Package ‘lpcde’

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basis_vec

*Unit basis vector*

**Description**

Function to generate unit basis vector according to polynomial order and derivative order. This function returns unit vector that is the same size as the vector returned by `poly_base(x, p)`.

**Usage**

```r
basis_vec(x, p, mu)
```

**Arguments**

- `x`: sample input scalar or vector.
- `p`: polynomial order.
- `mu`: derivative order.

**Value**

Vector of appropriate length with ones corresponding to entries of order `mu`.

**Examples**

```r
basis_vec(x = 2, p = 5, mu = 1)
```

---

**coef.lpbwcede**

*Coef Method for Local Polynomial Density Bandwidth Selection*

**Description**

The coef method for local polynomial density bandwidth selection objects.

**Usage**

```r
## S3 method for class 'lpbwcede'
coef(object, ...)
```
**coef.lpcde**

**Arguments**
- **object**: Class "lpcde" object, obtained by calling `lpcde`.
- ... Other arguments.

**Value**
- **Estimates**: A matrix containing y_grid points and selected bandwidths.

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**See Also**
- `lpcde` for data-driven bandwidth selection.
- Supported methods: `coef.lpcde`, `print.lpcde`, `summary.lpcde`.

---

**Description**
The `coef` method for local polynomial conditional density objects.

**Usage**
```r
## S3 method for class 'lpcde'
coef(object, ...)
```

**Arguments**
- **object**: Class "lpcde" object, obtained by calling `lpcde`.
- ... Additional options.

**Details**
Coef Method for Local Polynomial Density Conditional Estimation and Inference

**Value**
- **outputs**: A matrix containing the estimates
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Xinwei Ma, University of California San Diego. <x1ma@ucsd.edu>.

See Also

lpcde for local polynomial conditional density estimation.
Supported methods: coef.lpcde, confint.lpcde, plot.lpcde, print.lpcde, summary.lpcde, vcov.lpcde

confint.lpcde

Confint Method for Local Polynomial Density Conditional Estimation and Inference

Description

The confint method for local polynomial conditional density objects.

Usage

## S3 method for class 'lpcde'
confint(
  object,
  parm = NULL,
  level = NULL,
  CIuniform = FALSE,
  CIsimul = 2000,
  alpha = 0.05,
  ...
)

Arguments

object Class "lpdensity" object, obtained by calling lpcde.
parm Integer, indicating which parameters are to be given confidence intervals.
level Numeric scalar between 0 and 1, the significance level for computing confidence intervals.
CIuniform TRUE or FALSE (default), plotting either pointwise confidence intervals (FALSE) or uniform confidence bands (TRUE).
CIsimul Positive integer, specifies the number of simulations used to construct critical values (default is 2000). This option is ignored if CIuniform=FALSE.
**lpbwced**: Data-driven Bandwidth Selection for Local Polynomial Conditional Density Estimators

**Description**

**lpbwced** implements the bandwidth selection methods for local polynomial based conditional density (and derivatives) estimation proposed and studied in Cattaneo, Chandak, Jansson and Ma (2021).

Companion command: **lpced** for estimation and robust bias-corrected inference.

Related Stata and R packages useful for nonparametric estimation and inference are available at [https://nppackages.github.io/](https://nppackages.github.io/).

**alpha**

Numeric scalar between 0 and 1, specifies the significance level for plotting confidence intervals/bands.

**Value**

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>A matrix containing grid points, estimates and confidence interval end points using p- and q-th order local polynomials as well as bias-corrected estimates and corresponding confidence intervals.</td>
</tr>
<tr>
<td>crit_val</td>
<td>the critical value used in computing the confidence interval end points.</td>
</tr>
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**See Also**

**lpced** for local polynomial conditional density estimation.

Usage

lpbwcde(
  y_data,
  x_data,
  x,
  y_grid = NULL,
  p = NULL,
  q = NULL,
  mu = NULL,
  nu = NULL,
  kernel_type = c("epanechnikov", "triangular", "uniform"),
  bw_type = c("mse-rot", "imse-rot")
)

Arguments

y_data Numeric matrix/data frame, the raw data of independent.

x_data Numeric matrix/data frame, the raw data of covariates.

x Numeric, specifies the evaluation point in the x-direction. Default is median of
the dataset.

y_grid Numeric, specifies the grid of evaluation points. When set to default, grid points
will be chosen as 0.05-0.95 percentiles of the data, with a step size of 0.05.

p Nonnegative integer, specifies the order of the local polynomial for Y used to
construct point estimates. (Default is 2.)

q Nonnegative integer, specifies the order of the local polynomial for X used to
construct point estimates. (Default is 1.)

mu Nonnegative integer, specifies the derivative with respect to Y of the distribu-
tion function to be estimated. 0 for the distribution function, 1 (default) for the
density function, etc.

nu Nonnegative integer, specifies the derivative with respect to X of the distribution
function to be estimated.

kernel_type String, specifies the kernel function, should be one of "triangular", "uniform"
or "epanechnikov".

bw_type String, specifies the method for data-driven bandwidth selection. This option
will be ignored if bw is provided. Implementable with "mse-rot" (default, mean
squared error-optimal bandwidth selected for each grid point)

Value

BW A matrix containing (1) y_grid (grid point), (2) bw (bandwidth)

opt A list containing options passed to the function.
lpdce

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Examples

# Generate a random sample
set.seed(42);
x_data = rnorm(2000)
y_data = rnorm(2000, mean=x_data)
x = 0

# Construct bandwidth
bw1 <- lpbwcde(y_data=y_data, x_data=x_data, x=x, bw_type="mse-rot")
summary(bw1)

# Display bandwidths for a subset of y_grid points
summary(bw1, y_grid=bw1$BW[2:5, "y_grid"])

Description
lpdce implements the local polynomial regression based conditional density (and derivatives). The estimator proposed in Chandak, Cattaneo, Jansson and Ma. Robust bias-corrected inference methods, both pointwise (confidence intervals) and uniform (confidence bands), are also implemented.

Usage
lpdce(
  x_data,
  y_data,
  y_grid = NULL,
  x = NULL,
  bw = NULL,
  p = NULL,
  q = NULL,
  p_RBC = NULL,
  q_RBC = NULL,
  mu = NULL,
  nu = NULL,
  rbc = TRUE,
  ng = NULL,
)
kernel_type = c("epanechnikov", "triangular", "uniform"),
bw_type = "mse-rot"
)

Arguments

x_data  Numeric matrix/data frame, the raw data of covariates.
y_data  Numeric matrix/data frame, the raw data of independent.
y_grid  Numeric, specifies the grid of evaluation points in the y-direction. When set to default, grid points will be chosen as 0.05-0.95 percentiles of the data, with a step size of 0.05 in y-direction.
x      Numeric, specifies the grid of evaluation points in the x-direction. When set to default, the evaluation point will be chosen as the median of the x data.
bw     Numeric, specifies the bandwidth used for estimation. Can be (1) a positive scalar (common bandwidth for all grid points); or (2) a positive numeric vector/matrix specifying bandwidths for each grid point (should be the same dimension as grid).
p      Nonnegative integer, specifies the order of the local polynomial for Y used to construct point estimates. (Default is 2.)
q      Nonnegative integer, specifies the order of the local polynomial for X used to construct point estimates. (Default is 1.)
p_RBC  Nonnegative integer, specifies the order of the local polynomial for Y used to construct bias-corrected point estimates. (Default is p+1.)
q_RBC  Nonnegative integer, specifies the order of the local polynomial for X used to construct bias-corrected point estimates. (Default is q+1.)
mu     Nonnegative integer, specifies the derivative with respect to Y of the distribution function to be estimated. 0 for the distribution function, 1 (default) for the density function, etc.
u       Nonnegative integer, specifies the derivative with respect to X of the distribution function to be estimated. Default value is 0.
rbc    Boolean. TRUE (default) for rbc calculations, required for valid uniform inference.
ng     int. number of grid points to be used. generates evenly space points over the support of the data.
kernel_type String, specifies the kernel function, should be one of "triangular", "uniform", and "epanechnikov"(default).
bw_type String, specifies the method for data-driven bandwidth selection. This option will be ignored if bw is provided. Implementable with "mse-dpi" (default, mean squared error-optimal bandwidth selected for each grid point)

Details

Bias correction is only used for the construction of confidence intervals/bands, but not for point estimation. The point estimates, denoted by est, are constructed using local polynomial estimates of
order $p$ and $q$, while the centering of the confidence intervals/bands, denoted by \( \text{est\_RBC} \), are constructed using local polynomial estimates of order $p\text{\_RBC}$ and $q\text{\_RBC}$. The confidence intervals/bands take the form: $[\text{est\_RBC} - \text{cv} \times \text{SE(\text{est\_RBC})}, \text{est\_RBC} + \text{cv} \times \text{SE(\text{est\_RBC})}]$, where \( \text{cv} \) denotes the appropriate critical value and \( \text{SE(\text{est\_RBC})} \) denotes an standard error estimate for the centering of the confidence interval/band. As a result, the confidence intervals/bands may not be centered at the point estimates because they have been bias-corrected. Setting $p\text{\_RBC}$ equal to $p$ and $q\text{\_RBC}$ to $q$, results on centered at the point estimate confidence intervals/bands, but requires undersmoothing for valid inference (i.e., (I)MSE-optimal bandwidth for the density point estimator cannot be used). Hence the bandwidth would need to be specified manually when $q=p$, and the point estimates will not be (I)MSE optimal. See Cattaneo, Jansson and Ma (2020a, 2020b) for details, and also Calonico, Cattaneo, and Farrell (2018, 2020) for robust bias correction methods.

Sometimes the density point estimates may lie outside of the confidence intervals/bands, which can happen if the underlying distribution exhibits high curvature at some evaluation point(s). One possible solution in this case is to increase the polynomial order $p$ or to employ a smaller bandwidth.

### Value

**Estimate**
A matrix containing (1) grid (grid points),
(2) bw (bandwidths),
(3) est (point estimates with $p$-th and $q$-th order local polynomial),
(4) est\_RBC (point estimates with $p\text{\_RBC}$-th and $q\text{\_RBC}$-th order local polynomial),
(5) se (standard error corresponding to est). (6) se\_RBC (standard error corresponding to est\_RBC).

**CovMat**
The variance-covariance matrix corresponding to est.

**opt**
A list containing options passed to the function.

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Supported methods: coef.lpcde, confint.lpcde, plot.lpcde, print.lpcde, summary.lpcde, vcov.lpcde

---

**plot.lpcde**

*Plot Method for Local Polynomial Density Conditional Estimation and Inference*

### Description

The plot method for local polynomial density objects. A standard ggplot2 object is returned, hence can be used for further customization.
Usage

```r
## S3 method for class 'lpcde'
plot(
    ..., 
    alpha = NULL,
    type = NULL,
    lty = NULL,
    lwd = NULL,
    lcol = NULL,
    pty = NULL,
    pwd = NULL,
    pcol = NULL,
    y_grid = NULL,
    CItype = NULL,
    CIuniform = FALSE,
    CIsimul = 2000,
    CIshade = NULL,
    CIcol = NULL,
    title = NULL,
    xlabel = NULL,
    ylabel = NULL,
    legendTitle = NULL,
    legendGroups = NULL
)
```

Arguments

... Class "lpcde" object, obtained from calling `lpcde`.

alpha Numeric scalar between 0 and 1, specifies the significance level for plotting confidence intervals/bands.

type String, one of "line" (default), "points" and "both", specifies how the point estimates are plotted. If more than one is provided, they will be applied to each data series accordingly.

lty Line type for point estimates, only effective if type is "line" or "both". 1 for solid line, 2 for dashed line, 3 for dotted line. For other options, see the instructions for `ggplot2`. If more than one is provided, they will be applied to each data series accordingly.

lwd Line width for point estimates, only effective if type is "line" or "both". Should be strictly positive. For other options, see the instructions for `ggplot2`. If more than one is provided, they will be applied to each data series accordingly.

lcol Line color for point estimates, only effective if type is "line" or "both". 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for `ggplot2`. If more than one is provided, they will be applied to each data series accordingly.

pty Scatter plot type for point estimates, only effective if type is "points" or "both". For options, see the instructions for `ggplot2`. If more than one is provided, they will be applied to each data series accordingly.
plot.lpcde

pwd
Scatter plot size for point estimates, only effective if type is "points" or "both". Should be strictly positive. If more than one is provided, they will be applied to each data series accordingly.

pcol
Scatter plot color for point estimates, only effective if type is "points" or "both". 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for ggplot2. If more than one is provided, they will be applied to each data series accordingly.

y_grid
Numeric vector, specifies a subset of grid points to plot point estimates. This option is effective only if type is "points" or "both"; or if CItype is "ebar" or "all".

CItype
String, one of "region" (shaded region, default), "line" (dashed lines), "ebar" (error bars), "all" (all of the previous) or "none" (no confidence region), how the confidence region should be plotted. If more than one is provided, they will be applied to each data series accordingly.

CIuniform
TRUE or FALSE (default), plotting either pointwise confidence intervals (FALSE) or uniform confidence bands (TRUE).

CIsimul
Positive integer, specifies the number of simulations used to construct critical values (default is 2000). This option is ignored if CIuniform=FALSE.

CIshade
Numeric, specifies the opaqueness of the confidence region, should be between 0 (transparent) and 1. Default is 0.2. If more than one is provided, they will be applied to each data series accordingly.

CIcol
Color of the confidence region. 1 for black, 2 for red, 3 for green, 4 for blue. For other options, see the instructions for ggplot2. If more than one is provided, they will be applied to each data series accordingly.

title, xlabel, ylabel
Strings, specifies the title of the plot and labels for the x- and y-axis.

legendTitle
String, specifies the legend title.

legendGroups
String vector, specifies the group names used in legend.

Value

Figure
A standard ggplot2 object is returned, hence can be used for further customization.

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See Also
lpcde for local polynomial density estimation. Supported methods: coef.lpcde, confint.lpcde, plot.lpcde, print.lpcde, summary.lpcde, vcov.lpcde
**poly_base**  
*Polynomial basis vector expansion*

**Description**

Generate polynomial basis vector up to order p. has multivariate functionality as described in the main paper normalized by factorials in denominator. NOTE: currently works only up to 4th degree polynomial expansion for multivariate x.

**Usage**

```r
poly_base(x, p)
```

**Arguments**

- `x` a number or vector.
- `p` a number (integer).

**Value**

polynomial basis of x up to degree p.

**Examples**

```r
poly_base(x = 2, p = 5)
```

---

**print.lpbwcde**  
*Print Method for Local Polynomial Conditional Density Bandwidth Selection*

**Description**

The print method for local polynomial conditional density bandwidth selection objects.

**Usage**

```r
## S3 method for class 'lpbwcden'
print(x, ...)
```

**Arguments**

- `x` Class "lpbwcden" object, obtained by calling `lpbwcden`.
- `...` Other arguments.
**print.lpcde**

**Value**

Display output  A list of specified options provided to the function.

**Author(s)**

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**See Also**

lpbwcd for data-driven bandwidth selection.

Supported methods: coef.lpbwcd, print.lpbwcd, summary.lpbwcd.

---

**print.lpcde**

*Print Method for Local Polynomial Conditional Density Estimation and Inference*

**Description**

The print method for local polynomial conditional density objects.

**Usage**

```r
## S3 method for class 'lpcde'
print(x, ...)
```

**Arguments**

- `x`  Class "lpcde" object, obtained from calling `lpcde`.
- `...`  Additional options.

**Value**

Display output  summary of inputs to `lpcde`

**Author(s)**

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Xinwei Ma, University of California San Diego. <x1ma@ucsd.edu>.
See Also

lpdce for local polynomial conditional density estimation. Supported methods: coef.lpdce, confint.lpdce, plot.lpdce, print.lpdce, summary.lpdce, vcov.lpdce

summary.lpbwcdedefinition

Summary Method for Local Polynomial Conditional Density Bandwidth Selection

Description

The summary method for local polynomial conditional density bandwidth selection objects.

Usage

## S3 method for class 'lpbwcdedefinition'
summary(object, ...)

Arguments

object Class "lpbwcdedefinition" object, obtained by calling lpbwcdedefinition.
...
Additional options, including (i) y_grid specifies a subset of y_grid points to display the bandwidth; (ii) gridIndex specifies the indices of y_grid points to display the bandwidth.

Value

Display output A list of specified options and a matrix of grid points, bandwidth, and effective sample size.

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See Also

lpbwcdedefinition for data-driven bandwidth selection.

Supported methods: coef.lpbwcdedefinition, print.lpbwcdedefinition, summary.lpbwcdedefinition.
Examples

\begin{verbatim}
n=100
x_data = as.matrix(rnorm(n, mean=0, sd=1))
y_data = as.matrix(rnorm(n, mean=0, sd=1))
y_grid = stats::quantile(y_data, seq(from=0.1, to=0.9, by=0.1))
# bandwidth selection
y_grid = stats::quantile(y_data, seq(from=0.1, to=0.9, by=0.1))
model2 = lpcde::lpbwcde(y_data=y_data, x_data=x_data, x=0, y_grid = y_grid, bw_type = "mse-rot")
summary(model2)
\end{verbatim}

---

**summary.lpcde**

*Summary Method for Local Polynomial Density Conditional Estimation and Inference*

**Description**

The summary method for local polynomial conditional density objects.

**Usage**

```r
## S3 method for class 'lpcde'
summary(object, ...)
```

**Arguments**

- **object**: Class "lpcde" object, obtained from calling `lpcde`.
- **...**: Additional options, including (i) `y_grid` specifies a subset of grid points in y-directions to display results; (ii) `gridIndex` specifies the indices of grid points to display results; (iii) `alpha` specifies the significance level; (iv) `CIuniform` specifies whether displaying pointwise confidence intervals (FALSE, default) or the uniform confidence band (TRUE); (v) `CIsimul` specifies the number of simulations used to construct critical values (default is 2000).

**Value**

- **Display output**: A list of specified options and a matrix of grid points and estimates.

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Xinwei Ma, University of California San Diego. `<x1ma@ucsd.edu>`.
See Also

{lpcde} for local polynomial conditional density estimation. Supported methods: {coef.lpcde},
{confint.lpcde}, {plot.lpcde}, {print.lpcde}, {summary.lpcde}, {vcov.lpcde}

Examples

```r
n = 100
x_data = as.matrix(rnorm(n, mean=0, sd=1))
y_data = as.matrix(rnorm(n, mean=0, sd=1))
y_grid = stats::quantile(y_data, seq(from=0.1, to=0.9, by=0.1))
# density estimation
model1 = lpcde::lpcde(x_data=x_data, y_data=y_data, y_grid=y_grid, x=0, bw=0.5)
summary(model1)
```

---

### vcov.lpcde

<table>
<thead>
<tr>
<th>Variance-Covariance</th>
</tr>
</thead>
</table>

**Description**

The vcov method for local polynomial conditional density objects.

**Usage**

```r
## S3 method for class 'lpcde'
vcov(object, ...)
```

**Arguments**

- **object**: Class "lpdensity" object, obtained by calling {lpcde}.
- **...**: Additional options.

**Details**

Vcov Method for Local Polynomial Density Conditional Estimation and Inference

**Value**

- **stdErr**: A matrix containing grid points and standard errors using p- and q-th order local polynomials.
- **CovMat**: The variance-covariance matrix corresponding to est.
- **CovMat_RBC**: The variance-covariance matrix corresponding to est_RBC.
vcov.lpcde

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See Also

lpcde for local polynomial conditional density estimation.
Supported methods: plot.lpcde, print.lpcde, summary.lpcde,
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