Uniform Lightness Scales

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Introduction

A *Uniform Lightness Scale* is a reparameterization of luminance factor, so that equal numerical steps in lightness produce equal perceptual steps. The goal of this *munsellinterpol* vignette is to treat Munsell Value (V) as a Uniform Lightness Scale, compare the different definitions of Value, and compare Munsell Value with CIE Lightness. Featured functions in this vignette are: `VfromY()` and `YfromV()`.

```r
library( munsellinterpol )
```

Munsell Value and Luminance Factor

In the search for a perceptually uniform Munsell Value scale, there have been many refinements: from [Priest et al., 1920], to [Munsell et al., 1933], to [Newhall et al., 1943] to [D1535-08, 2008]. For definitions of these, see the man page for `VfromY()`.

```r
par( omi=c(0,0,0,0), mai=c(0.5,0.5,0.1,0.1) )
plot( c(0,100), c(0,10), type='n', xlab='', ylab='', las=1, tcl=0,
     lab=c(10,8,7), mgp=c(3,0.25,0) )
title( xlab='Y', line=1.5 ) ; title( ylab='Value', line=1.5 )
grid( lty=1 ) ; abline( h=0, v=0 )
V = seq( 0, 10, by=0.125 )
color = unlist( list(ASTM='black',OSA='black',MgO='black',Munsell='red',Priest='blue') )
for( w in names(color) )
    lines( YfromV(V,w), V, col=color[w], lty=ifelse(w=='MgO',2,1), lwd=0.75 )
lty = ifelse( names(color)=='MgO', 2, 1 )
legend( "bottomright", names(color), bty='n', lty=lty, lwd=1.5, col=color, inset=0.1 )
```
Figure 1: Value V vs. Luminance Factor Y

Note that for MgO, $Y=102.568$ when $V=10$. This luminance factor for this curve is relative to MgO instead of the perfect reflecting diffuser.

The curves for ASTM and OSA quintics are indistinguishable at this scale. Plot the difference between them.

```r
par( omi=c(0,0,0,0), mai=c(0.5,1,0.1,0.1) )
Y = seq( 0, 100, by=0.5 )
delta = VfromY(Y,'OSA') - VfromY(Y,'ASTM')
plot( range(Y), range(delta), type='n', xlab='', ylab='', las=1, tcl=0,
     lab=c(10,8,7), mgp=c(3,0.25,0) )
title( xlab='Y', line=1.5 ) ; title( ylab='{OSA Value} - {ASTM Value}', line=3 )
grid( lty=1 ) ; abline( h=0, v=0 )
lines( Y, delta, lwd=0.75 )
```
This difference is negligible.

**CIE Lightness and Munsell Value**

Most modern work uses the CIE Lightness function, which is actually a simplified version of Munsell Value, but on different domains. The linear domain for $Y$ is $[0,1]$ and the perceptually uniform domain for Lightness is $[0,100]$. Here is the definition:

```r
Lightness_from_linear <- function( Y ) {
    ifelse( Y < (24/116)^3, (116/12)^3 * Y, 116*Y^(1/3) - 16 )
}
```

And here is the comparison plot, properly scaled for the difference in domains.

```r
par( omi=c(0,0,0,0), mai=c(0.5,0.75,0.1,0.1) )
Y = (0:100)/100
L = Lightness_from_linear( Y )
plot( range(Y), range(L), type='n', xlab='', ylab='', las=1, tcl=0,
     lab=c(10,8,7), mgp=c(3,0.25,0) )
title( xlab='Y (luminance factor)', line=1.5 ); title( ylab='Lightness', line=2 )
grid( lty=1 ); abline( h=0, v=0 )
lines( Y, L, lwd=0.75 )
V = VfromY( 100 * Y, 'ASTM' )
lines( Y, 10*V, lty=2 )
legend( "bottomright", c("Lightness (CIE)","10*Value (ASTM)") , lty=c(1,2),
        bty='n', inset=0.1 )
```
The agreement is very good. Let’s examine the ratio.

```
par( omi=c(0,0,0,0), mai=c(0.5,0.75,0.1,0.1) )
quotient = L / V
plot( range(Y), range(quotient,na.rm=T), type='n', xlab='', ylab='', las=1, tcl=0,
      lab=c(10,8,7), mgp=c(3,0.25,0) )
title( xlab='Y (luminance factor)', line=1.5 )
title( ylab='Lightness / Value', line=3 )
grid( lty=1 ); abline( h=0, v=0 )
lines( Y, quotient )
```
So Value ≃ Lightness/10 is a good approximation.

References


Appendix

This document was prepared July 19, 2019  with the following configuration:

- R version 3.6.1 (2019-07-05), x86_64-w64-mingw32
- Running under: Windows 7 x64 (build 7601) Service Pack 1
- Matrix products: default
- Base packages: base, datasets, grDevices, graphics, methods, stats, utils
- Other packages: flextable 0.5.4, knitr 1.22, munsellinterpol 2.5-1, spacesRGB 1.2-2, spacesXYZ 1.0-4
- Loaded via a namespace (and not attached): R6 2.4.0, Rcpp 1.0.1, base64enc 0.1-3, compiler 3.6.1, data.table 1.12.2, digest 0.6.18, evaluate 0.13, gdtools 0.1.8, highr 0.8, htmltools 0.3.6, magrittr 1.5, microbenchmark 1.4-6, officer 0.3.4, rlang 0.3.4,
rmarkdown 1.12, rootSolve 1.7, stringi 1.4.3, stringr 1.4.0, tools 3.6.1, uuid 0.1-2, xfun 0.7, xml2 1.2.0, yaml 2.2.0, zip 2.0.1