Package ‘partialCI’

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Title    Partial Cointegration
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Description
A collection of time series is partially cointegrated if a linear combination of these time series can be found so that the residual spread is partially autoregressive - meaning that it can be represented as a sum of an autoregressive series and a random walk. This concept is useful in modeling certain sets of financial time series and beyond, as it allows for the spread to contain transient and permanent components alike. Partial cointegration has been introduced by Clegg and Krauss (2017) <doi:10.1080/14697688.2017.1370122>, along with a large-scale empirical application to financial market data. The partialCI package comprises estimation, testing, and simulation routines for partial cointegration models in state space. Clegg et al. (2017) <https://hdl.handle.net/10419/150014> provide an in-depth discussion of the package functionality as well as illustrating examples in the fields of finance and macroeconomics.

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Description

A collection of time series is said to be partially cointegrated if they have a linear combination that is partially autoregressive, e.g., that can be represented as a sum of an autoregressive series and a random walk. This may be useful in modeling certain sets of financial time series.

To find the partially cointegrated model that best fits two series X and Y, use:

```r
> fit.pci(Y, X)
```

An interface to Yahoo! Finance permits you to find the best fits for two particular stocks of interest:

```r
> yfit.pci("RDS-B", "RDS-A")
```

Fitted values for PCI model

\[
Y[t] = X[t]
\]

\[
M[t] = \rho M[t-1] + \varepsilon_M[t], \quad \varepsilon_M[t] \sim N(0, \sigma_M^2)
\]

\[
R[t] = R[t-1] + \varepsilon_R[t], \quad \varepsilon_R[t] \sim N(0, \sigma_R^2)
\]

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Std. Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta_RDS-A</td>
<td>1.0427</td>
</tr>
<tr>
<td>rho</td>
<td>0.1770</td>
</tr>
<tr>
<td>sigma_M</td>
<td>0.0825</td>
</tr>
<tr>
<td>sigma_R</td>
<td>0.1200</td>
</tr>
</tbody>
</table>

\[-LL = -255.97, \quad R^2[MR] = 0.446\]

This example was run on 29/9/2018. RDS-A and RDS-B are two classes of shares offered by Royal Dutch Shell that differ slightly in aspects of their tax treatment. The above fit shows that the spread between the two shares is mostly mean-reverting but that it contains a small random walk.
component. The mean-reverting component accounts for 44.6% of the variance of the daily returns. The value of 0.1770 for \( \rho \) corresponds to a half-life of mean reversion of about 4 trading days.

To test the goodness of fit, the `test.pci` function can be used:

```r
> h <- yfit.pci("RDS-B", "RDS-A")
> test.pci(Y=h)
```

R test of [RW or CI(1)] vs Almost PCI(1) (joint penalty, wilk)

data:  RDS-B / RDS-A

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Statistic</th>
<th>p-value</th>
<th>alpha</th>
<th>alpha_bonf</th>
<th>alpha_holm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Walk</td>
<td>-8.30</td>
<td>0.000</td>
<td>0.050</td>
<td>0.025</td>
<td>0.050</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-7.68</td>
<td>0.000</td>
<td>0.050</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

The `test.pci` function tests each of two different null hypotheses: (a) the residual series is purely a random walk, and (b) the residual series is purely autoregressive. Only if both null hypothesis can be rejected a time series is classified as partially cointegrated. The two p-values of 0.000 indicate that RDS-A and RDS-B are indeed partially cointegrated. This still holds true if a Bonferroni corrected significance level (alpha_bonf) is considered.

The partialCI package also contains a function for searching for hedging portfolios. Given a particular stock (or time series), a search can be conducted to find the set of stocks that best replicate the target stock. In the following example, a hedge is sought for SPY using sector ETF’s.

The partialCI package also contains a function for searching for hedging portfolios. Given a particular stock (or time series), a search can be conducted to find the set of stocks that best replicate the target stock. In the following example, a hedge is sought for SPY using sector ETF’s.

```
# Partially Cointegrated Model with Sector ETFs

1307.48 -2.9362  0.0531  0.2847  0.9722  0.8362  XLK | 3.1505
747.43  -3.6282  0.0266  0.0353  0.9112  0.5960  XLI | 1.8554  1.6029
548.63  -5.9474  0.0026  0.0007  0.6764  0.4098  XLY | 1.3450  1.2436  0.6750

Fitted values for PCI model

\[ Y[t] = X[t] \]
\[ M[t] = \rho \times M[t-1] + \text{eps}_M[t], \text{eps}_M[t] \sim N(0, \sigma_M^2) \]
\[ R[t] = R[t-1] + \text{eps}_R[t], \text{eps}_R[t] \sim N(0, \sigma_R^2) \]

Estimate Std. Err
beta_XLK  1.3450  0.0419
beta_XLI  1.2436  0.0352
beta_XLY  0.6750  0.0310
rho       0.6764  0.1502
sigma_M   0.2323  0.0429
sigma_R   0.3045  0.0331

-LL = 548.63, R^2[MR] = 0.410
The top table displays the quality of the fit that is found as each new factor is added to the fit. The best fit consisting of only one factor is found by using XLK (the technology sector). The negative log likelihood score for this model is 1307.48. However, the random walk hypothesis (p[rw]) cannot be rejected at the 5% level. When adding XLI (the industrial sector), the negative log likelihood drops to 747.43 and the random walk and the purely autoregressive hypothesis can not be rejected at the 5% level if we account for multiple testing. The best overall fit is obtained by also adding XLY (consumer discretionary) to the hedging portfolio. The final fit is

\[
\text{SPY} = 1.3450 \times \text{XLK} + 1.2436 \times \text{XLI} + 0.6750 \times \text{XLY}
\]

For this fit, the proportion of variance attributable to the mean reverting component is 41.0%. In addition, for the best fit we can reject the random walk and the purely autoregressive hypothesis at the 5% level. The later holds even if we account for multiple testing.

Please feel free to contact us if you have questions or suggestions.

Jonas Rende, Matthew Clegg and Christopher Krauss
Oktober 02, 2018

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**See Also**

fit.pci yfit.pci test.pci hedge.pci yhedge.pci

---

**fit.pci**

*Fits the partial cointegration model to a collection of time series*

**Description**

Fits the partial cointegration model to a collection of time series

**Usage**

```r
fit.pci(Y, X,  
  pci_opt_method = c("jp", "twostep"),  
  par_model = c("par", "ar1", "rw"),  
  lambda = 0,  
  robust = FALSE, nu = 5)
```
Arguments

Y  The time series that is to be modeled. A plain or zoo vector of length n.
X  A (possibly zoo) matrix of dimensions n x k. If k=1, then this may be a plain or zoo vector.

pci_opt_method Specifies the method that will be used for finding the best fitting model. One of the following:
  • "jp" The joint-penalty method (see below)
  • "twostep" The two-step method (see below)
Default: jp

par_model The model used for the residual series. One of the following:
  • "par" The residuals are assumed to follow a partially autoregressive model.
  • "ar1" The residuals are assumed to be autoregressive of order one.
  • "rw" The residuals are assumed to follow a random walk.
Default: par

lambda The penalty parameter to be used in the joint-penalty (jp) estimation method.
Default: 0.

robust If TRUE, then the residuals are assumed to follow a t-distribution with nu degrees of freedom. Default: FALSE.

nu The degrees-of-freedom parameter to be used in robust estimation. Default: 5.

Details

The partial cointegration model is given by the equations:

\[ Y_t = \beta_1 * X_{t,1} + beta_2 * X_{t,2} + ... + beta_k * X_{t,k} + M_t + R_t \]

\[ M_t = \rho M_{t-1} + \epsilon_{M,t} \]

\[ R_t = R_{t-1} + \epsilon_{R,t} \]

\[-1 < \rho < 1\]

\[ \epsilon_{M,t} \sim N(0, \sigma^2_M) \]

\[ \epsilon_{R,t} \sim N(0, \sigma^2_R) \]

Given the input series Y and X, this function searches for the parameter values beta, rho that give the best fit of this model when using a Kalman filter.

If pci_opt_method is twostep, then a two-step procedure is used. In the first step, a linear regression is performed of X on Y to determine the parameter beta. From this regression, a series of residuals is determined. In the second step, a model is fit to the residual series. If par_model is par, then a partially autoregressive model is fit to the residual series. If par_model is ar1, then an autoregressive model is fit to the residual series. If par_model is rw then a random walk model is fit to the residual series. Note that if pci_opt_method is twostep and par_model is ar1, then this reduces to the Engle-Granger two-step procedure.

If pci_opt_method is jp, then the joint-penalty procedure is used. In this method, the parameter beta are estimated jointly with the parameter rho using a gradient-search optimization function.
In addition, a penalty value of $\lambda \cdot \sigma_R^2$ is added to the Kalman filter likelihood score when searching for the optimum solution. By choosing a positive value for $\lambda$, you can drive the solution towards a value that places greater emphasis on the mean-reverting component.

Because the joint-penalty method uses gradient search, the final parameter values found are dependent upon the starting point. There is no guarantee that a global optimum will be found. However, the joint-penalty method chooses several different starting points, so as to increase the chance of finding a global optimum. One of the chosen starting points consists of the parameters found through the two-step procedure. Because of this, the joint-penalty method is guaranteed to find parameter values which give a likelihood score at least as good as those found using the two-step procedure. Sometimes the improvement over the two-step procedure is substantial.

Value

An object of class `pci.fit` containing the fit that was found. The following components may be of interest:

- `beta`: The vector of weights
- `beta.se`: The standard errors of the components of `beta`
- `rho`: The estimated coefficient of mean reversion
- `rho.se`: The standard error of `rho`
- `negloglik`: The negative of the log likelihood
- `pvmr`: The proportion of variance attributable to mean reversion

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Christopher Krauss <christopher.krauss@fau.de>
Jonas Rende <jonas.rende@fau.de>

References


See Also

- `egcm`: Engle-Granger cointegration model
- `partialAR`: Partially autoregressive models
**Examples**

```r
# Should be DIRECTLY executable !! ----
#-- Define data, use random,
#-- or do help(data=index) for the standard data sets.

YX <- rpci(n=1000, beta=c(2,3,4), sigma_C=c(1,1,1), rho=0.9, sigma_M=0.1, sigma_R=0.2)
fit.pci(YX[,1], YX[,2:ncol(YX)])
```

---

**hedge.pci**

*Searches for a partially cointegrated hedge for a given time series*

**Description**

Given a time series and a collection of possible factors, finds a subset of the factors that provides the best fit to the given time series using the partially cointegrated model.

**Usage**

```r
hedge.pci(Y, X,
  maxfact = 10,
  lambda = 0,
  use.multicore = TRUE,
  minimum.stepsize = 0,
  verbose = TRUE,
  exclude.cols = c(),
  search_type = c("lasso", "full", "limited"),
  pci_opt_method=c("jp", "twostep"),
  ...
)
```

**Arguments**

- **Y**  An N x 1 column vector or data.frame, representing the series that is to be hedged.
- **X**  An N x L data.frame, where each column represents a possible factor to be used in a partially cointegrated fit.
- **maxfact**  The maximum number of columns from X that will be selected for modeling Y. Default: 10
- **lambda**  A penalty to be applied to the random walk portion of the partialAR model. A positive value for lambda will drive the model towards a solution with a smaller random walk component. Default: 0
- **use.multicore**  If TRUE, parallel processing will be used to improve performance. See parallel:mclapply Default: TRUE
- **minimum.stepsize**  If this is non-NA, then the search stops if an improvement cannot be found of at least this much. Default: 0
verbose

If TRUE, then detailed information is printed about the execution. Default: TRUE

exclude.cols

A list of column indexes specifying columns from \( X \) which should be excluded from consideration. Alternatively, the list of excluded columns may be given as a list of strings, in which case they are interpreted as column names. Default: c()

search_type

If "lasso", then the lasso algorithm (see \texttt{glmnet}) is used to identify the factors that provide the best linear fit to the target sequence. If "full", then a greedy algorithm is used to search for factors to be used in the hedge. At each step, all possible additions to the portfolio are considered, and the best one is chosen for inclusion. If "limited", then at each iteration, a preliminary screening step is performed to identify the securities with the highest correlations to the residuals of the currently selected portfolio. The top securities from this list are then checked for whether they would improve the portfolio, and the best one included.

pci_opt_method

Specifies the method that will be used for finding the best fitting model. One of the following:

- "jp" The joint-penalty method (see \texttt{fit.pci})
- "twostep" The two-step method (see \texttt{fit.pci})

Default: \texttt{jp}

Other parameters to be passed onto the search function. See the source code.

Details

The hedge is constructed by searching for column indices \( i_1, i_2, \ldots, i_N \) from among the columns of \( X \) which yield the best fit to the partially cointegrated fit:

\[
Y_t = \beta_1 * X_{t,i_1} + \beta_2 * X_{t,i_2} + \ldots + \beta_N * X_{t,i_N} + M_t + R_t
\]

\[
M_t = \rho M_{t-1} + \epsilon_{M,t}
\]

\[
R_t = R_{t-1} + \epsilon_{R,t}
\]

\[-1 < \rho < 1 \]

\[
\epsilon_{M,t} \sim \mathcal{N}(0, \sigma_M^2)
\]

\[
\epsilon_{R,t} \sim \mathcal{N}(0, \sigma_R^2)
\]

if \texttt{search_type="lasso"} is specified, then the lasso algorithm (see \texttt{glmnet}) is used to search for the factors that give the best linear fit to the target sequence \( Y \). Having determined the list of factors, the cutoff point is determined based on successive improvements to the likelihood score of the fitted model.

Otherwise, a greedy algorithm (\texttt{search_type="full"}) or a modified greedy algorithm (\texttt{search_type="limited"}) is used. This proceeds by searching through all columns of \( X \) (except those listed in \texttt{exclude.cols}) to find the column that gives the best fit to \( Y \), as determined by the likelihood score of the partially cointegrated model. This column becomes the initial hedging portfolio. Having selected columns \( i_1, i_2, \ldots, i_K \), the next column is found by searching through all remaining columns of \( X \) (except those listed in \texttt{exclude.cols}) for the column which gives the best improvement to the partially cointegrated fit. However, if the best improvement is less than \texttt{minimum_stepsize}, or if \texttt{maxfact} columns have already been added, then the search terminates.
In the case of the modified greedy algorithm (search.type="limited"), a preprocessing step is used at the beginning of each iteration. In this preprocessing step, the correlation is computed between each unused column of \(X\) and the residual series of the currently computed best fit. The top \(B\) choices are then considered for inclusion in the portfolio, where \(B\) is a branching factor. The branching factor can be controlled by setting the value of the optional parameter max.branch. Its default value is 10.

The lasso algorithm is by far the fastest, followed by the limited greedy search. So, the best strategy is probably to start by using the lasso. If it fails to produce acceptable results, then move on to the limited greedy algorithm and finally the full search.

Value

Returns an S3 object of class \(pci.hedge\) containing the following fields:

- \(pci\): The best partially cointegrated fit that was found
- \(indexes\): The indexes of the columns from \(X\) that were selected
- \(index_names\): The names of the columns from \(X\) that were selected

Author(s)

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Christopher Krauss <christopher.krauss@fau.de>
Jonas Rende <jonas.rende@fau.de>

See Also

- \(fit.pci\) Fitting of partially cointegrated models
- \(partialAR\) Partially autoregressive models
- \(egcm\) Engle-Granger cointegration model

Examples

```r
# Should be DIRECTLY executable !!! ----
## --> Define data, use random,
##---or do help(data=index) for the standard data sets.

## Not run: YX <- rpci(n=1000, beta=c(2,3,4,5,6),
##     sigma.C=c(0.1,0.1,0.1,0.1,0.1), rho=0.9, sigma.M=1, sigma.R=1)
YX <- cbind(YX, matrix(rnorm(5000), ncol=5))
hedge.pci(YX[,1], YX[,2:ncol(YX)])
hedge.pci(YX[,1], YX[,2:ncol(YX)])
## End(Not run)
```
likelihood_ratio.pci  Computes the likelihood ratio of the partially cointegrated model vs the null model

Description

Computes the likelihood ratio of the partially cointegrated model vs the null model

Usage

likelihood_ratio.pci(Y, X,
  robust = FALSE,
  null_model = c("rw", "ar1"),
  pci_opt_method = c("jp", "twostep"),
  nu = 5)

Arguments

Y  The time series that is to be modeled. A plain or zoo vector of length \( n \).
X  A (possibly zoo) matrix of dimensions \( n \times k \). If \( k=1 \), then this may be a plain or zoo vector.
robust  If TRUE, then the residuals are assumed to follow a t-distribution with \( nu \) degrees of freedom. Default: FALSE.
null_model  This specifies the model that is assumed under the null hypothesis.
  • rw  Random walk. Assumes sigma_M = rho = 0. Default.
  • ar1  Autoregressive of order one. Assumes sigma_R=0.
pci_opt_method  Method to be used for fitting \( Y \) to \( X \).
  • jp  The coefficients of \( Y \) are jointly optimized with the parameters of the AAR fit of the residuals. Default.
  • twostep  A modified Engle-Granger procedure is used, where the coefficients of \( Y \) are first estimated, and then an AAR model is fit to the residuals.
nu  If robust is TRUE, then this is the degrees of freedom parameter used in fitting the t-distribution. Default: 5.

Details

First searches for the optimal fit under the null model, and computes the log of the likelihood score of this fit. Then, searches for the optimal fit under the full model, and computes the log of the likelihood score of this fit. Returns the difference of the two likelihood scores. Since the null model is nested in the full model, the log likelihood ratio score is guaranteed to be negative.

Value

The log of the ratio of the likelihoods of the two models.
Author(s)
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Christopher Krauss <christopher.krauss@fau.de>
Jonas Rende <jonas.rende@fau.de>

References

See Also
fit.pci Fitting partially cointegrated models

Examples
YX <- rpci(n=1000, beta=c(2,3,4), sigma_C=c(1,1,1), rho=0.9, sigma_M=0.1, sigma_R=0.2)
likelihood_ratio.pci(YX[,1], YX[,2:ncol(YX)])

loglik.pci

Computes the log likelihood of a partially cointegrated model

Description
Computes the log likelihood of a partially cointegrated model

Usage
loglik.pci(Y, X, beta, rho, sigma_M, sigma_R,
M0 = 0, R0 = 0,
calc_method = c("css", "kfas", "ss", "sst", "csst"),
nu = pci.nu.default())

Arguments
Y The time series that is to be modeled. A plain or zoo vector of length n.
X A (possibly zoo) matrix of dimensions n x k. If k=1, then this may be a plain or zoo vector.
beta A vector of length k representing the weightings to be given to the components of X.
rho The coefficient of mean reversion.
sigma_M The standard deviation of the innovations of the mean-reverting component of the model.
sigma_R The standard deviation of the innovations of the random walk component of the model.
$\mathbf{M0}$  The initial value of the mean-reverting component. Default = 0.

$\mathbf{R0}$  The initial value of the random walk component. Default = 0.

$\text{calc\_method}$  Specifies the Kalman filter implementation that will be used for computing the likelihood score:

- "ss" Steady-state Kalman filter
- "css" C++ implementation of steady-state Kalman filter
- "kfks" Kalman filter implementation of the KFAS package
- "sst" Steady-state Kalman filter using t-distributed innovations
- "csst" C++ implementation of steady-state Kalman filter using t-distributed innovations

Default: css

$\nu$  The degrees-of-freedom parameter to be used if $\text{calc\_method}$ is "sst" or "csst".

Details

The partial cointegration model is given by the equations:

\[
Y_t = \beta_1 X_{t,1} + \beta_2 X_{t,2} + \ldots + \beta_k X_{t,k} + M_t + R_t
\]

\[
M_t = \rho M_{t-1} + \epsilon_{M,t}
\]

\[
R_t = R_{t-1} + \epsilon_{R,t}
\]

\[-1 < \rho < 1\]

\[
\epsilon_{M,t} \sim N(0, \sigma_M^2)
\]

\[
\epsilon_{R,t} \sim N(0, \sigma_R^2)
\]

Given the input series $Y$ and $X$, and given the parameter values $\beta$, $\rho$, $\mathbf{M0}$ and $\mathbf{R0}$, the innovations $\epsilon_{M,t}$ and $\epsilon_{R,t}$ are calculated using a Kalman filter. Based upon these values, the log-likelihood score is then computed and returned.

Value

The log of the likelihood score of the Kalman filter

Author(s)

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Jonas Rende <jonas.rende@fau.de>

References

multigetYahooPrices

See Also

egcm Engle-Granger cointegration model
partialAR Partially autoregressive models

Examples

```r
##---- Should be DIRECTLY executable !! ----
##-- Define data, use random, 
##--or do help(data=index) for the standard data sets.

set.seed(1)
YX <- rpci(n=500, beta=c(2,3,4), sigma_C=c(1,1,1), rho=0.9, sigma_M=0.1, sigma_R=0.2)
loglik.pci(YX[,1], YX[,2:ncol(YX)], beta=c(2,3,4), rho=0.9, sigma_M=0.1, sigma_R=0.2)
```

multigetYahooPrices  Fetches closing prices of multiple stock tickers

Description

Fetches a `zoo` data frame of daily closing prices of multiple stock tickers.

Usage

```r
multigetYahooPrices(components, start, end, quiet = FALSE, adjust = TRUE)
```

Arguments

- `components` Character vector of Yahoo ticker symbols
- `start` First date of desired data in YYYYMMDD format. Default is earliest date of all series
- `end` Last date of desired data in YYYYMMDD format. Default is the last date for which data is available
- `quiet` If FALSE, then information is printed about the progress of the fetch operation
- `adjust` If TRUE, then adjusted closing prices are returned. Otherwise, unadjusted prices are returned.

Value

Returns a `zoo` data frame containing the closing prices of the series listed in the `components` parameter, one column per price series.

Author(s)

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Christopher Krauss <christopher.krauss@fau.de>
Jonas Rende <jonas.rende@fau.de>
Generates a random instance of a partial cointegration model

Usage

```r
rpci(n, beta, sigma_C, rho, sigma_M, sigma_R,
     include.state = FALSE, robust = FALSE, nu = 5)
```

Arguments

- `n` - Number of observations to generate
- `beta` - A vector of factor loadings
- `sigma_C` - A vector of standard deviations
- `rho` - The coefficient of mean reversion
- `sigma_R` - The standard deviation of the innovations of the random walk portion of the residual series
- `sigma_M` - The standard deviation of the innovations of the mean-reverting portion of the residual series
- `include.state` - If TRUE, then the output data.frame contains the innovations to the factors and residual series, as well as the state of the residual series. Default: FALSE
- `robust` - If TRUE, then a t-distribution is used to generate the innovations. Otherwise, the innovations are normally distributed. Default: FALSE.
- `nu` - The degrees of freedom parameter used for t-distributed innovations. Default: 5.
**Details**

Generates a random set of partially cointegrated vectors. On input, \( n \) is the length of the sequence to be generated. \( \text{beta} \) is a vector of length \( k \) representing the coefficients of the factor loadings, and \( \text{sigma}_C \) is a vector of length \( k \) representing the standard deviations of the increments of the factor loadings.

Generates a random realization of the sequence

\[
Y_t = \beta_1 F_{1,t} + \beta_2 F_{2,t} + ... + \beta_k F_{k,t} + M_t + R_t
\]

\[
F_{i,j} = F_{i,j-1} + \delta_{i,j}
\]

\[
M_t = \rho m_{t-1} + \epsilon_{M,t}
\]

\[
R_t = r_{t-1} + \epsilon_{R,t}
\]

\[
\delta_{i,j} \sim \mathcal{N}(0, \sigma_{C,i}^2)
\]

\[
\epsilon_{M,t} \sim \mathcal{N}(0, \sigma_{M}^2)
\]

\[
\epsilon_{R,t} \sim \mathcal{N}(0, \sigma_{R}^2)
\]

**Value**

A data.frame of \( n \) rows representing the realization of the partially cointegrated sequence.

If `include.state` is FALSE, returns an \( n \times (k+1) \) matrix whose columns are \( y, F_1, F_2, \ldots, F_k \).

If `include.state` is TRUE, returns an \( n \times (2k + 6) \) matrix whose columns are \( y, F_1, F_2, \ldots, F_k, x, M, R, \delta_1, \delta_k \).

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**See Also**

`fit.pci`

**Examples**

```r
rpci(10, beta=1, sigma_C=1, rho=0.9, sigma_R=1, sigma_M=1)
```
statehistory.pci  
*Generates the sequence of inferred states of a partial cointegration model*

### Description
Generates the sequence of inferred states of a partial cointegration model.

### Usage

```r
statehistory.pci(A, data = A$data, basis = A$basis)
```

### Arguments
- `A`  
  An object returned by `fit.pci` representing a partial cointegration fit.
- `data`  
  The data history for which the inferred states are to be computed. This should be a `(k+1) x n` matrix, where \( k \) is the number of independent variables and \( n \) is the number of observations. If this is omitted, then uses the data history that was used in fitting the model \( A \).
- `basis`  
  The coefficients of the independent variables. This is a vector of length \( k \). If this is omitted, then uses the coefficients that were computed in fitting the model \( A \).

### Details
Computes the expected internal states of the model over the course of the data history.

### Value
Returns a `data.frame` with the following columns:
- `Y`  
  The variable being modeled
- `X1, \ldots, X_N`  
  The independent variables
- `Z`  
  The residual series \( Y \sim \beta x \)
- `M`  
  The estimated state of the mean reverting component
- `R`  
  The estimated state of the random walk component
- `eps_M`  
  The innovation to the mean reverting component
- `eps_R`  
  The innovation to the random walk component

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See Also

egcm  Engle-Granger cointegration model
partialAR  Partially autoregressive models

Examples

```r
##----- Should be DIRECTLY executable !! -----
##-- => Define data, use random,
##--or do help(data=index) for the standard data sets.

YX <- rpci(n=1000, beta=c(2,3), sigma_C=c(0.1,0.1), rho=0.9, sigma_M=1, sigma_R=2)
f <- fit.pci(YX[,1], YX[,2:ncol(YX)])
statehistory.pci(f)
```

---

**test.pci**  
Tests the goodness of fit of a partial cointegration model

**Description**
Tests the goodness of fit of a partial cointegration model

**Usage**

```r
test.pci(Y, X, 
pci_opt_method=c("jp", "twostep"),
irobust=FALSE,
inu = 5,
null_hyp=c("par","rw", "ar1"),
imethod = c("wilk","boot"),
inrep = 999,
istart.seed= 1,
alpha = 0.05,
use.multicore = F)
```

**Arguments**

- **Y**  
The time series that is to be modeled. A plain or zoo vector of length n or a fit.pci object.

- **X**  
A (possibly zoo) matrix of dimensions n x k. If k=1, then this may be a plain or zoo vector.

- **pci_opt_method**  
The method that will be used for fitting a partially cointegrated model to X and Y. This can be either "jp" (joint penalty) or "twostep" (Engle-Granger two-step). See fit.pci for a complete explanation. Default: "jp".

- **irobust**  
If TRUE, then the residuals are assumed to follow a t-distribution. Default: FALSE.
inu  
The degrees-of-freedom parameter to be used in robust estimation. Default: 5.

null_hyp  
This specifies the null hypothesis. This can be either "rw", "ar1" or "par". If "rw", then the null hypothesis is a random walk. If "ar1", then the null hypothesis is an autoregressive process of order 1. (In this case, the null hypothesis calls for Y and X to be cointegrated.) If "par", then the null hypothesis is that the process can be modelled as a PAR process. Default: "par".

imethod  
The method used to calculate p-values associated with the likelihood ratio test. If set to "wilk" p-values are determined under the assumption that theorem of Wilks holds (Wilks, 1933). If set to "boot" a parametric bootstrap method is carried out following the instructions of MacKinnon (2009). For details see description below. Default: "wilk".

inrep  
The number of bootstrap iterations. Only han an effect if codeimethod = c("boot"). Default: inrep = 999.

istart.seed  
Sets the starting seed for the parametric bootstrap. Only han an effect if codeimethod = c("boot"). Default: istart.seed= 1.

alpha  
The global significance level used to determine the local significance level following (i) the conservative Bonferroni correction and (ii) the more liberal Holm correction to account for multiple testing. Only important if null_hyp="par". Default: 0.05.

use.multicore  
If TRUE, parallel processing will be used to improve performance. Only han an effect if codeimethod = c("boot"). Default: FALSE.

Details

The likelihood ratio test is used to determine whether the null hypothesis should be rejected or not, since the null models are nested in the partial cointegration alternative (Neymann, 1933). That is to say, a search is performed for the best fitting model under the null hypothesis, and the log likelihood score of this model is computed. Then a search is performed for the best fitting model under the alternative hypothesis of partial cointegration, and the log likelihood score of this model is computed. We consider two null hypotheses, namely the the pure random walk hypotheses and the pure stationary AR(1) hypothesis (refers to classic cointegration). Given that under the null hypothesis we are solely testing parameters at the boundaries of the corresponding parameter space the regularity condition underlying the theorem of Wilks that the true parameter has to be an inner point of the parameter space does not hold (Wilks, 1938). At boundaries standard asymptotics fail. The asymptotic distribution of the likelihood ratio test statistic is a mixture of $\chi^2$ distributions (Shapiro, 1988):

$$w[0] \times \chi[0]^2 + w[1] \times \chi[1]^2 + \ldots + w[m] \times \chi[m]^2 + \ldots + w[k] \times \chi[k]^2$$

where the weights $w[i]$, with $i = \{0, \ldots, k\}$ sum up to one. Let $m$ denote the degrees of freedom associated with the $\chi^2$ distribution of interest and $k$ is equal to the parameter difference of the full and the null model. Note that $\chi^2[0] = 0$, indicating a point mass at zero. Under Wilks theorem the weight for the $\chi^2[k]$ distribution is set to one. Therefore, the resulting p-values falsely assuming Wilks theorem holds are more conservative compared to the p-values associated with a mixed distribution. Therefore, we implement the latter as a conservative approximation (Stoel, 2006) for the true underlying distribution (imethod = "wilk"). If (imethod = "boot"), a parametric bootstrap following the instructions of MacKinnon (2009) is carried out calculate the p-value of the corresponding null hypothesis.
Value

An object of class "pcitest" containing the results of the hypothesis test.

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References

Neyman, J. and Pearson, E. S.,1933. On the Problem of the Most Efficient Tests of Statistical Hypotheses. Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character,231.

See Also

fit.pci Fits a partially cointegrated model
likelihood_ratio.pci Computes the likelihood ratio of a PCI model versus a null model

Examples

# The following should reject both the random walk and AR(1) models

## Not run:
# Example using the very fast "wilk" method
# The following should be classified as PCI, i.e., both Nullhypothesis should be rejected even
# if we account for multiple testing (alpha_bonf, alpha_holm)
set.seed(1313)
YX <- rpci(n=1000,beta=c(1), sigma_C=c(0.1), rho=0.8, sigma_M=1, sigma_R=1)
test.pci(Y=YX[,1], X=YX[,2:ncol(YX)])

# Example using the parametric bootstrap "boot" method with 999 iterations (inrep = 999 and
# a starting seed of 1 (istart.seed= 1)
# The following should be classified as PCI, i.e., both Nullhypothesis should be rejected even
# if we account for multiple testing (alpha_bonf, alpha_holm)
# Results are very similar
set.seed(1313)

YX <- rpci(n=1000, beta=c(1), sigma_C=c(0.1), rho=0.8, sigma_M=1, sigma_R=1)
test.pci(Y=YX[,1], X=YX[,2:ncol(YX)], imethod = "boot", inrep = 999, istart.seed= 1)

### End(Not run)

---

### yfit.pci

*Fetch series from Yahoo and perform a partial cointegration fit.*

---

**Description**

Fetch series from Yahoo and perform a partial cointegration fit.

**Usage**

```r
yfit.pci(target, factors, start, end, na.rm=FALSE, ...)
```

**Arguments**

- `target` The ticker symbol of the stock price series that is to be modeled.
- `factors` A list of ticker symbols of stock price series to be used in modeling `target`
- `start` The starting date for which data is to be fetched, given in the format YYYYMMDD. Default: 2 years ago today.
- `end` The ending date for which data is to be fetched, given in the format YYYYMMDD. Default: today.
- `na.rm` If TRUE, then NA’s will be removed from the data.frame of fetched prices. A heuristic approach is used to decide between deleting securities versus deleting days.
- `...` Additional optional parameters to be passed to `fit.pci`

**Value**

An S3 object of class `pci.fit` representing the best fit that was found.

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**See Also**

`fit.pci`
Examples

# Compare a cointegration fit Coca-Cola and Pepsi to a partial cointegration fit.
# Note that yegcm(X, Y) has a different parameter ordering than yfit.pci(Y, X)
# yegcm("PEP", "KO", start=as.numeric(format(Sys.Date() - 365*2, "%Y%m%d")))
# yfit.pci("KO", "PEP")

**yhedge.pci**  
*Hedge portfolio for a stock price series*

**Description**

Computes the hedge of a stock price series fetched from Yahoo! using one or more other stock price series also fetched from Yahoo!

**Usage**

```r
yhedge.pci(target, factors, start, end, na.rm=FALSE, ...)
```

**Arguments**

- **target**: The ticker symbol of the stock price series that is to be modeled.
- **factors**: A list of ticker symbols of stock price series to be used in modeling target
- **start**: The starting date for which data is to be fetched, given in the format YYYYMMDD. Default: 2 years ago today.
- **end**: The ending date for which data is to be fetched, given in the format YYYYMMDD. Default: today.
- **na.rm**: If TRUE, then NA's will be removed from the data.frame of fetched prices. A heuristic approach is used to decide between deleting securities versus deleting days.
- **...**: Additional optional parameters to be passed to `fit.pci`

**Value**

An S3 object of class `pci.hedge` representing the best fit that was found.

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**See Also**

`fit.pci`
Examples

# Compute the best hedge of Coca-Cola using sector ETFS.
# sectorETFS <- c("XLB", "XLE", "XLF", "XLI", "XLK", "XLP", "XLU", "XLV", "XLY")
# hedge <- yhedge.pci("KO", sectorETFS)
# hedge
# test.pci(hedge$pci)
# plot(hedge)
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