Package ‘powerMediation’

February 16, 2017

Version 0.2.7
Date 2017-02-15
Title Power/Sample Size Calculation for Mediation Analysis
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Depends R (>= 3.1.0), stats
Description Functions to
calculate power and sample size for testing
(1) mediation effects;
(2) the slope in a simple linear regression;
(3) odds ratio in a simple logistic regression;
(4) mean change for longitudinal study with 2 time points;
(5) interaction effect in 2-way ANOVA; and
(6) the slope in a simple Poisson regression.
License GPL (>= 2)
Repository CRAN
Date/Publication 2017-02-16 07:13:36
NeedsCompilation no

R topics documented:

  minEffect.SLR .......................... 2
  minEffect.VSMc .......................... 4
  minEffect.VSMc.cox ..................... 5
  minEffect.VSMc.logistic ............... 7
  minEffect.VSMc.poisson ............... 9
  power.SLR ............................ 11
  power.SLR.rho ......................... 12
  powerInteract ......................... 13
  powerLogisticBin ....................... 14
  powerLogisticCon ....................... 15
  powerLong .......................... 16
minEffect.SLR

Description

Calculate minimal detectable slope given sample size and power for simple linear regression.

Usage

\[
\text{minEffect.SLR}(n, \\
\quad \text{power}, \\
\quad \text{sigma.x}, \\
\quad \text{sigma.y}, \\
\quad \text{alpha} = 0.05, \\
\quad \text{verbose} = \text{TRUE})
\]

Arguments

- **n**: sample size.
- **power**: power for testing if \( \lambda = 0 \) for the simple linear regression \( y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2) \).
- **sigma.x**: standard deviation of the predictor \( sd(x) \).
The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

### Value

- **lambda.a**: minimum absolute detectable effect.
- **res.uniroot**: results of optimization to find the optimal minimum absolute detectable effect.

### Note

The test is a two-sided test. Code for one-sided tests will be added later.

### Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

### References


### See Also

- `power.SLR`, `power.SLR.rho`, `ss.SLR`, `ss.SLR.rho`. 

### Examples

```r
mineffect.SLR(n=100, power=0.8, sigma.x=0.2, sigma.y=0.5,
alpha = 0.05, verbose = TRUE)
```
minEffect.VSMc

Minimum detectable slope for mediator in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate minimal detectable slope for mediator given sample size and power in simple linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

minEffect.VSMc(n, power, sigma.m, sigma.e, corr.xm, alpha = 0.05, verbose = TRUE)

Arguments

n sample size.
power power for testing $b_2 = 0$ for the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$.
sigma.m standard deviation of the mediator.
sigma.e standard deviation of the random error term in the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$.
corr.xm correlation between the predictor $x$ and the mediator $m$.
alpha type I error rate.
verbose logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation corr.xm between the primary predictor and mediator is non-zero.

The full model is

$$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$

The reduced model is

$$y_i = b_0 + b_1x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_\epsilon^2)$$
Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( \text{corr}_xm \) as the multiple correlation of the mediator with the confounders as well as the predictor.

**Value**

- **b2**: minimum absolute detectable effect.
- **res.uniroot**: results of optimization to find the optimal sample size.

**Note**

The test is a two-sided test. Code for one-sided tests will be added later.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

- `powerMediation.VSMc`
- `ssMediation.VSMc`

**Examples**

```r
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# minimum effect is =0.1
minEffect.VSMc(n = 863, power = 0.8, sigma.m = 1,
               sigma.e = 1, corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

---

**Description**

Calculate minimal detectable slope for mediator given sample size and power in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.
minEffect.VSMc.cox

Usage

minEffect.VSMc.cox(n,  
  power,  
  sigma.m,  
  psi,  
  corr.xm,  
  alpha = 0.05,  
  verbose = TRUE)

Arguments

n sample size.

power power for testing \( b_2 = 0 \) for the cox regression \( \log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i \), where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.

sigma.m standard deviation of the mediator.

psi the probability that an observation is uncensored, so that the number of event \( d = n \times \psi \), where \( n \) is the sample size.

corr.xm correlation between the predictor \( x \) and the mediator \( m \).

alpha type I error rate.

verbose logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the cox regressions:

\[
\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i
\]

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \), if the correlation \( \text{corr.xm} \) between the primary predictor and mediator is non-zero.

The full model is

\[
\log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i
\]

The reduced model is

\[
\log(\lambda) = \log(\lambda_0) + b_1 x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( \text{corr.xm} \) as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

b2 minimum absolute detectable effect.

res.unirouut results of optimization to find the optimal sample size.
Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

diffMediation.VSM.cox, ssMediation.VSM.cox

Examples

# example in section 6 (page 547) of Vittinghoff et al. (2009).
# minimum effect is = log(1.5) = 0.4054651

minEffect.VSM.cox(n = 1399, power = 0.7999916,
sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
alpha = 0.05, verbose = TRUE)

minEffect.VSM.c.logistic

Minimum detectable slope for mediator in logistic regression based on
Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate minimal detectable slope for mediator given sample size and power in logistic regression
based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

minEffect.VSM.c.logistic(n,
  power,
  sigma.m,
  p,
  corr.xm,
  alpha = 0.05,
  verbose = TRUE)
Arguments

n  sample size.

power  power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1 - p_i)) = b_0 + b_1x_i + b_2m_i$.

sigma.m  standard deviation of the mediator.

p  the marginal prevalence of the outcome.

corr.xm  correlation between the predictor $x$ and the mediator $m$.

alpha  type I error rate.

verbose  logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1x_i + b_2m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation corr.xm between the primary predictor and mediator is non-zero.

The full model is

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1x_i + b_2m_i$$

The reduced model is

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

b2  minimum absolute detectable effect.

res.unirout  results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

**See Also**

powerMediation.VSMc.logistic, ssMediation.VSMc.logistic

**Examples**

```r
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# minimum effect is log(1.5)= 0.4054651

minEffect.VSMc.logistic(n = 255, power = 0.8, sigma.m = 1, 
p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```

---

**minEffect.VSMc.poisson**

*Minimum detectable slope for mediator in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method*

**Description**

Calculate minimal detectable slope for mediator given sample size and power in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

**Usage**

```r
minEffect.VSMc.poisson(n, 
power, 
sigma.m, 
EY, 
corr.xm, 
alpha = 0.05, 
verbose = TRUE)
```

**Arguments**

- **n**  
  sample size.
- **power**  
  power for testing $b_2 = 0$ for the poisson regression $\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$.
- **sigma.m**  
  standard deviation of the mediator.
- **EY**  
  the marginal mean of the outcome
- **corr.xm**  
  correlation between the predictor $x$ and the mediator $m$.
- **alpha**  
  type I error rate.
- **verbose**  
  logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.
Details

The test is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the poisson regressions:

\[
\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i
\]

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \), if the correlation \( \text{corr}.xm \) between the primary predictor and mediator is non-zero.

The full model is

\[
\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i
\]

The reduced model is

\[
\log(E(Y_i)) = b_0 + b_1 x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( \text{corr}.xm \) as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- \( b2 \) minimum absolute detectable effect.
- \( \text{res.unirrot} \) results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- `powerMediation.VSMc.poisson`, `ssMediation.VSMc.poisson`

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# minimum effect is = log(1.35) = 0.3001046
minEffect.VSMc.poisson(n = 1239, power = 0.7998578,
sigma.m = sqrt(0.25 * (1 - 0.25)),
EY = 0.5, corr.xm = 0.5,
alpha = 0.05, verbose = TRUE)
```
power.SLR

**Power for testing slope for simple linear regression**

**Description**

Calculate power for testing slope for simple linear regression.

**Usage**

```r
power.SLR(n,
lambda.a,  
sigma.x,    
sigma.y,    
alpha = 0.05,
verbose = TRUE)
```

**Arguments**

- `n` sample size.
- `lambda.a` regression coefficient in the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$. 
- `sigma.x` standard deviation of the predictor $sd(x)$.
- `sigma.y` marginal standard deviation of the outcome $sd(y)$. (not the marginal standard deviation $sd(y|x)$) 
- `alpha` type I error rate.
- `verbose` logical. TRUE means printing power; FALSE means not printing power.

**Details**

The power is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

**Value**

- `power` power for testing if $b_2 = 0$.
- `delta` $\lambda \sigma_x \sqrt{n}/\sqrt{\sigma^2_y - (\lambda \sigma_x)^2}$. 
- `s` $\sqrt{\sigma^2_y - (\lambda \sigma_x)^2}$.
- `t.cr` $\Phi^{-1}(1 - \alpha/2)$, where $\Phi$ is the cumulative distribution function of the standard normal distribution.
- `rho` correlation between the predictor $x$ and outcome $y = \lambda \sigma_x / \sigma_y$. 

Note
The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also
minEffect.SLR, power.SLR.rho, ss.SLR.rho, ss.SLR.

Examples
```
power.SLR(n=100, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5, 
alpha = 0.05, verbose = TRUE)
```

**power.SLR.rho**

*Power for testing slope for simple linear regression*

Description
Calculate power for testing slope for simple linear regression.

Usage
```
power.SLR.rho(n, 
 rho2, 
 alpha = 0.05, 
 verbose = TRUE)
```

Arguments
- `n` : sample size.
- `rho2` : square of the correlation between the outcome and the predictor.
- `alpha` : type I error rate.
- `verbose` : logical. TRUE means printing power; FALSE means not printing power.

Details
The power is for testing the null hypothesis \( \lambda = 0 \) versus the alternative hypothesis \( \lambda \neq 0 \) for the simple linear regressions:

\[
y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)
\]
Power for detecting interaction effect in 2-way ANOVA

Description

Power for detecting interaction effect in 2-way ANOVA.

Usage

\[ \text{powerInteract}(\text{nTotal}, \text{a}, \text{b}, \text{effsize}, \text{alpha} = 0.05, \text{nTests} = 1) \]

Arguments

- \text{nTotal} number of observations in total
- \text{a} number of levels in factor 1
- \text{b} number of levels in factor 2
- \text{effsize} effect size
- \text{alpha} type I error rate
- \text{nTests} number of tests. (Family-wise type I error rate will be controlled based on Bonferroni’s correction)
Value
power

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

Examples
    powerInteact(nTotal=50, a=2, b=3, effsize=1.5, alpha=0.05, nTests=1)

powerLogisticBin    Calculating power for simple logistic regression with binary predictor

Description
Calculating power for simple logistic regression with binary predictor.

Usage
  powerLogisticBin(n,
                  p1,
                  p2,
                  B,
                  alpha = 0.05)

Arguments
  n    total number of sample size.
  p1   \(pr(diseased|X = 0)\), i.e. the event rate at \(X = 0\) in logistic regression \(\logit(p) = a + bX\), where \(X\) is the binary predictor.
  p2   \(pr(diseased|X = 1)\), the event rate at \(X = 1\) in logistic regression \(\logit(p) = a + bX\), where \(X\) is the binary predictor.
  B    \(pr(X = 1)\), i.e. proportion of the sample with \(X = 1\)
  alpha    Type I error rate.

Details
The logistic regression mode is
  \[
  \log(p/(1-p)) = \beta_0 + \beta_1 X
  \]
where \(p = \text{prop}(Y = 1)\), \(X\) is the binary predictor, \(p_1 = \text{prop}(diseased|X = 0)\), \(p_2 = \text{prop}(diseased|X = 1)\), \(B = \text{prop}(X = 1)\), and \(p = (1 - B)p_1 + Bp_2\). The sample size formula we used for testing if \(\beta_1 = 0\), is Formula (2) in Hsieh et al. (1998):
  \[
  n = (Z_{1-\alpha/2}[p(1-p)/B]^{1/2} + \text{Z_{power}}[p_1(1-p_1)+p_2(1-p_2)(1-B)/B]^{1/2})^2 /[(p_1-p_2)^2(1-B)]
  \]
where \(n\) is the required total sample size and \(Z_u\) is the \(u\)-th percentile of the standard normal distribution.
Value
Estimated power.

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also
powerlogisticbin

Examples

```r
## Example in Table I Design (Balanced design with high event rates)  
## of Hsieh et al. (1998)  
## the power = 0.95  
## powerLogisticBin(n = 1281, p1 = 0.4, p2 = 0.5, B = 0.5, alpha = 0.05)
```

Description
Calculating power for simple logistic regression with continuous predictor.

Usage

```r
powerLogisticCon(n,  
    p1,  
    OR,  
    alpha = 0.05)
```

Arguments

- **n**
  total sample size.

- **p1**
  the event rate at the mean of the continuous predictor X in logistic regression
  \( \text{logit}(p) = a + bX \).

- **OR**
  expected odds ratio. \( \log(OR) \) is the change in log odds for an increase of one unit in X.

- **alpha**
  Type I error rate.
Details

The logistic regression mode is

\[ \log\left(\frac{p}{1 - p}\right) = \beta_0 + \beta_1 X \]

where \( p = \text{prob}(Y = 1) \), \( X \) is the continuous predictor, and \( \beta_1 \) is the log odds ratio. The sample size formula we used for testing if \( \beta_1 = 0 \) or equivalently \( OR = 1 \), is Formula (1) in Hsieh et al. (1998):

\[ n = \left( \frac{Z_{\alpha/2} + Z_{\text{power}}}{\sqrt{p_1(1 - p_1)[\log(OR)]^2}} \right)^2 \]

where \( n \) is the required total sample size, \( OR \) is the odds ratio to be tested, \( p_1 \) is the event rate at the mean of the predictor \( X \), and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

Value

Estimated power.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

SSizeLogisticCon

Examples

```r
## Example in Table II Design (Balanced design (1)) of Hsieh et al. (1998 )
## the power is 0.95
powerLogisticCon(n=317, p1=0.5, OR=exp(0.405), alpha=0.05)
```

Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.
Usage

\[
powerLong(es, n, \\
  rho = 0.5, \\
  alpha = 0.05)
\]

Arguments

- **es**: effect size of the difference of mean change.
- **n**: sample size per group.
- **rho**: correlation coefficient between baseline and follow-up values within a treatment group.
- **alpha**: Type I error rate.

Details

The power formula is based on Equation 8.31 on page 336 of Rosner (2006).

\[
power = \Phi \left( -Z_{1-\alpha/2} + \frac{\delta \sqrt{n}}{\sigma_d \sqrt{2}} \right)
\]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2 \), \( \delta = |\mu_1 - \mu_2| \), \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

When \( \sigma_1 = \sigma_2 = \sigma \), then formula reduces to

\[
power = \Phi \left( -Z_{1-\alpha/2} + \frac{|d| \sqrt{n}}{2 \sqrt{1 - \rho}} \right)
\]

where \( d = \delta / \sigma \).

Value

Power for testing for difference of mean changes.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

\[ \text{sslLong, sslLongFull, powerLongFull} \]
Examples

# Example 8.34 on page 336 of Rosner (2006)
# power=0.75
powerLong(es=5/15, n=75, rho=0.7, alpha=0.05)

---

powerLong.multiTime  Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points

Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points.

Usage

powerLong.multiTime(es, m, nn, sx2, rho = 0.5, alpha = 0.05)

Arguments

<table>
<thead>
<tr>
<th>Argument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>es</td>
<td>effect size</td>
</tr>
<tr>
<td>m</td>
<td>number of subjects</td>
</tr>
<tr>
<td>nn</td>
<td>number of observations per subject</td>
</tr>
<tr>
<td>sx2</td>
<td>within subject variance</td>
</tr>
<tr>
<td>rho</td>
<td>within subject correlation</td>
</tr>
<tr>
<td>alpha</td>
<td>type I error rate</td>
</tr>
</tbody>
</table>

Details

We are interested in comparing the slopes of the 2 groups $A$ and $B$:

$$
\beta_{1A} = \beta_{1B}
$$

where

$$
Y_{ijA} = \beta_{0A} + \beta_{1A}x_{jA} + \epsilon_{ijA}, j = 1, \ldots, nn; i = 1, \ldots, m
$$

and

$$
Y_{ijB} = \beta_{0B} + \beta_{1B}x_{jB} + \epsilon_{ijB}, j = 1, \ldots, nn; i = 1, \ldots, m
$$

The power calculation formula is (Equation on page 30 of Diggle et al. (1994)):

$$
\text{power} = \Phi \left[ -z_{1-\alpha} + \sqrt{\frac{mnn^2es^2}{2(1-\rho)}} \right]
$$

where $es = d/\sigma$, $d$ is the meaningful difference of interest, $\sigma^2$ is the variance of the random error, $\rho$ is the within-subject correlation, and $s_x^2$ is the within-subject variance.
Value

`power`

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

`sxlong.multitime`

Examples

```r
# power=0.8
powerLong.multitime(es=0.5/10, m=196, nn=3, sx2=4.22, rho = 0.5, alpha = 0.05)
```

---

**powerLongFull**

*Power calculation for longitudinal study with 2 time point*

Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage

```r
powerLongFull(delta, sigma1, sigma2, n, rho = 0.5, alpha = 0.05)
```

Arguments

- `delta`: absolute difference of the mean changes between the two groups: \( \delta = |\mu_1 - \mu_2| \)
  where \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2.
- `sigma1`: the standard deviation of baseline values within a treatment group
- `sigma2`: the standard deviation of follow-up values within a treatment group
- `n`: sample size per group
rho  correlation coefficient between baseline and follow-up values within a treatment group.

alpha  Type I error rate.

Details
The power formula is based on Equation 8.31 on page 336 of Rosner (2006).

\[
\text{power} = \Phi \left( -Z_{1-\alpha/2} + \frac{\delta \sqrt{n}}{\sigma_{d} \sqrt{2}} \right)
\]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2 \), \( \delta = |\mu_1 - \mu_2| \), \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

Value
power for testing for difference of mean changes.

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also
\texttt{ssLong}, \texttt{ssLongFull}, \texttt{powerLong}.

Examples

```r
# Example 8.33 on page 336 of Rosner (2006)
# power=0.80
powerLongFull(delta=5, sigma1=15, sigma2=15, n=85, rho=0.7, alpha=0.05)
```
Description

Calculate power for testing mediation effect based on Sobel’s test.

Usage

\[
\text{powerMediation.Sobel}(n, \\
\theta_{1a}, \\
\lambda_{a}, \\
\sigma_x, \\
\sigma_m, \\
\sigma_{\epsilon}, \\
\alpha = 0.05, \\
\text{verbose} = \text{TRUE})
\]

Arguments

- \(n\): sample size.
- \(\theta_{1a}\): regression coefficient for the predictor in the linear regression linking the predictor \(x\) to the mediator \(m\) \((m_i = \theta_0 + \theta_{1a}x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon})\)).
- \(\lambda_{a}\): regression coefficient for the mediator in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) \((y_i = \gamma + \lambda_{a}m_i + \lambda_{2}x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon})\)).
- \(\sigma_x\): standard deviation of the predictor.
- \(\sigma_m\): standard deviation of the mediator.
- \(\sigma_{\epsilon}\): standard deviation of the random error term in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) \((y_i = \gamma + \lambda_{a}m_i + \lambda_{2}x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon})\)).
- \(\alpha\): type I error.
- \(\text{verbose}\): logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \(\theta_1 \lambda = 0\) versus the alternative hypothesis \(\theta_{1a}\lambda_a \neq 0\) for the linear regressions:

\[
m_i = \theta_0 + \theta_{1a}x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon})
\]

\[
y_i = \gamma + \lambda_{a}m_i + \lambda_{2}x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon})
\]

Test statistic is based on Sobel’s (1982) test:

\[
Z = \frac{\hat{\theta}_{1a}}{\hat{\sigma}_{\theta_{1a}}}
\]
where \( \hat{\sigma}_{\theta_1 \lambda_a} \) is the estimated standard deviation of the estimate \( \hat{\theta}_1 \hat{\lambda}_a \) using multivariate delta method:

\[
\sigma_{\theta_1 \lambda_a} = \sqrt{\frac{\theta_1^2 \sigma_{\hat{\theta}_1}^2 + \lambda_a^2 \sigma_{\hat{\lambda}_a}^2}{\sigma_{\hat{\theta}_1}^2}}
\]

and \( \sigma_{\hat{\theta}_1}^2 = \frac{\sigma_e^2}{(n\sigma_x^2)} \) is the variance of the estimate \( \hat{\theta}_1 \), and \( \sigma_{\hat{\lambda}_a}^2 = \frac{\sigma_m^2(1 - \rho_{mx}^2)}{n \sigma_m^2} \) is the variance of the estimate \( \hat{\lambda}_a \), \( \sigma_{\hat{\lambda}_a}^2 \) is the variance of the mediator \( m_i \).

From the linear regression \( m_i = \theta_0 + \theta_1x_i + e_i \), we have the relationship \( \sigma_e^2 = \sigma_m^2(1 - \rho_{mx}^2) \).

Hence, we can simply the variance \( \sigma_{\theta_1 \lambda_a} \) to

\[
\sigma_{\theta_1 \lambda_a} = \sqrt{\frac{\theta_1^2 \sigma_e^2}{n \sigma_m^2(1 - \rho_{mx}^2)} + \frac{\lambda_a^2 \sigma_m^2(1 - \rho_{mx}^2)}{n \sigma_x^2}}
\]

**Value**

- **power**: power of the test for the parameter \( \theta_1 \lambda_a \)
- **delta**: \( \theta_1 \lambda_a / (sd(\hat{\theta}_1) \cdot sd(\hat{\lambda}_a)) \)

**Note**

The test is a two-sided test. Code for one-sided tests will be added later.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

[ssMediation.Sobel](#), [testMediation.Sobel](#)

**Examples**

```r
powerMediation.Sobel(n=248, theta.1a=0.1701, lambda.a=0.1998,
sigma.x=0.57, sigma.m=0.61, sigma.epsilon=0.2,
alpha = 0.05, verbose = TRUE)
```
**powerMediation.VSMc**

*Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method*

---

**Description**

Calculate Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

**Usage**

```r
powerMediation.VSMc(n, 
  b2, 
  sigma.m, 
  sigma.e, 
  corr.xm, 
  alpha = 0.05, 
  verbose = TRUE)
```

**Arguments**

- `n`: sample size.
- `b2`: regression coefficient for the mediator `m` in the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
- `sigma.m`: standard deviation of the mediator.
- `sigma.e`: standard deviation of the random error term in the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
- `corr.xm`: correlation between the predictor `x` and the mediator `m`.
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing power; FALSE means not printing power.

**Details**

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$

The reduced model is

$y_i = b_0 + b_1x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.
Value

- **power**: power for testing if $b_2 = 0$.
- **delta**: $b_2 \sigma_m \sqrt{1 - \rho_{xm}^2}/\sigma_e$, where $\sigma_m$ is the standard deviation of the mediator $m$, $\rho_{xm}$ is the correlation between the predictor $x$ and the mediator $m$, and $\sigma_e$ is the standard deviation of the random error term in the linear regression.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- `minEffect.VSMc`, `ssMediation.VSMc`

Examples

```r
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# power=0.8
powerMediation.VSMc(n = 863, b2 = 0.1, sigma.m = 1, sigma.e = 1,
corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

Description

Calculate Power for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

```r
powerMediation.VSMc.cox(n, b2, sigma.m, psi, corr.xm, alpha = 0.05, verbose = TRUE)
```
powerMediation.VSMc.cox

Arguments

n sample size.
b2 regression coefficient for the mediator m in the cox regression \( \log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i \), where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.
sigma.m standard deviation of the mediator.
psi the probability that an observation is uncensored, so that the number of event \( d = n \times psi \), where \( n \) is the sample size.
corr.xm correlation between the predictor \( x \) and the mediator \( m \).
alpha type I error rate.
verbose logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the cox regressions:

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i
\]

where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \).

The full model is

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i
\]

The reduced model is

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( corr.xm \) as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

\[
power \quad \text{power for testing if} \ b_2 = 0.
delta \quad b_2\sigma_m \sqrt{(1 - \rho_{xm}^2)} \psi
\]

, where \( \sigma_m \) is the standard deviation of the mediator \( m \), \( \rho_{xm} \) is the correlation between the predictor \( x \) and the mediator \( m \), and \( \psi \) is the probability that an observation is uncensored, so that the number of event \( d = n \times \psi \), where \( n \) is the sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>
References


See Also

minEffect.VSM.cox, ssMediation.VSM.cox

Examples

# example in section 6 (page 547) of Vittinghoff et al. (2009).
# power = 0.7999916
powerMediation.VSM.cox(n = 1399, b2 = log(1.5),
sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
alpha = 0.05, verbose = TRUE)

powerMediation.VSMc.logistic

Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method

Description

Calculate Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

powerMediation.VSMc.logistic(n, b2, sigma.m, p, corr.xm, alpha = 0.05, verbose = TRUE)

Arguments

n sample size.
b2 regression coefficient for the mediator m in the logistic regression \( \log(p_i/(1 - p_i)) = b0 + b1x_i + b2m_i \).
sigma.m standard deviation of the mediator.
p the marginal prevalence of the outcome.
corr.xm correlation between the predictor x and the mediator m.
alpha type I error rate.
verbose logical. TRUE means printing power; FALSE means not printing power.
Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1 - p_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(p_i/(1 - p_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(p_i/(1 - p_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $corr.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- `power`: power for testing if $b_2 = 0$.
- `delta`: $b_2 \sigma_m \sqrt{(1 - \rho_{xm}^2)p(1 - p)}$

where $\sigma_m$ is the standard deviation of the mediator $m$, $\rho_{xm}$ is the correlation between the predictor $x$ and the mediator $m$, and $p$ is the marginal prevalence of the outcome.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- `minEffect.VSMc.logistic`
- `ssMediation.VSMc.logistic`

Examples

```r
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# power = 0.8005793
powerMediation.VSMc.logistic(n = 255, b2 = log(1.5), sigma.m = 1, p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```
Description

Calculate Power for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

```r
calculate_power(n, b2, sigma.m, EY, corr.xm, alpha = 0.05, verbose = TRUE)
```

Arguments

- `n`: sample size.
- `b2`: regression coefficient for the mediator \( m \) in the poisson regression \( \log(E(Y_i)) = b0 + b1x_i + b2m_i \).
- `sigma.m`: standard deviation of the mediator.
- `EY`: the marginal mean of the outcome.
- `corr.xm`: correlation between the predictor \( x \) and the mediator \( m \).
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \( b2 = 0 \) versus the alternative hypothesis \( b2 \neq 0 \) for the poisson regressions:

\[
\log(E(Y_i)) = b0 + b1x_i + b2m_i
\]

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b2 = 0 \) versus the alternative hypothesis \( H_a : b2 \neq 0 \).

The full model is

\[
\log(E(Y_i)) = b0 + b1x_i + b2m_i
\]

The reduced model is

\[
\log(E(Y_i)) = b0 + b1x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.
Value

- **power**: power for testing if $b_2 = 0$.
- **delta**: $b_2\sigma_m\sqrt{1 - \rho_{xm}^2}E_Y$

, where $\sigma_m$ is the standard deviation of the mediator $m$, $\rho_{xm}$ is the correlation between the predictor $x$ and the mediator $m$, and $E_Y$ is the marginal mean of the outcome.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

minEffect.VSMc.poisson, ssMediation.VSMc.poisson

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# power = 0.7998578
powerMediation.VSMc.poisson(n = 1239, b2 = log(1.35),
sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
alpha = 0.05, verbose = TRUE)
```

powerPoisson 

*Power calculation for simple Poisson regression*

Description

Power calculation for simple Poisson regression. Assume the predictor is normally distributed.

Usage

```r
powerPoisson(
  beta0,
  beta1,
  mu.x1,
  sigma2.x1,
  mu.T = 1,
  phi = 1,
  alpha = 0.05,
  N = 50)
```
Arguments
---
- beta0: intercept
- beta1: slope
- mu.x1: mean of the predictor
- sigma2.x1: variance of the predictor
- mu.T: mean exposure time
- phi: a measure of over-dispersion
- alpha: type I error rate
- N: sample size

Details
The simple Poisson regression has the following form:

\[ Pr(Y_i = y_i | \mu_i, t_i) = \exp(-\mu_i t_i)(\mu_i t_i)^{y_i}/(y_i!) \]

where

\[ \mu_i = \exp(\beta_0 + \beta_1 x_{1i}) \]

We are interested in testing the null hypothesis \( \beta_1 = 0 \) versus the alternative hypothesis \( \beta_1 = \theta_1 \). Assume \( x_1 \) is normally distributed with mean \( \mu_{x_1} \) and variance \( \sigma_{x_1}^2 \). The sample size calculation formula derived by Signorini (1991) is

\[ N = \phi \left[ z_{1 - \alpha/2} \sqrt{V(b_1 | \beta_1 = 0)} + z_{\text{power}} \sqrt{V(b_1 | \beta_1 = \theta_1)} \right]^2 / \mu_T \exp(\beta_0) \theta_1^2 \]

where \( \phi \) is the over-dispersion parameter (\( = \text{var}(y_i)/\text{mean}(y_i) \)), \( \alpha \) is the type I error rate, \( b_1 \) is the estimate of the slope \( \beta_1 \), \( \beta_0 \) is the intercept, \( \mu_T \) is the mean exposure time, \( z_a \) is the 100 \( * a \)-th lower percentile of the standard normal distribution, and \( V(b_1 | \beta_1 = \eta) \) is the variance of the estimate \( b_1 \) given the true slope \( \beta_1 = \eta \).

The variances are

\[ V(b_1 | \beta_1 = 0) = \frac{1}{\sigma_{x_1}^2} \]

and

\[ V(b_1 | \beta_1 = \theta_1) = \frac{1}{\sigma_{x_1}^2} \exp \left[ - \left( \theta_1 \mu_{x_1} + \theta_1^2 \sigma_{x_1}^2 / 2 \right) \right] \]

Value
- power

Author(s)
- Weiliang Qiu <stwxq@channing.harvard.edu>

References
sizePoisson

See Also

See Also as sizePoisson

Examples

```r
# power = 0.8090542
print(powerPoisson(
  beta0 = 0.1,
  beta1 = 0.5,
  mu.x1 = 0,
  sigma2.x1 = 1,
  mu.T = 1,
  phi = 1,
  alpha = 0.05,
  N = 28))
```

---

**sizePoisson**

*Sample size calculation for simple Poisson regression*

---

Description

Sample size calculation for simple Poisson regression. Assume the predictor is normally distributed.

Usage

```r
sizePoisson(
  beta0,  # intercept
  beta1,  # slope
  mu.x1,  # mean of the predictor
  sigma2.x1,  # variance of the predictor
  mu.T = 1,  # mean exposure time
  phi = 1,  # a measure of over-dispersion
  alpha = 0.05,  # type I error rate
  power = 0.8)  # power
```

Arguments

- `beta0`: intercept
- `beta1`: slope
- `mu.x1`: mean of the predictor
- `sigma2.x1`: variance of the predictor
- `mu.T`: mean exposure time
- `phi`: a measure of over-dispersion
- `alpha`: type I error rate
- `power`: power
Details

The simple Poisson regression has the following form:

\[ Pr(Y_i = y_i | m_i, t_i) = \exp(-\mu_i t_i) (\mu_i t_i)^{y_i} / (y_i!) \]

where

\[ \mu_i = \exp(\beta_0 + \beta_1 x_{1i}) \]

We are interested in testing the null hypothesis \( \beta_1 = 0 \) versus the alternative hypothesis \( \beta_1 = \theta_1 \).

Assume \( x_1 \) is normally distributed with mean \( \mu_{x_1} \) and variance \( \sigma^2_{x_1} \). The sample size calculation formula derived by Signorini (1991) is

\[
N = \phi \left( \frac{z_{1-\alpha/2} \sqrt{V(b_1|\beta_1 = 0) + z_{\text{power}} \sqrt{V(b_1|\beta_1 = \theta_1)}}}{\mu_T \exp(\beta_0) \theta_1^2} \right)^2
\]

where \( \phi \) is the over-dispersion parameter \( = \text{var}(y_i)/\text{mean}(y_i) \), \( \alpha \) is the type I error rate, \( b_1 \) is the estimate of the slope \( \beta_1 \), \( \beta_0 \) is the intercept, \( \mu_T \) is the mean exposure time, \( z_a \) is the 100 \( \times \) \( a \)-th lower percentile of the standard normal distribution, and \( V(b_1|\beta_1 = \theta_1) \) is the variance of the estimate \( b_1 \) given the true slope \( \beta_1 = \eta \).

The variances are

\[
V(b_1|\beta_1 = 0) = \frac{1}{\sigma^2_{x_1}}
\]

and

\[
V(b_1|\beta_1 = \theta_1) = \frac{1}{\sigma^2_{x_1}} \exp \left[ - (\theta_1 \mu_{x_1} + \theta_1^2 \sigma^2_{x_1}/2) \right]
\]

Value

sample size

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also
See Also as powerPoisson

Examples

```r
# sample size = 28
print(sizePoisson()
  beta0 = 0.1,
  beta1 = 0.5,
  mu.x1 = 0,
  sigma2.x1 = 1,
```

```
ss.SLR

Sample size for testing slope for simple linear regression

Description

Calculate sample size for testing slope for simple linear regression.

Usage

```r
ss.SLR(power,
    lambda.a,
    sigma.x,
    sigma.y,
    n.lower = 2.01,
    n.upper = 1e+30,
    alpha = 0.05,
    verbose = TRUE)
```

Arguments

- `power`: power for testing if $\lambda = 0$ for the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `lambda.a`: regression coefficient in the simple linear regression $y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `sigma.x`: standard deviation of the predictor $sd(x)$.
- `sigma.y`: marginal standard deviation of the outcome $sd(y)$. (not the marginal standard deviation $sd(y|x)$)
- `n.lower`: lower bound for the sample size.
- `n.upper`: upper bound for the sample size.
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$
ss.SLR.rho

Value

- **n** sample size.
- **res.unir** results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- `minEffect.SLR`
- `power.SLR`
- `power.SLR.rho`
- `ss.SLR.rho`

Examples

```r
ss.SLR(power=0.8, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5,
       alpha = 0.05, verbose = TRUE)
```

---

**ss.SLR.rho**

*Sample size for testing slope for simple linear regression based on R2*

Description

Calculate sample size for testing slope for simple linear regression based on R2.

Usage

```r
ss.SLR.rho(power,
           rho2,
           n.lower = 2.01,
           n.upper = 1e+30,
           alpha = 0.05,
           verbose = TRUE)
```
ss.SLR.rho

Arguments

- **power**: power.
- **rho**: square of the correlation between the outcome and the predictor.
- **n.lower**: lower bound of the sample size.
- **n.upper**: upper bound of the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis \( \lambda = 0 \) versus the alternative hypothesis \( \lambda \neq 0 \) for the simple linear regressions:

\[
y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)
\]

Value

- **n**: sample size.
- **res.unirout**: results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- `mineffect.SLR`
- `power.SLR`
- `power.SLR.rho`
- `ss.SLR`

Examples

```r
ss.SLR.rho(power=0.8, rho=0.6, alpha = 0.05, verbose = TRUE)
```
Calculating sample size for simple logistic regression with binary predictor.

**Usage**

```r
SSizeLogisticBin(p1, p2, B, 
alpha = 0.05,  
power = 0.8)
```

**Arguments**

- **p1**: `pr(diseased|X = 0)`, i.e. the event rate at $X = 0$ in logistic regression $\logit(p) = a + bX$, where $X$ is the binary predictor.
- **p2**: `pr(diseased|X = 1)`, the event rate at $X = 1$ in logistic regression $\logit(p) = a + bX$, where $X$ is the binary predictor.
- **B**: `pr(X = 1)`, i.e. proportion of the sample with $X = 1$
- **alpha**: Type I error rate.
- **power**: power for testing if the odds ratio is equal to one.

**Details**

The logistic regression mode is

$$\log(p/(1 - p)) = \beta_0 + \beta_1 X$$

where $p = \text{prob}(Y = 1)$, $X$ is the binary predictor, $p_1 = pr(diseased|X = 0)$, $p_2 = pr(diseased|X = 1)$, $B = pr(X = 1)$, and $p = (1 - B)p_1 + Bp_2$. The sample size formula we used for testing if $\beta_1 = 0$, is Formula (2) in Hsieh et al. (1998):

$$n = \left(\frac{Z_{1 - \alpha/2} [p(1 - p)/B]^{1/2} + Z_{\text{power}} [p_1(1 - p_1) + p_2(1 - p_2)(1 - B)/B]^{1/2}}{[(p_1 - p_2)^2(1 - B)]}\right)^2$$

where $n$ is the required total sample size and $Z_u$ is the $u$-th percentile of the standard normal distribution.

**Value**

total sample size required.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>
References


See Also

powerLogisticBin

Examples

```r
## Example in Table I Design (Balanced design with high event rates)
## of Hsieh et al. (1998)
## the sample size is 1281
SSSizeLogisticBin(p1 = 0.4, p2 = 0.5, b = 0.5, alpha = 0.05, power = 0.95)
```

SSSizeLogisticCon  Calculating sample size for simple logistic regression with continuous predictor

Description

Calculating sample size for simple logistic regression with continuous predictor.

Usage

```r
SSSizeLogisticCon(p1, 
    OR, 
    alpha = 0.05, 
    power = 0.8)
```

Arguments

- **p1**
  - the event rate at the mean of the continuous predictor \( X \) in logistic regression 
    \[
    \text{logit}(p) = a + bX,
    \]
  
- **OR**
  - expected odds ratio. \( \log(OR) \) is the change in log odds for an increase of one unit in \( X \).

- **alpha**
  - Type I error rate.

- **power**
  - power for testing if the odds ratio is equal to one.
Details

The logistic regression mode is

\[ \log\left(\frac{p}{1 - p}\right) = \beta_0 + \beta_1 X \]

where \( p = \text{prob}(Y = 1) \), \( X \) is the continuous predictor, and \( \beta_1 \) is the log odds ratio. The sample size formula we used for testing if \( \beta_1 = 0 \) or equivalently \( OR = 1 \), is Formula (1) in Hsieh et al. (1998):

\[ n = \frac{(Z_{1-\alpha/2} + Z_{\text{power}})^2}{\left| p_1(1 - p_1)[\log(OR)]^2 \right|} \]

where \( n \) is the required total sample size, \( OR \) is the odds ratio to be tested, \( p_1 \) is the event rate at the mean of the predictor \( X \), and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

Value

total sample size required.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

powerLogisticCon

Examples

```r
## Example in Table II Design (Balanced design (1)) of Hsieh et al. (1998)
## the sample size is 317
SSSizeLogisticCon(p1 = 0.5, OR = exp(0.405), alpha = 0.05, power = 0.95)
```

---

**ssLong**

Sample size calculation for longitudinal study with 2 time point

Description

Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.
Usage

```r
ssLong(es,
    rho = 0.5,
    alpha = 0.05,
    power = 0.8)
```  

Arguments

- `es`: effect size of the difference of mean change.
- `rho`: correlation coefficient between baseline and follow-up values within a treatment group.
- `alpha`: Type I error rate.
- `power`: power for testing for difference of mean changes.

Details

The sample size formula is based on Equation 8.30 on page 335 of Rosner (2006).

\[
 n = \frac{2\sigma^2 \left(Z_{1-\alpha/2} + Z_{\text{power}}\right)^2}{\delta^2}
\]

where \(\sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2\), \(\delta = |\mu_1 - \mu_2|\), \(\mu_1\) is the mean change over time \(t\) in group 1, \(\mu_2\) is the mean change over time \(t\) in group 2, \(\sigma_1^2\) is the variance of baseline values within a treatment group, \(\sigma_2^2\) is the variance of follow-up values within a treatment group, \(\rho\) is the correlation coefficient between baseline and follow-up values within a treatment group, and \(Z_u\) is the \(u\)-th percentile of the standard normal distribution.

We wish to test \(\mu_1 = \mu_2\).

When \(\sigma_1 = \sigma_2 = \sigma\), then formula reduces to

\[
 n = \frac{4(1 - \rho)(Z_{1-\alpha/2} + Z_{\beta})^2}{\delta^2}
\]

where \(d = \delta/\sigma\).

Value

- required sample size per group

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

`ssLongFull, powerLong, powerLongFull.`
Examples

```r
# Example 8.33 on page 336 of Rosner (2006)
# n=85
ssLong(es=5/15, rho=0.7, alpha=0.05, power=0.8)
```

---

**ssLong.multiTime**

*Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points*

---

**Description**

Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points.

**Usage**

```r
ssLong.multiTime(es, power, nn, sx2, rho = 0.5, alpha = 0.05)
```

**Arguments**

- `es`: effect size
- `power`: power
- `nn`: number of observations per subject
- `sx2`: within subject variance
- `rho`: within subject correlation
- `alpha`: type I error rate

**Details**

We are interested in comparing the slopes of the 2 groups $A$ and $B$:

$$
\beta_{1A} = \beta_{1B}
$$

where

$$
Y_{ijA} = \beta_{0A} + \beta_{1A}x_{jA} + \epsilon_{ijA}, j = 1, \ldots, nn; i = 1, \ldots, m
$$

and

$$
Y_{ijB} = \beta_{0B} + \beta_{1B}x_{jB} + \epsilon_{ijB}, j = 1, \ldots, nn; i = 1, \ldots, m
$$

The sample size calculation formula is (Equation on page 30 of Diggle et al. (1994)):

$$
m = \frac{2 (Z_{1-\alpha} + z_{power})^2 (1 - \rho)}{nn s_x^2 es^2}
$$

where $es = d/\sigma$, $d$ is the meaningful difference of interest, $\sigma^2$ is the variance of the random error, $\rho$ is the within-subject correlation, and $s_x^2$ is the within-subject variance.
Value

subject per group

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

powerLong.multiTime

Examples

# subject per group = 196
ssLong.multiTime(es=0.5/10, power=0.8, nn=3, sx2=4.22, rho = 0.5, alpha=0.05)

ssLongFull  Sample size calculation for longitudinal study with 2 time point

Description

Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage

ssLongFull(delta, 
    sigma1, 
    sigma2, 
    rho = 0.5, 
    alpha = 0.05, 
    power = 0.8)

Arguments

delta absolute difference of the mean changes between the two groups: \( \delta = |\mu_1 - \mu_2| \)
where \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2.

sigma1 the standard deviation of baseline values within a treatment group

sigma2 the standard deviation of follow-up values within a treatment group

rho correlation coefficient between baseline and follow-up values within a treatment group.
alpha  Type I error rate
power  power for testing for difference of mean changes.

Details

The sample size formula is based on Equation 8.30 on page 335 of Rosner (2006).

\[ n = \frac{2\sigma_d^2(Z_{1-\alpha/2} + Z_{power})^2}{\delta^2} \]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2, \delta = |\mu_1 - \mu_2|, \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

Value

required sample size per group

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

ssLong, powerLong, powerLongFull.

Examples

# Example 8.33 on page 336 of Rosner (2006)
# n=85
ssLongFull(delta=5, sigma1=15, sigma2=15, rho=0.7, alpha=0.05, power=0.8)

---

**ssMediation.Sobel**  *Sample size for testing mediation effect (Sobel's test)*

Description

Calculate sample size for testing mediation effect based on Sobel's test.
Usage

\texttt{ssMediation.Sobel(power, theta.1a, lambda.a, sigma.x, sigma.m, sigma.epsilon, n.lower = 1, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)}

Arguments

- \texttt{power}: power of the test.
- \texttt{theta.1a}: regression coefficient for the predictor in the linear regression linking the predictor $x$ to the mediator $m$ ($m_i = \theta_0 + \theta_{1a}x_i + e_i, e_i \sim N(0, \sigma^2_e)$).
- \texttt{lambda.a}: regression coefficient for the mediator in the linear regression linking the predictor $x$ and the mediator $m$ to the outcome $y$ ($y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e)$).
- \texttt{sigma.x}: standard deviation of the predictor.
- \texttt{sigma.m}: standard deviation of the mediator.
- \texttt{sigma.epsilon}: standard deviation of the random error term in the linear regression linking the predictor $x$ and the mediator $m$ to the outcome $y$ ($y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e)$).
- \texttt{n.lower}: lower bound of the sample size.
- \texttt{n.upper}: upper bound of the sample size.
- \texttt{alpha}: type I error rate.
- \texttt{verbose}: logical. TRUE means printing power; FALSE means not printing power.

Details

The sample size is for testing the null hypothesis $\theta_1 \lambda = 0$ versus the alternative hypothesis $\theta_{1a} \lambda_a \neq 0$ for the linear regressions:

\[ m_i = \theta_0 + \theta_{1a}x_i + e_i, e_i \sim N(0, \sigma^2_e) \]
\[ y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e) \]

Test statistic is based on Sobel's (1982) test:

\[ Z = \frac{\hat{\theta}_{1a} \hat{\lambda}_a}{\hat{\sigma}_{\theta_{1a} \lambda_a}} \]

where $\hat{\sigma}_{\theta_{1a} \lambda_a}$ is the estimated standard deviation of the estimate $\hat{\theta}_{1a} \hat{\lambda}_a$ using multivariate delta method:

\[ \hat{\sigma}_{\theta_{1a} \lambda_a} = \sqrt{\theta_{1a}^2 \sigma_{\lambda_a}^2 + \lambda_a^2 \sigma_{\theta_{1a}}^2} \]
\[ \sigma^2_{\theta_1} = \frac{\sigma_e^2}{n\sigma_x^2} \]
is the variance of the estimate \( \hat{\theta}_1 \), and \( \sigma^2_{\lambda_a} = \frac{\sigma_e^2}{n\sigma_m^2(1 - \rho_{mx}^2)} \) is the variance of the estimate \( \hat{\lambda}_a \). \( \sigma^2_{\sigma_m} \) is the variance of the mediator \( m_i \).

From the linear regression \( m_i = \theta_0 + \theta_1 a x_i + e_i \), we have the relationship \( \sigma^2_e = \sigma_m^2(1 - \rho_{mx}^2) \).

Hence, we can simplify the variance \( \sigma_{\theta_1, \lambda_a} \) to

\[
\sigma_{\theta_1, \lambda_a} = \sqrt{\theta_1^2 \frac{\sigma_e^2}{n\sigma_m^2(1 - \rho_{mx}^2)} + \lambda_a^2 \frac{\sigma_m^2(1 - \rho_{mx}^2)}{n\sigma_x^2}}
\]

**Value**

- **n** sample size.
- **res.unirout** results of optimization to find the optimal sample size.

**Note**

The test is a two-sided test. Code for one-sided tests will be added later.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

- `powerMediation.Sobel`
- `testMediation.Sobel`

**Examples**

```r
ssMediation.Sobel(power=0.8, theta.1a=0.1701, lambda.a=0.1998, 
               sigma.x=0.57, sigma.m=0.61, sigma.epsilon=0.2, 
               alpha = 0.05, verbose = TRUE)
```

**ssMediation.VSMc** Sample size for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method

**Description**

Calculate sample size for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.
Usage

```r
ssMediation.VSMc(power, 
  b2, 
  sigma.m, 
  sigma.e, 
  corr.xm, 
  n.lower = 1, 
  n.upper = 1e+30, 
  alpha = 0.05, 
  verbose = TRUE)
```

Arguments

- **power**: power for testing $b_2 = 0$ for the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
- **b2**: regression coefficient for the mediator $m$ in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
- **sigma.m**: standard deviation of the mediator.
- **sigma.e**: standard deviation of the random error term in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.
- **corr.xm**: correlation between the predictor $x$ and the mediator $m$.
- **n.lower**: lower bound for the sample size.
- **n.upper**: upper bound for the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

The reduced model is

$$y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.
Value

n sample size.

res.unirisset of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwuxq@channing.harvard.edu>

References


See Also

minEffect.VSMc, powerMediation.VSMc

Examples

```r
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# n=863
ssMediation.VSMc(cox=ssMediation.VSMC(power = 0.80, b2 = 0.1, sigma.m = 1, sigma.e = 1,
corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

---

**ssMediation.VSMc.cox**  
Sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method

### Description

Calculate sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

### Usage

```r
ssMediation.VSMc.cox(power,
b2,
sigma.m,
psi,
corr.xm,
n.lower = 1,
n.upper = 1e+30,
alpha = 0.05,
verbose=TRUE)
```
Arguments

power  power for testing $b_2 = 0$ for the cox regression $\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$, where $\lambda$ is the hazard function and $\lambda_0$ is the baseline hazard function.

$b2$  regression coefficient for the mediator $m$ in the cox regression $\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$, where $\lambda$ is the hazard function and $\lambda_0$ is the baseline hazard function.

sigma.m  standard deviation of the mediator.

psi  the probability that an observation is uncensored, so that the number of event $d = n \times psi$, where $n$ is the sample size.

corr.xm  correlation between the predictor $x$ and the mediator $m$.

n.lower  lower bound for the sample size.

n.upper  upper bound for the sample size.

alpha  type I error rate.

verbose  logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the cox regressions:

$$\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$$

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$$

The reduced model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $corr.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

$n$  sample size.

res.unirouter  results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>
References


See Also

minEffect.VSM.cox, powerMediation.VSM.cox

Examples

```r
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# n = 1399
ssMediation.VSM.cox(power = 0.7999996, b2 = log(1.5),
sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
alpha = 0.05, verbose = TRUE)
```

**ssMediation.VSMc.logistic**

*Sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

```r
ssMediation.VSMc.logistic(power,
b2, sigma.m, p,
corr.xm, n.lower = 1,
upper = 1e+30,
alpha = 0.05,
verbose = TRUE)
```

Arguments

- `power`: power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1 - p_i)) = b_0 + b_1 x_i + b_2 m_i$.
- `b2`: regression coefficient for the mediator $m$ in the logistic regression $\log(p_i/(1 - p_i)) = b_0 + b_1 x_i + b_2 m_i$.
- `sigma.m`: standard deviation of the mediator.
- `p`: the marginal prevalence of the outcome.
The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $corr.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- n : sample size.
- res.uniroot : results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

Examples

# example in section 4 (page 545) of Vittinghoff et al. (2009).
# n=255

ssMediation.VSMc.logistic(power = 0.80, b2 = log(1.5), sigma.m = 1, p = 0.5,
corr.xm = 0.5, alpha = 0.05, verbose = TRUE)

ssMediation.VSMc.poisson

Sample size for testing mediation effect in poisson regression based on
Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate sample size for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

ssMediation.VSMc.poisson(power, 
b2, 
sigma.m, 
EY, 
corr.xm, 
n.lower = 1, 
n.upper = 1e+30, 
alpha = 0.05, 
verbose = TRUE)

Arguments

power power for testing $b_2 = 0$ for the poisson regression $\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$.
b2 regression coefficient for the mediator $m$ in the poisson regression $\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$.
sigma.m standard deviation of the mediator.
EY the marginal mean of the outcome.
corr.xm correlation between the predictor $x$ and the mediator $m$.
n.lower lower bound for the sample size.
n.upper upper bound for the sample size.
alpha type I error rate.
verbose logical. TRUE means printing sample size; FALSE means not printing sample size.
Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $corr.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- **n** sample size.
- **res.uniroot** results of optimization to find the optimal sample size.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

- minEffect.VSMc.poisson
- powerMediation.VSMc.poisson

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# n = 1239
ssMediation.VSMc.poisson(power = 0.7998578, b2 = log(1.35),
  sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
  alpha = 0.05, verbose = TRUE)
```
**Description**

Calculate p-value and confidence interval for testing mediation effect based on Sobel’s test.

**Usage**

```r
testMediation.Sobel(theta.1.hat, lambda.hat, sigma.theta1, sigma.lambda, alpha = 0.05)
```

**Arguments**

- `theta.1.hat`: estimated regression coefficient for the predictor in the linear regression linking the predictor \( x \) to the mediator \( m \) \( (m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e)) \).
- `lambda.hat`: estimated regression coefficient for the mediator in the linear regression linking the predictor \( x \) and the mediator \( m \) to the outcome \( y \) \( (y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)) \).
- `sigma.theta1`: standard deviation of \( \hat{\theta}_1 \) in the linear regression linking the predictor \( x \) to the mediator \( m \) \( (m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e)) \).
- `sigma.lambda`: standard deviation of \( \hat{\lambda} \) in the linear regression linking the predictor \( x \) and the mediator \( m \) to the outcome \( y \) \( (y_i = \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)) \).
- `alpha`: significance level of a test.

**Details**

The test is for testing the null hypothesis \( \theta_1 \lambda = 0 \) versus the alternative hypothesis \( \theta_1 \lambda_a \neq 0 \) for the linear regressions:

\[
\begin{align*}
    m_i &= \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e) \\
    y_i &= \gamma + \lambda m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)
\end{align*}
\]

Test statistic is based on Sobel’s (1982) test:

\[
Z = \frac{\hat{\theta}_1 \hat{\lambda}}{\hat{\sigma}_{\theta_1 \lambda}}
\]

where \( \hat{\sigma}_{\theta_1 \lambda} \) is the estimated standard deviation of the estimate \( \hat{\theta}_1 \hat{\lambda} \) using multivariate delta method:

\[
\sigma_{\theta_1 \lambda} = \sqrt{\hat{\theta}_1^2 \sigma^2_{\lambda} + \lambda^2 \sigma^2_{\theta_1}}
\]

and \( \hat{\sigma}_{\theta_1} \) is the estimated standard deviation of the estimate \( \hat{\theta}_1 \), and \( \hat{\sigma}_{\lambda} \) is the estimated standard deviation of the estimate \( \hat{\lambda} \).
testMediation.Sobel

Value

pval p-value for testing the null hypothesis $\theta_1 \lambda = 0$ versus the alternative hypothesis $\theta_1 \lambda \neq 0$.

CI.low Lower bound of the $100(1 - \alpha)\%$ confidence interval for the parameter $\theta_1 \lambda$.

CI.upp Upper bound of the $100(1 - \alpha)\%$ confidence interval for the parameter $\theta_1 \lambda$.

Note

The test is a two-sided test. Code for one-sided tests will be added later.

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References


See Also

powerMediation.Sobel, ssMediation.Sobel

Examples

testMediation.Sobel(theta.1.hat=0.1701, lambda.hat=0.1998, sigma.theta=0.01, sigma.lambda=0.02, alpha=0.05)
Index

*Topic method
  powerPoison, 29
  sizePoisson, 31
*Topic test
  minEffect.SLR, 2
  power.SLR, 3, 11, 13, 34, 35
  power.SLR.rho, 3, 12, 12, 34, 35
  powerInteract, 13
  powerLogisticBin, 14, 15, 37
  powerLogisticCon, 15, 38
  powerLong, 16, 20, 39, 42
  powerLong.multiTime, 18, 41
  powerLongFull, 17, 19, 39, 42
  powerMediation.Sobel, 21, 44, 53
  ss.SLR, 3, 12, 13, 33, 35
  ss.SLR.rho, 3, 12, 13, 34, 34
  ssSizeLogisticBin, 16, 37
  ssSizeLogisticCon, 16, 37
  ssLong, 17, 20, 38, 42
  ssLong.multiTime, 19, 40
  ssLongFull, 17, 20, 39, 41
  ssMediation.Sobel, 22, 42, 53
  ssMediation.VSMc, 5, 24, 44
  ssMediation.VSMc.cox, 17, 26, 46
  ssMediation.VSMc.logistic, 9, 27, 48
  ssMediation.VSMc.poisson, 10, 29, 50
  testMediation.Sobel, 22, 44, 52

powerNslr, 11
powerNslr.rho, 12
powerInteract, 13
powerLogisticBin, 14
powerLogisticCon, 15
powerLong, 16
powerLong.multiTime, 18
powerLongFull, 17
powerMediation.Sobel, 21
powerMediation.VSMc, 5
powerMediation.VSMc.cox, 17
powerMediation.VSMc.logistic, 9
powerMediation.VSMc.poisson, 10
powerPoison, 29
sizePoison, 31
ss.SLR, 3, 12, 13, 33, 35
ss.SLR.rho, 3, 12, 13, 34, 34
ssSizeLogisticBin, 16
ssSizeLogisticCon, 16
ssLong, 17
ssLong.multiTime, 19
ssLongFull, 17
ssMediation.Sobel, 22
ssMediation.VSMc, 5
ssMediation.VSMc.cox, 17
ssMediation.VSMc.logistic, 9
ssMediation.VSMc.poisson, 10
testMediation.Sobel, 22

minEffect.SLR, 2, 12, 13, 34, 35
minEffect.VSMc, 4, 24, 46
minEffect.VSMc.cox, 5, 26, 48
minEffect.VSMc.logistic, 7, 27, 49
minEffect.VSMc.poisson, 9, 29, 51

power.SLR, 3, 11, 13, 34, 35
power.SLR.rho, 3, 12, 12, 34, 35
powerInteract, 13
powerLogisticBin, 14, 15, 37
powerLogisticCon, 15, 38
powerLong, 16, 20, 39, 42
powerLong.multiTime, 18, 41
powerLongFull, 17, 19, 39, 42
powerMediation.Sobel, 21, 44, 53
powerMediation.VSMc, 5, 23, 46
powerMediation.VSMc.cox, 7, 24, 48
powerMediation.VSMc.logistic, 9, 26, 49
powerMediation.VSMc.poisson, 10, 28, 51
powerPoison, 29, 32
sizePoison, 31, 31
ss.SLR, 3, 12, 13, 33, 35
ss.SLR.rho, 3, 12, 13, 34, 34
ssSizeLogisticBin, 16
ssSizeLogisticCon, 16
ssLong, 17
ssLong.multiTime, 19
ssLongFull, 17
ssMediation.Sobel, 22
ssMediation.VSMc, 5
ssMediation.VSMc.cox, 7
ssMediation.VSMc.logistic, 9
ssMediation.VSMc.poisson, 10

testMediation.Sobel, 22, 44, 52