Comparison of response functions in \textit{kitagawa}

Andrew J. Barbour\textsuperscript{1} \cdot Jonathan Kennel\textsuperscript{2}

1 - U.S. Geological Survey \cdot 2 - University of Guelph

January 27, 2024

Abstract

In this vignette we demonstrate the response functions found in the package \texttt{kitagawa}, which are appropriate for modeling the effect of harmonic volumetric strain or pressure-head fluctuations in sealed and open water wells. For sealed-wells there is only one response function, from Kitagawa et al. (2011), and this gives the complex frequency response of virtual water height $Z$ or pressure $P$ as a function of areal strain $\epsilon$. For open wells there is a suite of open-well response functions, from Cooper et al. (1965); Hsieh et al. (1987); Rojstaczer (1988); Liu et al. (1989); Wang et al. (2018); and these give the complex frequency response of water height as a function of aquifer head $H$ or pressure. Wang et al. (2018) allows for leakage from the aquifer.

Contents

1 Introduction \hfill 2
2 Preliminaries \hfill 2
3 Sealed well response \hfill 4
3.1 Strain: Kitagawa et al. (2011) \hfill 4
4 Open well response \hfill 6
4.1 Pressure head: Cooper et al. (1965) \hfill 6
4.2 Pressure head: Hsieh et al. (1987) \hfill 8
4.3 Pressure head: Liu et al. (1989) \hfill 10
4.4 Pressure head (with leakage): Wang et al. (2018) \hfill 12
4.4.1 Figure 2 from Wang et al. (2018) \hfill 13
4.5 Strain: Rojstaczer (1988) \hfill 14
5 Model Comparisons

5.1 Responses to strain .......................................................... 16
5.2 Responses to pressure head (all open) .............................. 17

1 Introduction

The underlying physical model of these response functions is based upon the assumption that fluid flows radially through an homogeneous, isotropic, confined aquifer. The underlying principle is as follows. When a harmonic wave induces strain in a confined aquifer (one having aquitards above and below it), fluid flows radially into, and out of a well penetrating the aquifer. The flow-induced drawdown, $s$, is governed by the following partial differential equation, expressed in radial coordinates ($r$):

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} - \frac{S}{T} \frac{\partial s}{\partial t} = 0$$

where $S$ and $T$ are the aquifer storativity and transmissivity respectively.

The solution to this PDE, with periodic discharge boundary conditions, gives the amplitude and phase response we wish to calculate. The solution for an open well was first presented by Cooper et al. (1965), and subsequently modified by Rojstaczer (1988); Liu et al. (1989). Kitagawa et al. (2011) adapted the solution of Hsieh et al. (1987) for the case of a sealed well. Wang et al. (2018) provides the leaky aquifer response for an open well.

These models are applicable to any quasi-static process involving harmonic, volumetric strain of an aquifer (e.g., passing Rayleigh waves, or changes in the Earth’s tidal potential).

In practice, however, the presence of permeable fractures can violate the assumption of isotropic permeability, which may substantially alter the response by introducing shear-strain coupling. Such complications are beyond the scope of these models.

2 Preliminaries

Load the necessary packages:

```r
library(RColorBrewer)
Set1 <- brewer.pal(8, "Set1")
library(signal, warn.conflicts = FALSE)
library(kitagawa)

## Loaded kitagawa (3.1) - Spectral response of water wells; see ?kitagawa
```
Setup some constants:

S. <- 1e-05  # Storativity [nondimensional]
T. <- 1e-04  # Transmissivity [m**2 / s]
D. <- T./S.  # Diffusivity [m**2 / s]
Ta <- 50     # Aquifer thickness [m]
Hw <- z <- 50 # Depth to water table [m]

# Using AN01 stats from Kit Tbl 1
Rc. <- 0.075 # Radius of cased portion of well [m]
Lc. <- 570   # Length of cased portion of well [m]
Rs. <- 0.135 # Radius of screened portion of well [m]
Ls. <- 15    # Length of screened portion of well [m]
Vw. <- sensing_volume(Rc., Lc., Rs., Ls.) # volume of fluid [m**3]

# parameters assumed by well_response: rho=1000 # density of rock
# [kg/m**3] Kf=2.2e9 # Bulk modulus of fluid [Pascals] grav=9.81 #
# gravitational acceleration [m/s**2]
rhog <- 9.81 * 1000

# Kitagawa Fig 7: Ku B / Kw Aw = 3 => Aw=4.8 at 40GPa
Ku. <- 4e+10  # Bulk modulus [Pascals]
B. <- 0.5     # Skemptions ratio [nondimensional]

And create the dimensionless frequencies, defined by $z^2 \omega / 2D$, where $D$ is the hydraulic diffusivity:

Q <- 10^seq(-5, 2, by = 0.05)  # [nondimensional]
IQ <- log10(Q)
omega <- omega_norm(Q, z, D., invert = TRUE)  # [Hz]

Phase <- function(Z) {
  Phs. <- Arg(Z)  # will wrap to -pi/pi
  uPhs. <- signal::unwrap(Phs., tol = pi/30)
  return(data.frame(Phs = Phs., uPhs = uPhs.))
}

# Responses converted to pressure if TRUE
asP <- FALSE
ZasP <- FALSE

And onto the response functions...
3 Sealed well response

3.1 Strain: Kitagawa et al. (2011)

```r
                         B. = B., Avs = 1, Aw = 1, as.pressure = asP)
plot(wrsp)  # uses plot.wrsp method
```

Sealed well−response (kitagwawa)

(a) Amplitude

(b) Phase

Frequency [Hz]

```r
crsp <- wrsp[["Response"]][, 2]  # Complex response
kGain <- Mod(crsp)/Ku./B.  # Amplitude (or Gain)
kP <- Phase(crsp)  # Phase
```
Sealed Well Response (KITAGAWA): Harmonic Strain

(a) Gain

(b) Phase

Figure 1: The response of a sealed well to harmonic areal strain using the Kitagawa model. The amplitude is normalized by Skempton’s coefficient $B$ and the undrained bulk modulus $\kappa_u$. Frequency is dimensionless, based on the well-depth $z$ and the diffusivity $D$. 
4 Open well response

4.1 Pressure head: Cooper et al. (1965)

\[
\text{wrsp} \leftarrow \text{open\_well\_response}(\text{omega, } T. = T., \ S. = S., \ Ta = Ta, \ Hw = Hw, \\
\text{model} = \text{"cooper"}, \ \text{as.pressure} = \text{ZasP})
\]

\[
\text{plot(wrsp)}
\]

![Open well-response (cooper)](image)

\[
\text{crsp} \leftarrow \text{wrsp}[\text{"Response"}][, \ 2]
\]

\[
\text{cGain} \leftarrow \text{Mod(crsp)}
\]

\[
\text{cP} \leftarrow \text{Phase(crsp)}
\]
Open Well Response (COOPER): Harmonic Strain

Figure 2: The response of an open well to harmonic areal strain using the Cooper model. Frequency is dimensionless, based on the well-depth $z$ and the diffusivity $D$. 
4.2 Pressure head: Hsieh et al. (1987)

```r
wrsp <- open_well_response(omega, T. = T., S. = S., Ta = Ta, Hw = Hw,
                         model = "hsieh", as.pressure = ZasP)
plot(wrsp)
```

![Open well–response (hsieh)](attachment:figure.png)

```r
crsp <- wrsp[['Response']][, 2]
hGain <- Mod(crsp)
hP <- Phase(crsp)
```
Figure 3: The response of an open well to harmonic areal strain using the Hsieh model. Frequency is dimensionless, based on the well-depth $z$ and the diffusivity $D$. 

$$Q = \frac{z^2 \omega}{2D}$$
4.3 Pressure head: Liu et al. (1989)

\[
\text{wrsp} \leftarrow \text{open\_well\_response}(\omega, T. = T., S. = S., Ta = Ta, Hw = Hw, \\
\text{model} = "liu", \text{as\_pressure} = \text{ZasP})
\]

\text{plot(wrsp)}

![Open well-response (liu)](image)

\text{crsp} \leftarrow \text{wrsp}[["Response"]][, 2]
\text{lGain} \leftarrow \text{Mod(crsp)}
\text{lP} \leftarrow \text{Phase(crsp)}
Figure 4: The response of an open well to harmonic areal strain using the Liu model. Frequency is dimensionless, based on the well-depth $z$ and the diffusivity $D$. 

$$Q = \frac{z^2 \omega}{2D}$$
4.4 Pressure head (with leakage): Wang et al. (2018)

```
wrsp <- open_well_response(omega, T. = T., S. = S., leak = 1e-08, model = "wang",
                           as.pressure = asP)
plot(wrsp)

Open well–response (wang)

(a) Amplitude

(b) Phase
```

```
crsp <- wrsp[["Response"]][, 2]
rGain <- Mod(crsp)
rP <- Phase(crsp)
```
4.4.1 Figure 2 from Wang et al. (2018)

\text{Transmiss} \leftarrow c(1, 0.01, 1e-04, 1e-06, 1e-08)
\text{Storativ} \leftarrow c(0.01, 1e-04, 1e-06, 1e-08)
\text{omeg} \leftarrow 1.9322736/86400 \quad \# \text{M2 in Hz}
\text{leak} \leftarrow 10^{-\text{seq}(-11, -3, 0.2)}

\textbf{Figure 5:} Amplitude and phase shift as a function of the specific leakage \((K'/b')\) using the Wang 2018 model for the M2 tide.
4.5 Strain: Rojstaczer (1988)

wrsp <- open_well_response(omega, T. = T., S. = S., z = z, model = "rojstaczer", as.pressure = asP)
plot(wrsp)

Open well–response (rojstaczer)

(a) Amplitude

(b) Phase

Frequency [Hz]

[degrees]

crsp <- wrsp["Response"], 2
rGain <- Mod(crsp)
rP <- Phase(crsp)
Open Well Response (ROJSTACZER): Harmonic Strain

(a) Gain

(b) Phase

Figure 6: The response of an open well to harmonic areal strain using the Rojstaczer model. In the phase curve, phase wrapping has been removed. Modified from Rojstaczer (1988, Fig. 3). Frequency is dimensionless, based on the well-depth $z$ and the diffusivity $D$. 

Dimensionless frequency, $Q = z^2 \omega / 2D$
5 Model Comparisons

5.1 Responses to strain

![Harmonic Strain Well Responses](image)

**Figure 7:** A comparison of well responses to harmonic strain. The phase of the water level is relative to $-180^\circ$ the phase of strain.
5.2 Responses to pressure head (all open)

Harmonic Pressure–head Well Responses (Open)

(a) Gain

Harmonic Pressure–head Well Responses (Open)

(b) Phase

Figure 8: A comparison of well responses to harmonic pressure-head, from Cooper et al. (1965); Hsieh et al. (1987); Liu et al. (1989) (all for unsealed).
References


