

Package ‘spNNGP’

July 22, 2017

Title Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes

Version 0.1.1

Date 2017-07-14

Maintainer Andrew Finley <finleya@msu.edu>

Author Andrew Finley [aut, cre],
Abhirup Datta [aut],
Sudipto Banerjee [aut],
Alexander Mckim [ctb]

Depends R (>= 2.10), coda, Formula, RANN

Description Fits Gaussian univariate Bayesian spatial regression models for large datasets using Nearest Neighbor Gaussian Processes (NNGP) detailed in Datta, Banerjee, Finley, and Gelfand (2016) <doi:10.1080/01621459.2015.1044091> and Finley, Datta, Cook, Morton, Andersen, and Banerjee (2017) <arXiv:1702.00434v2>.

License GPL (>= 2)

Encoding UTF-8

URL <http://blue.for.msu.edu/software.html>

Repository CRAN

NeedsCompilation yes

Date/Publication 2017-07-21 22:33:23 UTC

R topics documented:

CHM	2
spConjNNGP	2
spNNGP	6
spPredict	10

Index	13
--------------	-----------

CHM	<i>Canopy Height Model from NASA Goddard's LiDAR Hyperspectral and Thermal (G-LiHT)</i>
-----	---

Description

Canopy Height Model (CHM) from NASA Goddard's LiDAR Hyperspectral and Thermal (G-LiHT; Cook et al. 2013) Airborne Imager over a subset of Harvard Forest Simes Tract, MA, collected in Summer 2012.

The CHM matrix columns are longitude, latitude, and canopy height (m) from ground for 17,23,137 observations. Longitude and latitude are in UTM Zone 18 (proj4string "+proj=utm +zone=18 +datum=WGS84 +units=m +no_defs +ellps=WGS84 +towgs84=0,0,0").

Usage

```
data(CHM)
```

Format

A matrix containing 17,23,137 rows and 3 columns named longitude, latitude, and CHM.

Source

Data were downloaded from <https://gliht.gsfc.nasa.gov> with metadata available at ftp://fusionftp.gsfc.nasa.gov/G-LiHT/Simes_Jun2012/metadata/Simes_Jun2012_metadata.pdf.

References

Cook, B.D., L.W. Corp, R.F. Nelson, E.M. Middleton, D.C. Morton, J.T. McCorkel, J.G. Masek, K.J. Ranson, and V. Ly. (2013) NASA Goddard's Lidar, Hyperspectral and Thermal (G-LiHT) airborne imager. Remote Sensing 5:4045-4066.

spConjNNGP	<i>Function for fitting univariate Bayesian conjugate spatial regression models</i>
------------	---

Description

The function spConjNNGP fits Gaussian univariate Bayesian conjugate spatial regression models using Nearest Neighbor Gaussian Processes (NNGP).

Usage

```
spConjNNGP(formula, data = parent.frame(), coords, n.neighbors = 15,
  theta.alpha, sigma.sq.IG, cov.model = "exponential",
  k.fold = 5, score.rule = "crps",
  X.0, coords.0,
  n.omp.threads = 1, search.type = "tree",
  return.neighbors = FALSE, verbose = TRUE, ...)
```

Arguments

formula	a symbolic description of the regression model to be fit. See example below.
data	an optional data frame containing the variables in the model. If not found in data, the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>spConjNNGP</code> is called.
coords	an $n \times 2$ matrix of the observation coordinates in R^2 (e.g., easting and northing).
n.neighbors	number of neighbors used in the NNGP.
theta.alpha	a vector or matrix of parameter values for phi, nu, and alpha, where $\alpha = \tau^2/\sigma^2$ and nu is only required if <code>cov.model="matern"</code> . A vector is passed to run the model using one set of parameters. The vector elements must be named and hold values for phi, nu, and alpha. If a matrix is passed, columns must be named and hold values for phi, alpha, and nu. Each row in the matrix defines a set of parameters for which the model will be run.
sigma.sq.IG	a vector of length two that holds the hyperparameters, <i>shape</i> and <i>scale</i> respectively, for the inverse-Gamma prior on σ^2 .
cov.model	a quoted keyword that specifies the covariance function used to model the spatial dependence structure among the observations. Supported covariance model keywords are: "exponential", "matern", "spherical", and "gaussian". See below for details.
k.fold	specifies the number of k folds for cross-validation. If <code>theta.alpha</code> is a vector then cross-validation is not performed and <code>k.fold</code> and <code>score.rule</code> are ignored. In k -fold cross-validation, the data specified in <code>model</code> is randomly partitioned into k equal sized subsamples. Of the k subsamples, $k-1$ subsamples are used to fit the model and the remaining k samples are used for prediction. The cross-validation process is repeated k times (the folds). Root mean squared prediction error (RMSPE) and continuous ranked probability score (CRPS; Gneiting and Raftery, 2007) rules are averaged over the k fold prediction results and reported for the parameter sets defined by <code>theta.alpha</code> . The parameter set that yields the <i>best</i> performance based on the scoring rule defined by <code>score.rule</code> is used fit the final model that uses all the data and make predictions if <code>X.0</code> and <code>coords.0</code> are specified. Results from the k -fold cross-validation are returned in the <code>k.fold.scores</code> matrix.
score.rule	a quoted keyword "rmspe" or "crps" that specifies the scoring rule used to select the <i>best</i> parameter set, see argument definition for <code>k.fold</code> for more details.
X.0	the design matrix for prediction locations. An intercept should be provided in the first column if one is specified in <code>model</code> .

<code>coords.0</code>	the spatial coordinates corresponding to <code>X.0</code> .
<code>n.omp.threads</code>	a positive integer indicating the number of threads to use for SMP parallel processing. The package must be compiled for OpenMP support. For most Intel-based machines, we recommend setting <code>n.omp.threads</code> up to the number of hyperthreaded cores.
<code>search.type</code>	a quoted keyword that specifies type of nearest neighbor search algorithm. Supported method key words are: "tree" and "brute" both will yield the same solution but "tree" should be much faster.
<code>return.neighbors</code>	if TRUE, a list containing the indices for each location's nearest neighbors will be returned along with ordered data used to fit a NNGP model. This argument should typically be FALSE. See <code>n.indx</code> below for more details.
<code>verbose</code>	if TRUE, model specification and progress is printed to the screen. Otherwise, nothing is printed to the screen.
<code>...</code>	currently no additional arguments.

Value

An object of class `cNNGP`, which is a list comprising:

<code>theta.alpha</code>	the input <code>theta.alpha</code> vector, or <i>best</i> (according to the selected scoring rule) set of parameters in the <code>theta.alpha</code> matrix. All subsequent parameter estimates are based on this parameter set.
<code>beta.hat</code>	a matrix of regression coefficient estimates corresponding to the returned <code>theta.alpha</code> .
<code>beta.var</code>	<code>beta.hat</code> variance-covariance matrix.
<code>sigma.sq.hat</code>	estimate of σ^2 corresponding to the returned <code>theta.alpha</code> .
<code>sigma.sq.var</code>	<code>sigma.sq.hat</code> variance.
<code>k.fold.scores</code>	results from the k-fold cross-validation if <code>theta.alpha</code> is a matrix.
<code>y.0.hat</code>	prediction if <code>X.0</code> and <code>coords.0</code> are specified.
<code>y.0.var.hat</code>	<code>y.0.hat</code> variance.
<code>n.neighbors</code>	number of neighbors used in the NNGP.
<code>n.indx</code>	if <code>return.neighbors=TRUE</code> then <code>n.indx</code> will be a list of length <code>n</code> . The <i>i</i> -th element in the list corresponds to the <i>i</i> -th row in <code>coords.ord</code> matrix and the elements are the nearest neighbor indices for the given location.
<code>ord</code>	if <code>return.neighbors=TRUE</code> the vector <code>ord=order(coords[,1])</code> , which is the vector of indices used to order data necessary for fitting the NNGP model, is returned.
<code>coords.ord</code>	if <code>return.neighbors=TRUE</code> then <code>coords.ord = coords[ord,]</code> is returned.
<code>y.ord</code>	if <code>return.neighbors=TRUE</code> then <code>y.ord = y[ord]</code> is returned.
<code>X.ord</code>	if <code>return.neighbors=TRUE</code> then <code>X.ord = X[ord, ,drop=FALSE]</code> is returned.
<code>run.time</code>	execution time for building the nearest neighbor index and parameter estimation reported using <code>proc.time()</code> .

Author(s)

Andrew O. Finley <finleya@msu.edu>,
 Abhirup Datta <abhidatta@jhu.edu>,
 Sudipto Banerjee <sudipto@ucla.edu>

References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. *Journal of the American Statistical Association*, 111:800-812.

Finley, A.O., A. Datta, B.C. Cook, D.C. Morton, H.E. Andersen, and S. Banerjee (2017) Applying Nearest Neighbor Gaussian Processes to massive spatial data sets: Forest canopy height prediction across Tanana Valley Alaska, <https://arxiv.org/abs/1702.00434v2>.

Gneiting, T and A.E. Raftery. (2007) Strictly proper scoring rules, prediction, and estimation. *Journal of the American Statistical Association*, 102:359-378.

Examples

```
rmvn <- function(n, mu=0, V = matrix(1)){
  p <- length(mu)
  if(any(is.na(match(dim(V),p))))
    stop("Dimension problem!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}

##Make some data
set.seed(1)
n <- 2000
coords <- cbind(runif(n,0,1), runif(n,0,1))

x <- cbind(1, rnorm(n))

B <- as.matrix(c(1,5))

sigma.sq <- 5
tau.sq <- 1
phi <- 3/0.5

D <- as.matrix(dist(coords))
R <- exp(-phi*D)
w <- rmvn(1, rep(0,n), sigma.sq*R)
y <- rnorm(n, x%*%B + w, sqrt(tau.sq))

ho <- sample(1:n, 1000)

y.ho <- y[ho]
x.ho <- x[ho,,drop=FALSE]
w.ho <- w[ho]
```

```

coords.ho <- coords[ho,]

y <- y[-ho]
x <- x[-ho,,drop=FALSE]
w <- w[-ho,,drop=FALSE]
coords <- coords[-ho,]

##Fit a Conjugate NNGP model and predict for the holdout
sigma.sq.IG <- c(2, sigma.sq)

cov.model <- "exponential"

g <- 10
theta.alpha <- cbind(seq(phi,30,length.out=g), seq(tau.sq/sigma.sq,5,length.out=g))

colnames(theta.alpha) <- c("phi", "alpha")

##one thread
m.c <- spConjNNGP(y~x-1, coords=coords, n.neighbors = 10,
                 X.0 = x.ho, coords.0 = coords.ho,
                 k.fold = 5, score.rule = "crps",
                 n.omp.threads = 1,
                 theta.alpha = theta.alpha, sigma.sq.IG = sigma.sq.IG, cov.model = cov.model)

m.c$beta.hat
m.c$theta.alpha.sigmaSq
m.c$k.fold.scores

##two threads
m.c <- spConjNNGP(y~x-1, coords=coords, n.neighbors = 10,
                 X.0 = x.ho, coords.0 = coords.ho,
                 k.fold = 5, score.rule = "crps",
                 n.omp.threads = 2,
                 theta.alpha = theta.alpha, sigma.sq.IG = sigma.sq.IG, cov.model = cov.model)

m.c$beta.hat
m.c$sigmaSq.hat
m.c$k.fold.scores

```

spNNGP

Function for fitting univariate Bayesian spatial regression models

Description

The function spNNGP fits Gaussian univariate Bayesian spatial regression models using Nearest Neighbor Gaussian Processes (NNGP).

Usage

```
spNNGP(formula, data = parent.frame(), coords, method = "response", n.neighbors = 15,
        starting, tuning, priors, cov.model = "exponential",
        n.samples, n.omp.threads = 1, search.type = "tree",
        return.neighbors = FALSE, verbose = TRUE, n.report = 100, ...)
```

Arguments

formula	a symbolic description of the regression model to be fit. See example below.
data	an optional data frame containing the variables in the model. If not found in data, the variables are taken from <code>environment(formula)</code> , typically the environment from which <code>spNNGP</code> is called.
coords	an $n \times 2$ matrix of the observation coordinates in R^2 (e.g., easting and northing).
method	a quoted keyword that specifies the NNGP sampling algorithm. Supported method keywords are: "response" and "sequential". When n is large, e.g., greater than 100k, the "response" algorithm should be faster. In general, unless estimates of spatial random effects are needed, the "response" algorithm should be used. See below for details.
n.neighbors	number of neighbors used in the NNGP.
starting	a list with each tag corresponding to a parameter name. Valid tags are <code>beta</code> , <code>sigma.sq</code> , <code>tau.sq</code> , <code>phi</code> , and <code>nu</code> . <code>nu</code> is only specified if <code>cov.model="matern"</code> . The value portion of each tag is the parameter's starting value.
tuning	a list with each tag corresponding to a parameter name. Valid tags are <code>sigma.sq</code> , <code>tau.sq</code> , <code>phi</code> , and <code>nu</code> . If <code>method="sequential"</code> then only <code>phi</code> and <code>nu</code> need to be specified. The value portion of each tag defines the variance of the Metropolis sampler Normal proposal distribution.
priors	a list with each tag corresponding to a parameter name. Valid tags are <code>sigma.sq.ig</code> , <code>tau.sq.ig</code> , <code>phi.unif</code> , and <code>nu.unif</code> . Variance parameters, <code>sigma.sq</code> and <code>tau.sq</code> , are assumed to follow an inverse-Gamma distribution, whereas the spatial decay <code>phi</code> and smoothness <code>nu</code> parameters are assumed to follow Uniform distributions. The hyperparameters of the inverse-Gamma are passed as a vector of length two, with the first and second elements corresponding to the <i>shape</i> and <i>scale</i> , respectively. The hyperparameters of the Uniform are also passed as a vector of length two with the first and second elements corresponding to the lower and upper support, respectively.
cov.model	a quoted keyword that specifies the covariance function used to model the spatial dependence structure among the observations. Supported covariance model keywords are: "exponential", "matern", "spherical", and "gaussian". See below for details.
n.samples	the number of posterior samples to collect.
n.omp.threads	a positive integer indicating the number of threads to use for SMP parallel processing. The package must be compiled for OpenMP support. For most Intel-based machines, we recommend setting <code>n.omp.threads</code> up to the number of hyperthreaded cores.

<code>verbose</code>	if TRUE, model specification and progress of the sampler is printed to the screen. Otherwise, nothing is printed to the screen.
<code>search.type</code>	a quoted keyword that specifies type of nearest neighbor search algorithm. Supported method key words are: "tree" and "brute" both will yield the same solution but "tree" should be much faster.
<code>return.neighbors</code>	if TRUE, a list containing the indices for each location's nearest neighbors will be returned along with ordered data used to fit a NNGP model. This argument should typically be FALSE. See <code>n.indx</code> below for more details.
<code>n.report</code>	the interval to report Metropolis sampler acceptance and MCMC progress.
<code>...</code>	currently no additional arguments.

Details

Model parameters can be fixed at their starting values by setting their tuning values to zero. The *no nugget* model is specified by setting `tau.sq` to zero in the starting and tuning lists.

Value

An object of class `rNNGP` or `sNNGP` depending on the method, which is a list comprising:

<code>p.beta.samples</code>	a coda object of posterior samples for the regression coefficients.
<code>p.theta.samples</code>	a coda object of posterior samples for covariance parameters.
<code>p.w.samples</code>	is a matrix of posterior samples for the spatial random effects, where rows correspond to locations in <code>coords</code> and columns hold the <code>n.samples</code> posterior samples. This is only returned if <code>method="sequential"</code> .
<code>n.indx</code>	if <code>return.neighbors=TRUE</code> then <code>n.indx</code> will be a list of length n . The i -th element in the list corresponds to the i -th row in <code>coords.ord</code> matrix and the elements are the nearest neighbor indices for the given location.
<code>ord</code>	the vector <code>order(coords[, 1])</code> , which is the vector of indices used to order data necessary for fitting the NNGP model.
<code>coords.ord</code>	the matrix <code>coords[ord,]</code> .
<code>y.ord</code>	the vector <code>y[ord]</code> .
<code>X.ord</code>	the matrix <code>X[ord, , drop=FALSE]</code> .
<code>run.time</code>	execution time for building the nearest neighbor index and MCMC sampler reported using <code>proc.time()</code> .

The return object will include additional objects used for subsequent prediction and/or model fit evaluation.

Author(s)

Andrew O. Finley <finleya@msu.edu>
 Abhirup Datta <abhidatta@jhu.edu>
 Sudipto Banerjee <sudipto@ucla.edu>

References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. *Journal of the American Statistical Association*, 111:800-812

Finley, A.O., A. Datta, B.C. Cook, D.C. Morton, H.E. Andersen, and S. Banerjee (2017) Applying Nearest Neighbor Gaussian Processes to massive spatial data sets: Forest canopy height prediction across Tanana Valley Alaska, <https://arxiv.org/abs/1702.00434v2>.

Examples

```
rmvn <- function(n, mu=0, V = matrix(1)){
  p <- length(mu)
  if(any(is.na(match(dim(V),p))))
    stop("Dimension problem!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}

##Make some data
set.seed(1)
n <- 100
coords <- cbind(runif(n,0,1), runif(n,0,1))

x <- cbind(1, rnorm(n))

B <- as.matrix(c(1,5))

sigma.sq <- 5
tau.sq <- 1
phi <- 3/0.5

D <- as.matrix(dist(coords))
R <- exp(-phi*D)
w <- rmvn(1, rep(0,n), sigma.sq*R)
y <- rnorm(n, x%*%B + w, sqrt(tau.sq))

##Fit a Response and Sequential NNGP model
n.samples <- 500

starting <- list("phi"=phi, "sigma.sq"=5, "tau.sq"=1)

tuning <- list("phi"=0.5, "sigma.sq"=0.5, "tau.sq"=0.5)

priors <- list("phi.Unif"=c(3/1, 3/0.01), "sigma.sq.IG"=c(2, 5), "tau.sq.IG"=c(2, 1))

cov.model <- "exponential"

m.s <- spNNGP(y~x-1, coords=coords, starting=starting, method="sequential", n.neighbors=10,
              tuning=tuning, priors=priors, cov.model=cov.model,
              n.samples=n.samples, n.omp.threads=2)
```

```

round(summary(m.s$p.beta.samples)$quantiles[,c(3,1,5)],2)
round(summary(m.s$p.theta.samples)$quantiles[,c(3,1,5)],2)
plot(apply(m.s$p.w.samples, 1, median), w)

m.r <- spNNGP(y~x-1, coords=coords, starting=starting, method="response", n.neighbors=10,
             tuning=tuning, priors=priors, cov.model=cov.model,
             n.samples=n.samples, n.omp.threads=2)

round(summary(m.r$p.beta.samples)$quantiles[,c(3,1,5)],2)
round(summary(m.r$p.theta.samples)$quantiles[,c(3,1,5)],2)

```

spPredict

Function for prediction at new locations using spNNGP models.

Description

The function `spPredict` collects posterior predictive samples for a set of new locations given a `spNNGP` object.

Usage

```

spPredict(sp.obj, X.0, coords.0, start=1, end, thin=1, n.omp.threads = 1,
          verbose=TRUE, n.report=100, ...)

```

Arguments

<code>sp.obj</code>	an object returned by <code>spNNGP</code> .
<code>X.0</code>	the design matrix for prediction locations. An intercept should be provided in the first column if one is specified in <code>sp.obj</code> model.
<code>coords.0</code>	the spatial coordinates corresponding to <code>X.0</code> .
<code>start</code>	specifies the first sample included in the composition sampling.
<code>end</code>	specifies the last sample included in the composition. The default is to use all posterior samples in <code>sp.obj</code> .
<code>thin</code>	a sample thinning factor. The default of 1 considers all samples between <code>start</code> and <code>end</code> . For example, if <code>thin = 10</code> then 1 in 10 samples are considered between <code>start</code> and <code>end</code> .
<code>n.omp.threads</code>	a positive integer indicating the number of threads to use for SMP parallel processing. The package must be compiled for OpenMP support. For most Intel-based machines, we recommend setting <code>n.omp.threads</code> up to the number of hyperthreaded cores.
<code>verbose</code>	if TRUE, model specification and progress of the sampler is printed to the screen. Otherwise, nothing is printed to the screen.
<code>n.report</code>	the interval to report sampling progress.
<code>...</code>	currently no additional arguments.

Value

A list comprising:

p.y. θ	a matrix that holds the response variable posterior predictive samples where rows are locations corresponding to coords. θ and columns are samples.
p.w. θ	a matrix that holds the random effect posterior predictive samples where rows are locations corresponding to coords. θ and columns are samples. This is only returned if spNNGP method = "sequential".
run.time	execution time reported using proc.time().

Author(s)

Andrew O. Finley <finleya@msu.edu>,
 Abhirup Datta <abhidatta@jhu.edu>,
 Sudipto Banerjee <sudipto@ucla.edu>

References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. *Journal of the American Statistical Association*, 111:800-812.

Finley, A.O., A. Datta, B.C. Cook, D.C. Morton, H.E. Andersen, and S. Banerjee (2017) Applying Nearest Neighbor Gaussian Processes to massive spatial data sets: Forest canopy height prediction across Tanana Valley Alaska, <https://arxiv.org/abs/1702.00434v2>.

Examples

```
rmvn <- function(n, mu=0, V = matrix(1)){
  p <- length(mu)
  if(any(is.na(match(dim(V),p))))
    stop("Dimension problem!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}

##Make some data
set.seed(1)
n <- 100
coords <- cbind(runif(n,0,1), runif(n,0,1))

x <- cbind(1, rnorm(n))

B <- as.matrix(c(1,5))

sigma.sq <- 5
tau.sq <- 1
phi <- 3/0.5

D <- as.matrix(dist(coords))
```

```

R <- exp(-phi*D)
w <- rmv(1, rep(0,n), sigma.sq*R)
y <- rnorm(n, x%%B + w, sqrt(tau.sq))

ho <- sample(1:n, 50)

y.ho <- y[ho]
x.ho <- x[ho,,drop=FALSE]
w.ho <- w[ho]
coords.ho <- coords[ho,]

y <- y[-ho]
x <- x[-ho,,drop=FALSE]
w <- w[-ho,,drop=FALSE]
coords <- coords[-ho,]

##Fit a Response and Sequential NNGP model
n.samples <- 500

starting <- list("phi"=phi, "sigma.sq"=5, "tau.sq"=1)

tuning <- list("phi"=0.5, "sigma.sq"=0.5, "tau.sq"=0.5)

priors <- list("phi.Unif"=c(3/1, 3/0.01), "sigma.sq.IG"=c(2, 5), "tau.sq.IG"=c(2, 1))

cov.model <- "exponential"

n.report <- 500

##Predict for holdout set using both models
m.s <- spNNGP(y~x-1, coords=coords, starting=starting, method="sequential", n.neighbors=10,
             tuning=tuning, priors=priors, cov.model=cov.model,
             n.samples=n.samples, n.omp.threads=2, n.report=n.report)

m.r <- spNNGP(y~x-1, coords=coords, starting=starting, method="response", n.neighbors=10,
             tuning=tuning, priors=priors, cov.model=cov.model,
             n.samples=n.samples, n.omp.threads=2, n.report=n.report)

##Prediction for holdout data
p.s <- spPredict(m.s, X.0 = x.ho, coords.0 = coords.ho, n.omp.threads=2)

plot(apply(p.s$p.w.0, 1, mean), w.ho)
plot(apply(p.s$p.y.0, 1, mean), y.ho)

p.r <- spPredict(m.r, X.0 = x.ho, coords.0 = coords.ho, n.omp.threads=2)

points(apply(p.r$p.y.0, 1, mean), y.ho, pch=19, col="blue")

```

Index

*Topic **datasets**

CHM, [2](#)

*Topic **model**

spConjNNGP, [2](#)

spNNGP, [6](#)

spPredict, [10](#)

CHM, [2](#)

spConjNNGP, [2](#)

spNNGP, [6](#), [10](#), [11](#)

spPredict, [10](#)