Package ‘powerMediation’

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calculate power and sample size for testing
(1) mediation effects;
(2) the slope in a simple linear regression;
(3) odds ratio in a simple logistic regression;
(4) mean change for longitudinal study with 2 time points;
(5) interaction effect in 2-way ANOVA; and
(6) the slope in a simple Poisson regression.
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minEffect.SLR

**Description**

Calculate minimal detectable slope given sample size and power for simple linear regression.

**Usage**

```r
minEffect.SLR(n, power, sigma.x, sigma.y, alpha = 0.05, verbose = TRUE)
```

**Arguments**

- `n`: sample size.
- `power`: power for testing if \( \lambda = 0 \) for the simple linear regression \( y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\varepsilon) \).
- `sigma.x`: standard deviation of the predictor \( sd(x) = \sigma_x \).
The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

**Value**

- `lambda.a` minimum absolute detectable effect.
- `res.uniroot` results of optimization to find the optimal minimum absolute detectable effect.

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set `alpha=0.10` to obtain one-sided test at 5% significance level.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

`power.SLR`, `power.SLR.rho`, `ss.SLR`, `ss.SLR.rho`.

**Examples**

```r
minEffect.SLR(n = 100, power = 0.8, sigma.x = 0.2, sigma.y = 0.5, alpha = 0.05, verbose = TRUE)
```
**Description**

Calculate minimal detectable slope for mediator given sample size and power in simple linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

**Usage**

```r
minEffect.VSMc(n, power, sigma.m, sigma.e, corr.xm, alpha = 0.05, verbose = TRUE)
```

**Arguments**

- `n`: sample size.
- `power`: power for testing $b_2 = 0$ for the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `sigma.m`: standard deviation of the mediator.
- `sigma.e`: standard deviation of the random error term in the linear regression $y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `corr.xm`: correlation between the predictor $x$ and the mediator $m$.
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

**Details**

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation `corr.xm` between the primary predictor and mediator is non-zero.

The full model is

$$y_i = b_0 + b_1x_i + b_2m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

The reduced model is

$$y_i = b_0 + b_1x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$
Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $\text{corr.xm}$ as the multiple correlation of the mediator with the confounders as well as the predictor.

**Value**

- **b2**: minimum absolute detectable effect.
- **res.uniroot**: results of optimization to find the optimal sample size.

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $\text{alpha}=0.10$ to obtain one-sided test at 5% significance level.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

- `powerMediation.VSMc`
- `ssMediation.VSMc`

**Examples**

```r
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# minimum effect is =0.1
minEffect.VSMc(n = 863, power = 0.8, sigma.m = 1,
              sigma.e = 1, corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

---

**Description**

Calculate minimal detectable slope for mediator given sample size and power in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.
Usage

\texttt{minEffect.VSMc.cox(}n, power, sigma.m, psi, corr.xm, alpha = 0.05, verbose = \texttt{TRUE})

Arguments

\begin{itemize}
  \item \texttt{n} sample size.
  \item \texttt{power} power for testing \( b_2 = 0 \) for the cox regression \( \log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i \), where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.
  \item \texttt{sigma.m} standard deviation of the mediator.
  \item \texttt{psi} the probability that an observation is uncensored, so that the number of event \( d = n \times psi \), where \( n \) is the sample size.
  \item \texttt{corr.xm} correlation between the predictor \( x \) and the mediator \( m \).
  \item \texttt{alpha} type I error rate.
  \item \texttt{verbose} logical. \texttt{TRUE} means printing minimum absolute detectable effect; \texttt{FALSE} means not printing minimum absolute detectable effect.
\end{itemize}

Details

The test is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the cox regressions:

\[ \log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i \]

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0: b_2 = 0 \) versus the alternative hypothesis \( H_a: b_2 \neq 0 \), if the correlation \texttt{corr.xm} between the primary predictor and mediator is non-zero.

The full model is

\[ \log(\lambda) = \log(\lambda_0) + b_1 x_i + b_2 m_i \]

The reduced model is

\[ \log(\lambda) = \log(\lambda_0) + b_1 x_i \]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \texttt{corr.xm} as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

\begin{itemize}
  \item \texttt{b2} minimum absolute detectable effect.
  \item \texttt{res.unirrot} results of optimization to find the optimal sample size.
\end{itemize}
minEffect.VSMc.logistic

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

powerMediation.VSMc.cox, ssMediation.VSMc.cox

Examples

# example in section 6 (page 547) of Vittinghoff et al. (2009).
# minimum effect is = log(1.5) = 0.4054651

minEffect.VSMc.cox(n = 1399, power = 0.7999916,
                 sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
                 alpha = 0.05, verbose = TRUE)

minEffect.VSMc.logistic

Minimum detectable slope for mediator in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method

Description

Calculate minimal detectable slope for mediator given sample size and power in logistic regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

minEffect.VSMc.logistic(n,
                         power,
                         sigma.m,
                         p,
                         corr.xm,
                         alpha = 0.05,
                         verbose = TRUE)
Arguments

- **n**: sample size.
- **power**: power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1 - p_i)) = b_0 + b_1x_i + b_2m_i$.
- **sigma.m**: standard deviation of the mediator.
- **p**: the marginal prevalence of the outcome.
- **corr.xm**: correlation between the predictor $x$ and the mediator $m$.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1 - p_i)) = b_0 + b_1x_i + b_2m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation corr.xm between the primary predictor and mediator is non-zero.

The full model is

$$\log(p_i/(1 - p_i)) = b_0 + b_1x_i + b_2m_i$$

The reduced model is

$$\log(p_i/(1 - p_i)) = b_0 + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- **b2**: minimum absolute detectable effect.
- **res.uniroot**: results of optimization to find the optimal sample size.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also

powerMediation.VSMc.logistic, ssMediation.VSMc.logistic

Examples

# example in section 4 (page 545) of Vittinghoff et al. (2009).
# minimum effect is log(1.5)= 0.4054651

minEffect.VSMc.logistic(n = 255, power = 0.8, sigma.m = 1,
p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)

Description

Calculate minimal detectable slope for mediator given sample size and power in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

minEffect.VSMc.poisson(n,
  power,
  sigma.m,
  EY,
  corr.xm,
  alpha = 0.05,
  verbose = TRUE)

Arguments

n sample size.
power power for testing $b_2 = 0$ for the poisson regression $\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$.
sigma.m standard deviation of the mediator.
EY the marginal mean of the outcome
corr.xm correlation between the predictor $x$ and the mediator $m$.
alpha type I error rate.
verbose logical. TRUE means printing minimum absolute detectable effect; FALSE means not printing minimum absolute detectable effect.
Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$, if the correlation $\text{corr}.xm$ between the primary predictor and mediator is non-zero.

The full model is

$$\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1 x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $\text{corr}.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- **b2** minimum absolute detectable effect.
- **res.uniroot** results of optimization to find the optimal sample size.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $\alpha=0.10$ to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

- `powerMediation.VSMc.poisson`
- `ssMediation.VSMc.poisson`

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# minimum effect is log(1.35) = 0.3001046
minEffect.VSMc.poisson(n = 1239, power = 0.7998578,
sigma.m = sqrt(0.25 * (1 - 0.25)),
EY = 0.5, corr.xm = 0.5,
alpha = 0.05, verbose = TRUE)
```
Description

Calculate power for testing slope for simple linear regression.

Usage

```r
power.SLR(n, lambda.a, sigma.x, sigma.y, alpha = 0.05, verbose = TRUE)
```

Arguments

- `n` sample size.
- `lambda.a` regression coefficient in the simple linear regression \( y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2) \).
- `sigma.x` standard deviation of the predictor \( sd(x) \).
- `sigma.y` marginal standard deviation of the outcome \( sd(y) \). (not the marginal standard deviation \( sd(y|x) \)).
- `alpha` type I error rate.
- `verbose` logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \( \lambda = 0 \) versus the alternative hypothesis \( \lambda \neq 0 \) for the simple linear regressions:

\[
y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)
\]

Value

- `power` power for testing if \( b_2 = 0 \).
- `delta` \( \lambda \sigma_x \sqrt{n}/ \sqrt{\sigma_y^2 - (\lambda \sigma_x)^2} \).
- `s` \( \sqrt{\sigma_y^2 - (\lambda \sigma_x)^2} \).
- `t.cr` \( \Phi^{-1}(1 - \alpha/2) \), where \( \Phi \) is the cumulative distribution function of the standard normal distribution.
- `rho` correlation between the predictor \( x \) and outcome \( y = \lambda \sigma_x / \sigma_y \).
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

minEffect.SLR, power.SLR.rho, ss.SLR.rho, ss.SLR.

Examples

```r
power.SLR(n=100, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5, alpha = 0.05, verbose = TRUE)
```

---

**power.SLR.rho**

Power for testing slope for simple linear regression

Description

Calculate power for testing slope for simple linear regression.

Usage

```r
power.SLR.rho(n, 
  rho2, 
  alpha = 0.05, 
  verbose = TRUE)
```

Arguments

- `n` sample size.
- `rho2` square of the correlation between the outcome and the predictor.
- `alpha` type I error rate.
- `verbose` logical. TRUE means printing power; FALSE means not printing power.
Details

The power is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

Value

- **power**: power for testing if $b_2 = 0$.
- **delta**: $\sqrt{n}/\sqrt{1/\rho^2 - 1}$.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $\text{alpha}=0.10$ to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

`minEffect.SLR, power.SLR, ss.SLR.rho, ss.SLR`

Examples

```r
power.SLR.rho(n=100, rho2=0.6, alpha = 0.05, verbose = TRUE)
```
Arguments

- **n**: integer. Number of subjects per group.
- **tauBetaSigma**: Effect sizes $\frac{(τβ)_{ij}}{σ}, i = 1, \ldots, a, j = 1, \ldots, b$, where $a = b = 2$ and $σ$ is the standard deviation of random error. Rows are for factor 1 and columns are for factor 2. Note that $\sum_{i=1}^{a} (τβ)_{ij} = \sum_{j=1}^{b} (τβ)_{ij} = 0$. We can get $(τβ)_{11} = \theta$, $(τβ)_{12} = -θ$, $(τβ)_{21} = -θ$, $(τβ)_{22} = θ$. So $\text{tauBetaSigma}=\frac{θ}{σ}$
- **alpha**: family-wise type I error rate.
- **nTests**: integer. For high-throughput omics study, we perform two-way ANOVA for each of ‘nTests’ probes. We use Bonferroni correction to control for family-wise type I error rate. That is, for each probe, type I error rate would be $\alpha/nTests$.
- **verbose**: logical. Indicating if intermediate results should be printed out.

Details

We assume the following model:

$$y_{ijk} = µ + τ_i + β_j + (τβ)_{ij} + ε_{ijk},$$

where $i = 1, \ldots, a, j = 1, \ldots, b, k = 1, \ldots, n, \sum_{i=1}^{a} τ_i = 0, \sum_{j=1}^{b} β_j = 0, \sum_{i=1}^{a} (τβ)_{ij} = 0, \sum_{j=1}^{b} (τβ)_{ij} = 0,$$ and $ε_{ijk} \overset{i.i.d}{\sim} N(0, σ^2)$.

The group means are

$$µ_{ij} = µ + τ_i + β_j + (τβ)_{ij}, i = 1, \ldots, a, j = 1, \ldots, b.$$  

Note that $µ = \sum_{i=1}^{a} \sum_{j=1}^{b} µ_{ij} / (ab), τ_i = \sum_{j=1}^{b} µ_{ij} / b - µ, \text{and } β_j = \sum_{i=1}^{a} µ_{ij} / a - µ.$

The null hypothesis $H_0$: all $(τβ)_{ij}, i = 1, \ldots, a, j = 1, \ldots, b$ are equal to zero. The alternative hypothesis $H_a$: at least one $(τβ)_{ij}$ is different from zero.

The F test statistic is

$$F = MS_{AB}/MS_E \overset{H_a}{\sim} F_{(a-1)(b-1),ab(n-1),ncp},$$

where $ncp$ is the non-centrality parameter of the F test statistic:

$$ncp = n \sum_{i=1}^{a} \sum_{j=1}^{b} \left[\frac{(τβ)_{ij}}{σ}\right]^2.$$

For the scenario $a = b = 2$, we have $(τβ)_{11} = \theta$, $(τβ)_{12} = -θ$, $(τβ)_{21} = -θ$, $(τβ)_{22} = θ$. Hence, the non-centrality parameter can be simplified to

$$ncp = 4n \left(\frac{θ}{σ}\right)^2.$$

The power for testing the null hypothesis $H_0$ versus the alternative hypothesis $H_a$ is

$$power = Pr(F > F_0|H_a),$$

where the rejection region boundary $F_0$ satisfies:

$$Pr(F > F_0|H_0) = \alpha/nTests.$$
powerLogisticBin

Value

A list with 5 elements:

- **power**: the power of the two-way ANOVA test
- **df1**: the first degree of freedom of the F test statistic \((df_1=(a-1)(b-1))\)
- **df2**: the second degree of freedom of the F test statistic \((df_1=ab(n-1))\)
- **F0**: the rejection region boundary
- **ncp**: the non-centrality parameter

Author(s)

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References


Examples

\[
\begin{align*}
n &= 25 \\
\text{tauBetaSigma} &= 0.3
\end{align*}
\]

\[
\text{# power} = 0.8437275
\]

\[
\text{res2} = \text{powerInteract2by2}(n = n, \text{tauBetaSigma} = \text{tauBetaSigma},
\text{alpha} = 0.05, nTests = 1, \text{verbose} = \text{TRUE})
\]

---

**powerLogisticBin**: **Calculating power for simple logistic regression with binary predictor**

Description

Calculating power for simple logistic regression with binary predictor.

Usage

\[
\text{powerLogisticBin}(n, p1, p2, B, \text{alpha} = 0.05)
\]
Arguments

- **n**: total number of sample size.
- **p1**: \( pr(diseased|X = 0) \), i.e. the event rate at \( X = 0 \) in logistic regression \( \text{logit}(p) = a + bX \), where \( X \) is the binary predictor.
- **p2**: \( pr(diseased|X = 1) \), the event rate at \( X = 1 \) in logistic regression \( \text{logit}(p) = a + bX \), where \( X \) is the binary predictor.
- **B**: \( pr(X = 1) \), i.e. proportion of the sample with \( X = 1 \)
- **alpha**: Type I error rate.

Details

The logistic regression model is

\[
\log(p/(1-p)) = \beta_0 + \beta_1 X
\]

where \( p = \text{prob}(Y = 1) \), \( X \) is the binary predictor, \( p_1 = pr(diseased|X = 0) \), \( p_2 = pr(diseased|X = 1) \), \( B = pr(X = 1) \), and \( p = (1 - B)p_1 + Bp_2 \). The sample size formula we used for testing if \( \beta_1 = 0 \), is Formula (2) in Hsieh et al. (1998):

\[
n = (Z_{\alpha/2}[p(1-p)/B]^{1/2} + Z_{\text{power}}[p_1(1-p_1) + p_2(1-p_2)(1-B)/B]^{1/2})^2/[(p_1-p_2)^2(1-B)]]
\]

where \( n \) is the required total sample size and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

Value

Estimated power.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \text{alpha}=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

`powerLogisticBin`
## Description
Calculating power for simple logistic regression with continuous predictor.

### Usage
```
powerLogisticCon(n, p1, OR, alpha = 0.05)
```

### Arguments
- `n`: total sample size.
- `p1`: the event rate at the mean of the continuous predictor $X$ in logistic regression $\text{logit}(p) = a + bX$.
- `OR`: Expected odds ratio. $\log(OR)$ is the change in log odds for the difference between at the mean of $X$ and at one SD above the mean.
- `alpha`: Type I error rate.

### Details
The logistic regression mode is

$$\log(p/(1-p)) = \beta_0 + \beta_1 X$$

where $p = \text{prob}(Y = 1)$, $X$ is the continuous predictor, and $\log(OR)$ is the the change in log odds for the difference between at the mean of $X$ and at one SD above the mean. The sample size formula we used for testing if $\beta_1 = 0$ or equivalently $OR = 1$, is Formula (1) in Hsieh et al. (1998):

$$n = (Z_{1-\alpha/2} + Z_{\text{power}})^2 / [p_1(1-p_1)[\log(OR)]^2]$$

where $n$ is the required total sample size, $OR$ is the odds ratio to be tested, $p_1$ is the event rate at the mean of the predictor $X$, and $Z_u$ is the $u$-th percentile of the standard normal distribution.

### Value
Estimated power.
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \alpha = 0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

SSizeLogisticCon

Examples

```r
## Example in Table II Design (Balanced design (1)) of Hsieh et al. (1998 )
## the power is 0.95
powerLogisticCon(n=317, p1=0.5, OR=exp(0.405), alpha=0.05)
```

---

**powerLong**

*Power calculation for longitudinal study with 2 time point*

Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage

```r
powerLong(es, 
  n, 
  rho = 0.5, 
  alpha = 0.05)
```

Arguments

- `es`  
  effect size of the difference of mean change.
- `n`  
  sample size per group.
- `rho`  
  correlation coefficient between baseline and follow-up values within a treatment group.
- `alpha`  
  Type I error rate.
The power formula is based on Equation 8.31 on page 336 of Rosner (2006).

\[
\text{power} = \Phi \left( -Z_{1-\alpha/2} + \frac{\delta \sqrt{n}}{\sigma_d \sqrt{2}} \right)
\]

where \(\sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2\), \(\delta = |\mu_1 - \mu_2|\), \(\mu_1\) is the mean change over time \(t\) in group 1, \(\mu_2\) is the mean change over time \(t\) in group 2, \(\sigma_1^2\) is the variance of baseline values within a treatment group, \(\sigma_2^2\) is the variance of follow-up values within a treatment group, \(\rho\) is the correlation coefficient between baseline and follow-up values within a treatment group, and \(Z_u\) is the \(u\)-th percentile of the standard normal distribution.

We wish to test \(\mu_1 = \mu_2\).

When \(\sigma_1 = \sigma_2 = \sigma\), then formula reduces to

\[
\text{power} = \Phi \left( -Z_{1-\alpha/2} + \frac{|d| \sqrt{n}}{2\sqrt{1 - \rho}} \right)
\]

where \(d = \delta / \sigma\).

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \(\text{alpha}=0.10\) to obtain one-sided test at 5% significance level.

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See Also

`ssLong`, `ssLongFull`, `powerLongFull`.

# Example 8.34 on page 336 of Rosner (2006)
# power=0.75
powerLong(es=5/15, n=75, rho=0.7, alpha=0.05)
Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points.

Usage

\[ \text{powerLong.multiTime(es, m, nn, sx2, rho = 0.5, alpha = 0.05)} \]

Arguments

- \( es \) : effect size
- \( m \) : number of subjects
- \( nn \) : number of observations per subject
- \( sx2 \) : within subject variance
- \( rho \) : within subject correlation
- \( alpha \) : type I error rate

Details

We are interested in comparing the slopes of the 2 groups \( A \) and \( B \):

\[ \beta_{1A} = \beta_{1B} \]

where

\[ Y_{ijA} = \beta_{0A} + \beta_{1A}x_{jA} + \epsilon_{ijA}, j = 1, \ldots, nn; i = 1, \ldots, m \]

and

\[ Y_{ijB} = \beta_{0B} + \beta_{1B}x_{jB} + \epsilon_{ijB}, j = 1, \ldots, nn; i = 1, \ldots, m \]

The power calculation formula is (Equation on page 30 of Diggle et al. (1994)):

\[ \text{power} = \Phi \left[ -z_{1-\alpha} + \sqrt{\frac{\text{mnns}_2 \text{es}^2}{2(1 - \rho)}} \right] \]

where \( es = d/\sigma \), \( d \) is the meaningful difference of interest, \( sigma^2 \) is the variance of the random error, \( \rho \) is the within-subject correlation, and \( s_x^2 \) is the within-subject variance.

Value

\( \text{power} \)
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

sslLong.multiTime

Examples

# power=0.8
powerLong.multiTime(es=0.5/10, m=196, nn=3, sx2=4.22, rho = 0.5, alpha = 0.05)

powerLongFull

Power calculation for longitudinal study with 2 time point

Description

Power calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage

powerLongFull(delta, 
               sigma1, 
               sigma2, 
               n, 
               rho = 0.5, 
               alpha = 0.05)

Arguments

delta absolute difference of the mean changes between the two groups: $\delta = |\mu_1 - \mu_2|$ where $\mu_1$ is the mean change over time $t$ in group 1, $\mu_2$ is the mean change over time $t$ in group 2.

sigma1 the standard deviation of baseline values within a treatment group

sigma2 the standard deviation of follow-up values within a treatment group

n sample size per group
rho correlation coefficient between baseline and follow-up values within a treatment group.

alpha Type I error rate.

Details

The power formula is based on Equation 8.31 on page 336 of Rosner (2006).

\[
\text{power} = \Phi \left( -Z_{1-\alpha/2} + \frac{\delta \sqrt{n}}{\sigma_d \sqrt{2}} \right)
\]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2 \), \( \delta = |\mu_1 - \mu_2| \), \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

Value

power for testing for difference of mean changes.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \text{alpha}=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

ssLong, ssLongFull, powerLong.

Examples

```
# Example 8.33 on page 336 of Rosner (2006)
# power=0.80
powerLongFull(delta=5, sigma1=15, sigma2=15, n=85, rho=0.7, alpha=0.05)
```
powerMediation.Sobel

Power for testing mediation effect (Sobel’s test)

Description

Calculate power for testing mediation effect based on Sobel’s test.

Usage

```r
powerMediation.Sobel(n,
  theta.1a,
  lambda.a,
  sigma.x,
  sigma.m,
  sigma.epsilon,
  alpha = 0.05,
  verbose = TRUE)
```

Arguments

- `n` sample size.
- `theta.1a` regression coefficient for the predictor in the linear regression linking the predictor `x` to the mediator `m` \( (m_i = \theta_0 + \theta_{1a}x_i + e_i, e_i \sim N(0, \sigma^2_e)) \).
- `lambda.a` regression coefficient for the mediator in the linear regression linking the predictor `x` and the mediator `m` to the outcome `y` \( (y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)) \).
- `sigma.x` standard deviation of the predictor.
- `sigma.m` standard deviation of the mediator.
- `sigma.epsilon` standard deviation of the random error term in the linear regression linking the predictor `x` and the mediator `m` to the outcome `y` \( (y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)) \).
- `alpha` type I error.
- `verbose` logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \( \theta_1 \lambda = 0 \) versus the alternative hypothesis \( \theta_{1a} \lambda_a \neq 0 \) for the linear regressions:

- \( m_i = \theta_0 + \theta_{1a}x_i + e_i, e_i \sim N(0, \sigma^2_e) \)
- \( y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon) \)

Test statistic is based on Sobel’s (1982) test:

\[
Z = \frac{\hat{\theta}_{1a} \hat{\lambda}_a}{\hat{\sigma}_{\theta_{1a} \lambda_a}}
\]
where $\hat{\sigma}_{\theta_1 \lambda a}$ is the estimated standard deviation of the estimate $\hat{\theta}_{1a} \hat{\lambda}_a$ using multivariate delta method:

$$
\sigma_{\theta_1 \lambda a} = \sqrt{\theta_{1a}^2 \sigma_{\hat{\theta}_{1a}}^2 + \lambda_{aa}^2 \sigma_{\hat{\lambda}_{1a}}^2}
$$

and $\sigma_{\theta_{1a}}^2 = \sigma_e^2 / (n \sigma_x^2)$ is the variance of the estimate $\hat{\theta}_{1a}$, and $\sigma_{\lambda_a}^2 = \sigma_e^2 / (n \sigma_m^2 (1 - \rho_{mx}^2))$ is the variance of the estimate $\hat{\lambda}_a$. $\sigma_m^2$ is the variance of the mediator $m_i$.

From the linear regression $m_i = \theta_0 + \theta_{1a} x_i + e_i$, we have the relationship $\sigma_e^2 = \sigma_m^2 (1 - \rho_{mx}^2)$. Hence, we can simply the variance $\sigma_{\theta_1 \lambda a}$ to

$$
\sigma_{\theta_1 \lambda a} = \sqrt{\theta_{1a}^2 \frac{\sigma_e^2}{n \sigma_m^2 (1 - \rho_{mx}^2)} + \lambda_{aa}^2 \frac{\sigma_m^2 (1 - \rho_{mx}^2)}{n \sigma_x^2}}
$$

**Value**

- **power**: power of the test for the parameter $\theta_{1a} \lambda_a$
- **delta**: $\theta_{1a} \lambda / (sd(\hat{\theta}_{1a}) \cdot sd(\hat{\lambda}_a))$

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $\text{alpha}=0.10$ to obtain one-sided test at 5% significance level.

**Author(s)**

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**References**


**See Also**

- `ssMediation.Sobel`
- `testMediation.Sobel`

**Examples**

```r
powerMediation.Sobel(n=248, theta.1a=0.1701, lambda.a=0.1998, 
                      sigma.x=0.57, sigma.m=0.61, sigma.epsilon=0.2, 
                      alpha = 0.05, verbose = TRUE)
```
powerMediation.VSMc

powerMediation.VSMc  Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate Power for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

powerMediation.VSMc(n,
  b2,
  sigma.m,
  sigma.e,
  corr.xm,
  alpha = 0.05,
  verbose = TRUE)

Arguments

n  sample size.

b2  regression coefficient for the mediator m in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.

sigma.m  standard deviation of the mediator.

sigma.e  standard deviation of the random error term in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$.

corr.xm  correlation between the predictor $x$ and the mediator $m$.

alpha  type I error rate.

verbose  logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

The reduced model is

$$y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.
Value

power  

power for testing if $b_2 = 0$.

delta  

$b_2 \sigma_m \sqrt{1 - \rho_{xm}^2} / \sigma_e$, where $\sigma_m$ is the standard deviation of the mediator $m$, $\rho_{xm}$ is the correlation between the predictor $x$ and the mediator $m$, and $\sigma_e$ is the standard deviation of the random error term in the linear regression.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $alpha=0.10$ to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

minEffect.VSMc, ssMediation.VSMc

Examples

```r
# example in section 3 (page 544) of Vittinghoff et al. (2009).
# power=0.8
powerMediation.VSMc(n = 863, b2 = 0.1, sigma.m = 1, sigma.e = 1, 
corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
```

powerMediation.VSMc.cox

*Power for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method*

Description

Calculate Power for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.
Usage

`powerMediation.VSMc.cox(n, b2, sigma.m, psi, corr.xm, alpha = 0.05, verbose = TRUE)`

Arguments

- `n`: sample size.
- `b2`: regression coefficient for the mediator `m` in the cox regression \( \log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i \), where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.
- `sigma.m`: standard deviation of the mediator.
- `psi`: the probability that an observation is uncensored, so that the number of event \( d = n \times \psi \), where \( n \) is the sample size.
- `corr.xm`: correlation between the predictor `x` and the mediator `m`.
- `alpha`: type I error rate.
- `verbose`: logical. `TRUE` means printing power; `FALSE` means not printing power.

Details

The power is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the cox regressions:

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i
\]

where \( \lambda \) is the hazard function and \( \lambda_0 \) is the baseline hazard function.

Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \).

The full model is

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i
\]

The reduced model is

\[
\log(\lambda) = \log(\lambda_0) + b_1x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining `corr.xm` as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- `power`: power for testing if \( b_2 = 0 \).
- `delta`: \( b_2\sigma_m \sqrt{(1 - \rho_{xm}^2)\psi} \)

, where \( \sigma_m \) is the standard deviation of the mediator `m`, \( \rho_{xm} \) is the correlation between the predictor `x` and the mediator `m`, and \( \psi \) is the probability that an observation is uncensored, so that the number of event \( d = n \times \psi \), where \( n \) is the sample size.
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

`minEffect.VSMc.cox`, `ssMediation.VSMc.cox`

Examples

```r
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# power = 0.7999916
powerMediation.VSMc.cox(n = 1399, b2 = log(1.5),
                       sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
                       alpha = 0.05, verbose = TRUE)
```

powerMediation.VSMc.logistic

*Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method*

Description

Calculate Power for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch's (2009) method.

Usage

```r
powerMediation.VSMc.logistic(n, b2, sigma.m, p, corr.xm, alpha = 0.05, verbose = TRUE)
```
Arguments

n  sample size.
b2  regression coefficient for the mediator m in the logistic regression \( \log(p_i/(1 - p_i)) = b0 + b1x_i + b2m_i \).
sigma.m  standard deviation of the mediator.
p  the marginal prevalence of the outcome.
corr.xm  correlation between the predictor x and the mediator m.
alpha  type I error rate.
verbose  logical. TRUE means printing power; FALSE means not printing power.

Details

The power is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the logistic regressions:

\[
\log(p_i/(1 - p_i)) = b0 + b1x_i + b2m_i
\]

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \).

The full model is

\[
\log(p_i/(1 - p_i)) = b0 + b1x_i + b2m_i
\]

The reduced model is

\[
\log(p_i/(1 - p_i)) = b0 + b1x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( corr.xm \) as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

power  power for testing if \( b_2 = 0 \).
delta  \( b_2\sigma_m\sqrt{(1 - \rho_{xm}^2)p(1 - p)} \), where \( \sigma_m \) is the standard deviation of the mediator m, \( \rho_{xm} \) is the correlation between the predictor x and the mediator m, and \( p \) is the marginal prevalence of the outcome.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( alpha=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@ching.hvd.edu>

References

powerMediation.VSMc.poisson

See Also

minEffect.VSMc.logistic, ssMediation.VSMc.logistic

Examples

# example in section 4 (page 545) of Vittinghoff et al. (2009).
# power = 0.8005793
powerMediation.VSMc.logistic(n = 255, b2 = log(1.5), sigma.m = 1,
p = 0.5, corr.xm = 0.5, alpha = 0.05, verbose = TRUE)

Usage

powerMediation.VSMc.poisson(n, 
b2, 
sigma.m, 
EY, 
corr.xm, 
alpha = 0.05, 
verbose = TRUE)

Arguments

n sample size.
b2 regression coefficient for the mediator $m$ in the poisson regression $\log(E(Y_i)) = b0 + b1x_i + b2m_i$.
sigma.m standard deviation of the mediator.
EY the marginal mean of the outcome.
corr.xm correlation between the predictor $x$ and the mediator $m$.
alpha type I error rate.
verbose logical. TRUE means printing power; FALSE means not printing power.
Details

The power is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the poisson regressions:

$$\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$$

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$.

The full model is

$$\log(E(Y_i)) = b_0 + b_1x_i + b_2m_i$$

The reduced model is

$$\log(E(Y_i)) = b_0 + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- **power**: power for testing if $b_2 = 0$.
- **delta**: $b_2\sigma_m\sqrt{(1 - \rho^2_{xm})EY}$, where $\sigma_m$ is the standard deviation of the mediator $m$, $\rho_{xm}$ is the correlation between the predictor $x$ and the mediator $m$, and $EY$ is the marginal mean of the outcome.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $alpha=0.10$ to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

- `minEffect.VSMc.poisson`
- `ssMediation.VSMc.poisson`

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# power = 0.7998578
powerMediation.VSMc.poisson(n = 1239, b2 = log(1.35),
    sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
    alpha = 0.05, verbose = TRUE)
```
Power calculation for simple Poisson regression

Usage

powerPoisson(
  beta0,  
  beta1,  
  mu.x1,  
  sigma2.x1,  
  mu.T = 1,  
  phi = 1,  
  alpha = 0.05,  
  N = 50)

Arguments

beta0 intercept
beta1 slope
mu.x1 mean of the predictor
sigma2.x1 variance of the predictor
mu.T mean exposure time
phi a measure of over-dispersion
alpha type I error rate
N total sample size

Details

The simple Poisson regression has the following form:

\[ \Pr(Y_i = y_i | \mu_i, t_i) = \exp(-\mu_i t_i) (\mu_i t_i)^{y_i} / (y_i!) \]

where

\[ \mu_i = \exp(\beta_0 + \beta_1 x_{1i}) \]

We are interested in testing the null hypothesis \( \beta_1 = 0 \) versus the alternative hypothesis \( \beta_1 = \theta_1 \). Assume \( x_1 \) is normally distributed with mean \( \mu_{x_1} \) and variance \( \sigma_{x_1}^2 \). The sample size calculation formula derived by Signorini (1991) is

\[
N = \phi \left[ z_{1-\alpha/2} \sqrt{V(b_1 | \beta_1 = 0)} + z_{\text{power}} \sqrt{V(b_1 | \beta_1 = \theta_1)} \right]^2 \frac{\mu_T \exp(\beta_0) \theta_1^2}{\mu_T \exp(\beta_0)}
\]
where $\phi$ is the over-dispersion parameter ($= \text{var}(y_i)/\text{mean}(y_i)$), $\alpha$ is the type I error rate, $b_1$ is the estimate of the slope $\beta_1$, $\beta_0$ is the intercept, $\mu_T$ is the mean exposure time, $z_a$ is the $100 \cdot a$-th lower percentile of the standard normal distribution, and $V(b_1|\beta_1 = \theta)$ is the variance of the estimate $b_1$ given the true slope $\beta_1 = \theta$.

The variances are

$$V(b_1|\beta_1 = 0) = \frac{1}{\sigma_{x1}^2}$$

and

$$V(b_1|\beta_1 = \theta_1) = \frac{1}{\sigma_{x1}^2} \exp \left[-\left(\theta_1 \mu_{x1} + \theta_1^2 \sigma_{x1}^2 / 2\right)\right]$$

Value

power

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $\alpha = 0.10$ to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

See Also as sizePoisson

Examples

```r
# power = 0.8090542
print(powerPoisson(
    beta0 = 0.1,
    beta1 = 0.5,
    mu.x1 = 0,
    sigma2.x1 = 1,
    mu.T = 1,
    phi = 1,
    alpha = 0.05,
    N = 28))
```
Sample size calculation for simple Poisson regression

Description
Sample size calculation for simple Poisson regression. Assume the predictor is normally distributed. Two-sided test is used.

Usage
sizePoisson(
  beta0,
  beta1,
  mu.x1,
  sigma2.x1,
  mu.T = 1,
  phi = 1,
  alpha = 0.05,
  power = 0.8)

Arguments
- beta0: intercept
- beta1: slope
- mu.x1: mean of the predictor
- sigma2.x1: variance of the predictor
- mu.T: mean exposure time
- phi: a measure of over-dispersion
- alpha: type I error rate
- power: power

Details
The simple Poisson regression has the following form:

\[ \Pr(Y_i = y_i | \mu_i, t_i) = \exp(-\mu_i t_i) (\mu_i t_i)^{y_i} / (y_i!) \]

where

\[ \mu_i = \exp(\beta_0 + \beta_1 x_{1i}) \]

We are interested in testing the null hypothesis \( \beta_1 = 0 \) versus the alternative hypothesis \( \beta_1 = \theta_1 \). Assume \( x_1 \) is normally distributed with mean \( \mu_x \) and variance \( \sigma^2 \). The sample size calculation formula derived by Signorini (1991) is

\[ N = \phi \left[ z_{1-\alpha/2} \sqrt{V(b_1 \beta_1 = 0)} + z_{\text{power}} \sqrt{V(b_1 \beta_1 = \theta_1)} \right]^2 \mu_T \exp(\beta_0) \theta^2_1 \]
where $\phi$ is the over-dispersion parameter (\(= \text{var}(y_i)/\text{mean}(y_i)\)), $\alpha$ is the type I error rate, $b_1$ is the estimate of the slope $\beta_1$, $\beta_0$ is the intercept, $\mu_T$ is the mean exposure time, $z_a$ is the $100 \times a$-th lower percentile of the standard normal distribution, and $V(b_1 | \beta_1 = \theta)$ is the variance of the estimate $b_1$ given the true slope $\beta_1 = \theta$.

The variances are

$$V(b_1 | \beta_1 = 0) = \frac{1}{\sigma_{x_1}^2}$$

and

$$V(b_1 | \beta_1 = \theta_1) = \frac{1}{\sigma_{x_1}^2} \exp \left[ - \left( \theta_1 \mu x_1 + \theta_1^2 \sigma_{x_1}^2 / 2 \right) \right]$$

**Value**

total sample size

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set `alpha=0.10` to obtain one-sided test at 5% significance level.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

See Also as `powerPoisson`

**Examples**

```r
# sample size = 28
print(sizePoisson(
  beta0 = 0.1,
  beta1 = 0.5,
  mu.x1 = 0,
  sigma2.x1 = 1,
  mu.T = 1,
  phi = 1,
  alpha = 0.05,
  power = 0.8))
```
ss.SLR

Sample size for testing slope for simple linear regression

Description

Calculate sample size for testing slope for simple linear regression.

Usage

ss.SLR(power,
lambda.a,
sigma.x,
sigma.y,
n.lower = 2.01,
n.upper = 1e+30,
alpha = 0.05,
verbose = TRUE)

Arguments

- **power**: power for testing if \( \lambda = 0 \) for the simple linear regression \( y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2) \).
- **lambda.a**: regression coefficient in the simple linear regression \( y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2) \).
- **sigma.x**: standard deviation of the predictor \( sd(x) \).
- **sigma.y**: marginal standard deviation of the outcome \( sd(y) \). (not the marginal standard deviation \( sd(y|x) \))
- **n.lower**: lower bound for the sample size.
- **n.upper**: upper bound for the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis \( \lambda = 0 \) versus the alternative hypothesis \( \lambda \neq 0 \) for the simple linear regressions:

\[
y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma_e^2)
\]

Value

- **n**: sample size.
- **res.uniopt**: results of optimization to find the optimal sample size.
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

minEffect.SLR, power.SLR, power.SLR.rho, ss.SLR.rho.

Examples

```r
ss.SLR(power=0.8, lambda.a=0.8, sigma.x=0.2, sigma.y=0.5,
    alpha = 0.05, verbose = TRUE)
```

<table>
<thead>
<tr>
<th>ss.SLR.rho</th>
<th>Sample size for testing slope for simple linear regression based on R2</th>
</tr>
</thead>
</table>

Description

Calculate sample size for testing slope for simple linear regression based on R2.

Usage

```r
ss.SLR.rho(power, rho2, n.lower = 2.01, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)
```

Arguments

- **power**: power.
- **rho2**: square of the correlation between the outcome and the predictor.
- **n.lower**: lower bound of the sample size.
- **n.upper**: upper bound of the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing sample size; FALSE means not printing sample size.
Details
The test is for testing the null hypothesis $\lambda = 0$ versus the alternative hypothesis $\lambda \neq 0$ for the simple linear regressions:

$$y_i = \gamma + \lambda x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

Value
n  sample size.
res.unirroot  results of optimization to find the optimal sample size.

Note
The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also
minEffect.SLR, power.SLR, power.SLR.rho, ss.SLR.

Examples
ss.SLR.rho(power=0.8, rho2=0.6, alpha = 0.05, verbose = TRUE)

SSizeLogisticBin

Calculating sample size for simple logistic regression with binary predictor

Description
Calculating sample size for simple logistic regression with binary predictor.

Usage
SSizeLogisticBin(p1,
                  p2,
                  B,
                  alpha = 0.05,
                  power = 0.8)
SSizeLogisticBin

Arguments

- **p1**: \( \text{pr}(\text{diseased} | X = 0) \), i.e. the event rate at \( X = 0 \) in logistic regression \( \text{logit}(p) = a + bX \), where \( X \) is the binary predictor.
- **p2**: \( \text{pr}(\text{diseased} | X = 1) \), the event rate at \( X = 1 \) in logistic regression \( \text{logit}(p) = a + bX \), where \( X \) is the binary predictor.
- **B**: \( \text{pr}(X = 1) \), i.e. proportion of the sample with \( X = 1 \)
- **alpha**: Type I error rate.
- **power**: power for testing if the odds ratio is equal to one.

Details

The logistic regression mode is

\[
\log(p/(1-p)) = \beta_0 + \beta_1 X
\]

where \( p = \text{prob}(Y = 1) \), \( X \) is the binary predictor, \( p_1 = \text{pr}(\text{diseased} | X = 0) \), \( p_2 = \text{pr}(\text{diseased} | X = 1) \), \( B = \text{pr}(X = 1) \), and \( p = (1-B)p_1 + Bp_2 \). The sample size formula we used for testing if \( \beta_1 = 0 \), is Formula (2) in Hsieh et al. (1998):

\[
n = \left( Z_{1-\alpha/2} \frac{p(1-p)}{B} \right)^{1/2} + Z_{\text{power}} \left[ p_1(1-p_1) + p_2(1-p_2)(1-B)/B \right]^{1/2} / \left[ (p_1-p_2)^2(1-B) \right]^{1/2}
\]

where \( n \) is the required total sample size and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

Value

total sample size required.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \text{alpha}=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

powerLogisticBin
Examples

## Example in Table I Design (Balanced design with high event rates)
## of Hsieh et al. (1998 )
## the sample size is 1281

SSizeLogisticBin(p1 = 0.4, p2 = 0.5, B = 0.5, alpha = 0.05, power = 0.95)

SSizeLogisticCon  Calculating sample size for simple logistic regression with continuous predictor

Description

Calculating sample size for simple logistic regression with continuous predictor.

Usage

SSizeLogisticCon(p1, OR, 
alpha = 0.05, 
power = 0.8)

Arguments

p1
the event rate at the mean of the continuous predictor \( X \) in logistic regression
\[
\logit(p) = a + bX
\]

OR  
Expected odds ratio. \( \log(OR) \) is the change in log odds for the difference between at the mean of \( X \) and at one SD above the mean.

alpha  
Type I error rate.

power  
power for testing if the odds ratio is equal to one.

Details

The logistic regression mode is

\[
\log(p/(1-p)) = \beta_0 + \beta_1 X
\]

where \( p = \text{prob}(Y = 1) \), \( X \) is the continuous predictor, and \( \log(OR) \) is the change in log odds for the difference between at the mean of \( X \) and at one SD above the mean. The sample size formula we used for testing if \( \beta_1 = 0 \) or equivalently \( OR = 1 \), is Formula (1) in Hsieh et al. (1998):

\[
n = \left( Z_{1-\alpha/2} + Z_{\text{power}} \right)^2 / \left[ p_1 (1 - p_1) [\log(OR)]^2 \right]
\]

where \( n \) is the required total sample size, \( OR \) is the odds ratio to be tested, \( p_1 \) is the event rate at the mean of the predictor \( X \), and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

Value

total sample size required.
Note
The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)
Weiliang Qiu <stwxq@channing.harvard.edu>

References

See Also
powerLogisticCon

Examples
## Example in Table II Design (Balanced design (1)) of Hsieh et al. (1998 )
## the sample size is 317
SSizeLogisticCon(p1 = 0.5, OR = exp(0.405), alpha = 0.05, power = 0.95)

### ssLong
Sample size calculation for longitudinal study with 2 time point

Description
Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage
ssLong(es, 
  rho = 0.5, 
  alpha = 0.05, 
  power = 0.8)

Arguments
es
  effect size of the difference of mean change.
rho
  correlation coefficient between baseline and follow-up values within a treatment group.
alpha
  Type I error rate.
power
  power for testing for difference of mean changes.
Details

The sample size formula is based on Equation 8.30 on page 335 of Rosner (2006).

\[
n = \frac{2\sigma_d^2(Z_{1-\alpha/2} + Z_{\text{power}})^2}{\delta^2}
\]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 \), \( \delta = |\mu_1 - \mu_2| \), \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

When \( \sigma_1 = \sigma_2 = \sigma \), then formula reduces to

\[
n = \frac{4(1 - \rho)(Z_{1-\alpha/2} + Z_\beta)^2}{\delta^2}
\]

where \( d = \delta/\sigma \).

Value

required sample size per group

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \text{alpha}=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

ssLongFull, powerLong, powerLongFull.

Examples

# Example 8.33 on page 336 of Rosner (2006)
# n=85
ssLong(es=5/15, rho=0.7, alpha=0.05, power=0.8)
Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with more than 2 time points.

Usage

ssLong.multiTime(es, power, nn, sx2, rho = 0.5, alpha = 0.05)

Arguments

es effect size
power power
nn number of observations per subject
sx2 within subject variance
rho within subject correlation
alpha type I error rate

Details

We are interested in comparing the slopes of the 2 groups A and B:

$$\beta_{1A} = \beta_{1B}$$

where

$$Y_{ijA} = \beta_{0A} + \beta_{1A}x_{jA} + \epsilon_{ijA}, j = 1, \ldots, nn; i = 1, \ldots, m$$

and

$$Y_{ijB} = \beta_{0B} + \beta_{1B}x_{jB} + \epsilon_{ijB}, j = 1, \ldots, nn; i = 1, \ldots, m$$

The sample size calculation formula is (Equation on page 30 of Diggle et al. (1994)):

$$m = \frac{2(Z_{1-\alpha} + z_{\text{power}})^2(1 - \rho)}{nn\sigma^2 es^2}$$

where $es = d/\sigma$, $d$ is the meanful differnce of interest, $\sigma^2$ is the variance of the random error, $\rho$ is the within-subject correlation, and $s^2_{\epsilon}$ is the within-subject variance.

Value

subject per group
Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

powerLong.multiTime

Examples

# subject per group = 196
ssLong.multiTime(es=0.5/10, power=0.8, nn=3, sx2=4.22, rho = 0.5, alpha=0.05)

ssLongFull

Sample size calculation for longitudinal study with 2 time point

Description

Sample size calculation for testing if mean changes for 2 groups are the same or not for longitudinal study with 2 time point.

Usage

ssLongFull(delta, 
    sigma1, 
    sigma2, 
    rho = 0.5, 
    alpha = 0.05, 
    power = 0.8)

Arguments

delta absolute difference of the mean changes between the two groups: $\delta = |\mu_1 - \mu_2|$
where $\mu_1$ is the mean change over time $t$ in group 1, $\mu_2$ is the mean change over time $t$ in group 2.

sigma1 the standard deviation of baseline values within a treatment group

sigma2 the standard deviation of follow-up values within a treatment group
rho  correlation coefficient between baseline and follow-up values within a treatment group.
alpha  Type I error rate
power  power for testing for difference of mean changes.

Details

The sample size formula is based on Equation 8.30 on page 335 of Rosner (2006).

\[ n = \frac{2\sigma_d^2(Z_{1-\alpha/2} + Z_{\text{power}})^2}{\delta^2} \]

where \( \sigma_d = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 \), \( \delta = |\mu_1 - \mu_2| \), \( \mu_1 \) is the mean change over time \( t \) in group 1, \( \mu_2 \) is the mean change over time \( t \) in group 2, \( \sigma_1^2 \) is the variance of baseline values within a treatment group, \( \sigma_2^2 \) is the variance of follow-up values within a treatment group, \( \rho \) is the correlation coefficient between baseline and follow-up values within a treatment group, and \( Z_u \) is the \( u \)-th percentile of the standard normal distribution.

We wish to test \( \mu_1 = \mu_2 \).

Value

required sample size per group

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

ssLong, powerLong, powerLongFull.

Examples

# Example 8.33 on page 336 of Rosner (2006)
# n=85
ssLongFull(delta=5, sigma1=15, sigma2=15, rho=0.7, alpha=0.05, power=0.8)
ssMediation.Sobel  Sample size for testing mediation effectd (Sobel's test)

Description

Calculate sample size for testing mediation effect based on Sobel’s test.

Usage

ssMediation.Sobel(power, theta.1a, lambda.a, sigma.x, sigma.m, sigma.epsilon, n.lower = 1, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)

Arguments

- **power**: power of the test.
- **theta.1a**: regression coefficient for the predictor in the linear regression linking the predictor \(x\) to the mediator \(m\) (\(m_i = \theta_0 + \theta_{1a}x_i + e_i, e_i \sim N(0, \sigma^2_e)\)).
- **lambda.a**: regression coefficient for the mediator in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) (\(y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)\)).
- **sigma.x**: standard deviation of the predictor.
- **sigma.m**: standard deviation of the mediator.
- **sigma.epsilon**: standard deviation of the random error term in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) (\(y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_\epsilon)\)).
- **n.lower**: lower bound of the sample size.
- **n.upper**: upper bound of the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing power; FALSE means not printing power.

Details

The sample size is for testing the null hypothesis \(\theta_1 \lambda = 0\) versus the alternative hypothesis \(\theta_{1a} \lambda_a \neq 0\) for the linear regressions:

\[ m_i = \theta_0 + \theta_{1a} x_i + e_i, e_i \sim N(0, \sigma^2_e) \]
\[ y_i = \gamma + \lambda_a m_i + \lambda_2 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2) \]

Test statistic is based on Sobel’s (1982) test:

\[ Z = \frac{\hat{\theta}_{1a} \hat{\lambda}_a}{\hat{\sigma}_{\theta_{1a}\lambda_a}} \]

where \( \hat{\sigma}_{\theta_{1a}\lambda_a} \) is the estimated standard deviation of the estimate \( \hat{\theta}_{1a} \hat{\lambda}_a \) using multivariate delta method:

\[ \hat{\sigma}_{\theta_{1a}\lambda_a} = \sqrt{\sigma^2_{\theta_{1a}} + \lambda_a^2 \sigma^2_{\lambda_a}} \]

and \( \sigma^2_{\theta_{1a}} = \sigma^2_{\epsilon}/(n \sigma^2_x) \) is the variance of the estimate \( \hat{\theta}_{1a} \), and \( \sigma^2_{\lambda_a} = \sigma^2_{\epsilon}/(n \sigma^2_m(1 - \rho^2_{m.x})) \) is the variance of the estimate \( \hat{\lambda}_a \). \( \sigma^2_m \) is the variance of the mediator \( m_i \).

From the linear regression \( m_i = \theta_0 + \theta_{1a} x_i + e_i \), we have the relationship \( \sigma^2_{\epsilon} = \sigma^2_m(1 - \rho^2_{m.x}) \). Hence, we can simply the variance \( \sigma_{\theta_{1a}, \lambda_a} \) to

\[ \sigma_{\theta_{1a}, \lambda_a} = \sqrt{\frac{\sigma^2_{\epsilon}}{n \sigma^2_m(1 - \rho^2_{m.x})} + \frac{\lambda_a^2 \sigma^2_m(1 - \rho^2_{m.x})}{n \sigma^2_x}} \]

**Value**

- **n** sample size.
- **res.uniroot** results of optimization to find the optimal sample size.

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

- powerMediation.Sobel
- testMediation.Sobel

**Examples**

```r
ssMediation.Sobel(power=0.8, theta.1a=0.1701, lambda.a=0.1998, sigma.x=0.57, sigma.m=0.61, sigma.epsilon=0.2, alpha = 0.05, verbose = TRUE)
```
Description

Calculate sample size for testing mediation effect in linear regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

```r
ssMediation.VSMc(power, b2, sigma.m, sigma.e, corr.xm, n.lower = 1, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)
```

Arguments

- `power`: power for testing $b_2 = 0$ for the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `b2`: regression coefficient for the mediator $m$ in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `sigma.m`: standard deviation of the mediator.
- `sigma.e`: standard deviation of the random error term in the linear regression $y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$.
- `corr.xm`: correlation between the predictor $x$ and the mediator $m$.
- `n.lower`: lower bound for the sample size.
- `n.upper`: upper bound for the sample size.
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the linear regressions:

$$y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_e)$$

Vittinghoff et al. (2009) showed that for the above linear regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$. 

```r
ssMediation.VSMc(power, b2, sigma.m, sigma.e, corr.xm, n.lower = 1, n.upper = 1e+30, alpha = 0.05, verbose = TRUE)
```
The full model is
\[ y_i = b_0 + b_1 x_i + b_2 m_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon}) \]

The reduced model is
\[ y_i = b_0 + b_1 x_i + \epsilon_i, \epsilon_i \sim N(0, \sigma^2_{\epsilon}) \]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining corr.xm as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

\[ n \] sample size.

res.uniroot results of optimization to find the optimal sample size.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set alpha=0.10 to obtain one-sided test at 5% significance level.

Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

References


See Also

minEffect.VSMc, powerMediation.VSMc

Examples

# example in section 3 (page 544) of Vittinghoff et al. (2009).
# n=863
ssMediation.VSMc(power = 0.80, b2 = 0.1, sigma.m = 1, sigma.e = 1, 
corr.xm = 0.3, alpha = 0.05, verbose = TRUE)
Sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate sample size for testing mediation effect in cox regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

ssMediation.VSMc.cox(power, b2, sigma.m, psi, corr.xm, n.lower = 1, n.upper = 1e+30, alpha = 0.05, verbose=TRUE)

Arguments

- **power**: power for testing $b_2 = 0$ for the cox regression $\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$, where $\lambda$ is the hazard function and $\lambda_0$ is the baseline hazard function.
- **b2**: regression coefficient for the mediator $m$ in the cox regression $\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$, where $\lambda$ is the hazard function and $\lambda_0$ is the baseline hazard function.
- **sigma.m**: standard deviation of the mediator.
- **psi**: the probability that an observation is uncensored, so that the number of event $d = n * psi$, where $n$ is the sample size.
- **corr.xm**: correlation between the predictor $x$ and the mediator $m$.
- **n.lower**: lower bound for the sample size.
- **n.upper**: upper bound for the sample size.
- **alpha**: type I error rate.
- **verbose**: logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the cox regressions:

$$\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$$
Vittinghoff et al. (2009) showed that for the above cox regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0: b_2 = 0$ versus the alternative hypothesis $H_a: b_2 \neq 0$.

The full model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i + b_2m_i$$

The reduced model is

$$\log(\lambda) = \log(\lambda_0) + b_1x_i$$

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining $corr.xm$ as the multiple correlation of the mediator with the confounders as well as the predictor.

### Value

- **n**: sample size.
- **res.unir0ot**: results of optimization to find the optimal sample size.

### Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set $alpha=0.10$ to obtain one-sided test at 5% significance level.

### Author(s)

Weiliang Qiu <stwxq@channing.harvard.edu>

### References


### See Also

- minEffect.VSMc.cox
- powerMediation.VSMc.cox

### Examples

```r
# example in section 6 (page 547) of Vittinghoff et al. (2009).
# n = 1399
ssMediation.VSMc.cox(power = 0.7999916, b2 = log(1.5),
  sigma.m = sqrt(0.25 * (1 - 0.25)), psi = 0.2, corr.xm = 0.3,
  alpha = 0.05, verbose = TRUE)
```
Sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch’s (2009) method

Description

Calculate sample size for testing mediation effect in logistic regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

Usage

```r
ssMediation.VSMc.logistic(power, 
  b2, 
  sigma.m, 
  p, 
  corr.xm, 
  n.lower = 1, 
  n.upper = 1e+30, 
  alpha = 0.05, 
  verbose = TRUE)
```

Arguments

- `power` power for testing $b_2 = 0$ for the logistic regression $\log(p_i/(1-p_i)) = b_0 + b_1x_i + b_2m_i$.
- `b2` regression coefficient for the mediator $m$ in the logistic regression $\log(p_i/(1-p_i)) = b_0 + b_1x_i + b_2m_i$.
- `sigma.m` standard deviation of the mediator.
- `p` the marginal prevalence of the outcome.
- `corr.xm` correlation between the predictor $x$ and the mediator $m$.
- `n.lower` lower bound for the sample size.
- `n.upper` upper bound for the sample size.
- `alpha` type I error rate.
- `verbose` logical. TRUE means printing sample size; FALSE means not printing sample size.

Details

The test is for testing the null hypothesis $b_2 = 0$ versus the alternative hypothesis $b_2 \neq 0$ for the logistic regressions:

$$\log(p_i/(1-p_i)) = b_0 + b_1x_i + b_2m_i$$

Vittinghoff et al. (2009) showed that for the above logistic regression, testing the mediation effect is equivalent to testing the null hypothesis $H_0 : b_2 = 0$ versus the alternative hypothesis $H_a : b_2 \neq 0$. 
The full model is
\[
\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 x_i + b_2 m_i
\]

The reduced model is
\[
\log\left(\frac{p_i}{1 - p_i}\right) = b_0 + b_1 x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \(corr.xm\) as the multiple correlation of the mediator with the confounders as well as the predictor.

**Value**

- **n** sample size.
- **res.uniroot** results of optimization to find the optimal sample size.

**Note**

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \(alpha=0.10\) to obtain one-sided test at 5% significance level.

**Author(s)**

Weiliang Qiu <stwxq@channing.harvard.edu>

**References**


**See Also**

- minEffect.VSMc.logistic
- powerMediation.VSMc.logistic

**Examples**

```r
# example in section 4 (page 545) of Vittinghoff et al. (2009).
# n=255

ssMediation.VSMc.logistic(power = 0.80, b2 = log(1.5), sigma.m = 1, p = 0.5,
corr.xm = 0.5, alpha = 0.05, verbose = TRUE)
```
**ssMediation.VSMc.poisson**

Sample size for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method

---

**Description**

Calculate sample size for testing mediation effect in poisson regression based on Vittinghoff, Sen and McCulloch’s (2009) method.

**Usage**

```r
ssMediation.VSMc.poisson(power,
                          b2,
                          sigma.m,
                          EY,
                          corr.xm,
                          n.lower = 1,
                          n.upper = 1e+30,
                          alpha = 0.05,
                          verbose = TRUE)
```

**Arguments**

- `power`: power for testing \( b_2 = 0 \) for the poisson regression \( \log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i \).
- `b2`: regression coefficient for the mediator \( m \) in the poisson regression \( \log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i \).
- `sigma.m`: standard deviation of the mediator.
- `EY`: the marginal mean of the outcome.
- `corr.xm`: correlation between the predictor \( x \) and the mediator \( m \).
- `n.lower`: lower bound for the sample size.
- `n.upper`: upper bound for the sample size.
- `alpha`: type I error rate.
- `verbose`: logical. TRUE means printing sample size; FALSE means not printing sample size.

**Details**

The test is for testing the null hypothesis \( b_2 = 0 \) versus the alternative hypothesis \( b_2 \neq 0 \) for the poisson regressions:

\[
\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i
\]

Vittinghoff et al. (2009) showed that for the above poisson regression, testing the mediation effect is equivalent to testing the null hypothesis \( H_0 : b_2 = 0 \) versus the alternative hypothesis \( H_a : b_2 \neq 0 \).
The full model is
\[
\log(E(Y_i)) = b_0 + b_1 x_i + b_2 m_i
\]

The reduced model is
\[
\log(E(Y_i)) = b_0 + b_1 x_i
\]

Vittinghoff et al. (2009) mentioned that if confounders need to be included in both the full and reduced models, the sample size/power calculation formula could be accommodated by redefining \( \text{corr}.xm \) as the multiple correlation of the mediator with the confounders as well as the predictor.

Value

- \( n \) sample size.
- \( \text{res.unirroot} \) results of optimization to find the optimal sample size.

Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set \( \text{alpha}=0.10 \) to obtain one-sided test at 5% significance level.

Author(s)

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References


See Also

- `minEffect.VSMc.poisson`
- `powerMediation.VSMc.poisson`

Examples

```r
# example in section 5 (page 546) of Vittinghoff et al. (2009).
# n = 1239
ssMediation.VSMc.poisson(power = 0.7998578, b2 = log(1.35),
sigma.m = sqrt(0.25 * (1 - 0.25)), EY = 0.5, corr.xm = 0.5,
alpha = 0.05, verbose = TRUE)
```
**Description**

Calculate p-value and confidence interval for testing mediation effect based on Sobel’s test.

**Usage**

```r
testMediation.Sobel(theta.1.hat, 
lambda.hat, 
sigma.theta1, 
sigma.lambda, 
alpha = 0.05)
```

**Arguments**

- `theta.1.hat`: estimated regression coefficient for the predictor in the linear regression linking the predictor \(x\) to the mediator \(m\) \((m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e))\).
- `lambda.hat`: estimated regression coefficient for the mediator in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) \((y_i = \gamma + \lambda m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e))\).
- `sigma.theta1`: standard deviation of \(\hat{\theta}_1\) in the linear regression linking the predictor \(x\) to the mediator \(m\) \((m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e))\).
- `sigma.lambda`: standard deviation of \(\hat{\lambda}\) in the linear regression linking the predictor \(x\) and the mediator \(m\) to the outcome \(y\) \((y_i = \gamma + \lambda m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e))\).
- `alpha`: significance level of a test.

**Details**

The test is for testing the null hypothesis \(\theta_1 \lambda = 0\) versus the alternative hypothesis \(\theta_1 \lambda_{\alpha} \neq 0\) for the linear regressions:

- \(m_i = \theta_0 + \theta_1 x_i + e_i, e_i \sim N(0, \sigma^2_e)\)
- \(y_i = \gamma + \lambda m_i + \lambda_2 x_i + e_i, e_i \sim N(0, \sigma^2_e)\)

Test statistic is based on Sobel’s (1982) test:

\[
Z = \frac{\hat{\theta}_1 \hat{\lambda}}{\hat{\sigma}_{\theta_1 \lambda}}
\]

where \(\hat{\sigma}_{\theta_1 \lambda}\) is the estimated standard deviation of the estimate \(\hat{\theta}_1 \hat{\lambda}\) using multivariate delta method:

\[
\hat{\sigma}_{\theta_1 \lambda} = \sqrt{\theta^2_1 \hat{\lambda}^2 + \lambda^2 \sigma^2_{\theta_1}}
\]

and \(\hat{\sigma}_{\theta_1}\) is the estimated standard deviation of the estimate \(\hat{\theta}_1\), and \(\hat{\sigma}_{\lambda}\) is the estimated standard deviation of the estimate \(\hat{\lambda}\).
### Value

- **pval**: p-value for testing the null hypothesis $\theta_1\lambda = 0$ versus the alternative hypothesis $\theta_1\lambda \neq 0$.
- **CI.low**: Lower bound of the $100(1 - \alpha)$% confidence interval for the parameter $\theta_1\lambda$.
- **CI.upp**: Upper bound of the $100(1 - \alpha)$% confidence interval for the parameter $\theta_1\lambda$.

### Note

The test is a two-sided test. For one-sided tests, please double the significance level. For example, you can set `alpha=0.10` to obtain one-sided test at 5% significance level.

### Author(s)

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### References


### See Also

- `powerMediation.Sobel`
- `ssMediation.Sobel`

### Examples

```r
testMediation.Sobel(theta.1.hat=0.1701, lambda.hat=0.1998, 
                    sigma.theta=0.01, sigma.lambda=0.02, alpha=0.05)
```
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