Benchmarks for Discrete Fourier Transform (DFT) calculations in R

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Abstract

The DFT calculator in R, \texttt{stats::fft}, uses the Mixed-Radix algorithm of Singleton (1969). In this vignette we show how this calculator compares to FFT in the \texttt{fftw} package (Krey et al., 2011), which uses the FFTW algorithm of Frigo and Johnson (2005). For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (1) the series to be transformed is very long (10^6 terms), and especially (2) when the series length is not highly composite. In both exceptions the algorithm \texttt{FFT} outperforms \texttt{fft}.

Update: I have decided that (for now) \texttt{psd} will not use \texttt{fftw::FFT}, despite its advantage over \texttt{stats::fft} for large-n 'NHC' series, simply because the binaries on CRAN have not been reliably built for some time now. If they do become reliable, I may consider using \texttt{fftw::FFT} instead.

Contents

1 Benchmarking function 2
2 Highly composite (HC) series 2
3 Non highly composite (NHC) series 3
4 Visualization 3
5 Conclusion 5
1 Benchmarking function

We use both functions in their default state, and ask them to transform the same univariate random series. Benchmark information comes from the rbenchmark program, and the versatile plyr and reshape2 packages are used to manipulate the information for this presentation: ggplot2 is used for plotting. First we load the libraries needed:

```r
rm(list=ls())
library(fftw)
library(rbenchmark)
library(plyr)
library(reshape2)
library(ggplot2)
```

and create a benchmark function:

```r
reps <- 10
dftbm <- function(nd, repls=reps){
  set.seed(1234)
  x <- rnorm(nd, mean=0, sd=1)
  bmd <- benchmark(replications=repls, fftw::FFT(x), stats::fft(x))
  bmd$num_dat <- nd
  bmd$relative[is.na(bmd$relative)] <- 1  # NA happens.
  return(bmd)
}
```

2 Highly composite (HC) series

It’s well known that DFT algorithms are most efficient for “Highly Composite Numbers”\(^1\), specifically multiples of (2,3,5).

So, we create a vector of series lengths we wish to benchmark

```r
nterms.even <- round(2**seq.int(from=4,to=20,by=1))
```

<table>
<thead>
<tr>
<th></th>
<th>[1]</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
</tr>
</thead>
<tbody>
<tr>
<td>[10]</td>
<td>8192</td>
<td>16384</td>
<td>32768</td>
<td>65536</td>
<td>131072</td>
<td>262144</td>
<td>524288</td>
<td>1048576</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and use it with lapply and the benchmark function previously defined. These data are further distilled into a usable format with ldply:

\(^1\)This is the reason for the stats::nextn function.
3 Non highly composite (NHC) series

DFT algorithms can have drastically reduced performance if the series length is not highly composite (NHC). We now test NHC series by adding one to the HC series-length vector (also restricting the total length for sanity’s sake):

```r
nterms.odd <- nterms.even + 1
nterms.odd <- nterms.odd[nterms.odd < 50e3] # painfully long otherwise!
```

and performing the full set of benchmarks again:

```r
bench.odd <- function(){
  benchdat.o <- plyr::ldply(lapply(X=nterms.odd, FUN=dftbm))
}
bench.odd() # FAIR WARNING: this can take a while!!
```

4 Visualization

In order to plot the results, we need to perform some map/reduce operations on the data (Wickham, 2011). We intend to show faceted ggplot2-based figures with row-wise summary information\(^2\) so we can easily intercompare the benchmark data. The benchmark data we will show are user.self, sys.self, elapsed, and relative. The results are shown in Figure 1.

```r
pltbench <- function(lentyp=c("even","odd")){
  benchdat <- switch(match.arg(lentyp), even=benchdat.e, odd=benchdat.o)
  stopifnot(exists("benchdat"))
  tests <- unique(benchdat$test)
  ## subset only information we care about
  allbench.df.drp <- subset(benchdat,
    select=c(test, num_dat, user.self, sys.self, elapsed, relative))
}
```

\(^2\)Based on this post:
## reduce data.frame with melt

```r
allbench.df.mlt <- reshape2::melt(allbench.df.drp,
                                   id.vars=c("test","num_dat"))
```

## calculate the summary information to be plotted:

```r
tmpd <- plyr::ddply(allbench.df.mlt,
                    .(variable, num_dat),
                    summarise,
                    summary="medians",
                    value=ggplot2::mean_cl_normal(value)[1,1])
```

## create copies for each test and map to data.frame

```r
allmeds <<- plyr::ldply(lapply(X=tests,
                             FUN=function(x,df=tmpd){
                               df$test <- x;
                               return(df)
                             }))
```

## plot the benchmark data

```r
# 1/sqrt(n) standard errors [assumes N(0,1)]
g <- ggplot(data=allbench.df.mlt,
            aes(x=log10(num_dat),
                y=log2(value),
                ymin=log2(value*(1-1/sqrt(reps))),
                ymax=log2(value*(1+1/sqrt(reps))),
                colour=test,
                group=test)) +
    scale_colour_discrete(guide="none") +
    theme_bw() +
    ggtitle(sprintf("DFT benchmarks of %s length series", toupper(lentyp))) +
    ylim(c(-11,11)) +
    xlim(c(0.5,6.5))
```

## add previous summary curves if exist

```r
if (exists("allmeds.prev")){
  g <- g + geom_path(size=1.5, colour="dark grey", data=allmeds.prev,
                     aes(group=test))
}
```

## create a facetted version

```r
g2 <- g + facet_grid(variable~test) #, scales="free_y"
```

## add the summary data as a line

```r
g3 <- g2 + geom_path(colour="black", data=allmeds, aes(group=test))
```

## and finally the data

```r
print(g4 <<- g3 + geom_pointrange())
```
pltbench("even")
allmeds.prev <- allmeds
pltbench("odd")

Figure 1: DFT benchmark results for HC series lengths (left), and NHC series lengths (right) as a function of logarithmic series length. In each figure, the left facet-column is for results from `fftw::FFT` and the right column is for `stats::fft`. We also show the summary curves from the HC results in the NHC frames (thick grey curve) to highlight the drastic degradation in performance.

5 Conclusion

Figure 1 compares the DFT calculations for HC and NHC length series. For univariate DFT computations, the methods are nearly equivalent with two exceptions which are not mutually exclusive: (A) the series to be transformed is very long, and especially (B) when the series length is not highly composite. In both exceptions the algorithm FFT outperforms fft. In the case of exception (B), both methods have drastically increased computation times; hence, zero padding should be done to ensure the length does not adversely affect the efficiency of the DFT calculator.
Session Info

utils::sessionInfo()

## R version 4.1.2 (2021-11-01)
## Platform: x86_64-apple-darwin17.0 (64-bit)
## Running under: macOS Mojave 10.14.6
##
## Matrix products: default
## BLAS: /Library/Frameworks/R.framework/Versions/4.1/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/4.1/Resources/lib/libRlapack.dylib
##
## locale:
## [
##]
##
## attached base packages:
## [
## [1] stats     graphics  grDevices utils     datasets  methods   base
##]
##
## loaded via a namespace (and not attached):
## [
## [1] compiler_4.1.2 magrittr_2.0.2 tools_4.1.2       stringi_1.7.6 highr_0.9
## [6] knitr_1.37    stringr_1.4.0 xfun_0.29        evaluate_0.14
##]

References


