Risk Parity Portfolios with riskParityPortfolio

Prof. Daniel P. Palomar  
(Joint work with Zé Vinícius)  
Hong Kong University of Science and Technology (HKUST)

R/Finance 2019  
University of Illinois at Chicago (UIC), Chicago, IL, USA  
17 May 2019
Markowitz portfolio

- Let us denote the returns of $N$ assets at time $t$ with the vector $\mathbf{r}_t$.
- Suppose that $\mathbf{r}_t$ follows an i.i.d. distribution (not totally accurate but widely adopted) with mean $\mu$ and covariance matrix $\Sigma$.
- The portfolio vector $\mathbf{w}$ denotes the normalized dollar weights of the $N$ assets ($\mathbf{1}^T \mathbf{w} = 1$).
- Portfolio return is $r_{\text{portf}}^t = \mathbf{w}^T \mathbf{r}_t$.
- Markowitz proposed in his seminar 1952 paper\textsuperscript{1} to find a trade-off between the portfolio expected return $\mathbf{w}^T \mu$ and its risk measured by the variance $\mathbf{w}^T \Sigma \mathbf{w}$:

\[
\begin{align*}
\text{maximize} \quad & \mathbf{w}^T \mu - \lambda \mathbf{w}^T \Sigma \mathbf{w} \\
\text{subject to} \quad & \mathbf{w} \geq 0, \quad \mathbf{1}^T \mathbf{w} = 1,
\end{align*}
\]

where $\lambda$ is a parameter that controls how risk-averse the investor is.

**Drawbacks of Markowitz portfolio:** Markowitz’s portfolio has been heavily critized for over half a century and has never been fully embraced by practitioners for many reasons:
- variance is not a good measure of risk,
- portfolio is highly sensitive to parameter estimation errors,
- only considers the risk as a whole and ignores the risk diversification.

**Risk parity** is an approach to portfolio management that focuses on allocation of risk rather than allocation of capital.

Some of its theoretical components were developed in the 1950s and 1960s but the **first risk parity fund, called the “All Weather” fund**, was pioneered by Bridgewater Associates LP in 1996.

**Some portfolio managers have expressed skepticism** but others point to its performance during the financial crisis of 2007-2008 as an indication of its potential success.
From “dollar” to risk diversification

Equally weighted portfolio (aka uniform portfolio) vs risk parity portfolio:

Portfolio allocation of EWP

Relative risk contribution of EWP

Portfolio allocation of RPP

Relative risk contribution of RPP
From Euler’s theorem, the volatility can be decomposed as

$$\sigma (w) = \sum_{i=1}^{N} RC_i$$

where $RC_i$ is the risk contribution (RC) from the $i$th asset to the total risk $\sigma(w)$:

$$RC_i = \frac{w_i (\Sigma w)_i}{\sqrt{w^T \Sigma w}}.$$

The risk parity portfolio (RPP) attempts to “equalize” the risk contributions:

$$RC_i = \frac{1}{N} \sigma(w).$$

More generally, the risk budgeting portfolio (RBP) attempts to allocate the risk according to the risk profile determined by the weights $b$ (with $1^T b = 1$ and $b \geq 0$):

$$RC_i = b_i \sigma(w).$$
1 **Naive diagonal formulation**: pretend that $\Sigma$ is diagonal and simply use the volatilities $\sigma = \sqrt{\text{diag}(\Sigma)}$, obtaining:

$$w = \frac{\sigma^{-1}}{1^T \sigma^{-1}}.$$  

2 **Vanilla convex formulation**: suppose we only have the constraints $1^T w = 1$ and $w \geq 0$, then after some change of variable the problem reduced to solving

$$\Sigma x = b/x.$$  

3 **General nonconvex formulation** (there are many reformulations possible):

minimize $\sum_{i,j=1}^{N} \left( w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2 - F(w)$

subject to $w \geq 0$, $1^T w = 1$, $w \in \mathcal{W}$. 
Some R packages contain functions to compute the RPP, e.g., PortfolioAnalytics, FRAPO, cccp, and FinCovRegularization. But they are based on general-purpose solvers and may not be efficient.

riskParityPortfolio is the first package specifically devised for the computation of different versions of RPP in an efficient way: https://CRAN.R-project.org/package=riskParityPortfolio

Published on Christmas of 2018 and somehow was well-received by the community (600 downloads in 2 days).

Authors: Zé Vinícius and Daniel P. Palomar.
Using `riskParityPortfolio`

- Load Package:

```r
library(riskParityPortfolio)
?riskParityPortfolio  # to get help for the function
```

- The simplest use is for the vanilla RPP:

```r
rpp_vanilla <- riskParityPortfolio(Sigma)
names(rpp_vanilla)
```

```
R>> [1] "w"                   "risk_contribution"
```

```r
print(rpp_vanilla$w, digits = 2)
```

```
R>>  AAPL  AMD  ADI  ABBV  AEZS  A  APD  AA  CF
R>>  0.156 0.068 0.125 0.133 0.045 0.129 0.158 0.085 0.101
```
Using `riskParityPortfolio`

- Naive diagonal formulation:

```r
rpp_naive <- riskParityPortfolio(Sigma,
                                 formulation = "diag")
```

- Unified nonconvex formulation including expected return in objective and box constraints:

\[
\begin{align*}
\text{minimize} \quad & \sum_{i,j=1}^{N} \left( w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2 - \lambda w^T \mu \\
\text{subject to} \quad & w \geq 0, \quad 1^T w = 1, \quad l \leq w \leq u.
\end{align*}
\]

```r
rpp_mu <- riskParityPortfolio(Sigma,
                              mu = mu, lmd_mu = 1e-3,
                              w_ub = 0.16)
```
Risk concentration terms

Many formulations included in the package:

\[ R(\mathbf{w}) = \sum_{i,j=1}^{N} \left( w_i (\mathbf{\Sigma w})_i - w_j (\mathbf{\Sigma w})_j \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i=1}^{N} \left( w_i (\mathbf{\Sigma w})_i - \theta \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i=1}^{N} \left( \frac{w_i(\mathbf{\Sigma w})_i}{\mathbf{w}^T\mathbf{\Sigma w}} - b_i \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i,j=1}^{N} \left( \frac{w_i(\mathbf{\Sigma w})_i}{b_i} - \frac{w_j(\mathbf{\Sigma w})_j}{b_j} \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i=1}^{N} \left( w_i (\mathbf{\Sigma w})_i - b_i \mathbf{w}^T\mathbf{\Sigma w} \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i=1}^{N} \left( \frac{w_i(\mathbf{\Sigma w})_i}{\sqrt{\mathbf{w}^T\mathbf{\Sigma w}}} - b_i \sqrt{\mathbf{w}^T\mathbf{\Sigma w}} \right)^2 \]

\[ R(\mathbf{w}) = \sum_{i=1}^{N} \left( \frac{w_i(\mathbf{\Sigma w})_i}{b_i} - \theta \right)^2 \]
Using `riskParityPortfolio`

**Risk contribution**

- Markowitz
- RPP (naive)
- RPP (vanilla)
- RPP + μ

stocks: AAPL, AMD, ADI, ABBV, AEZS, A, APD, AA, CF

D. Palomar (HKUST)
Using riskParityPortfolio

Illustration of the **expected return vs risk concentration** trade-off:
Using riskParityPortfolio

Illustration of the volatility vs risk concentration trade-off:
References

- Standard textbooks:

- Vanilla formulations:

- Unified formulation and advanced algorithms:
Thanks

For more information visit:

https://www.danielppalomar.com