is not expected to change this setting. The fwd.rtn.duration parameter corresponds to $\Delta T_f$, which is set to 10 for the 10-year forecast.

The package comes with static data going back 100-200 years. The static data will be only updated by the infrequent releases of the package. The user can choose to amend the most recent data from the internet by specifying online=TRUE. This will allow you to make timely forecasts.

The mapping between the mathematical notations and the columns in the data tables is described as follows:

- $t = dj$fraction: Time in years. Each month is in the 1/12 unit. Note that we follow Shiller’s “middle of the month” convention since he averages the quantity. But when we download the monthly data from FRED, we use the monthly data as is.
- $X(t) = dj$log.tri: The logarithm of the total return index for SPX.
- $r_{10}^{nom}(t) = dj$eqty.logr.f10: The 10-year forward returns of $X(t)$.
- $r_{20}^{nom}(t) = dj$eqty.logr.f20: The 20-year forward returns of $X(t)$.
- CPI$_{10}(t) = dj$cpi.logr.10: The 10-year (backward) log-return of CPI.
- CPI$_{20}(t) = dj$cpi.logr.20: The 20-year (backward) log-return of CPI.
- $R(t) = dj$eqty.lm.r: The channel return.
- $Y(t) = dj$eqty.lm.y: The channel deviation.
- $a(t) = dj$eqty.lm.a: The trend-following channel moving average.
- $\log(CAPE_{10}(t)) = dj$log.cape.10: The 10-year log-CAPE.
- $\log(CAPE_{20}(t)) = dj$log.cape.20: The 20-year log-CAPE.

As a starter, the following R code demonstrates a plot on the channel deviation $Y(t)$ via the corresponding column dj$eqty.lm.y in the data table. $Y(t)$ is the major analytical quantity that enhances the CAPE model. In Figure 1, we observe four bottoms (1896, 1932, 1974, 2009) approximately 40 years apart. Such long cycle is essential for the long-term forecast model. But it is too long to work with for the business cycle forecast, which we will address in Section “Short-Horizon Mean Reversion”.
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> J <- which(dj$fraction > 1881)  # defines the in-sample window
> plot(dj$fraction[J], dj$eqty.lm.y[J], col="blue", type="l",
+       xlab="$t$", ylab="$Y(t)$",
+       main="The channel deviation $Y(t)$")
> abline(h=0, col="red", lty=2)
>
> lines(dj$fraction, dj$usrec.nber*0.25-1.0, col="gray", lty=1)

Figure 1: The channel deviation $Y(t)$. It is mean-reverting with an approximate 40-year long-term cycle. The NBER recession indicator is drawn in gray line to illustrate the relative scale to the short-term business cycles.

Regression of The 10-Year Returns

The fault lines \( \{ (t_{adj}^i, \Delta \log \text{CAPE}_i^T) \}, \forall i = 1 \cdots N \} \) have been pre-packaged into data sets inside the function jubilee.std_fault_line(). The user only needs to choose which data set to use. Here we use "r_nom_f10_5ftr_4fl", which means “for nominal return regression, forward 10 years, 5-factors, 4 fault lines”. Its content is shown below:

> fl.10 <- jubilee.std_fault_line("r_nom_f10_5ftr_4fl")
> fl.10

<table>
<thead>
<tr>
<th>fraction</th>
<th>shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1907.06</td>
</tr>
<tr>
<td>2</td>
<td>1935.61</td>
</tr>
<tr>
<td>3</td>
<td>1944.48</td>
</tr>
<tr>
<td>4</td>
<td>1985.85</td>
</tr>
</tbody>
</table>

The following R code shows the procedure described in Eq. (Introduction) and Eq. (Introduction) to adjust log-CAPE with the fault lines, perform the regression, and generate the prediction:

> dj$log.cape10.adj <- jubilee.adj_fault_line(dj$fraction, dj$log.cape10, fl.10)
> J <- which(dj$fraction > 1881)  # defines the in-sample window
> df <- dj[J,]  # defines the in-sample data set
> lm.10 <- lm(eqty.logr.f10 ~ eqty.lm.y + eqty.lm.r + log.cape10.adj + cpi.logr.10 + cpi.logr.20,
+             data=df)
> pred.10 <- jubilee.predict(ju, lm.10, df)
> summary(lm.10)
Call:
\[
\text{lm(formula = eqty.logr.f10 ~ eqty.lm.y + eqty.lm.r + log.cape10.adj + cpi.logr.10 + cpi.logr.20, data = df)}
\]

Residuals:

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.109464</td>
<td>-0.012396</td>
<td>-0.000075</td>
<td>0.013694</td>
<td>0.066083</td>
</tr>
</tbody>
</table>

Coefficients:

|                     | Estimate | Std. Error | t value | Pr(>|t|) |
|---------------------|----------|------------|---------|---------|
| (Intercept)         | 0.4059376| 0.0064490  | 62.95   | <2e-16 *** |
| eqty.lm.y           | -0.0885762| 0.0018667  | -47.45  | <2e-16 *** |
| eqty.lm.r           | -2.7814138| 0.0595289  | -46.72  | <2e-16 *** |
| log.cape10.adj      | -0.0432007| 0.0008091  | -53.40  | <2e-16 *** |
| cpi.logr.10         | -0.0575684| 0.0340685  | -1.69   | 0.0913 .  |
| cpi.logr.20         | 0.9964247| 0.0441279  | 22.58   | <2e-16 *** |

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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02039 on 1541 degrees of freedom
(121 observations deleted due to missingness)
Multiple R-squared: 0.8139, Adjusted R-squared: 0.8133
F-statistic: 1348 on 5 and 1541 DF, p-value: < 2.2e-16

The summary statistics shows \( R^2 \) above 80%, indicating the regression has very high explanatory power. The prediction logic is encapsulated in the jubilee.predict() function, which produces the pred.10 data table, containing many useful columns in a single table. The some column definitions in pred.10 are described below:

- \( X_{\text{pred}}(t) = \text{pred.10$log.tri} \): The predicted log-index.
- \( [r_{\text{nom}}^{10}(t)]_{\text{pred}} = \text{pred.10$logr} \): The predicted nominal returns.
- \( p_{\text{pred}}(t) = \text{pred.10$price} \): The predicted SPX total-return price, rebased according to the SPX price of the most recent month.
- \( Y_{\text{pred}}(t) = \text{pred.10$eqty.lm.y} \): The predicted \( Y(t) \).
- \( X(t) = \text{pred.10$log.tri.source} \): The realized log-index. (An interpolated copy.)
- \( p(t) = \text{pred.10$tri.source} \): The realized SPX total-return price, rebased according to the SPX price of the most recent month. (An interpolated copy.)
- \( a(t) = \text{pred.10$eqty.lm.a.source} \): The realized \( a(t) \). (An interpolated copy.)

The forward-return regression can be visualized in Figure 2 by the following R code:
Figure 2: The 10-year nominal return forecast by 5-factor tectonic CAPE model. The blue line is the realized returns. The red line is the predicted returns. The dotted blue lines are the locations of the faults.

We can also visualize the log-index as well as the price-level forecast since 1997 via the following plots in Figure 3. We observe that the predicted index is much smoother than the actual index. The prediction is of the long-term nature, forecasting SPX will exceed 5000 in ten years (dividend included). But it leaves out the short-horizon fluctuations unexplained, such as the late-1990 rally, the dot-com crash, and the 2008-2009 crash and rebound.
> J10 <- which(pred.10$fraction >= 1997)
> par(mfcol=c(2,1))
> plot(pred.10$fraction[J10], pred.10$log.tri[J10], col="red", type="l",
+     xlab="t", ylab="$X(t)$",
+     main="Forecast of log-index X(t) in 10y model")
> lines(dj$fraction, dj$log.tri, cex=0.1, col="blue")
> legend(x=2000, y=17, bg="transparent",
+     legend=c("Actual", "10y"),
+     lwd=c(1,1), lty=c(1,1), cex=0.6,
+     col=c("blue", "red"))
> plot(pred.10$fraction[J10], pred.10$price[J10], col="red", type="l",
+     xlab="t", ylab="$p(t)$",
+     main="Forecast of price index p(t) in 10y Model")
> lines(pred.10$fraction, pred.10$tri.source, cex=0.1, col="blue")
> abline(h=3000, col="black", lty=3)

**Figure 3:** The log-index $X(t)$ and the price-level $p(t)$ forecasts since 1997 in the 10-year model. The blue lines are the realized values. The red lines are the predicted values.

**Forecast in The 20-Year Model**

It was mentioned in the introduction that the 20-year period matches the natural frequency of the long-term mean-reversion cycles. Thus the 20-year regression requires the least tectonic adjustments. We also find that the 20-year model is more suitable to describe the short-horizon fluctuation. This gives us the incentive to explore it in depth here in the tutorial.
We repeat the data loading procedure, except that the \texttt{fwd.rtn.duration} parameter $\Delta T_f$ is set to 20 years:

```r
\texttt{\textgreater \repo <- jubilee.repo(online=FALSE)}
```

[1] "Maximum date in raw ie.data is 2019.12 and SPX average at 3223.38"
[1] "Maximum date for unrate is 2019-12-16 and for GDP, 2019-08-16"

```r
\texttt{\ju <- jubilee(repo@ie, lookback.channel=45, fwd.rtn.duration=20)}
\texttt{\dt <- ju@dtb}
\texttt{\dj <- ju@reg.db}
```

Next, we use the fault line data set \textit{"r_nom_f20_5ftr_2fl_ramp5y"}, which means “for nominal return regression, forward 20 years, 5-factors, 2 fault lines, 5-year ramp-up", shown below. The 5-year ramp-up time smooths out the two jumps caused by the fault lines.

```r
\texttt{\fl.20 <- jubilee.std_fault_line("r_nom_f20_5ftr_2fl_ramp5y")}
\texttt{\fl.20}
```

<table>
<thead>
<tr>
<th>fraction shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1902.42 -1.077</td>
</tr>
<tr>
<td>2 1929.20 -1.499</td>
</tr>
</tbody>
</table>

Note that the two fault lines are before WWII. We like the fact that they are not intrusive to the modern history after WWII. This indicates that, from the perspective of 20-year window, the modern economic history has been quite stable. The forward-return regression is performed as following:

```r
\texttt{\dj$log.cape20.adj <- jubilee.adj_fault_line(dj$fraction, dj$log.cape20, fl.20, months=12*5)}
\texttt{\J <- which(dj$fraction > 1881)}
\texttt{\df <- \dj[J,]}
\texttt{\lm.20 <- lm(eqty.logr.f20 ~ eqty.lm.y + eqty.lm.r + log.cape20.adj + cpi.logr.10 + cpi.logr.20, data=df)}
\texttt{\pred.20 <- jubilee.predict(ju, lm.20, df)}
\texttt{\summary(lm.20)}
```

```
Call:
\texttt{lm(formula = eqty.logr.f20 ~ eqty.lm.y + eqty.lm.r + log.cape20.adj +}
\texttt{cpi.logr.10 + cpi.logr.20, data = df)}

Residuals:
\texttt{Min 1Q Median 3Q Max}
\texttt{-0.042841 -0.007351 0.000205 0.006378 0.046092}

Coefficients:
\texttt{Estimate Std. Error t value Pr(>|t|)}
\texttt{(Intercept) 0.0951878 0.0020242 47.03 <2e-16 ***}
\texttt{eqty.lm.y -0.0262233 0.0010905 -24.05 <2e-16 ***}
\texttt{eqty.lm.r 0.3891438 0.0247056 15.75 <2e-16 ***}
\texttt{log.cape20.adj -0.0248452 0.0003942 -63.02 <2e-16 ***}
\texttt{cpi.logr.10 0.3253286 0.0164986 19.72 <2e-16 ***}
\texttt{cpi.logr.20 -0.8882677 0.0263884 -33.66 <2e-16 ***}

---
Signif. codes: \texttt{0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1}

Residual standard error: 0.0109 on 1302 degrees of freedom
(360 observations deleted due to missingness)
Multiple R-squared: 0.8624, Adjusted R-squared: 0.8618
F-statistic: 1631 on 5 and 1302 DF, p-value: < 2.2e-16
```

The summary statistics shows $R^2$ above 85%, indicating the regression has very high explanatory power. All the factors are highly significant. The return regression can be visualized in Figure 4 by the following R code:
> plot(pred.20$fraction[J], pred.20$logr[J]*100, col="red", type="l", + xlab="t", ylab="r_{\text{f20}}^{\text{nom}}(t)"", ylim=c(2,17), + main="20-year nominal forecast, 5-factor tectonic CAPE model")
> lines(df$fraction, df$eqty.logr.f20*100, col="blue")
> for (i in 1:NROW(fl.20)) { abline(v=fl.20[i,1], col="blue", lty=2) } # faults

**Figure 4:** The 20-year nominal return forecast by 5-factor tectonic CAPE model. The blue line is the realized returns. The red line is the predicted returns. The dotted blue lines are the locations of the faults.

**Prediction of X(t) and Y(t)**

The log-index forecast from the 20-year model is shown in the upper panel of Figure 5. The blue line is the realized $X(t)$. The solid red line is the predicted path from the 20-year model. The dotted red line is the predicted path from the 10-year model. We observe that the two red lines generally agree. But the 20-year model produces slightly more optimistic forecast than the 10-year model for the next decade. Such optimism looks more pronounced in the price level forecast in the lower panel.
We notice again that the predicted path is much smoother than the realized path in the in-sample
> par(mfcol=c(1,1))
> JY <- which(dj$fraction > 1912)
> plot(dj$fraction[JY], dj$eqty.lm.y[JY], type="l", col="blue",
+ xlab="$t$", ylab="$Y(t)$",
+ main="Predicted and realized $Y(t)$ in 20y model")
> lines(pred.20$fraction[JY], pred.20$eqty.lm.y[JY], col="red")
> lines(pred.10$fraction[JY], pred.10$eqty.lm.y[JY], col="red", lty=2)
> rect(1937, 0.5, 1945, 0.7, col="orange") # WWII

Figure 6: The blue line is the realized $Y(t)$, since 1912. Since $Y(t)$ is mean-reverting, a clear historical plot can be presented. The solid red line is the predicted path $Y_{pred}(t)$ from the 20-year model. The predicted path is much smoother than the realized path. As a comparison, the dotted red line is $Y_{pred}(t)$ from the 10-year model. The orange box represents WWII.

period. This difference can be best illustrated by the $Y(t)$ plot in Figure 6. The blue line is the realized $Y(t)$. The solid red line is $Y_{pred}(t)$ from the 20-year model. Their difference leads us to define the next important quantity: the "short-horizon mean reversion" index, which captures the impact from the business cycles. It will be explored in the rest of this paper.

Short-Horizon Mean Reversion

The long-term model leaves ample room for short-horizon fluctuations. If you are truly a buy-and-hold long-term investor, this fluctuation should not trouble you. However, individuals and organizations have utilities in limited time frame. Asset bubble forces you to buy at expensive prices. Steep downside fluctuation becomes an acute problem when you have to withdraw funds from your investment. In this section, we provide a preliminary framework to understand such fluctuation. This topic is still under research, thus the exploration here is quite open-ended.

An interesting outcome of the 20-year model is that the "short-horizon mean reversion" (SMR) index is least noisy and most meaningful. It is mainly due to the fact that there is no fault line adjustment after WWII.

$$
Z_{20}(t) \equiv X(t) - X_{pred}(t) \Big|_{\Delta T_f=20},
$$

$$
\equiv Y(t) - Y_{pred}(t) \Big|_{\Delta T_f=20}
$$

$Z_{20}(t)$ is stored in the log.tri.smr column in the pred.20 data table. Its style statistics is shown below:

```r
> mean(pred.20$log.tri.smr, na.rm=TRUE)
[1] -1.449178e-05
> sd(pred.20$log.tri.smr, na.rm=TRUE)
```
As one can see, its mean is zero, thus a mean-reverting process. As shown in Figure 7, whenever $Z_{20}(t)$ swings to 2-stdev, it usually indicates the market is in extreme conditions, either over-sold or over-bought.

> J <- which(dj$fraction > 1912)
> plot(pred.20$fraction[J], pred.20$log.tri.smr[J], type="l", col="blue",
+        ylab="$Z_{20}(t)$", xlab="$t$",
+        main="Short-horizon mean reversion in 20y model")
> abline(h=0, col="red", lty=2)
> sd2 <- 2*sd(pred.20$log.tri.smr, na.rm=TRUE)
> abline(h=sd2, col="blue", lty=2)
> abline(h=-sd2, col="blue", lty=2)
> rect(1937, 0.5, 1945, 0.7, col="orange") # WWII
> lines(dj$fraction, dj$usrec.nber*0.25-0.9, col="gray", lty=1)

![Short-horizon mean reversion in 20y model](image)

**Figure 7:** The short-horizon mean reversion (SMR) index $Z_{20}(t)$, drawn in the blue line. The dotted blue lines are the ±2-stdev of $Z_{20}(t)$. The orange box represents WWII. The NBER recession indicator is drawn in gray line to illustrate the short-term business cycles.

The log-index $X(t)$ can be viewed as a decomposition of the long-term component $X_{pred}(t)$ and the short-term component $Z_{20}(t)$. The long-term component can be predicted by our enhanced “Tectonic CAPE” model. What drives the short-term component? Our view is that $Z_{20}(t)$ is mainly influenced by the interest rate policy, barring major world wars. We will also examine the unemployment rate and inflation.

**Interest Rate Policy and Yield Spread**

Yield curve inversion has been found to be one of the most reliable leading indicators of the past several large-scale bear markets. Yield spread is the most powerful policy tool that affects all aspects of macro-economics.
The economic reasoning of yield curve inversion is quite appealing: The stock market can be viewed as a large financial operation that borrows on the short-term rate and lends on the long-term rate. Companies such as banks can build leverage to enhance their returns from the spread. But some companies can use excessive leverage to “game the system”, so to speak. Thus, modulating the yield spread is a key policy tool for FED to tame the speculation. When the yield spread turns negative, there is no profit to be made from the spread, causing dismal earnings for several quarters, and the stock price crashes due to lower valuation. A few of them with poor balance sheet may go into bankruptcy. This chilling effect usually lingers for about 12-24 months.

Furthermore, the narrowing of yield spread creates a regime switching scenario. In a red hot economy, as the yield spread becomes smaller, some executives would not think of reducing their balance sheet risk. Instead, many of them would choose to increase the leverage, in order to generate equal or more profit and justify the rising stock price. Suddenly, the yield curve is inverted, the mathematics of leveraging reaches a singularity. And the balance sheets of these companies collapse.

Later, we will introduce the concept of “unemployment rate as high-yield bond”. This extends the influence of interest rate policy far beyond the financial sector. In the internet age, the largest companies are all in the consumer sector selling goods and products to the vast consumers. Employment rate is their borrowing.

In some occasions, FED’s tightening was in a straight path to inversion. But in other times, FED might swing in short cycles of tightening and easing before touching down on the ultimate inversion. The current trend points to next inversion around year 2020, but it is totally up to the FED to make it happen or not.

Given the following attributes:

- \( G_{10}(t) = \text{dj$rate.gs10} \): the yield of 10-year Treasury,
- \( G_{3m}(t) = \text{dj$rate.tb3ms} \): the yield of 3-month Treasury bill,
- \( YS(t) \equiv G_{10}(t) - G_{3m}(t) = \text{dj$rate.spread} \): the yield spread,

we define the centered yield spread normalized by the 10-year yield:

\[
\hat{YS}(t) = \frac{YS(t) - \langle YS(t) \rangle}{G_{10}(t)}.
\]

It is calculated and stored in \text{dj$rate.spread.norm} column. Here \( \langle YS(t) \rangle \) is the mean of \( YS(t) \), which is approximately 1.5% as shown by the variable \text{rate.spread.mean} below, which is also a slot in the \text{ju} object. It is the equilibrium level of yield spread.

Our first finding is that \( Z_{20}(t) \) is deeply related to \( \hat{YS}(t) \). This is illustrated in Figure 8. The two quantities have nearly the same magnitude, that is, \( Z_{20}(t) \approx \hat{YS}(t) \). They often move in tandem. Every yield curve inversion (shown as vertical dotted red lines) coincides with the market top, and the following easing matches the bottom of the selloff. For the equity market, yield spread inversion is the decisive turning point that causes regime switching from a bull market to a bear market. On the contrary, the market switches back from a bear market to a bull market expeditiously once the yield spread is widened large enough.
> rate.spread.mean <- mean(dj$rate.spread, na.rm=TRUE)
> rate.spread.mean
[1] 1.452224

> J <- which(dj$fraction > 1912)
> ma <- function(x,n,sides=1){filter(x,rep(1/n,n), sides=sides)}
> plot(pred.20$fraction[J], pred.20$log.tri.smr[J], type="l", lwd=1, col="blue",
+ ylab="$$Z_{20}(t)$$ vs $$\widehat{YS}(t)$$", xlab="$$t$$",
+ main="Short-horizon mean reversion and yield spread inversion")
> lines(dj$fraction[J], ma(-dj$rate.spread.norm,3)[J], type="l", col="red")
> for (t in repo@yield.inversion) { abline(v=t, lty=2, col="red") }
> abline(h=0, col="blue", lty=2)
> rect(1937, 0.5, 1945, 0.7, col="orange") # WWII

Figure 8: Overlay of normalized yield spread $-\widehat{YS}(t)$ (red line) with $Z_{20}(t)$ (blue line). The vertical dotted red lines are the locations of maximum yield spread inversion in each business cycle. Here $\widehat{YS}(t)$ is smoothed by 3-month moving average to make it less noisy in the plot. The orange box represents WWII.

The Uncertainty Principle

How wide of the yield spread is “wide enough”? Is the fluctuation of the stock market affected by the interest rate environment it is in? We think this is related to the following quantity. Define the modified inverse of 10-year yield $GSI(t)$ as

$$GSI(t) \equiv \frac{\langle YS(t) \rangle}{GSI_{10}(t)}.$$  \hspace{1cm} (8)

It is calculated and stored in $dj$rate.gs10.modinv column. The fact that $Z_{20}(t) \approx \widehat{YS}(t)$ implies that the maximum amplitude of $Z_{20}(t)$ is approximately $GSI(t)$, which associates the volatility of the equity market with the inverse of the long-term interest rate. We call this relation the Uncertainty Principle. This explains that, on one hand, in a low interest rate environment, the equity market is susceptible to high volatility. On the other hand, the equity market is fairly dormant in a high interest rate environment, e.g. Dow was flat from 1966 to 1981.

Figure 9 adds the overlay of $GSI(t+74/12)$ with $Z_{20}(t)$ and $\widehat{YS}(t)$. The 74-month shift is described in Section 9.1 of Lihn (2018) as the equity market’s lead time over GS10. We draw $\pm GSI(t)$ in purple lines to form an envelope, and we smooth $GSI(t)$ by its 24-month moving average to remove the noise.

We observe that the $GSI(t)$ envelope indeed describes the fluctuation of $Z_{20}(t)$ quite well. The
equity market went through very large swings whenever the Treasury yield was trending into the low interest rate period. The first incident was marked by the big rally from 1921 to the 1929 peak, followed by a decade of turmoils, including WWII. The second incident was marked by the irrational exuberance leading to the 2000 peak, followed by two severe bear markets in the next decade.

Statistically speaking, it is far from certain to prove whether such statement can be true or not. The GS10 cycle is so long (80 years) that you won’t see the next low interest rate environment until the end of the century.

```r
> plot(pred.20$fraction[J], pred.20$log.tri.smr[J], type="l", lwd=1, col="blue",
+       ylab="Z_{20}(t)", xlab="t",
+       main="Uncertainty principle: Z(t) and inverse of GS10 (GSI)"
> rect(1937, 0.8, 1945, 0.95, col="orange") # WWII
> lines(dj$fraction[J], ma(-dj$rate.spread.norm,3)[J], type="l", col="red")
> abline(h=0, col="blue", lty=2)
> GSI <- dj$rate.gs10.modinv
> lines(dj$fraction[J]-74/12, ma(GSI,24)[J], col="purple", lty=2, lwd=2)
> lines(dj$fraction[J]-74/12, -ma(GSI,24)[J], col="purple", lty=2, lwd=2)
> lines(dj$fraction[J], GSI[J], col="purple", lty=3, lwd=1)
```

**Figure 9:** Overlay of $GSI(t+74/12)$ with $Z_{20}(t)$ (the blue line). The 74-month shift is the equity market’s lead time over Treasury market. We draw $\pm GSI(t+74/12)$ in thick purple dotted lines to form an envelope, which are smoothed by 24-month moving average to remove the noise. The red line is the yield spread. We draw $GSI(t)$ in thin purple dotted line to indicate where inversion occurs. The orange box represents WWII.

**Unemployment Rate as Alternative High-Yield Bond**

The unemployment rate (UNRATE) is highly correlated to the interest rate policy and is a major barometer of the health of the economy. We observe that, after WWII, the unemployment rate can be thought of as a high-yield bond. That is, in a low interest rate environment, the lowest unemployment rate tends to be very low. In a high interest rate environment, even the best unemployment rate is still quite high. This makes sense. Corporations have to compare the cost of employment to the cost of the capital.

In addition, the unemployment rate is a clear indicator how the economy responds to the yield spread tightening and easing. We find that the unemployment rate since 1950 (post-WWII) can be regressed in the following three-factor model:

$$UNRATE(t) \sim \beta_0 + \beta_1 GS10(t) + \beta_2 GSI(t) + \beta_3 \overset{\hat{}}{YS}(t) + \epsilon.$$  (9)

The role of the $\beta_2 GSI(t)$ term can be seen as modifying the weight of the mean $\langle YS(t) \rangle$ inside
\( \hat{YS}(t) \) from 1 to \((1 - \beta_2/\beta_3) \). This regression is shown below:

```r
> J <- which (dt$fraction >= 1950 & dt$fraction <= 2019)
> a <- lm(unrate ~ rate.gs10 + rate.gs10.modinv + rate.spread.norm, data=dj[J])
> summary(a)
```

```
Call: lm(formula = unrate ~ rate.gs10 + rate.gs10.modinv + rate.spread.norm, data = dj[J])

Residuals:
          Min           1Q       Median           3Q          Max
-1.991898  -0.822744  -0.018327   0.684437   3.261124

Coefficients:
                     Estimate Std. Error    t value Pr(>|t|)  
(Intercept)      2.3359225   0.2614605  8.934200  < 2e-16 ***  
rate.gs10         0.4054900   0.0240789 16.839000  < 2e-16 ***  
rate.gs10.modinv  3.3619275   0.4012514   8.378920  2.29e-16 ***  
rate.spread.norm  4.7459418   0.1645393 28.842940  < 2e-16 ***  

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 1 ' ' 0

Residual standard error: 1.027677 on 824 degrees of freedom
Multiple R-squared: 0.608381, Adjusted R-squared: 0.607110
F-statistic: 426.8563 on 3 and 824 DF, p-value: < 2.2e-16
```

All three factors are highly significant. The \( R^2 \) is above 60%, which is quite high. \( \beta_0 \approx 2.34 \) can be interpreted as a baseline unemployment rate. The remaining terms expand into a complicated nonlinear relation on \( GS_{10}(t) \) such as

\[
\text{UNRATE}(t) \sim \beta_0 + \beta_1 GS_{10}(t) + \beta_3 YS(t) - \left(1 - \frac{\beta_2}{\beta_3}\right) \langle YS(t) \rangle GS_{10}(t) + \epsilon.
\]

(10)

Figure 10 shows the result of this regression. \( \text{UNRATE}(t) \) is the blue line and the regression is the red line. At each peak of a business cycle when \( YS(t) \approx 0 \), we have \( GS_{3m}(t) \approx GS_{10}(t) \) and \( YS(t) \approx -GS_{10}(t) \). Then the lowest unemployment rate at that moment can be estimated by

\[
\text{UNRATE}_{\text{min}}(t) \equiv \beta_0 + \beta_1 GS_{10}(t) + \left(\beta_2 - \beta_3\right) \langle YS(t) \rangle GS_{10}(t) - \beta_3 GS_{3m}(t) GS_{10}(t) + \epsilon
\]

(11)

```
\( \text{UNRATE}_{\text{min}}(t) \) depends on \( GS_{10}(t) \) alone. We observe in Figure 10 that \( \text{UNRATE}_{\text{min}}(t) \) matches the lowest unemployment rate in each cycle peak quite well (with 1970 as a major exception). It provides a lower bound on the unemployment rate.
```

A cyclically increasing unemployment rate is the best confirmation of a bear market. There are unemployment spikes at the depths of some recessions, e.g. in 1974-75, in early 1980, and around 2010, that are not captured by the regression. These spikes, recorded by the residual \( \epsilon \), are what we call the “high-yield” elements in the unemployment rate.

In this business cycle, between 2013 and 2016, \( \text{UNRATE}_{\text{min}}(t) \) went below 2% briefly when \( GS_{10}(t) \) was below 1.85%. When the next yield spread will be inverted (in 2020?), if \( GS_{10}(t) \) stays below 3%, the minimum unemployment rate can go as low as 3%.

\(^1\)Note that it would yield similar \( R^2 \) if the first factor is replaced with \( \beta_1 GS_{3m}(t) \).
> J <- which (dt$fraction >= 1950 & is.finite(dt$unrate))
> plot(dj$fraction[J], dj$unrate[J], type="l", col="blue",
+   ylab="UNRATE", xlab="t", ylim=c(1.5,11),
+   main=sprintf("Unemployment rate as alternative high-yield bond"))
> lines(dj$fraction[J], predict(a, newdata=dj[J]), col="red")
> ac <- a$coefficients
> unrate.min <- ac[1]+ac[2]*dj$rate.gs10+(ac[3]-ac[4])*dj$rate.gs10.modinv
> lines(dj$fraction[J], ma(unrate.min,3)[J], col="black", lwd=2, lty=3)
> for (t in repo$yield.inversion) { abline(v=t, lty=2, col="red") }

Figure 10: Unemployment rate $\text{UNRATE}(t)$ (the blue line) and its 3-factor regression to the bond market (the red line). The dotted vertical red lines mark the locations of yield spread inversion. The dotted black line is $\text{UNRATE}_{\text{min}}(t)$ (smoothed by 3-month moving average).

One-Year Change of Unemployment Rate

Just as important as observing the yield spread inversion, we also want to observe the inversion of the unemployment rate. The 1-year change (log-return) of the unemployment rate $r_{\text{UNR}}(t)$ is defined as

$$r_{\text{UNR}}(t) \equiv \log \text{UNRATE}(t) - \log \text{UNRATE}(t-1).$$ (12)

We find that $r_{\text{UNR}}(t)$ shows striking correlation with $-\text{YS}(t)$ and $Z_{20}(t)$, as shown in Figure 11. The combination of a weakening employment and the yield spread inversion appears to provide very convincing signal for a coming bear market. After the yield spread inversion, if $r_{\text{UNR}}(t)$ also turn negative, the recession is around the corner. The economy will continue to shed jobs until the yield spread is wide enough to provide corporations incentive to hire again. This process usually takes 12-24 months from past experiences.
Figure 11: The reversal of $Z_{20}(t)$ (the blue line), $\text{r}_{\text{UNR}}(t)$ (the red line) and $-\hat{Y}_S(t)$ (the gray line). When $\text{r}_{\text{UNR}}(t)$ turns negative and $\hat{Y}_S(t)$ undergoes a yield spread inversion, it is a clear indicator that $Z_{20}(t)$ has entered a bear market.

**Inflation**

Lastly, in some circumstances, FED raises short-term rate in order to combat the rising inflation. Let $\Delta CPI_{f_1}(t)$ be the forward 1-year inflation rate $CPI_{f_1}(t)$ in excess of $GS_{10}(t)$. They are defined as

$$
\begin{align*}
CPI_{f_1}(t) & \equiv \log CPI(t+1) - \log CPI(t); \\
\Delta CPI_{f_1}(t) & \equiv CPI_{f_1}(t) - GS_{10}(t).
\end{align*}
$$

We find that $\Delta CPI_{f_1}(t)$ is correlated with the yield spread during some periods. This is illustrated in Figure 12. The policy of combating inflation is quite obvious in the 1970’s. $\Delta CPI_{f_1}(t)$ moves in tandem with $-2(Y_S(t) - \langle Y_S(t) \rangle)$. After 1980, $\Delta CPI_{f_1}(t)$ is mostly negative, there has been a fairly large premium between $GS_{10}(t)$ and one-year inflation. Thus such policy goal is much less clear. After 2000, FED was able to maintain $\Delta CPI_{f_1}(t)$ barely below zero. Only recently, such premium is disappearing gradually, $\Delta CPI_{f_1}(t)$ is inching up above zero. And FED is indeed raising the short-term rate to combat it.
Figure 12: The 1-year forward inflation rate in excess of GS10: $\Delta CPI_{f1}(t)$ (blue line). The red line is $-2(YS(t) - \langle YS(t) \rangle)$ indicating the strength of yield spread needed to suppress the runaway inflation. The top dotted blue line is the constant $2\langle YS(t) \rangle$, showing where the yield spread becomes inverted.

Discussion

We’ve shown that yield spread inversion is likely a major factor that causes the stock market to turn south. It can cause the stock market to drop 40-50% in 12-24 months. It also has a dire social consequence of causing the unemployment rate to spike. FED must invoke the inversion with extreme care.

We find that the best interpretation of FED’s motivation to invoke a yield spread inversion is to cure an asset bubble. In 1970’s, it was for the rising inflation, especially in gold and oil. This was consummated in 1980 with an ultimate peak of inflation, GS10, and the commodity bubble. In 1990, it was for the junk bond bubble and the Japanese stock market. Because the level of SPX was fairly subdued in terms of $Z_{20}(t)$ during this period, it didn’t cause a big drop in SPX. In 2000, it was for the dot-com mania. In 2007, it was for the sub-prime debt bubble.

Thus in order to figure out the next bear market in $Z_{20}(t)$, one will need to watch a few things:

1. $YS(t)$: When will the yield spread become negative?
   - https://fred.stlouisfed.org/series/T10Y3M
2. UNRATE(t) and rUNR(t): When will the unemployment rate begin to rise? When will the 1-year change turn negative?
   - https://fred.stlouisfed.org/series/UNRATE
3. $\Delta CPI_{f1}(t)$: Is the 1-year inflation rate expected to exceed the 10-year Treasury yield?
4. Ultimately, is there an asset bubble that makes FED uncomfortable? In SPX? In Nasdaq? In the bond market? In any major foreign stock market?
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Bibliography


